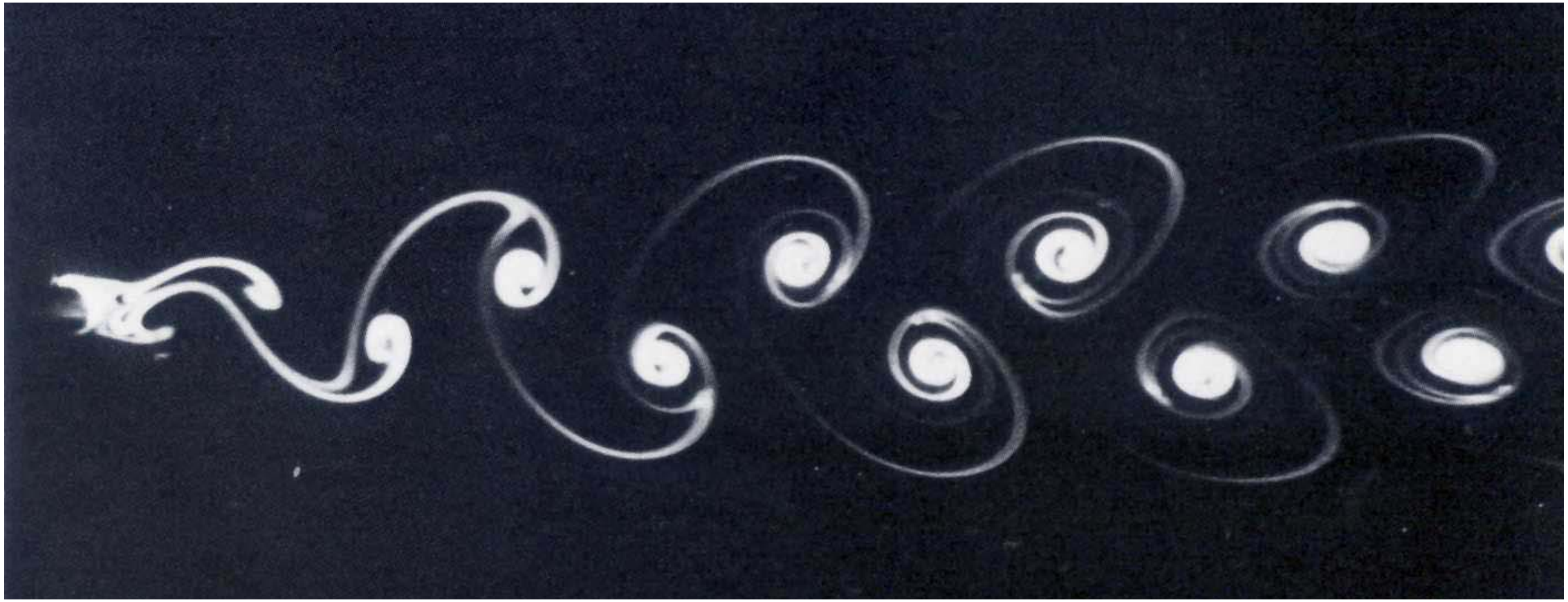
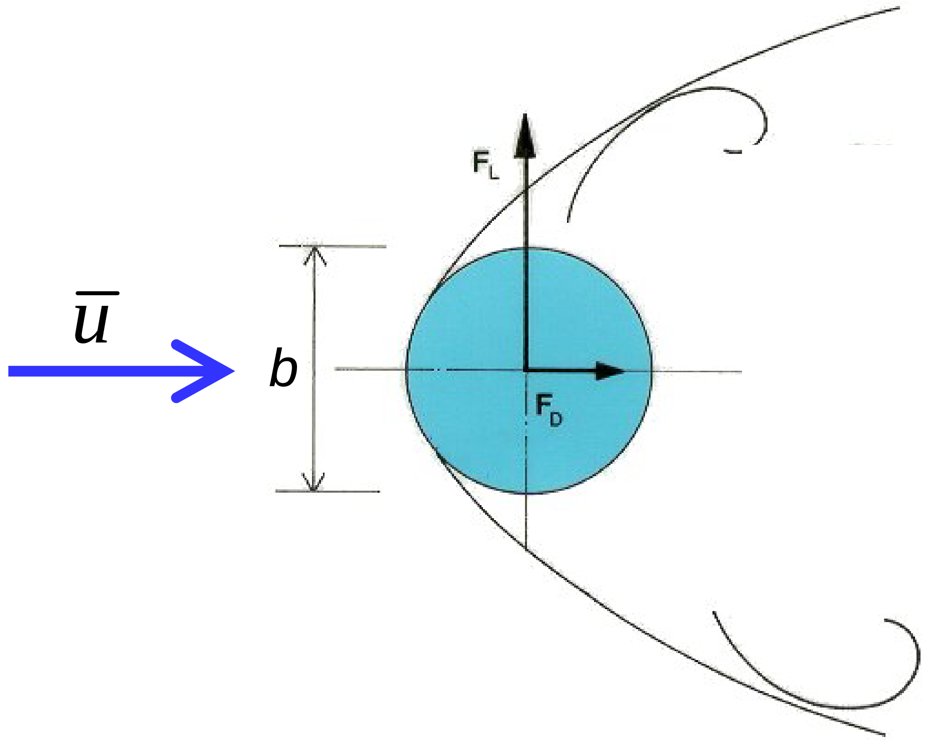
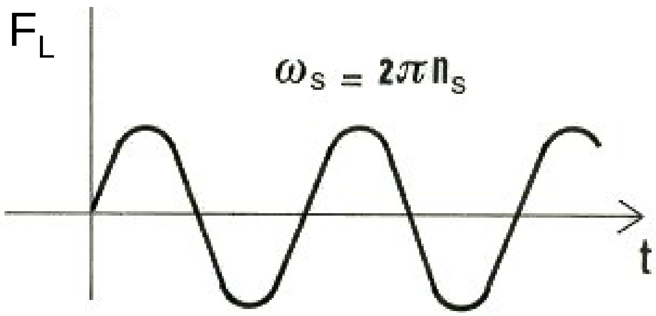


Vortex shedding from a circular cylinder

Vortex shedding from an island





Vortex shedding frequency

$$n_s = \frac{S\bar{u}}{b}$$

S = Strouhal number

Resonance condition

$$n_s = \frac{S\bar{u}}{b} = n_j$$

Critical velocity

$$\bar{u}_{cr,j} = \frac{n_j b}{S}$$

Vortex shedding

Equation of motion

$$m\ddot{y}(t)+c\dot{y}(t)+ky(t)=f(t)$$

$$f(t)=\frac{1}{2}\rho\overline{u}^2 dc_{L0}\sin(2\pi n_s t)+h_a y(t)+k_a\dot{y}(t)$$

$$n_s=n_0 \Rightarrow h_a \sim 0;\ k_a=4\pi n_0\rho d^2 K_{a0} \Rightarrow$$

$$\frac{\ddot{y}(t)}{d}+4\pi n_0\left(\xi_s-\frac{\rho d^2}{m}K_{a0}\right)\frac{\dot{y}(t)}{d}+(2\pi n_0)^2\frac{y(t)}{d}=\frac{1}{m}\frac{1}{2}\rho\overline{u}^2 c_{L0}\sin(2\pi n_0 t)$$

$$\frac{\ddot{y}(t)}{d}+4\pi n_0(\xi_s+\xi_a)\frac{\dot{y}(t)}{d}+(2\pi n_0)^2\frac{y(t)}{d}=\frac{1}{m}\frac{1}{2}\rho\overline{u}^2 c_{L0}\sin(2\pi n_0 t)$$

$\xi_s=$ structural damping

$\xi_a=-\dfrac{\rho d^2}{m}K_{a0}=$ aerodynamic damping

Scruton & Flint (1964)

Equation of motion

$$\frac{\ddot{y}(t)}{d} + 4\pi n_0 (\xi_s + \xi_a) \frac{\dot{y}(t)}{d} + (2\pi n_0)^2 \frac{y(t)}{d} = \frac{1}{m} \frac{1}{2} \rho \bar{u}^2 c_{L0} \sin(2\pi n_0 t)$$

ξ_s = structural damping

$\xi_a = -\dfrac{\rho d^2}{m} K_{a0}$ = aerodynamic damping

Equivalent damping

$$\xi_{eq} = \xi_s + \xi_a = \frac{\rho d^2}{4\pi m}\left(\frac{4\pi m \xi_s}{\rho d^2} - 4\pi K_{a0}\right) = \frac{\rho d^2}{4\pi m}(Sc - 4\pi K_{a0})$$

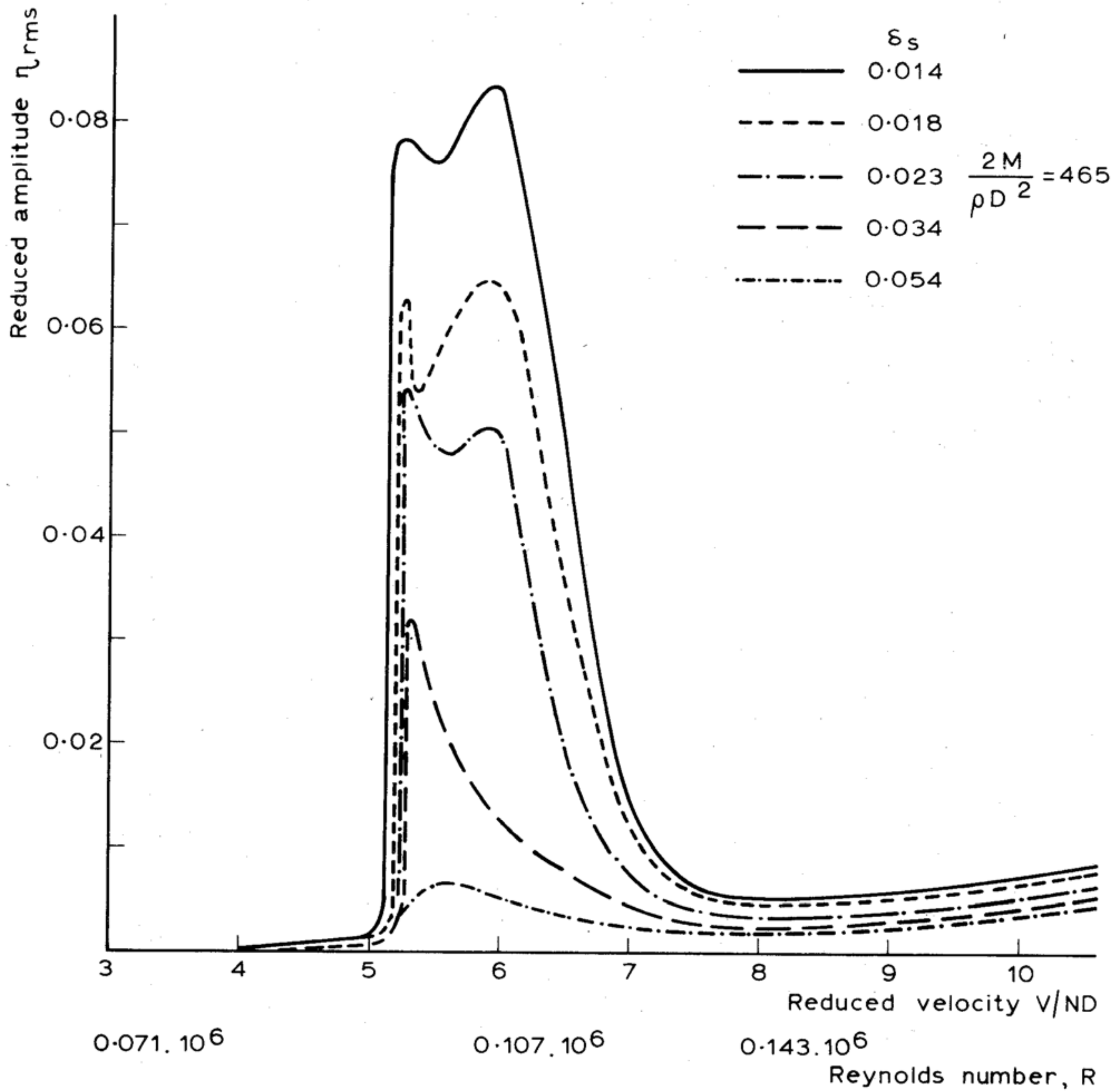
Scruton number (1983)

$$Sc = \frac{4\pi m \xi_s}{\rho d^2}$$

Scruton & Flint (1964)

Scruton number (circular cylinders) $\mathrm{Sc} = \dfrac{4\pi m \xi_s}{\rho d^2}$

Sc > 30	the probability of lock-in is quite low and vortex shedding is not the critical load case; it is nevertheless advisable to perform the checks;
5 < Sc < 30	vortex shedding is very sensitive to different parameters, first of all turbulence intensity. High values of turbulence intensity reduce the risk of strong vibrations; small values of turbulence intensity, usually associated with small values of critical velocities, may amplify the critical vortex shedding. In any case, specific analyses must be performed to ensure that vibrations do not induce large stresses and that fatigue limits are not exceeded;
Sc < 5	vibrations induced by vortex shedding may be very large and dangerous; it is thus advisable to address the problem with the utmost attention and prudence, or seek specialist advice.

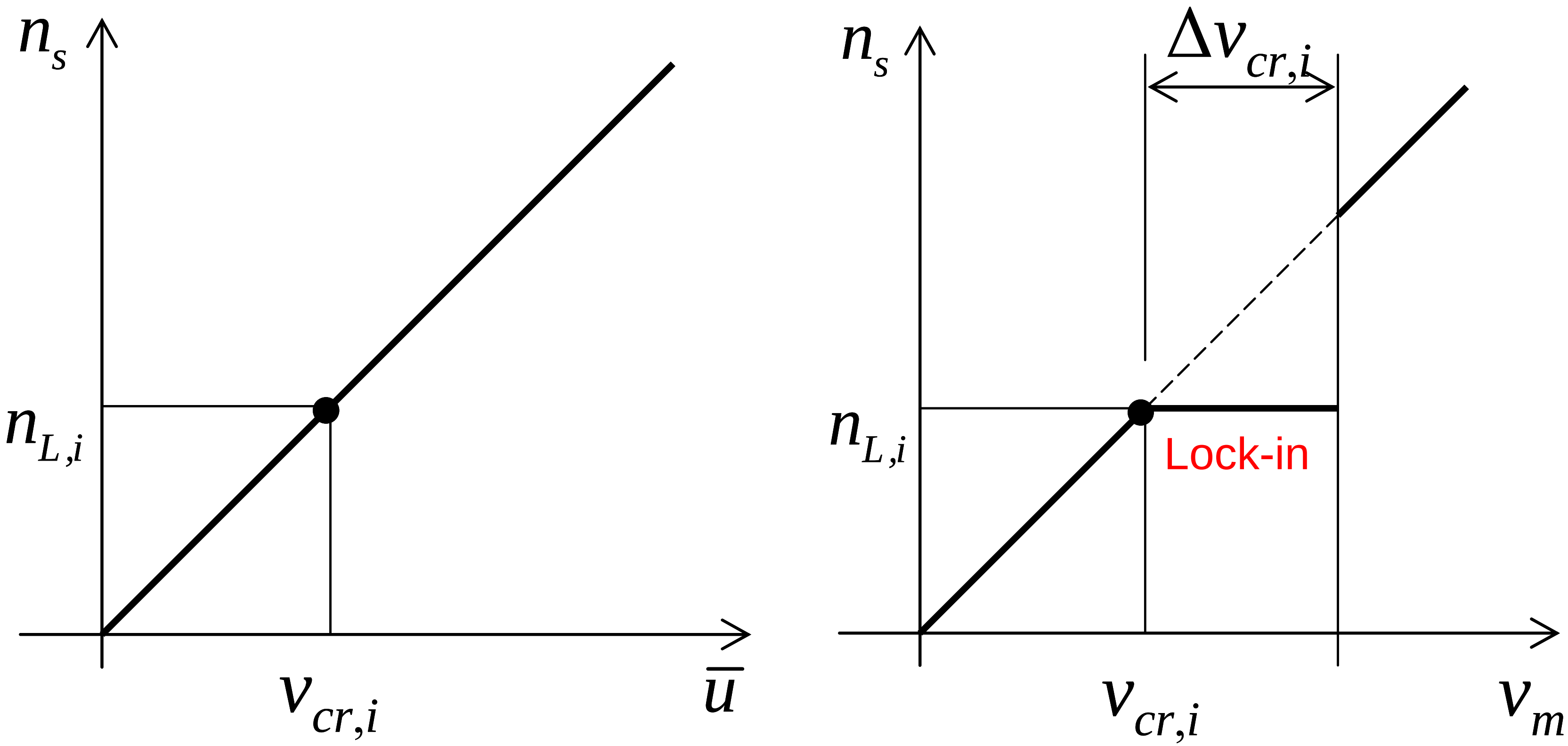


Wootton (1969)

