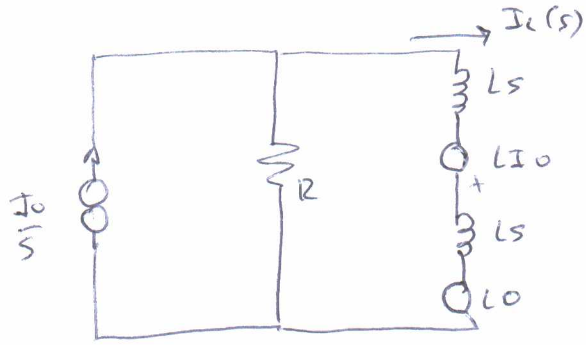
Problema 3 . a.   SISTEMAS LINEALES 2 . Dic 2015

TRAMO 1 , $t \in [0, T]$



$$\frac{R}{L} = \frac{1}{\tau}$$

$$I_L(s) = \frac{I_0}{s}\frac{R}{R+2Ls} + \frac{LI_0}{R+2Ls} = \frac{I_0}{2\tau}\frac{1}{s(s+1/2\tau)} + \frac{I_0}{2}\frac{1}{s+1/2\tau}$$

↑ superposición

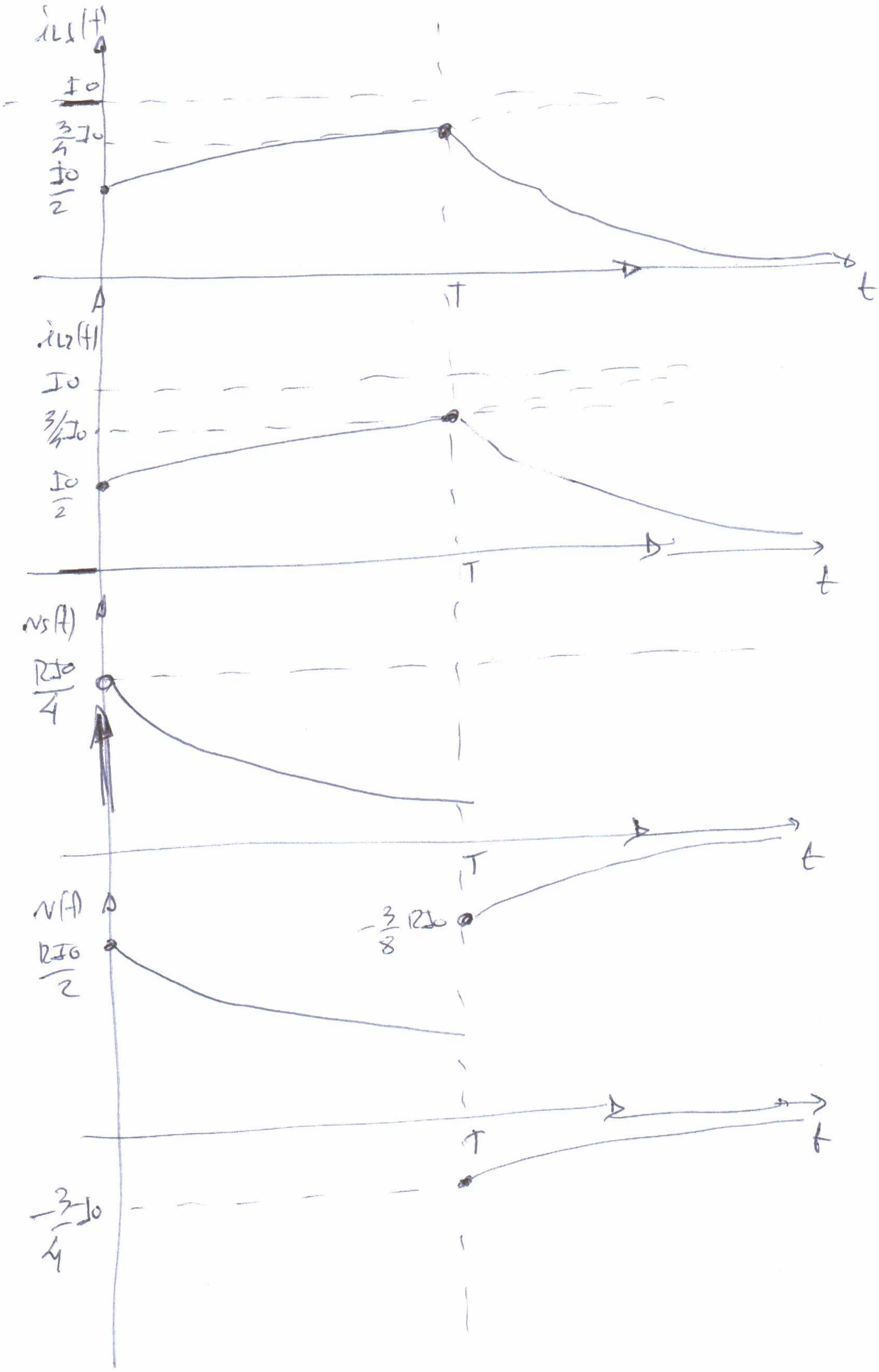
$$\Rightarrow \boxed{\begin{aligned} i_{L1}(t) = i_{L2}(t) = i_L(t) &= I_0\,Y(t)\left[1 - e^{-t/2\tau} + \frac{1}{2}e^{-t/2\tau}\right] \\ &= I_0\,Y(t)\left[1 - \frac{1}{2}e^{-t/2\tau}\right] \quad t \in [0, T] \end{aligned}}$$

$$v_s(t) = v_{L2}(t) = L\frac{d}{dt}i_{L2}(t) =$$

$$L\frac{I_0}{2}\delta(t) + \frac{LI_0}{4\tau}e^{-t/2\tau} \Rightarrow \boxed{v_s(t) = RI_0\left[\tau\,\delta(t) + \frac{1}{4}e^{-t/2\tau}\right] \quad t \in [0, T]}$$

$$V(s) = 2Ls\,I_L(s) - LI_0 = \dots = \frac{LI_0}{2\tau}\frac{1}{s+1/2\tau}$$

$$\Rightarrow \boxed{v(t) = \frac{RI_0}{2}e^{-t/2\tau} \quad t \in [0, T]}$$

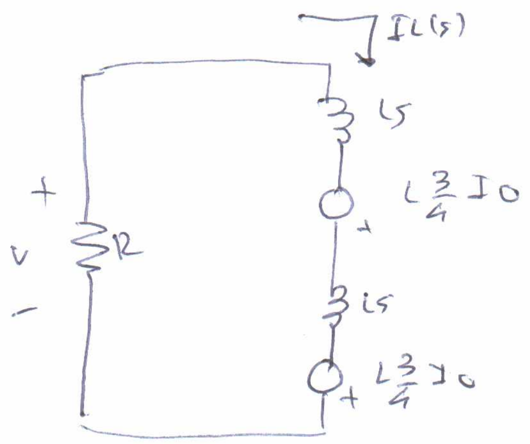


TRAMO 2 $\quad t \in [T, +\infty)$

$$\sigma = t - T$$

DP $\quad i_{L2}(\sigma=0^-) = i_{L1}(\sigma=0^-) = I_0\left[1 - \frac{1}{2}e^{-T/2\tau}\right] =$

$$= I_0\left[1 - \tfrac{1}{2}e^{-\ln 2}\right] = I_0\left[1 - \tfrac{1}{4}\right] = \frac{3}{4}I_0$$



$$I_L(s) = \frac{2L\frac{3}{4}I_0}{R + 2Ls} = \frac{\frac{3}{4}I_0}{s + \frac{1}{2\tau}} \Rightarrow \boxed{i_L(t) = \frac{3}{4}I_0\, e^{-t/2\tau} \quad t \in [0, +\infty)}$$

$$\boxed{i_{L1}(t) = i_{L2}(t) = i_L(t) \quad \forall t \in [0, +\infty)}$$

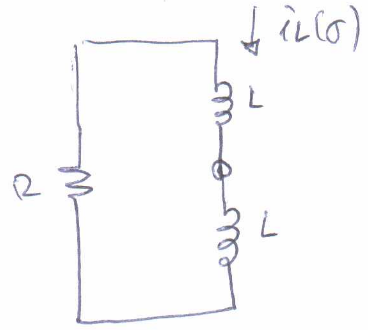
$$\boxed{v(t) = -R\, i_L(t) = -\frac{3}{4} R I_0\, e^{-t/2\tau}}$$

$$v_s(t) = L\frac{di_L(t)}{dt} = -\frac{\frac{3}{4}I_0 L}{2\tau} e^{-t/2\tau} = -\frac{3}{8} R I_0\, e^{-t/2\tau} \quad t \in [0, +\infty)$$

$t \in [0, +\infty)$

$$\Rightarrow \boxed{v_s(t) = -\frac{3}{8} R I_0\, e^{-t/2\tau} \quad t \in [0, +\infty)}$$

b



DP $i_L(0^-) = \frac{3}{4} I_0$

El estado inicial es

$$\boxed{i_{L2}(0^-) = i_{L1}(0^-) = \frac{3}{4} I_0}$$

El estado final es

$$\boxed{i_{L2}(+\infty) = i_{L1}(+\infty) = 0}$$

Por Tellegen

$$P_{L1}(\sigma) + P_{L2}(\sigma) + P_R(\sigma) = 0 \quad \forall \sigma \in [0, +\infty)$$

$$\Rightarrow \int_0^{+\infty} P_{L1}(\sigma)\, d\sigma + \int_0^{\infty} P_{L2}(\sigma)\, d\sigma + \underbrace{\int_0^{\infty} P_R(\sigma)\, d\sigma}_{E_R} = 0$$

$$\Rightarrow E_R = -E_{L1} - E_{L2} = -\left[\frac{1}{2} L\, i_{L1}^2 \Big|_0^{\infty}\right] - \left[\frac{1}{2} L\, i_{L2}^2 \Big|_0^{\infty}\right]$$

$$= \frac{1}{2} L \left(\frac{3}{4} I_0\right)^2 + \frac{1}{2} L \left(\frac{3}{4} I_0\right)^2$$

$$\Rightarrow \boxed{E_R = \frac{9}{16} L I_0^2}$$