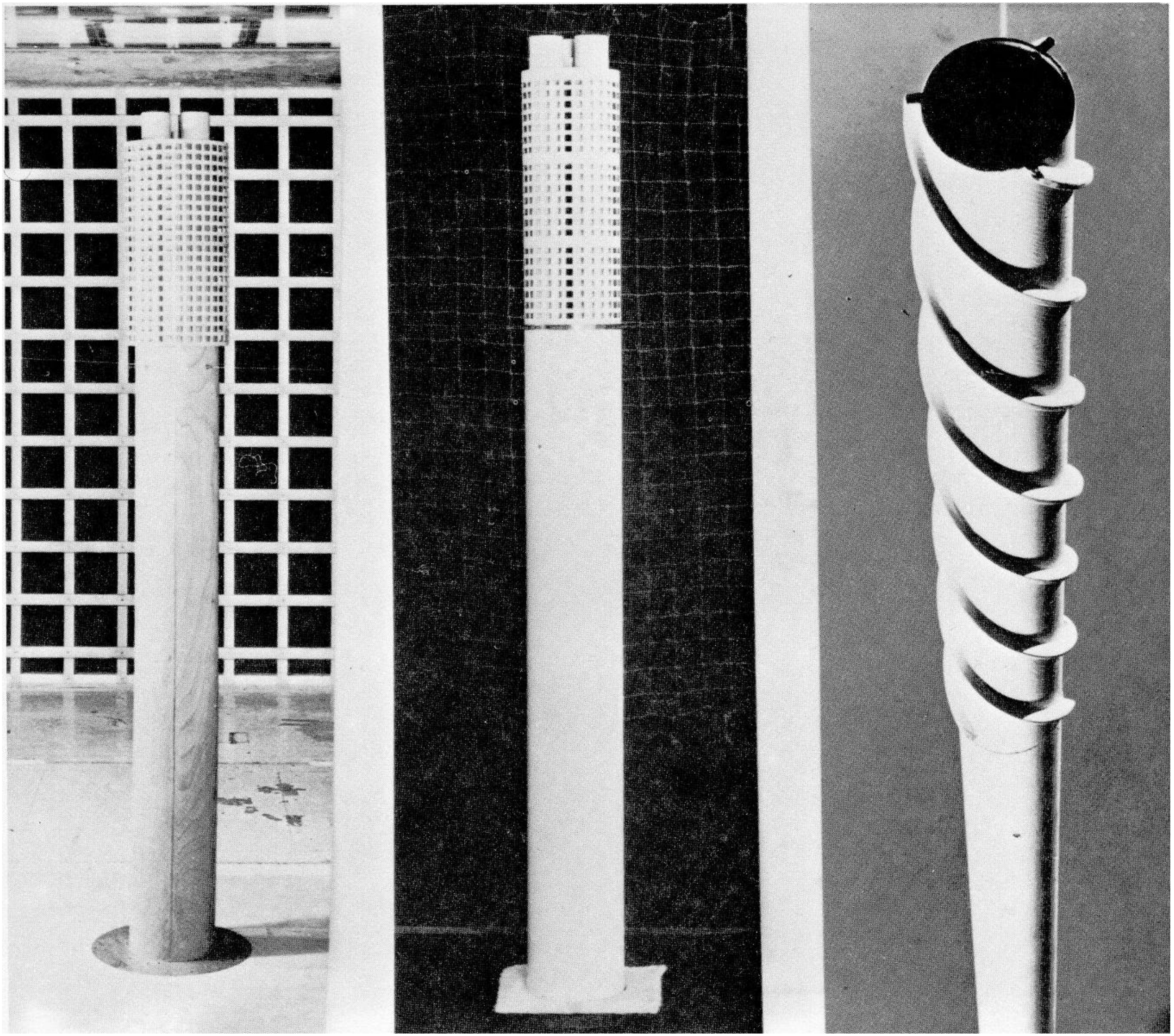
<u>Vortex shedding frequency</u> $n_s = \dfrac{S\bar{u}}{b}$

High Scruton number

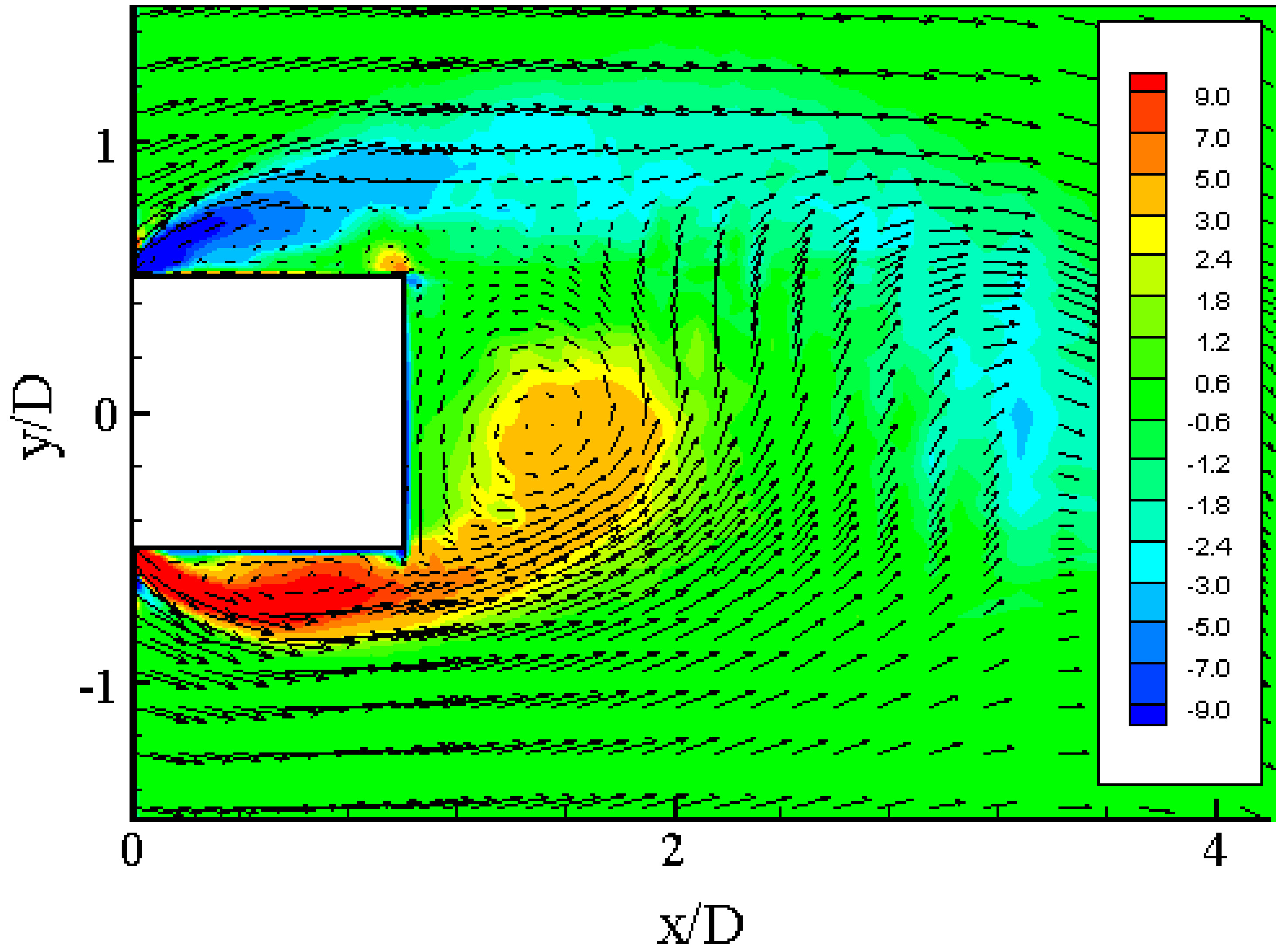
Small Scruton number



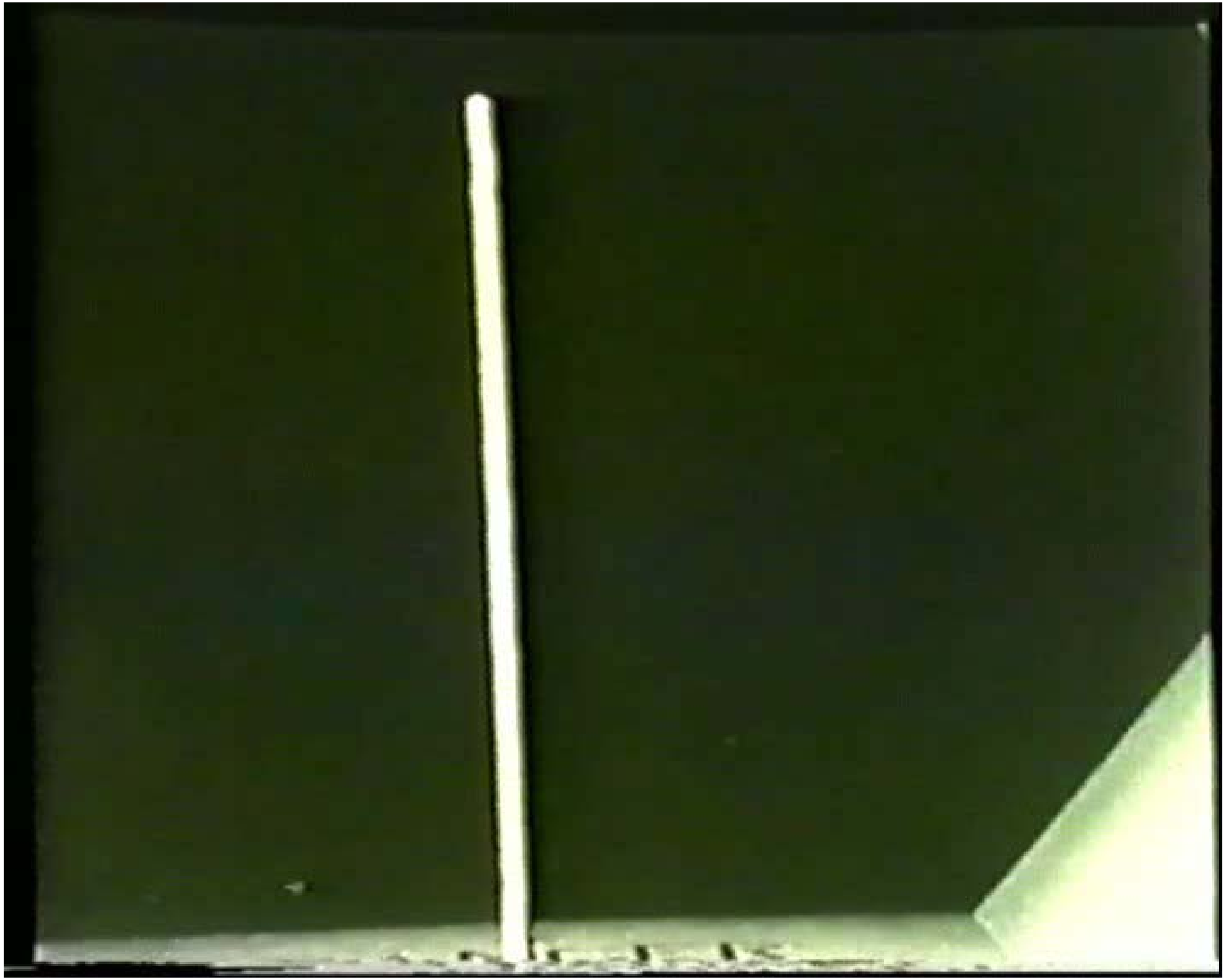
Scruton (1963, 1967), Wootton (1969)



Devices for the suppression of vortex-excited vibrations (1968)



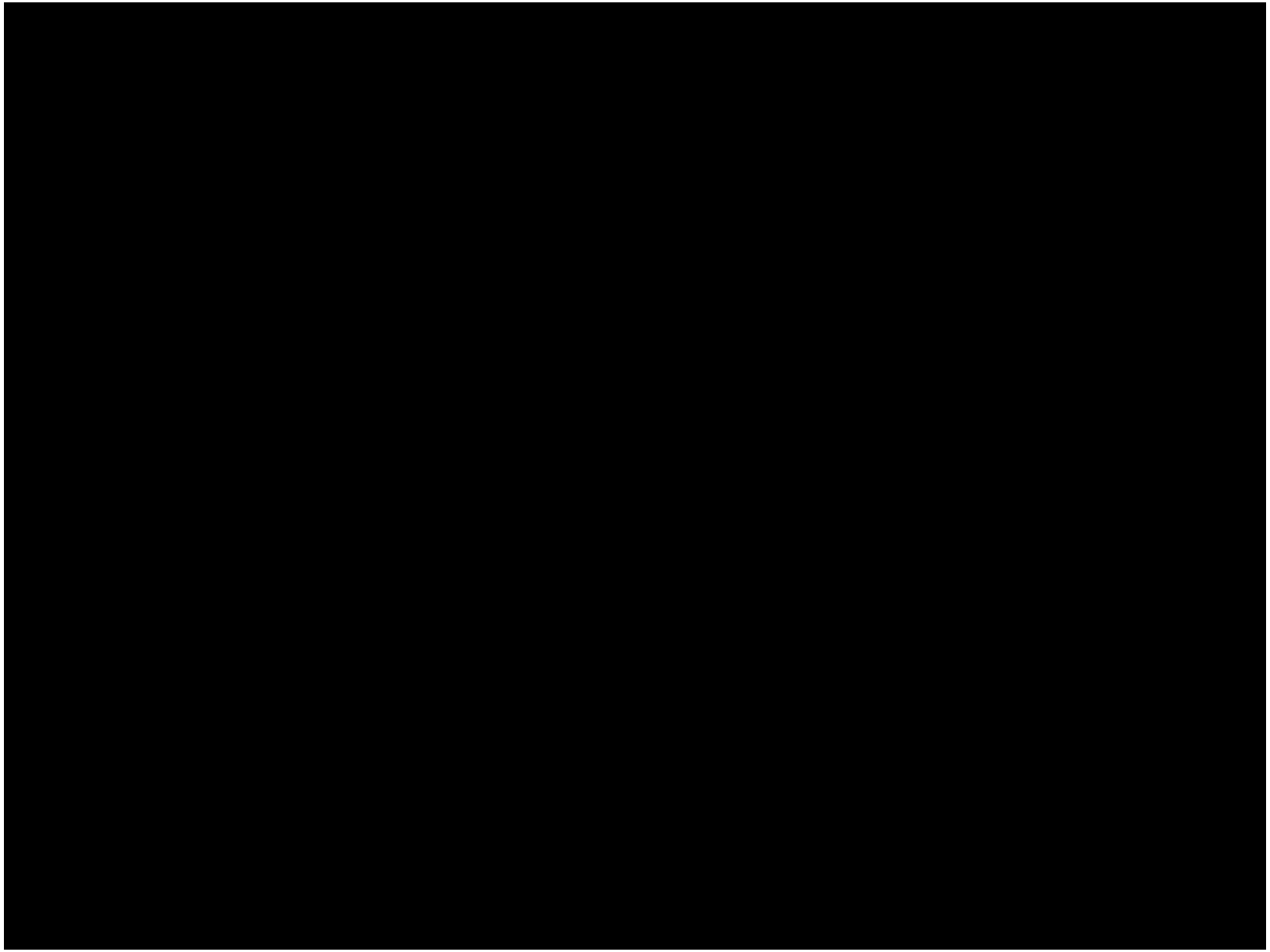
Vortex shedding



Vortex-induced vibrations

Vortex-induced vibrations

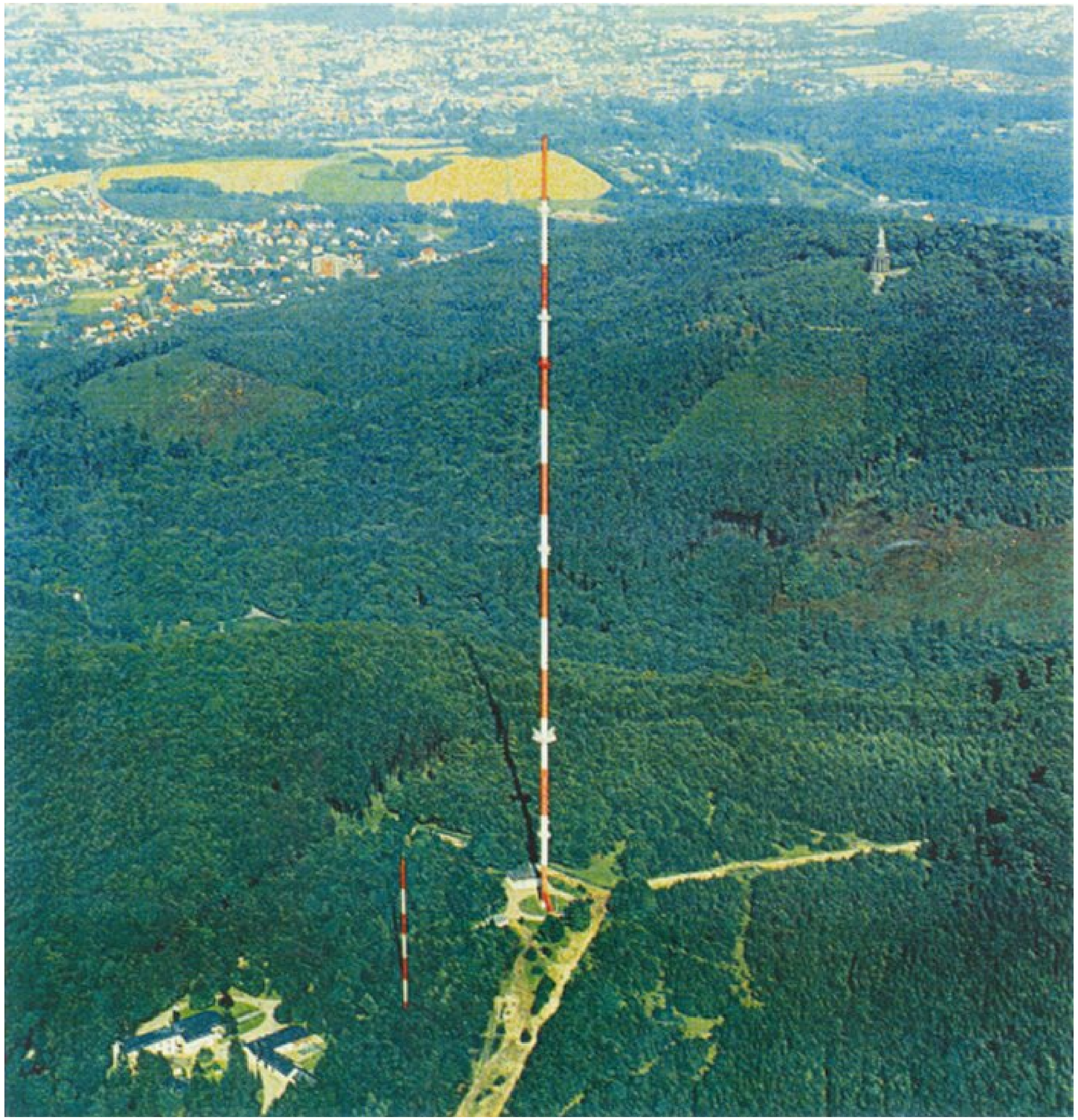
Vortex-induced vibrations

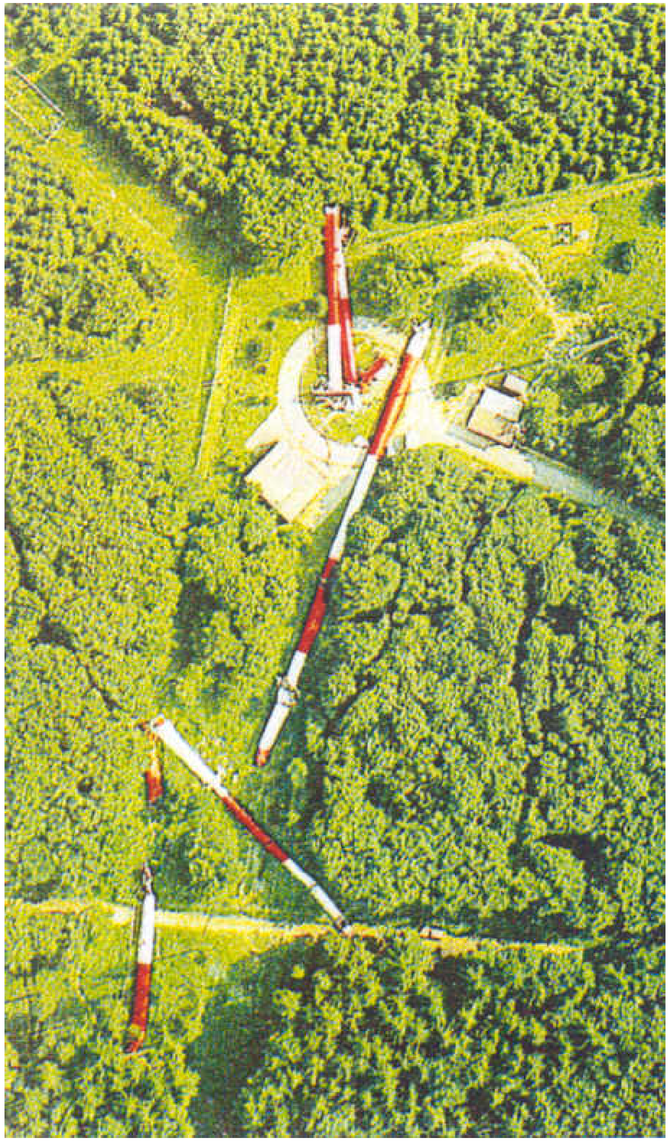


Vortex-induced vibrations

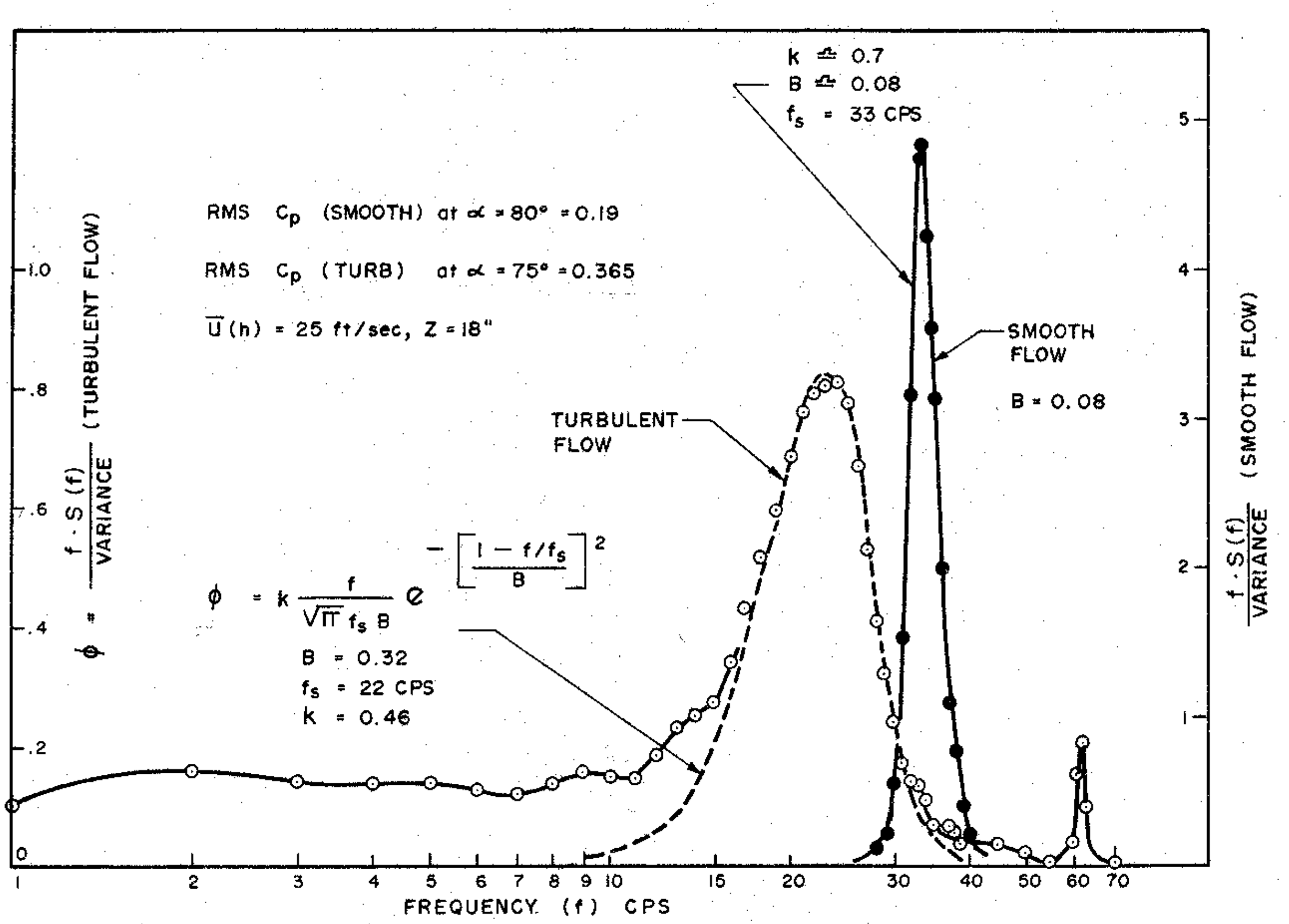
Steel chimney with helical strakes

Collapse due to vortex-induced vibrations



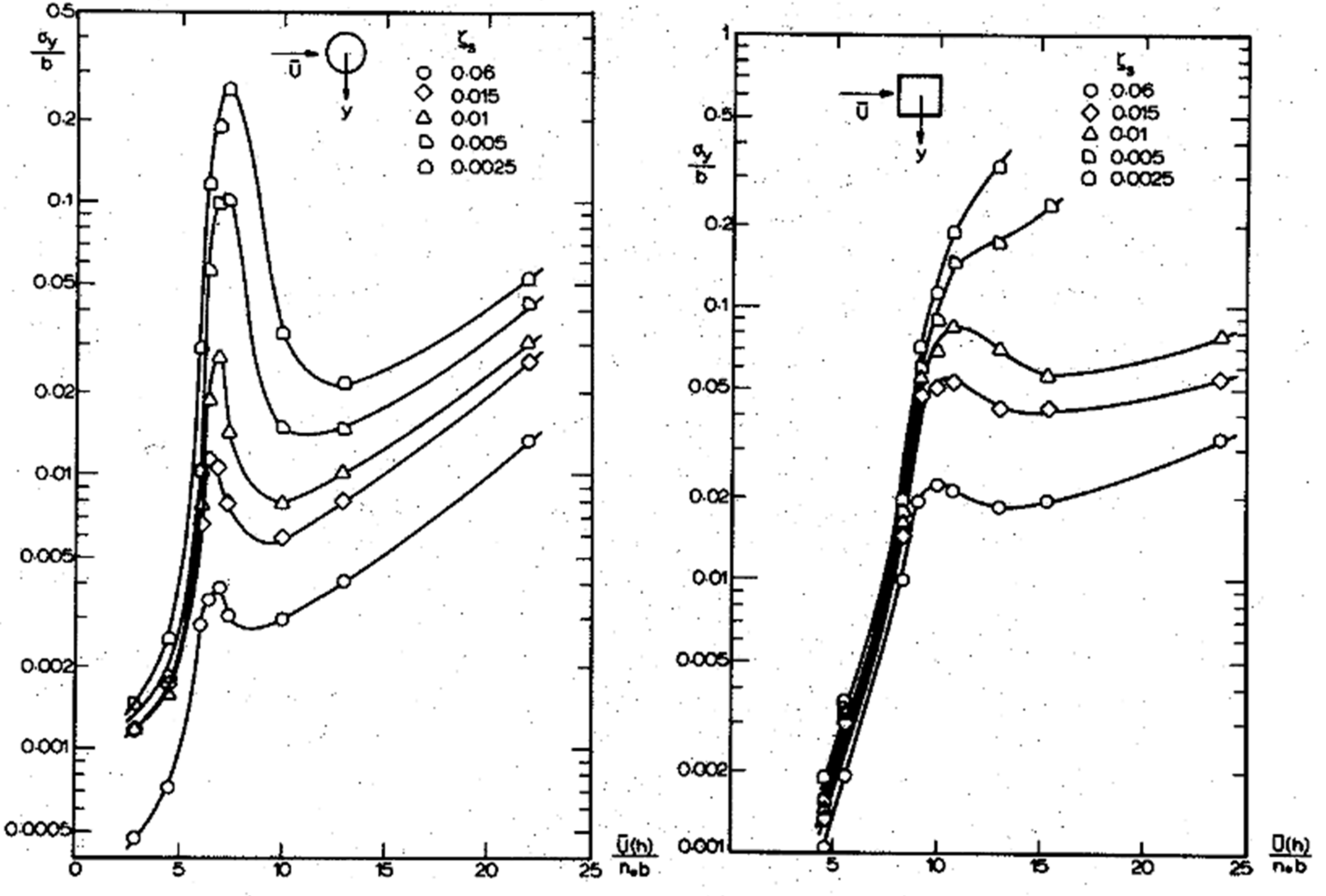


Collapse due to vortex-induced vibrations

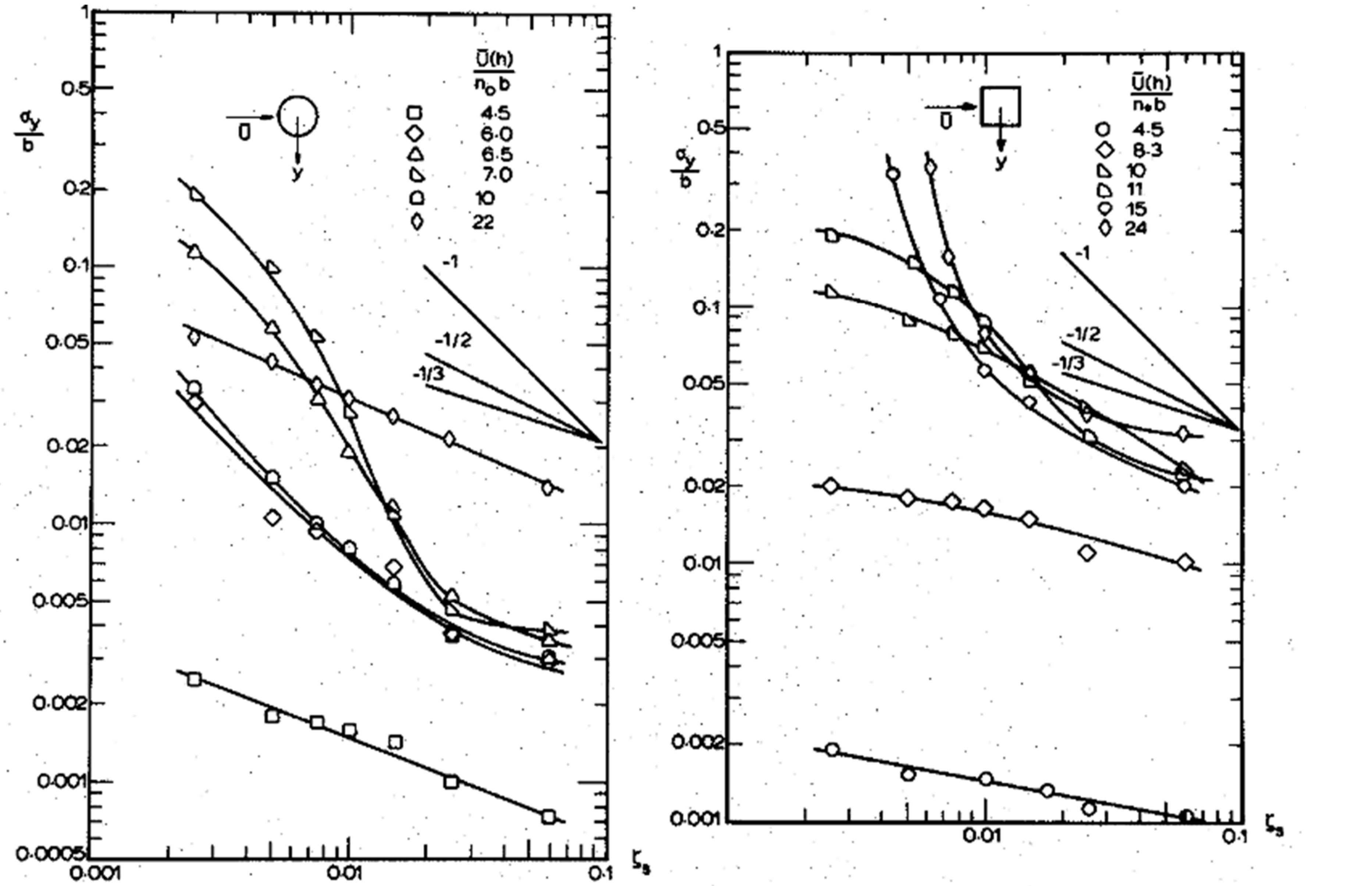


$$\frac{n\,S(n)}{\tilde{c}_L^2}=\frac{n}{\sqrt{\pi}\,\beta n_s}\exp\left\{-\frac{1}{\beta^2}\left(1-\frac{n}{n_s}\right)^2\right\};\ \beta=\sqrt{\beta_0^2+2I_u^2};\ \beta_0\approx 0,08$$

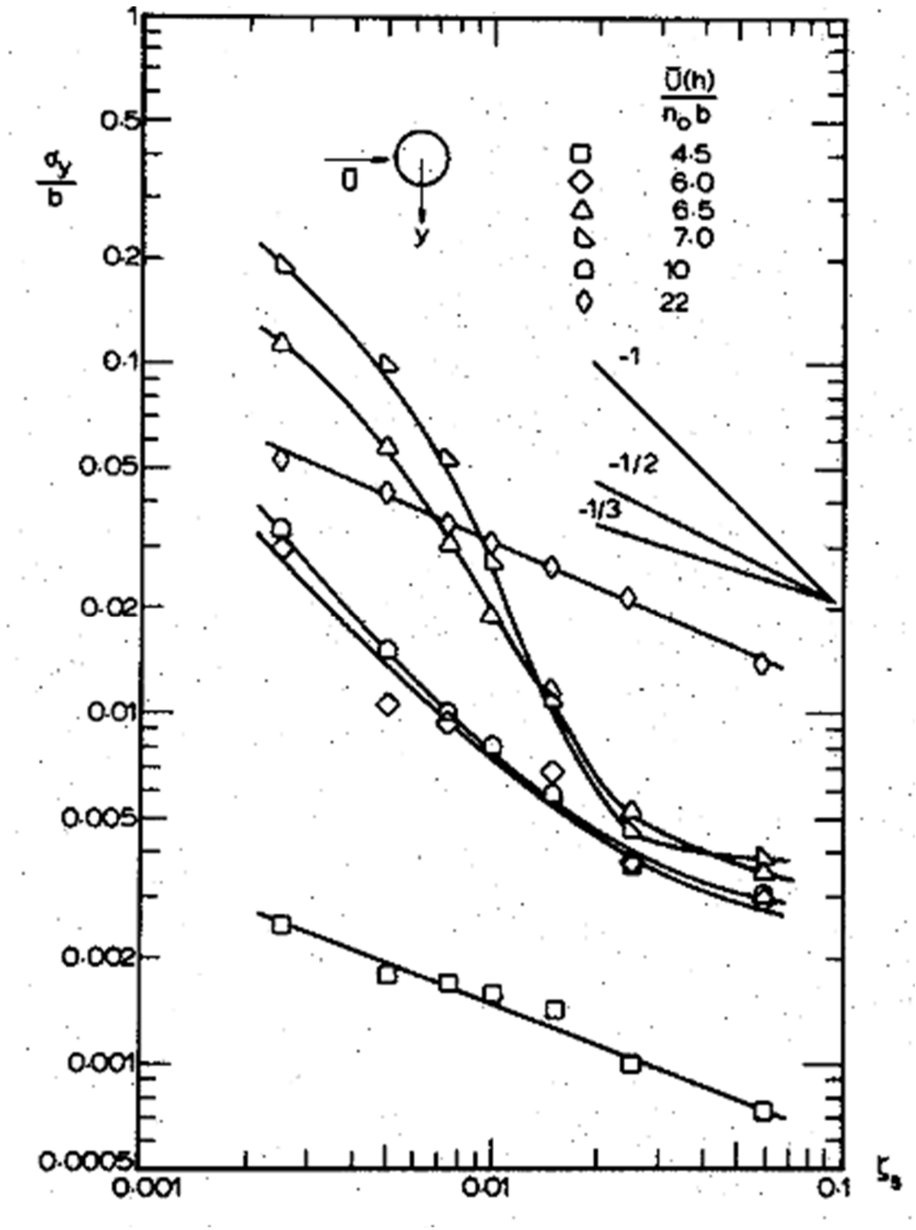
Vickery & Clark (1972) – Psdf of the vortex shedding



Kwok & Melbourne (1979, 1981)



Kwok & Melbourne (1979, 1981)



$$\xi_s \text{ high} \Rightarrow \frac{\sigma_y}{d} \propto \frac{1}{\sqrt{\xi_s}} \Rightarrow$$

random vibrations

$$\xi_s \text{ low} \Rightarrow \frac{\sigma_y}{d} \propto \frac{1}{\xi_s} \Rightarrow$$

deterministic vibrations

Kwok & Melbourne (1979, 1981)

Equation of motion

$$m\ddot{y}(t)+c\dot{y}(t)+ky(t)=f(t)$$

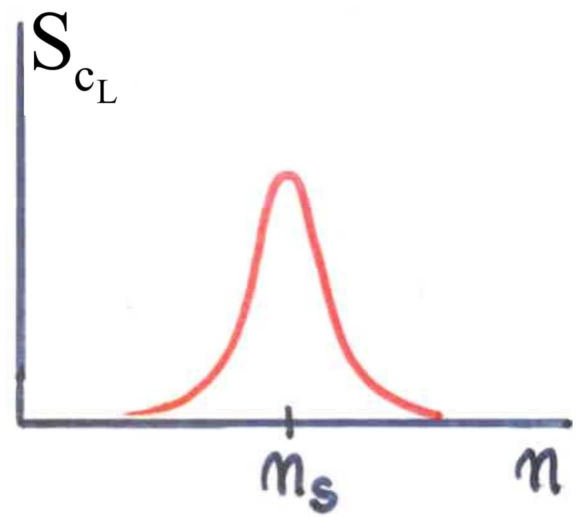
$$f(t)=f_s(t)+f_a(t)$$

Vortex – induced force on the stationary structure

$$f_s(t)=\frac{1}{2}\rho\bar{u}^2 d c_L(t)$$

$$\frac{nS_{c_L}(n)}{\sigma_{c_L}^2}=\frac{n}{\sqrt{\pi}\beta n_s}\exp\left\{-\frac{1}{\beta^2}\left(1-\frac{n}{n_s}\right)^2\right\}$$

$$n_s=\frac{S\bar{u}}{d}$$



Vickery & Basu (1983)

Equation of motion

$$m\ddot{y}(t)+c\dot{y}(t)+ky(t)=f(t)$$

$$f(t)=f_s(t)+f_a(t)$$

Motion – induced force (Marris 1964)

$$f_a(t)=4\pi n_s \rho d^2 K_{a0}\left[1-\frac{y^2(t)}{y_{lim}^2}\right]\dot{y}(t)$$

Limiting amplitude of the crosswind displacement

$$y_{lim}=\alpha d$$

Vickery & Basu (1983)

Equation of motion

$$m\ddot{y}(t)+c\dot{y}(t)+ky(t)=f(t)=f_s(t)+f_a(t)\Rightarrow$$

$$\ddot{y}(t)+4\pi n_0\left\{\xi_s-\frac{\rho d^2}{m}K_{a0}\left[1-\frac{y^2(t)}{\alpha^2 d^2}\right]\right\}\dot{y}(t)+$$

$$+(2\pi n_0)^2\,y(t)=\frac{1}{m}f_s(t)\Rightarrow$$

$$\ddot{y}(t)+4\pi n_0\underbrace{\frac{\rho d^2}{4\pi m}\left\{\underbrace{\frac{4\pi m\xi_s}{\rho d^2}}_{Sc}-4\pi K_{a0}\left[1-\frac{y^2(t)}{\alpha^2 d^2}\right]\right\}}_{\xi_{eq}}\dot{y}(t)+\ldots$$

Vickery & Basu (1983)

Equation of motion

$$\ddot{y}(t)+4\pi n_0\xi_{eq}\dot{y}(t)+(2\pi n_0)^2 y(t)=\frac{1}{m}f_s(t)$$

Equivalent damping

$$\xi_{eq}=\frac{\rho d^2}{4\pi m}\left\{Sc-4\pi K_{a0}\left[1-\frac{y^2(t)}{\alpha^2 d^2}\right]\right\}\Rightarrow$$

$$\xi_{eq}\simeq\frac{\rho d^2}{4\pi m}\left\{Sc-4\pi K_{a0}\left[1-\frac{\sigma_y^2}{\alpha^2 d^2}\right]\right\}$$

$$Sc=\frac{4\pi m\xi_s}{\rho d^2}$$

Vickery & Basu (1983)

Equation of motion

$$\ddot{y}(t) + 4\pi n_0 \xi_{eq} \dot{y}(t) + (2\pi n_0)^2 y(t) = \frac{1}{m} f_s(t)$$

Frequency domain random solution $(n_s = n_0; \bar{u} = \bar{u}_{cr})$

$$S_y(n) = |H(n)|^2 S_{f_s}(n)$$

$$H(n) = \frac{1}{m(2\pi n_0)^2} \frac{1}{1 - \frac{n^2}{n_0^2} + 2i\xi_{eq}\frac{n}{n_0}}$$

$$S_{F_s}(n) = \left(\frac{1}{2}\rho \bar{u}_{cr}^2 d\right)^2 \frac{\sigma_{c_L}^2}{\sqrt{\pi}\beta n_0} \exp\left\{-\frac{1}{\beta^2}\left(1 - \frac{n}{n_0}\right)^2\right\}$$

$$\sigma_y^2 = \int_0^\infty S_y(n)\,dn \simeq \sigma_{yR}^2 = \frac{1}{m^2(2\pi n_0)^4} \frac{\pi n_0}{4\xi_{eq}} S_{f_s}(n_0) \Rightarrow$$

$$\xi_{eq} \simeq \frac{\rho d^2}{4\pi m}\left\{Sc - 4\pi K_{a0}\left[1 - \frac{\sigma_y^2}{\alpha^2 d^2}\right]\right\}; \; Sc = \frac{4\pi m \xi_s}{\rho d^2}$$

Variance of the crosswind response

$$\sigma_y^2 = \frac{1}{m^2(2\pi n_0)^4}\frac{\pi n_0}{4\xi_{eq}} S_{f_s}(n_0) \Rightarrow$$

$$\sigma_y^2 = \frac{1}{m^2(2\pi n_0)^4}\frac{\pi n_0}{4\dfrac{\rho d^2}{4\pi m}\left\{Sc - 4\pi K_{a0}\left[1-\dfrac{\sigma_y^2}{\alpha^2 d^2}\right]\right\}}\left(\frac{1}{2}\rho\bar{u}_{cr}^2 d\right)^2 \frac{\sigma_{c_L}^2}{\sqrt{\pi}\beta n_0} \Rightarrow$$

$$\left(\frac{\sigma_y}{d}\right)^2 = \frac{\rho d^2 \sigma_{c_L}^2}{64\pi^2 m\beta S^4}\frac{1}{Sc - 4\pi K_{a0}\left[1-\dfrac{1}{\alpha^2}\left(\dfrac{\sigma_y}{d}\right)^2\right]} \Rightarrow$$

$$\left(\frac{\sigma_y}{d}\right)^2 = \frac{A^2}{Sc - 4\pi K_{a0}\left[1-\dfrac{1}{\alpha^2}\left(\dfrac{\sigma_y}{d}\right)^2\right]}$$

$$A = \frac{d\sigma_{c_L}}{8\pi S^2}\sqrt{\frac{\rho}{m\beta}}; \quad S = \frac{n_s d}{\bar{u}} = \frac{n_0 d}{\bar{u}_{cr}}$$

Variance of the crosswind response

$$\left(\frac{\sigma_y}{d}\right)^2 = \frac{A^2}{Sc - 4\pi K_{a0}\left[1 - \frac{1}{\alpha^2}\left(\frac{\sigma_y}{d}\right)^2\right]} \Rightarrow$$

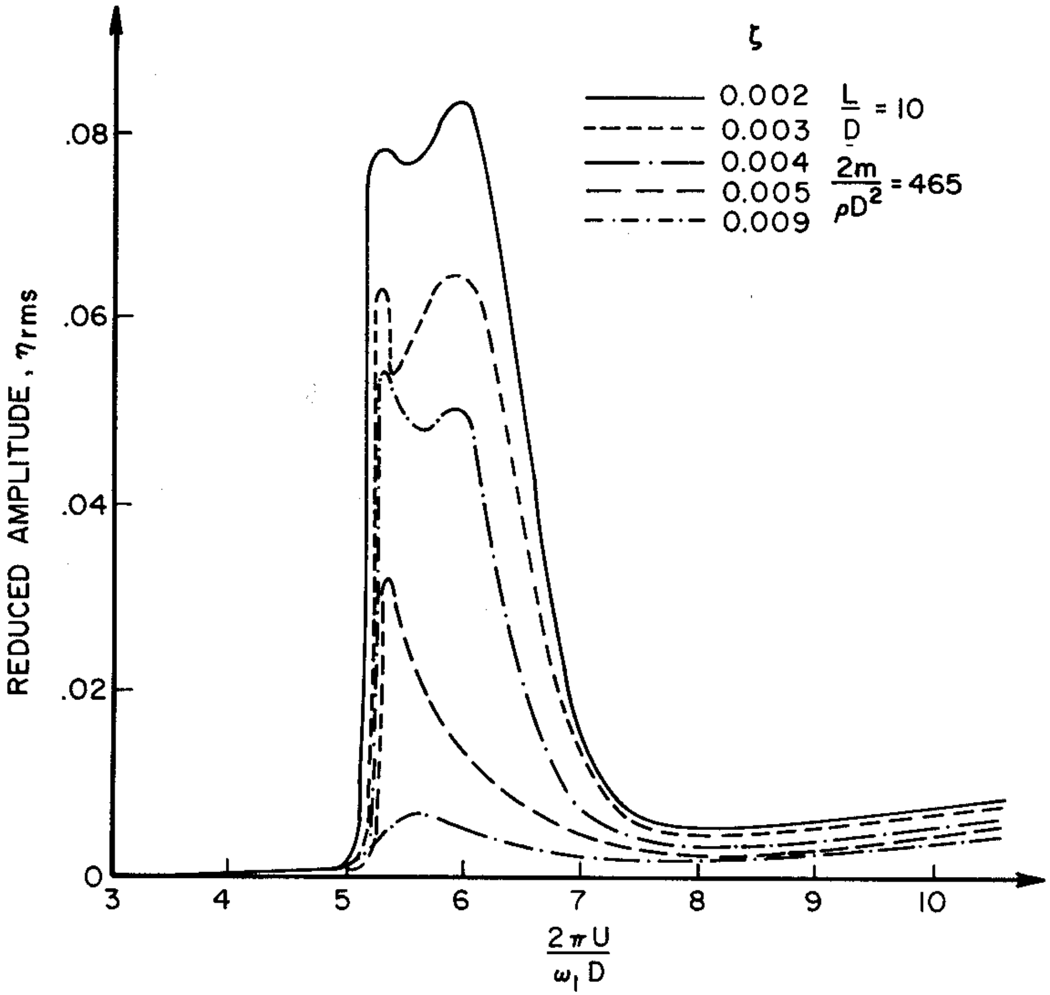
$$\frac{4\pi K_{a0}}{\alpha^2}\left(\frac{\sigma_y}{d}\right)^4 + \left(Sc - 4\pi K_{a0}\right)\left(\frac{\sigma_y}{d}\right)^2 - A^2 = 0 \Rightarrow$$

$$\left(\frac{\sigma_y}{d}\right)^2 = \frac{-\alpha^2\left(Sc - 4\pi K_{a0}\right) + \sqrt{\alpha^4\left(Sc - 4\pi K_{a0}\right)^2 + 16\pi K_{a0} A^2}}{8\pi K_{a0}} \Rightarrow$$

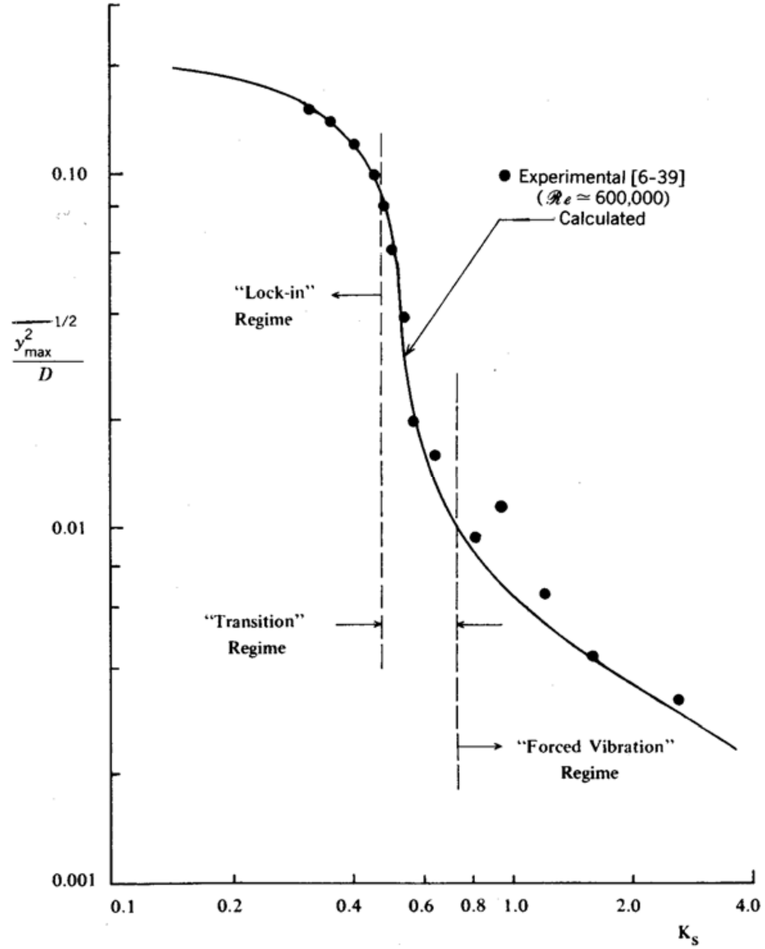
$$\frac{\sigma_y}{d} = \sqrt{\frac{-\alpha^2\left(Sc - 4\pi K_{a0}\right) + \sqrt{\alpha^4\left(Sc - 4\pi K_{a0}\right)^2 + 16\pi K_{a0} A^2}}{8\pi K_{a0}}}$$

Vickery & Basu (1983)

Wootton (1969)

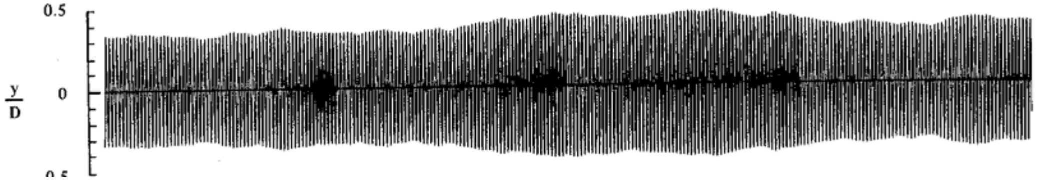


Vickery & Basu (1983)



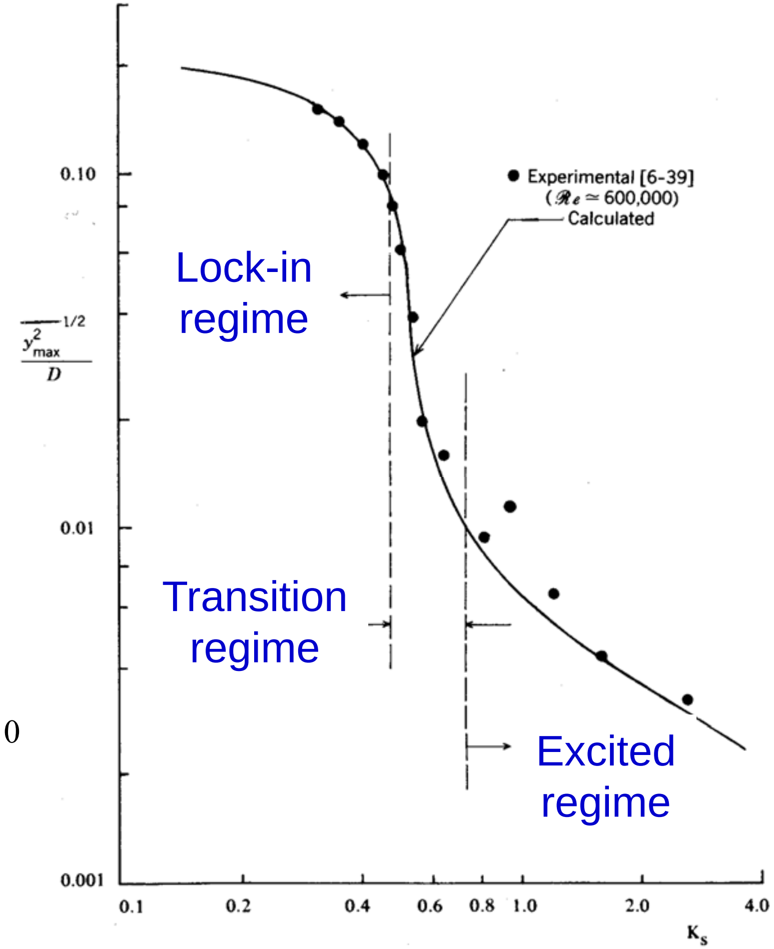
Vickery & Basu (1983)

Lock-in deterministic regime $Sc \ll 4\pi K_{a0}$

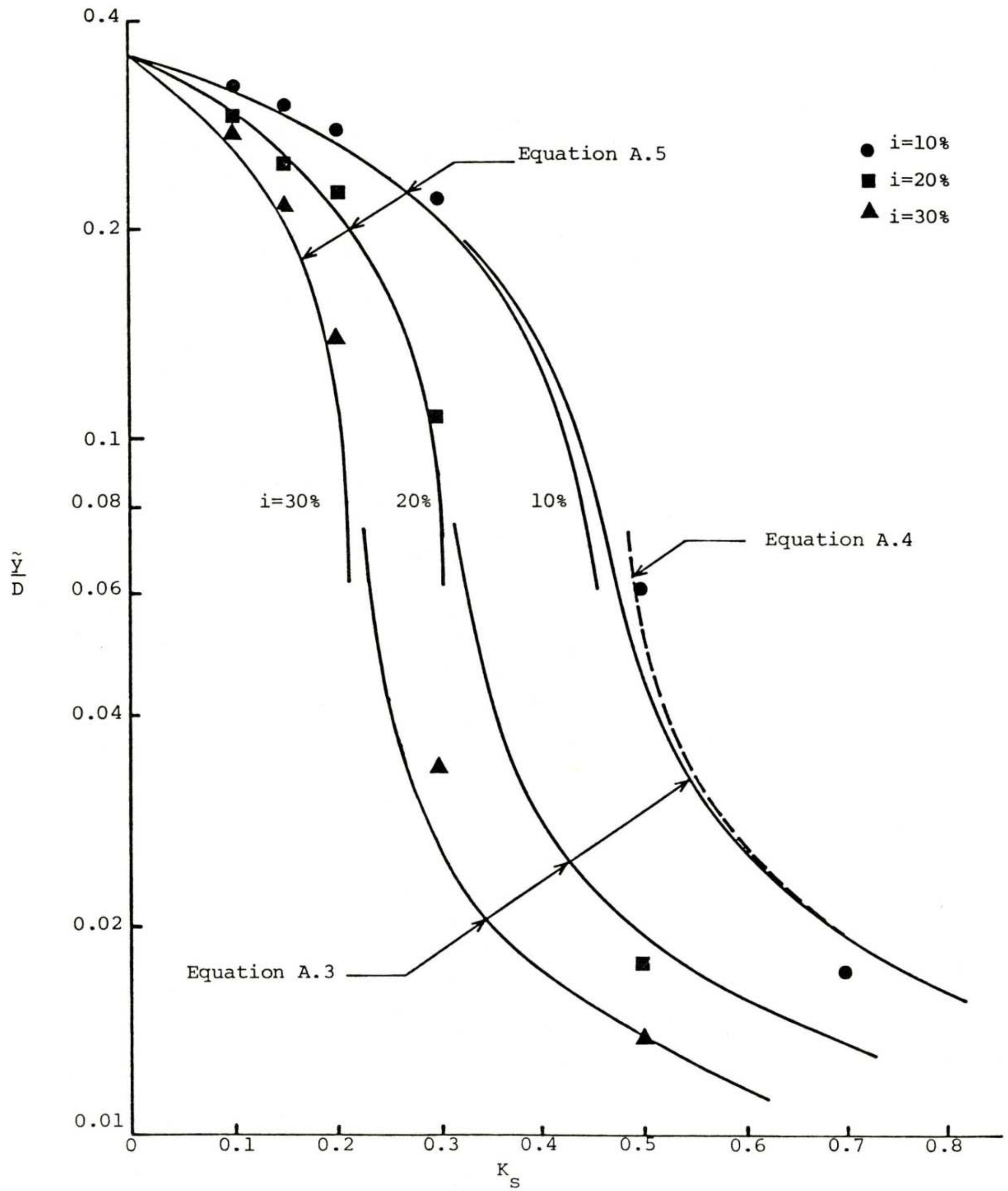


Transition regime $Sc \approx 4\pi K_{a0}$

Vortex-excited random regime $Sc \gg 4\pi K_{a0}$



Vickery & Basu (1983)



Vickery & Basu (1983)

Maximum crosswind response

$$\overline{y}_{max} = g_y \sigma_y$$

Vortex – excited random regime

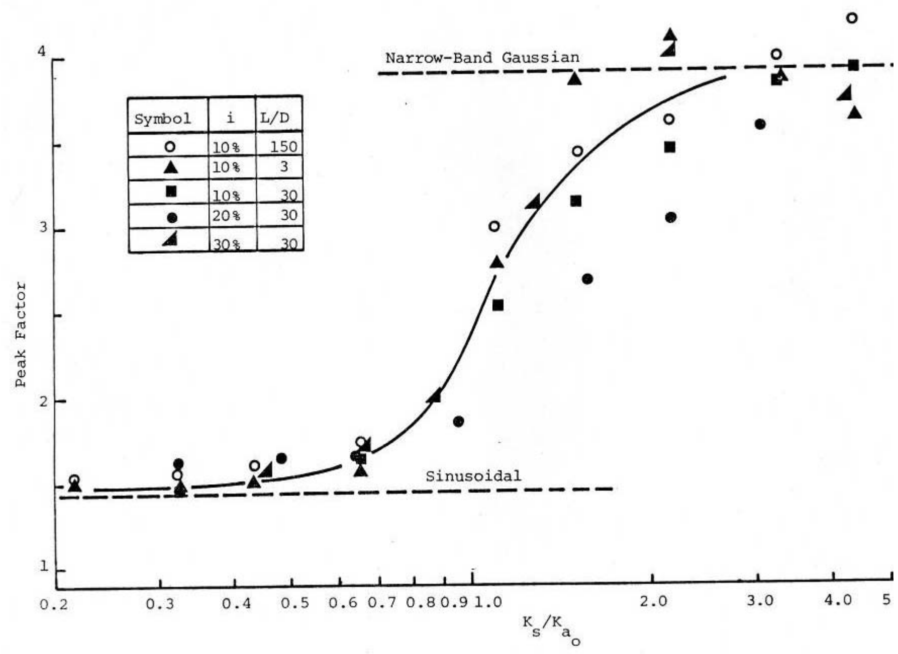
$$g_y = \sqrt{2\ell n(n_0 T)} + \frac{0.5772}{\sqrt{2\ell n(n_0 T)}}$$

Lock – in deterministic regime

$$g_y = \sqrt{2}$$

Transition equation

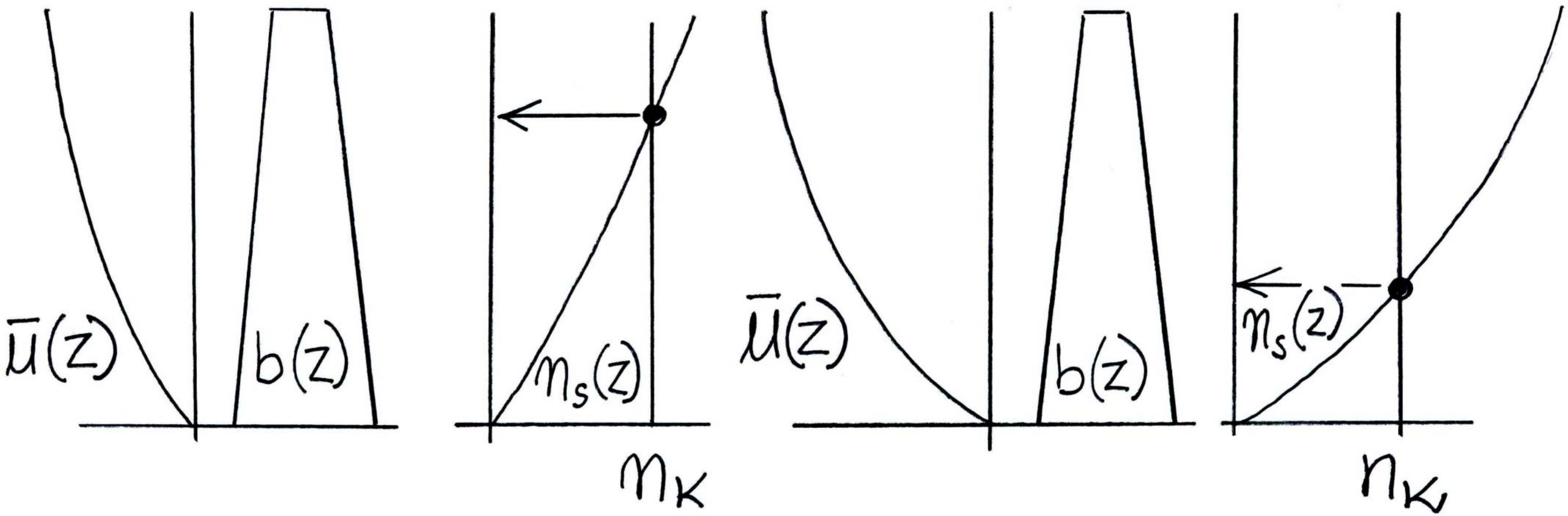
$$g_y = \sqrt{2} \cdot \left\{ 1 + 1,2 \cdot \mathrm{arctg}\left[\frac{0,75 \cdot Sc}{(4\pi K_{a0})^4} \right] \right\}$$



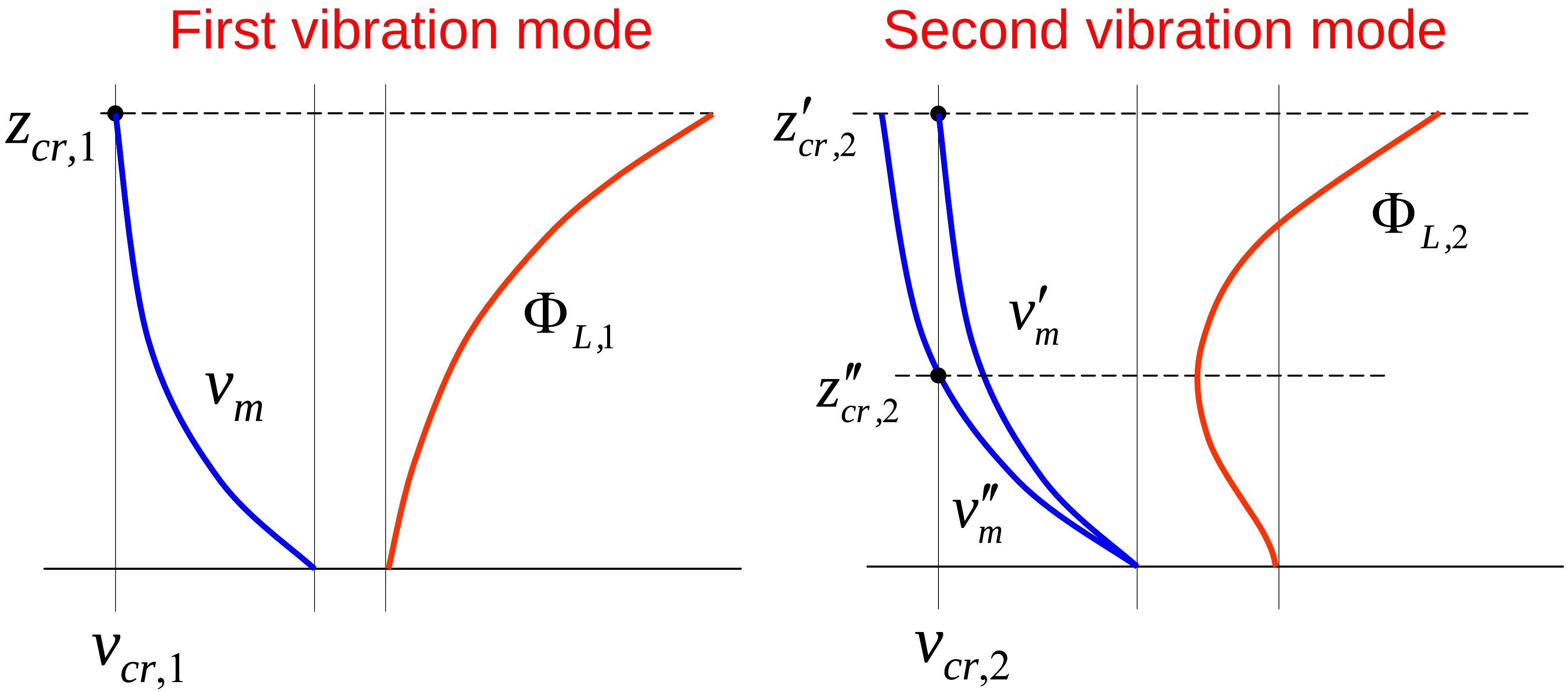
Vickery & Basu (1983)

Vortex shedding frequency

$$n_s(z) = \frac{\bar{u}(z)S(z)}{b(z)}$$



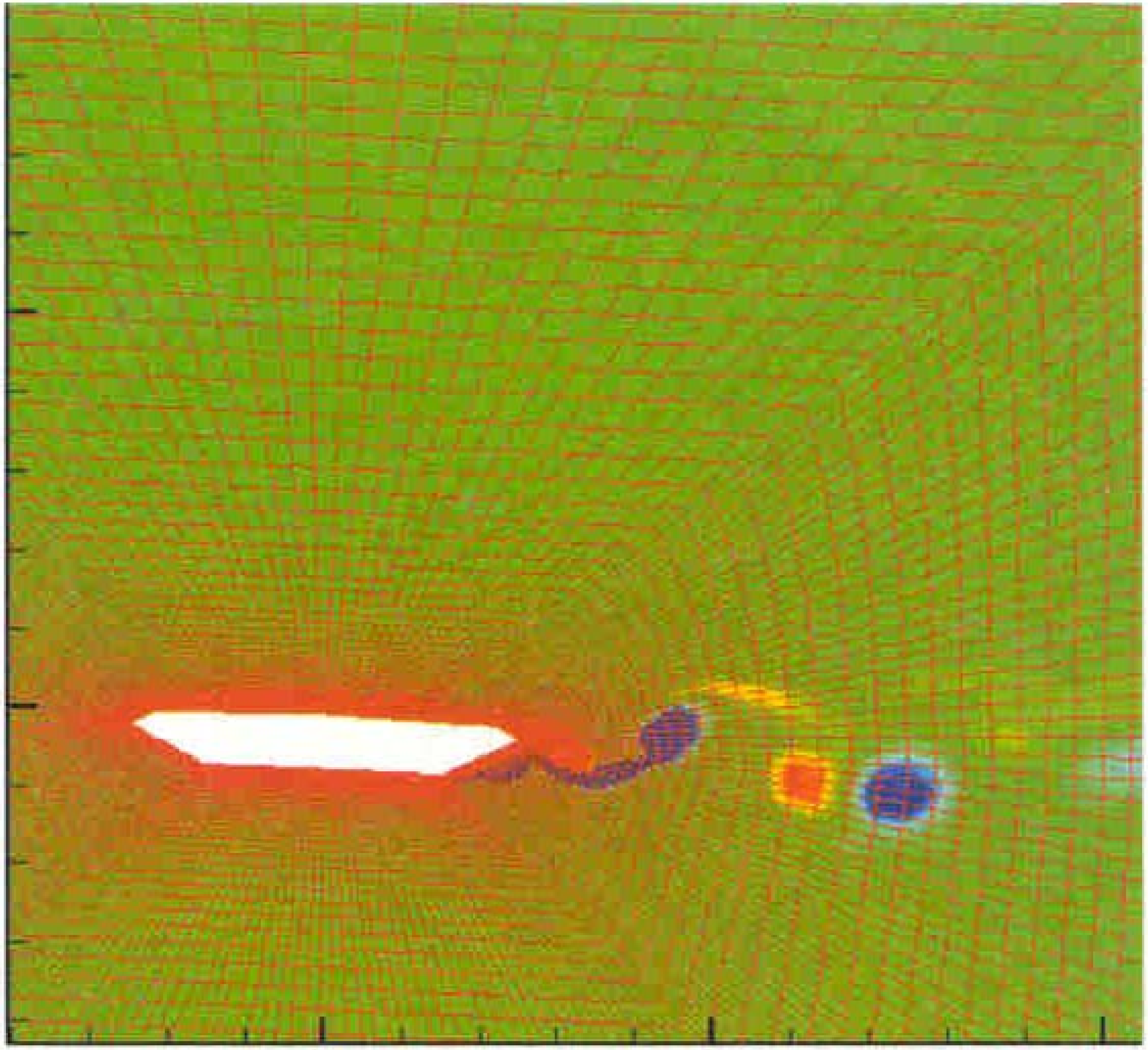
Variable positions at which the resonant vortex shedding occurs



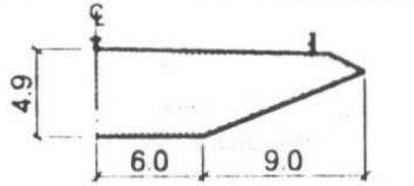
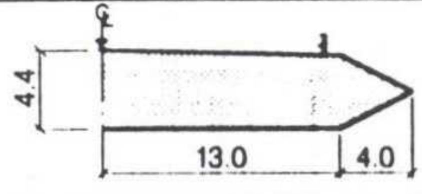
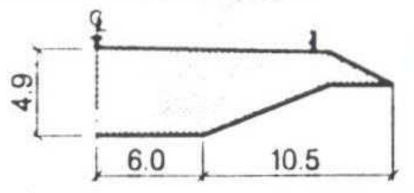
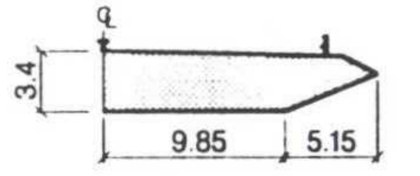
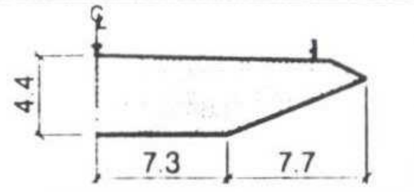
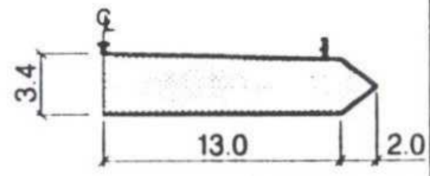
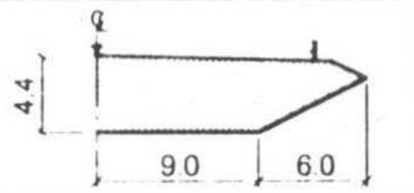
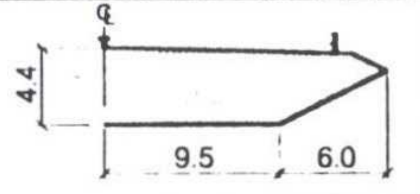
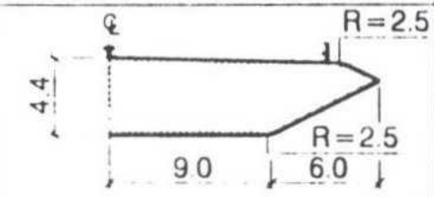
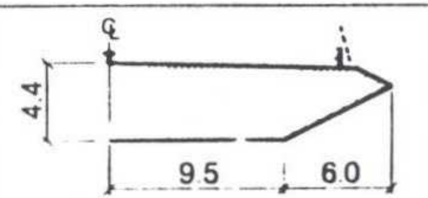
Critical positions at which the resonant vortex shedding occurs

Great Belt East Bridge, Denmark

Great Belt East Bridge, Denmark

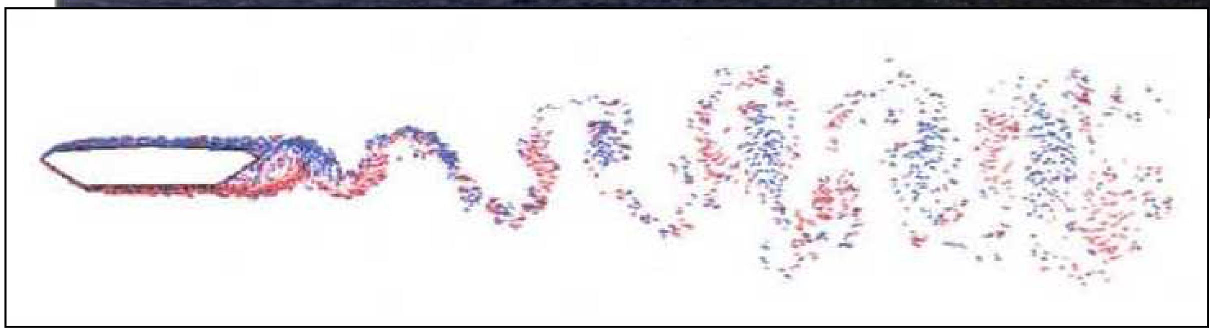


Great Belt East Bridge, Denmark

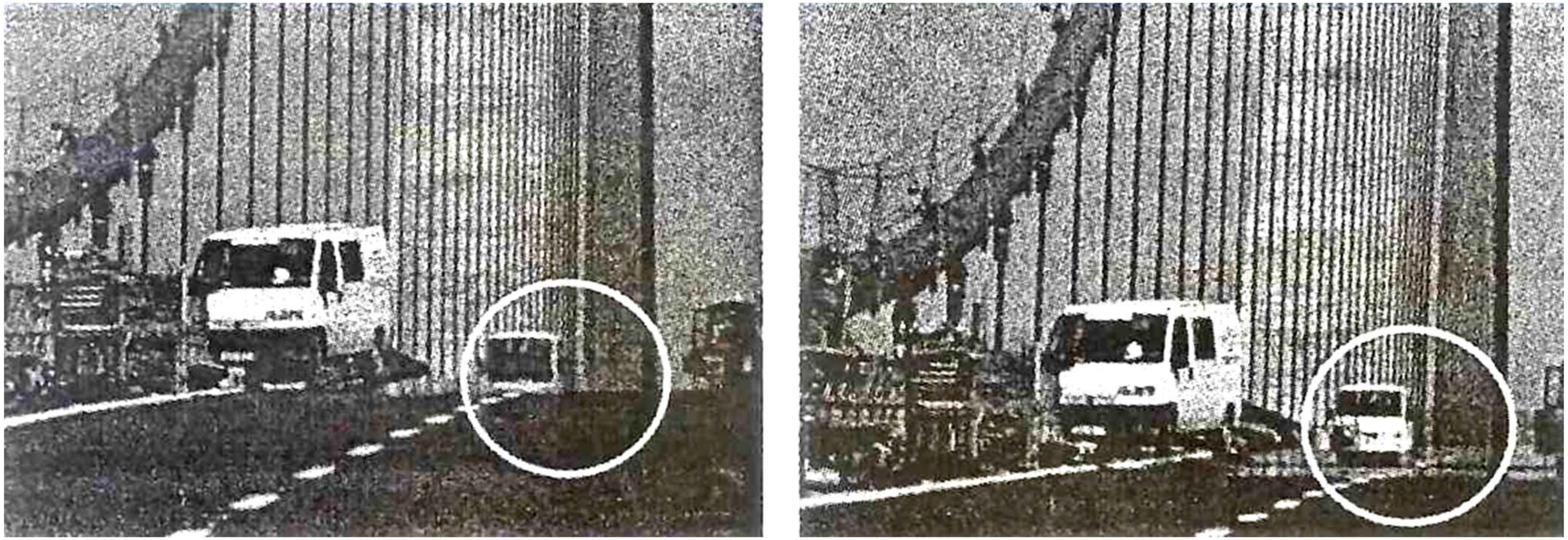
Critical Wind Speeds U_c for onset of Flutter							
Desig-nation	Cross Section	$\frac{U_c \text{ meas}}{U_c \text{ selb}}$	Flow Condition	Desig-nation	Cross Section	$\frac{U_c \text{ meas}}{U_c \text{ selb}}$	Flow Condition
H1.1		1.04	Smooth	H5.1		1.03 0.93	Smooth Turbulent
H1.2		1.03	Smooth	H6.1		1.05 0.94	Smooth Turbulent
H3.1		1.11 0.96	Smooth Turbulent	H7.1		1.03 0.95	Smooth Turbulent
H4.1		1.00 0.95	Smooth Turbulent	H9.1		0.99 0.94	Smooth Turbulent
H4.2		0.98	Turbulent	H9.2 Wind Screen Added		0.78	Smooth Turbulent

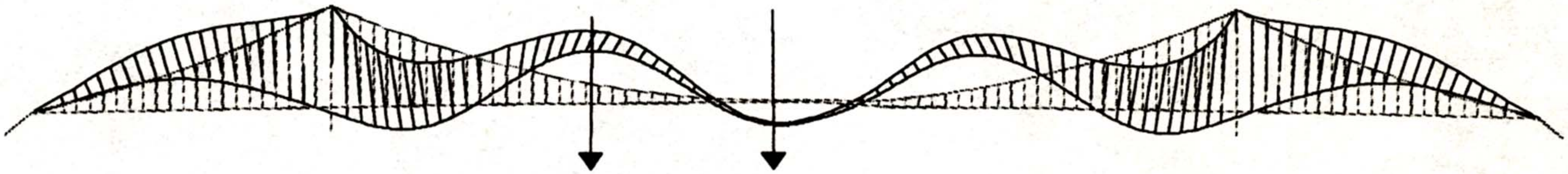
Great Belt East Bridge, Denmark

Great Belt East Bridge, Denmark



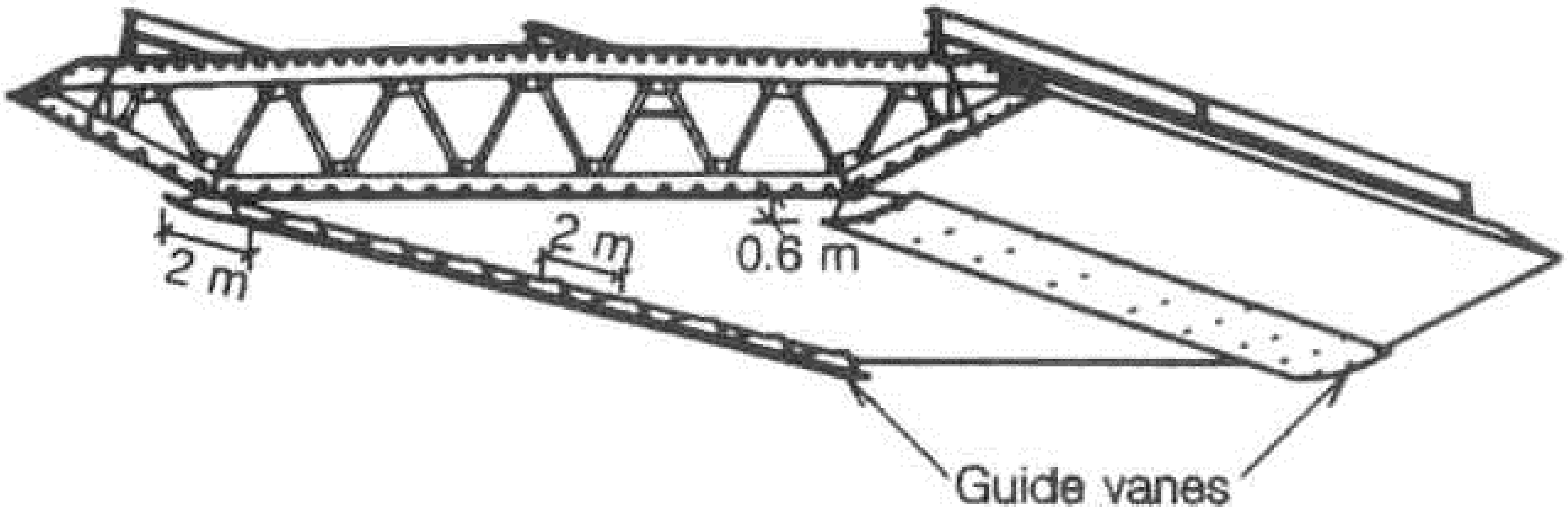
Great Belt East Bridge, Denmark





Great Belt East Bridge, Denmark

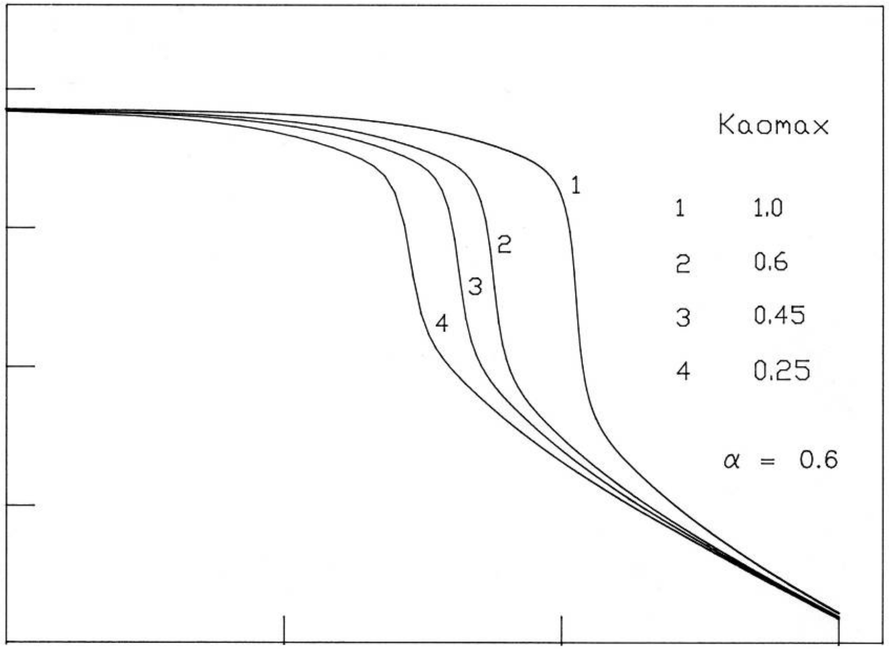
Great Belt East Bridge, Denmark

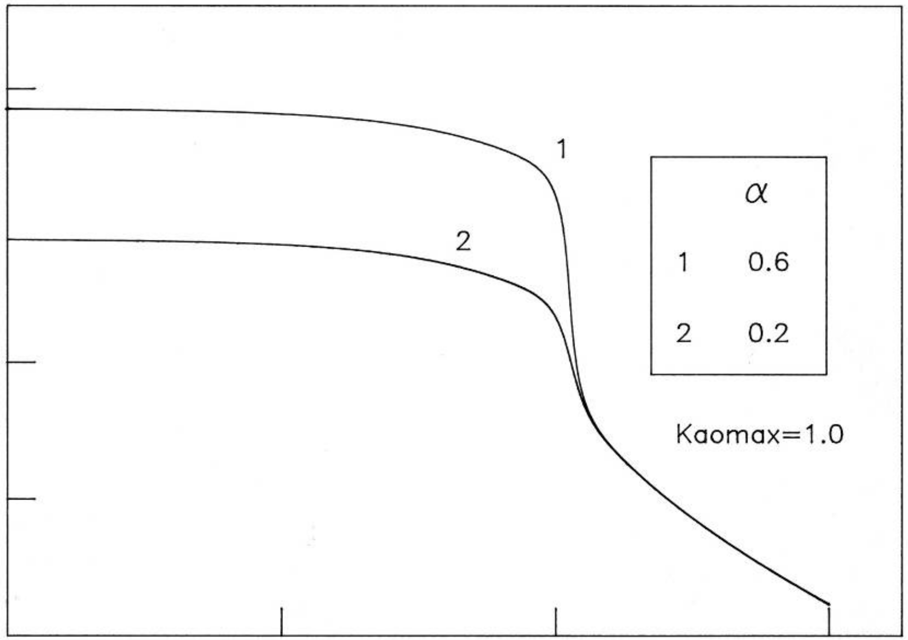


Great Belt East Bridge, Denmark – Guide vanes

Equivalent damping

$$\xi_{eq} \simeq \frac{\rho d^2}{4\pi m}\left\{Sc - 4\pi K_{a0}\left[1 - \frac{\sigma_y^2}{\alpha^2 d^2}\right]\right\}$$



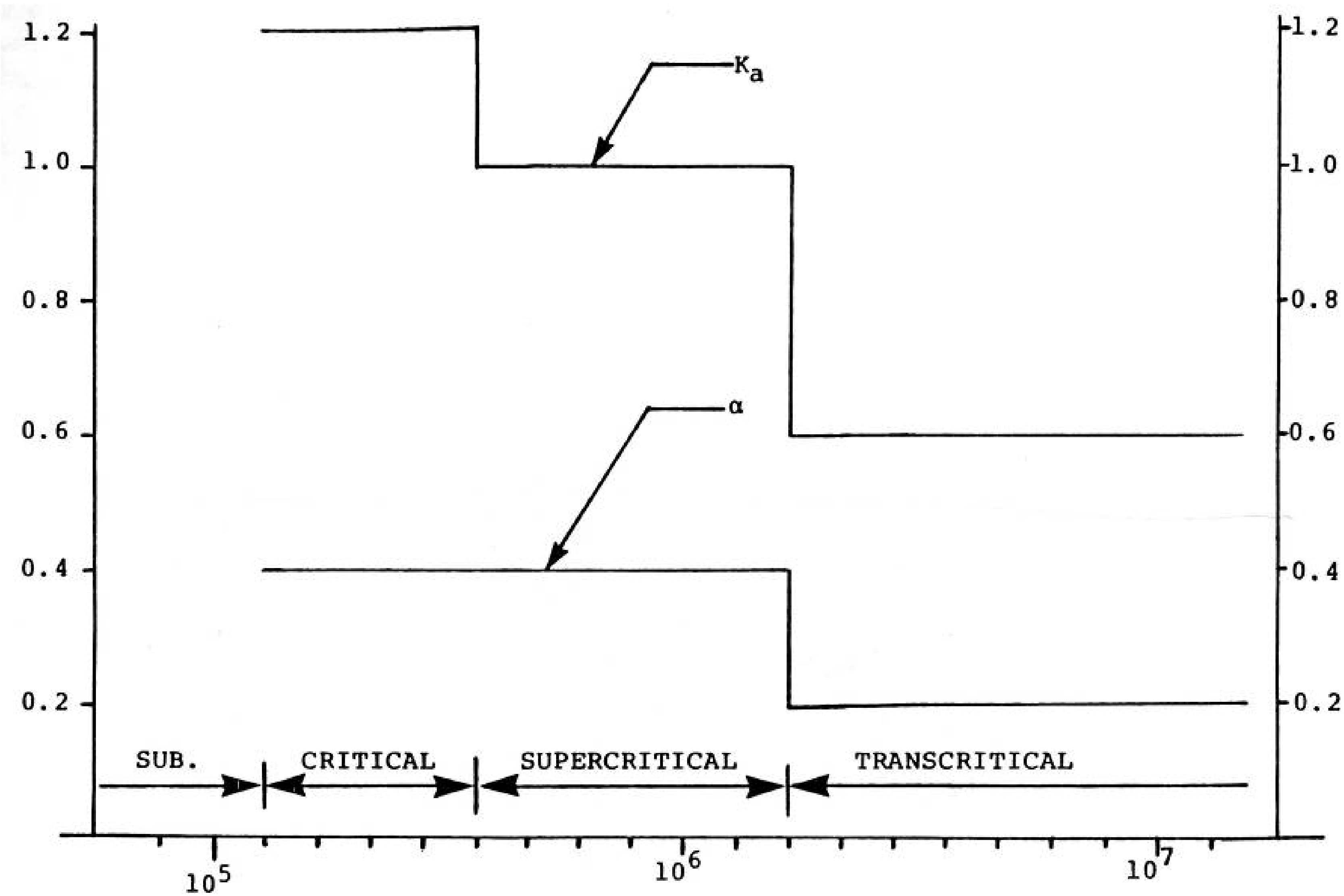


Vickery & Basu (1983)

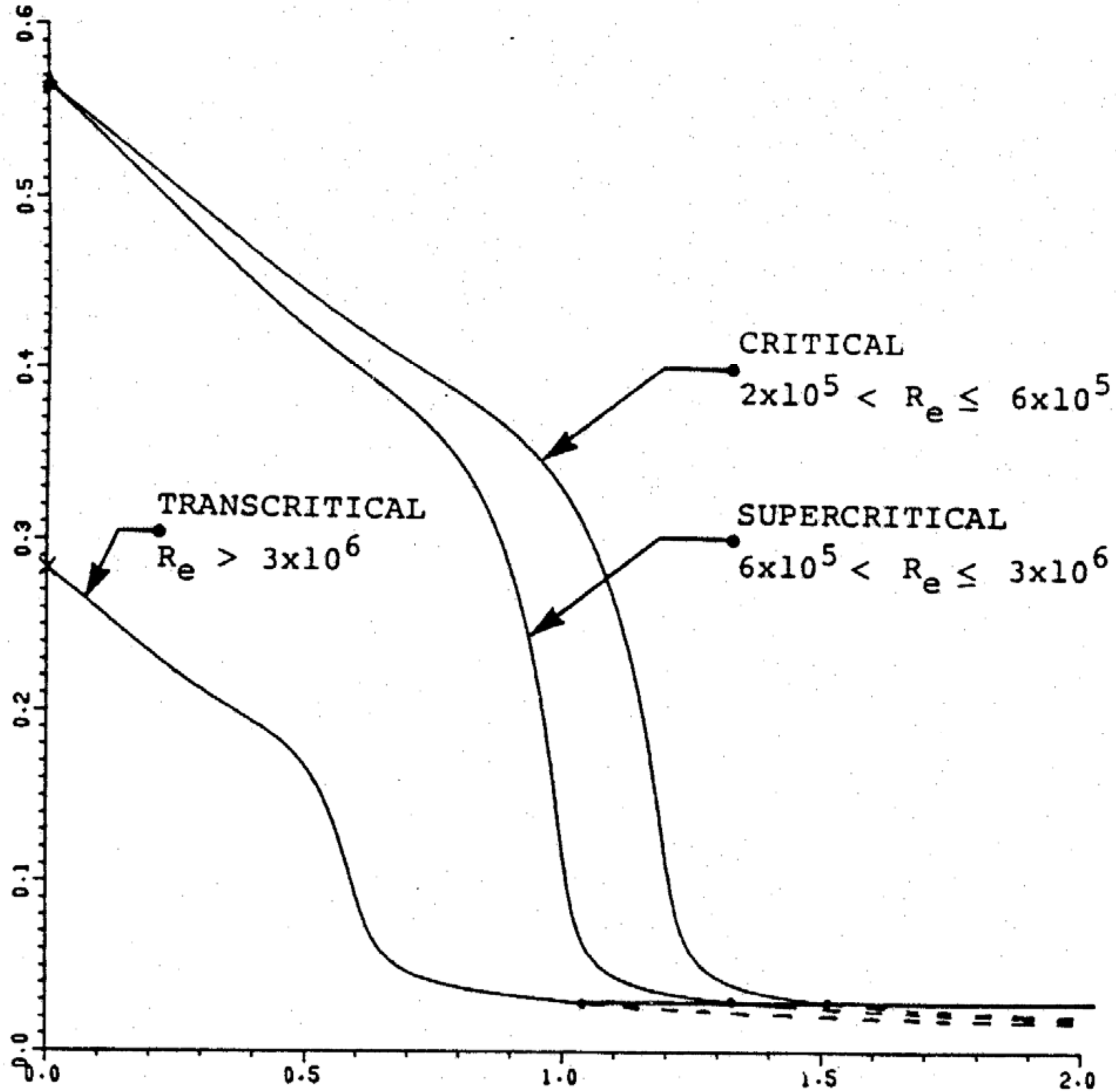
Table 1 Chimney data

Chimney identification			Chimney description										Derived or measured properties						Observations	
No	Location	Ref	Support	One of group	Lining	Shape	L (m)	LI/L	DT (m)	D_B/D_T	tb (mm)	w (kg/m)	C	f (Hz)	VCR (m/s)	$Re \times 10^{-6}$	Stiff's[1] (m³/kN)	Stiff's[2] (m³/kN)	Δ/L	Δ/D_T
1	Moss Landing	8	Piled	Yes	No	2	68.5	0.4	3.45	1.65	12	680	0.004^{m}	1.12^{m}	19	4.4	0.13	0.11	0.006^{f}	0.116^{f}
2	Moss Landing	8	Piled	Yes	50mm Gunite	2	68.5	0.4	3.45	1.65	12	1470	0.006^{m}	0.82^{m}	14	3.2	0.11	0.10	$0.003^{?}$	$0.06^{?}$
3	New Jersey	E	Piled	No	No	2	46	0.4	1.7	2.2	9.5	262	0.004						0.01^{d}	0.29^{d}
4	New Jersey	E	Piled	No	50mm Gunite	2	46	0.4	1.7	2.2	9.5	650	0.006						$0.002^{?}$	$0.05^{?}$
5	Detroit St. Clair No. 1	9	Piled	Yes	No	2	91.5	0.32	4.88	1.69	16	765	0.005	1.0^{m}	24	8.1	0.22	0.19	0.011^{f}	0.22^{f}
6	Detroit St. Clair No. 4	9, 11	Piled	Yes	60/120mm Gunite	2	76	0.38	4.9	1.7	15	2175	0.008^{m}	1.05^{m}	25	8.2	0.07	0.06	$0.002^{?}$	0.03
7	Mitzushima Japan	10	Piled	No	Yes$^{?}$	3	90	1.0	4.5	1.6	$15^{?}$	2090	0.005^{m}	0.75^{m}	27	8.2	0.15	0.12	0.003	0.055
8	No. 35	2	Piled	?	No	2	36	0.02	1.5	1.55	6	230	0.004	1.04^{m}	8	0.76	0.20	0.17	0.015	0.4
9	No. 36	2	Piled	?	35mm Gunite	2	49	0.31	2.9	1.8	6	955	0.006						0.007	0.12
10	No. 37	2	Piled	?	No	2	61	0.25	2.1	1.75	14	410	0.004	0.66	5.5^{m}	1.0	0.34	0.30	$0.008^{?}$	$0.24^{?}$
11	No. 38	2	Piled	?	No	2	30.5	0.25	1.4	1.75	9.5	275	0.004						0.003	0.07
12	Contra Costa	11	Piled	Yes	No	1	61	–	3.35	1.0	12	620	0.006^{m}	0.97^{m}	16	3.7	0.16	0.13	$0.008^{?}$	$0.15^{?}$
13	Contra Costa	11	Piled	Yes	75mm Gunite	1	61	–	3.35	1.0	12	2040	0.009^{m}	0.71^{m}	12	2.8	0.14	0.12	0.001	0.02
14	New London Connecticut	11	Shallow footing	No	60mm Gunite	2	74	0.25	3.5	1.5	$12^{?}$	1640	0.006						0.001	0.014
15	Chiba Japan	10	Shallow footing	No	50mm Refr.	3	91.5	1.0	4.38	1.58	$18^{?}$	2010	0.004^{m}	0.68^{m}	24	7.4	0.36	0.28	0.003	0.06
16	Aldermaston	12	Shallow footing	No	No	2	46	0.26	1.22	2.5	25	190	0.004	0.92^{m}	6	0.5	0.28	0.24	0.006	0.25
17	Wakayama Japan	10	Shallow footing	No	Yes$^{?}$	3	83	1.0	3.2	2.0	$18^{?}$	1360	0.006^{m}	1.15^{m}	29	6.6	0.2	0.15	0.0025	0.06
18	Sakai Japan	10	Shallow footing	No	Yes$^{?}$	3	77	1.0	3.2	1.73	$18^{?}$	1360	0.005^{m}	0.69^{m}	18	4.0	0.21	0.17	0.001	0.02
19	Germany	13	Shallow footing	No	No	2	145	0.24	6.0	1.68	30	1950	0.004	0.48^{m}	14	5.6	0.79	0.74	0.008^{f}	0.2^{f}
20	No 2	14	Shallow footing	?	No	2	30.5	0.25	1.4	1.75	$15^{?}$	330	0.004	1.6^{m}	11	1.1	0.08	0.07	0.004	0.09
21	No 32	2	Shallow footing	?	No	2	76	0.31	2.75	2.22	$15^{?}$	450	0.004					–	0.015	0.41
22	France*	E	Shallow footing	No	No	2	72	0.33	2.5	1.69	18	470	0.015					–	0.002	0.06
23	France	E	Shallow	No	No	2	72	0.33	2.5	1.69	18	470	0.004					–	0.02^{d}	0.4^{d}
24	France	E	Shallow footing	No	50mm Refr.	2	72	0.33	2.5	1.69	18	980	0.006					–	0.003	0.08
25	India	E	Shallow footing	No	50mm Refr.	2	46	0.29	1.4	2.0	9.5	450	0.006					–	0.007	0.21
26	Canada	E	Shallow footing	No	No	2	36	0.36	0.4	2.26	12	85	0.004					–	0.011	1.0
27	Florida	E	Shallow footing	No	30mm Refr.	3	49	1.0	0.85	2.73	9.5	245	0.006					–	0.006^{d}	0.35^{d}

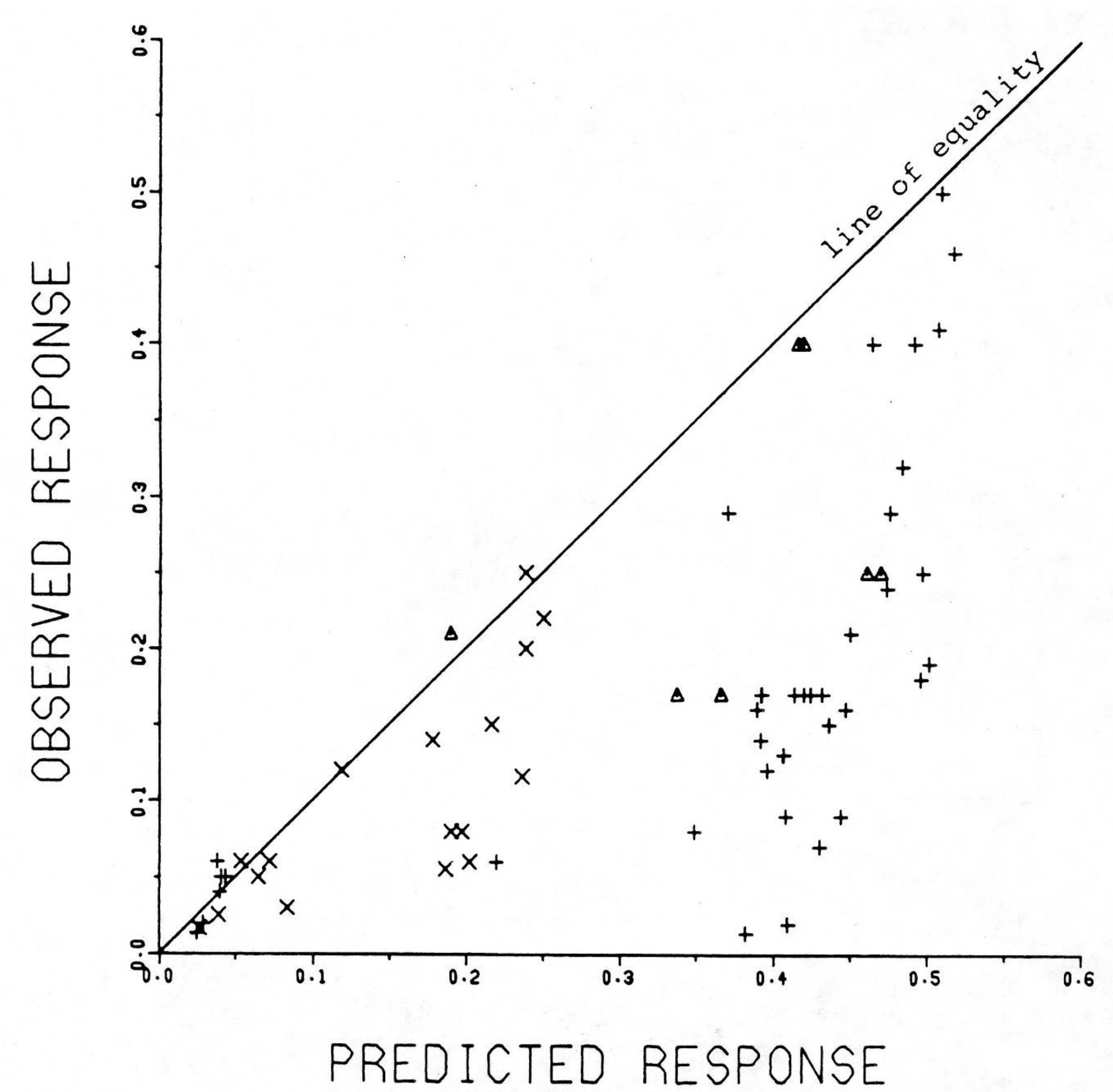
Pritchard (1984)



Pritchard (1984)



Pritchard (1984)


line of equality
OBSERVED RESPONSE
PREDICTED RESPONSE
0.0
0.1
0.2
0.3
0.4
0.5
0.6

Daly (1986)

Crosswind equation of motion

$$y(z,t) = \sum_j \psi_j(z) p_j(t)$$

Vortex shedding resonant with the k – th mode

$$y(z,t) \simeq y_k(z,t) = \psi_k(z) p_k(t)$$

$$\ddot{p}_k(t) + 2\xi_k(2\pi n_k)\dot{p}_k(t) + (2\pi n_k)^2 p_k(t) = \frac{1}{m_k}\int_0^h f(z,t)\psi_k(z)dz$$

$$m_k = \int_0^h m(z)\psi_k^2(z)dz$$

Ruscheweyh (1988)

$$y(z,t) = \sum_j \psi_j(z) p_j(t) \Rightarrow$$

Maximum crosswind displacement

$$\{y_k(z)\}_{max} = \psi_k(z) |p_k|_{max}$$

Equivalent static crosswind force

$$f_{k,eq}(z) = \alpha\, m(z) \psi_k(z) \Rightarrow$$

$$(2\pi n_k)^2 |p_k|_{max} = \frac{1}{m_k} \int_0^h f_{k,eq}(z) \psi_k(z) dz = \frac{1}{m_k} \int_0^h \alpha\, m(z) \psi_k^2(z) dz = \alpha$$

$$f_{keq}(z) = (2\pi n_k)^2 |p_k|_{max}\, m(z) \psi_k(z)$$

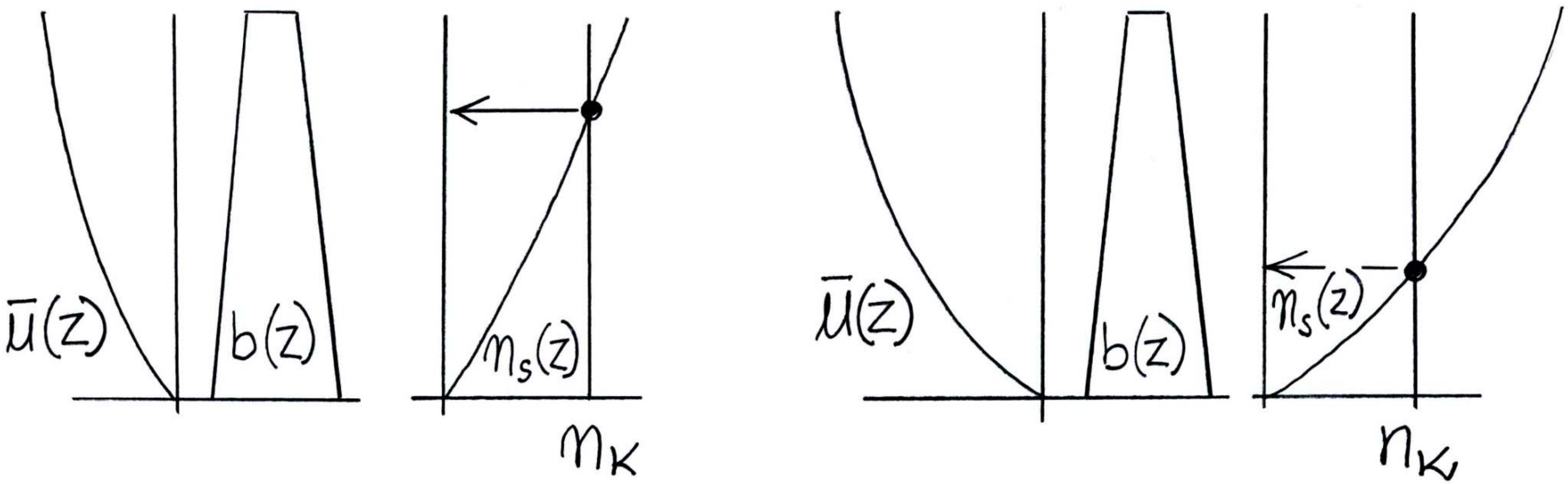
Ruscheweyh (1988)

$$\ddot{p}_k(t) + 2\xi_k(2\pi n_k)\dot{p}_k(t) + (2\pi n_k)^2 p_k(t) = \frac{1}{m_k}\int_0^h f(z,t)\psi_k(z)dz$$

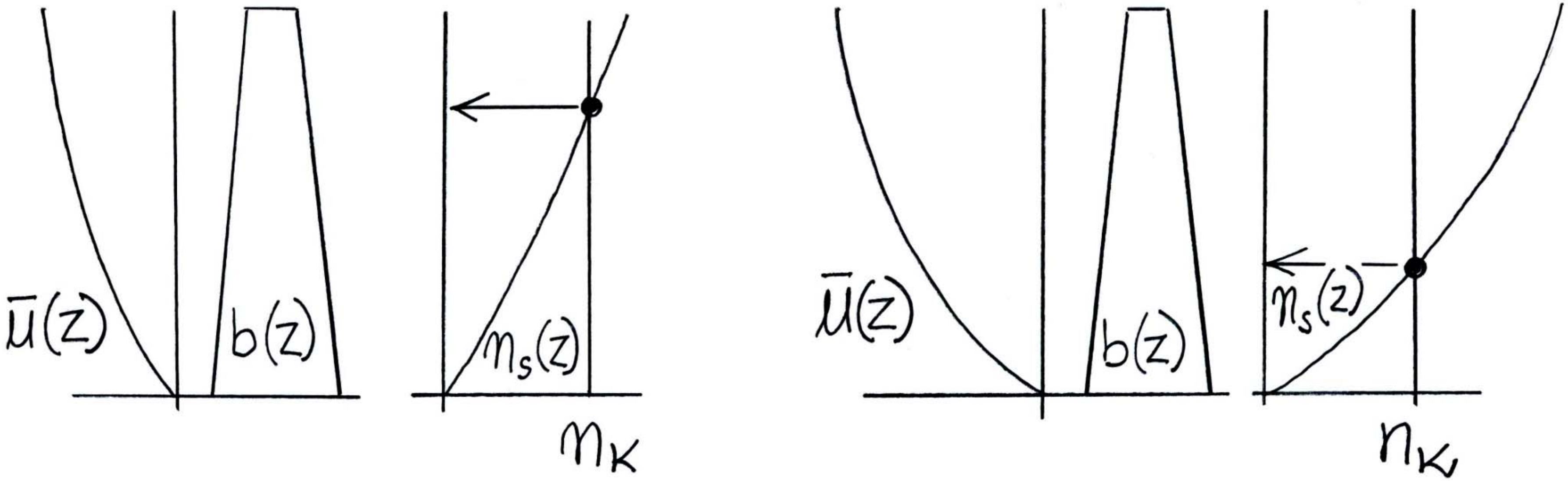
Harmonic crosswind force due to vortex shedding

$$f(z,t) = \frac{1}{2}\rho\bar{u}^2(z)b(z)c_L(z)\sin\left[2\pi n_s(z)t\right]$$

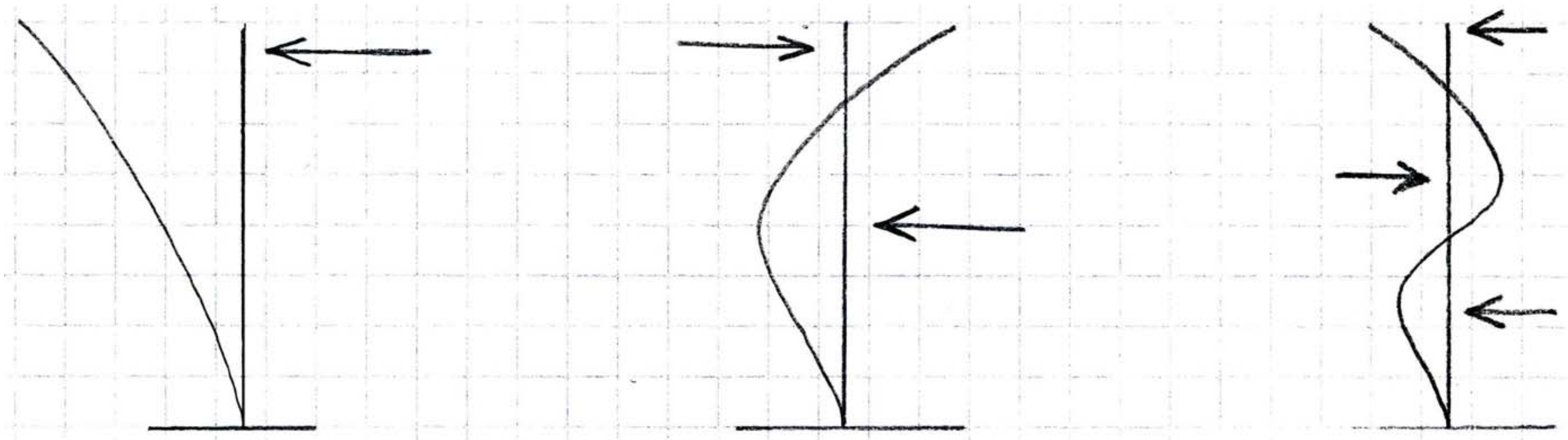
$$n_s(z) = \frac{\bar{u}(z)S(z)}{b(z)}$$



Ruscheweyh (1988)



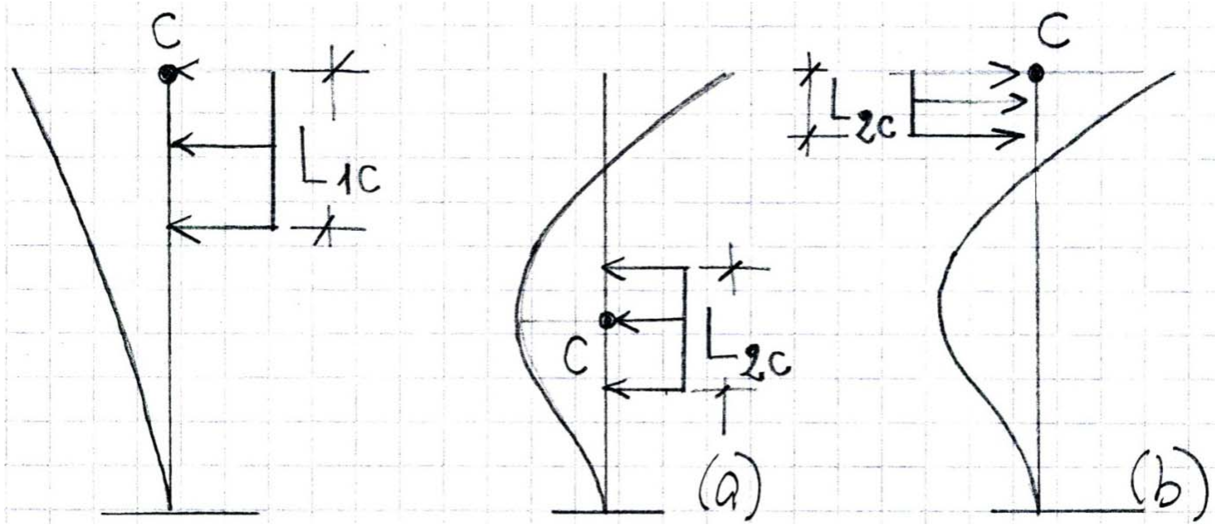
Worst resonant vortex shedding conditions



Ruscheweyh (1988)

Antinode position C at heigth z_{kc}

$$\bar{u}_{kc} = \bar{u}(z_{kc}), b_{kc} = b(z_{kc}) \Rightarrow \bar{u}_{kc} = \frac{n_k b_{kc}}{S}$$



Correlation length L_{kc}

$$f(z,t) = \frac{1}{2}\rho\bar{u}_{kc}^2 b_{kc} c_{Lc} \sin(2\pi n_k t) \quad \text{for } z \in L_{kc}$$

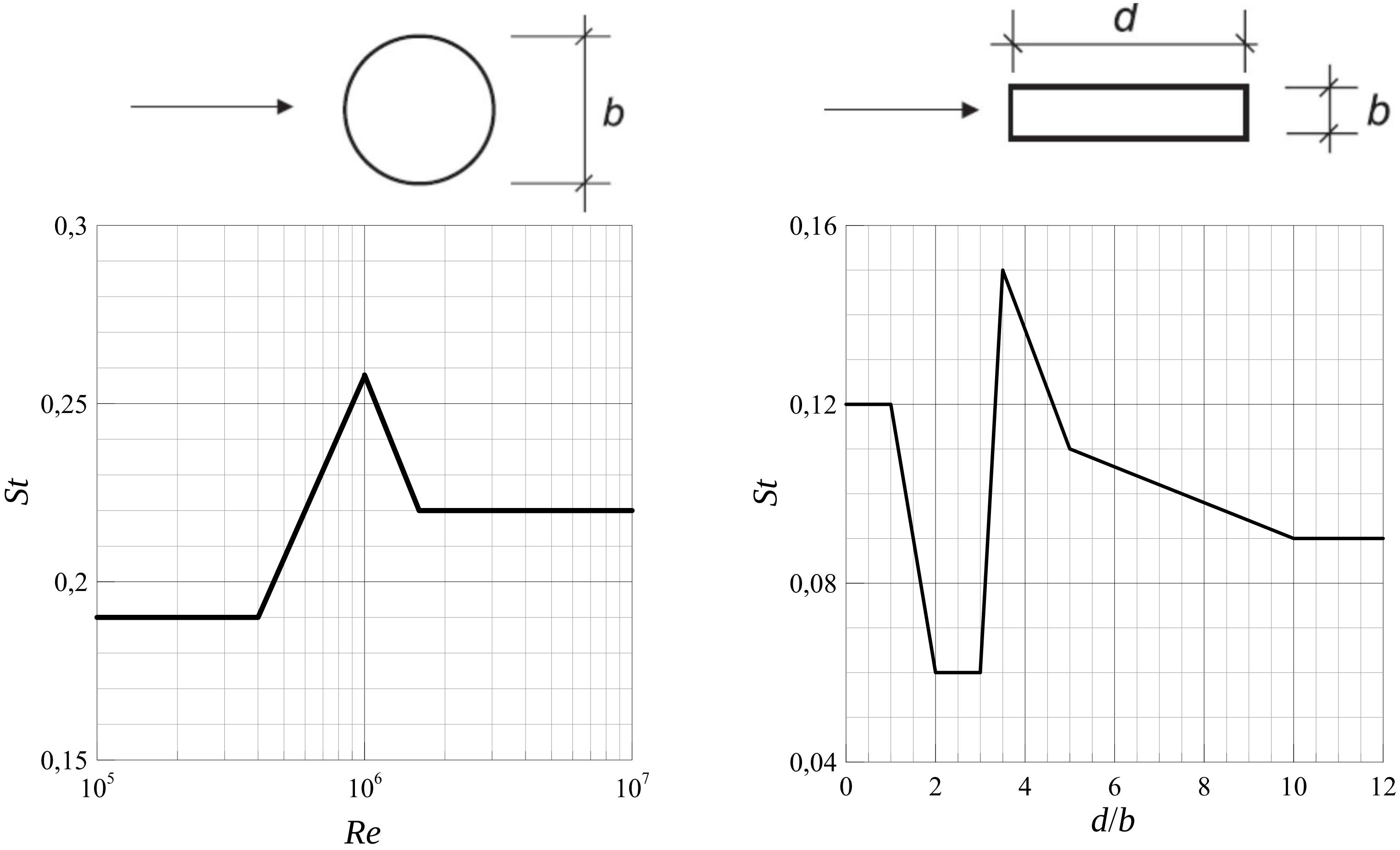
$$f(z,t) = 0 \quad \text{elsewhere}$$

$$c_{Lc} = c_L(z_{kc})$$

$$\boxed{\frac{\left(y_{kc}\right)_{max}}{b_{kc}} = K_k K_{wk} \frac{c_{Lc}}{Sc_k S^2}}$$

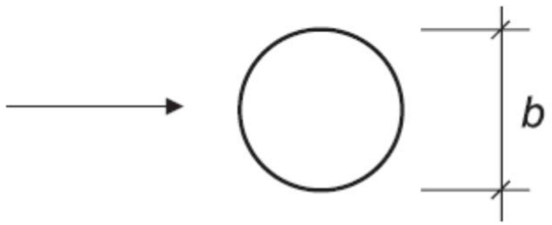
Maximum crosswind displacement

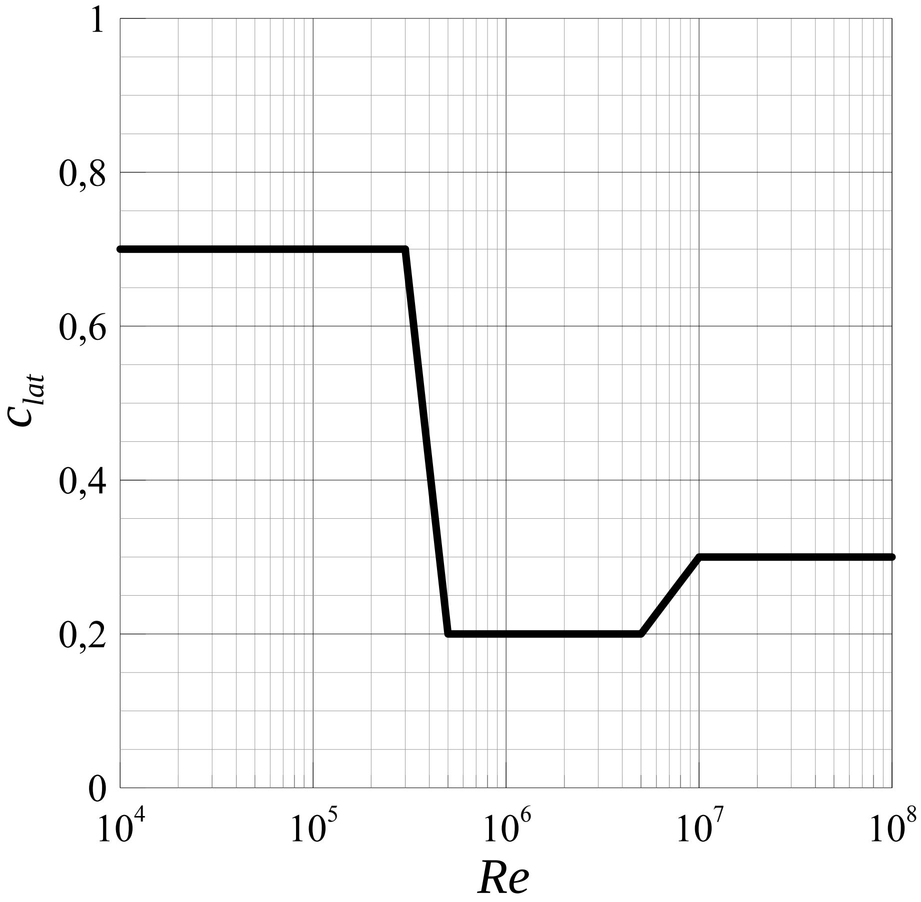
$$\boxed{\frac{\left(y_{kc}\right)_{max}}{b_{kc}} = K_k K_{wk} \frac{c_{Lc}}{Sc_k S^2}}$$



Strouhal number

$$\boxed{\frac{(y_{kc})_{max}}{b_{kc}} = K_k K_{wk} \frac{c_{Lc}}{Sc_k S^2}}$$

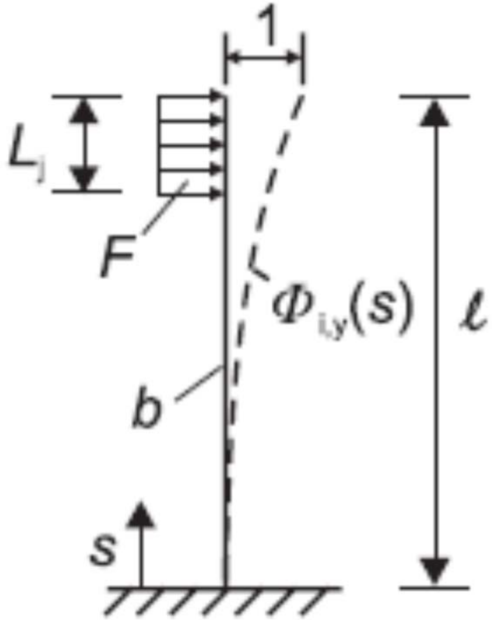




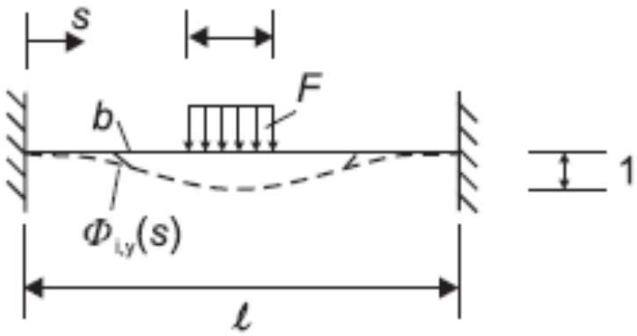
Wake lift coefficient

$$\boxed{\frac{\left(y_{kc}\right)_{max}}{b_{kc}} = K_k K_{wk} \frac{c_{Lc}}{Sc_k S^2}}$$

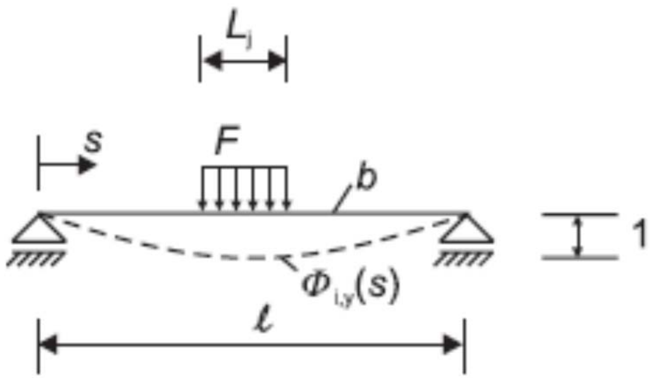
$$\boxed{K_k = \frac{\left|\psi_{kc}\right|}{4\pi} \frac{\int_0^h \left|\psi_k(z)\right| dz}{\int_0^h \psi_k^2(z) dz}}$$



$$\boxed{K_k = 0{,}13}$$



$$\boxed{K_k = 0{,}10}$$

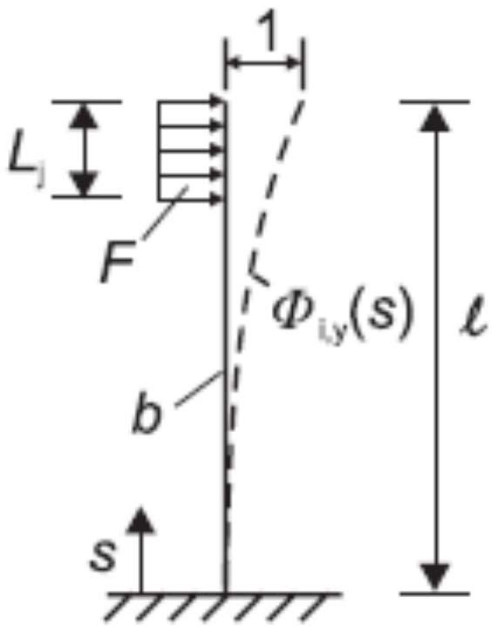


$$\boxed{K_k = 0{,}11}$$

Mode shape factor

$$\frac{(y_{kc})_{max}}{b_{kc}} = K_k K_{wk} \frac{c_{Lc}}{Sc_k S^2}$$

$$K_{wk} = \frac{\int_{L_{kc}} |\psi_k(z)| dz}{\int_0^h |\psi_k(z)| dz}$$

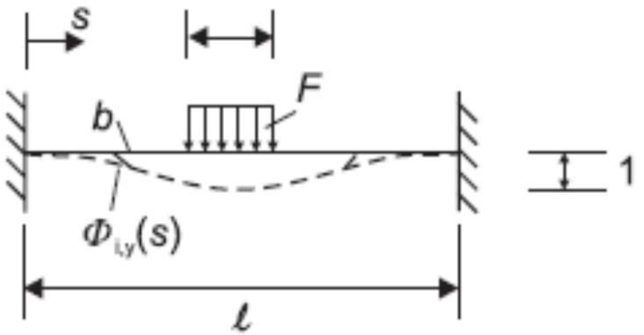


$$K_{wk} = 3 \cdot \frac{L_{kc}}{h}\left[1 - \frac{L_{kc}}{h} + \frac{1}{3} \cdot \left(\frac{L_{kc}}{h}\right)^2\right] \leq 0,6$$

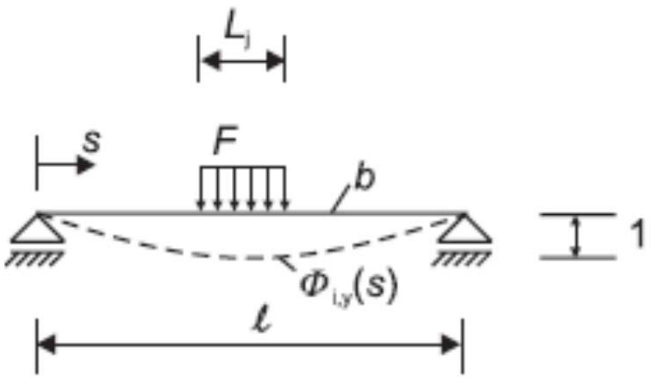
Correlation length factor

$$\frac{(y_{kc})_{max}}{b_{kc}} = K_k K_{wk} \frac{c_{Lc}}{Sc_k S^2}$$

$$K_{wk} = \frac{\int_{L_{kc}} |\psi_k(z)| dz}{\int_0^h |\psi_k(z)| dz}$$



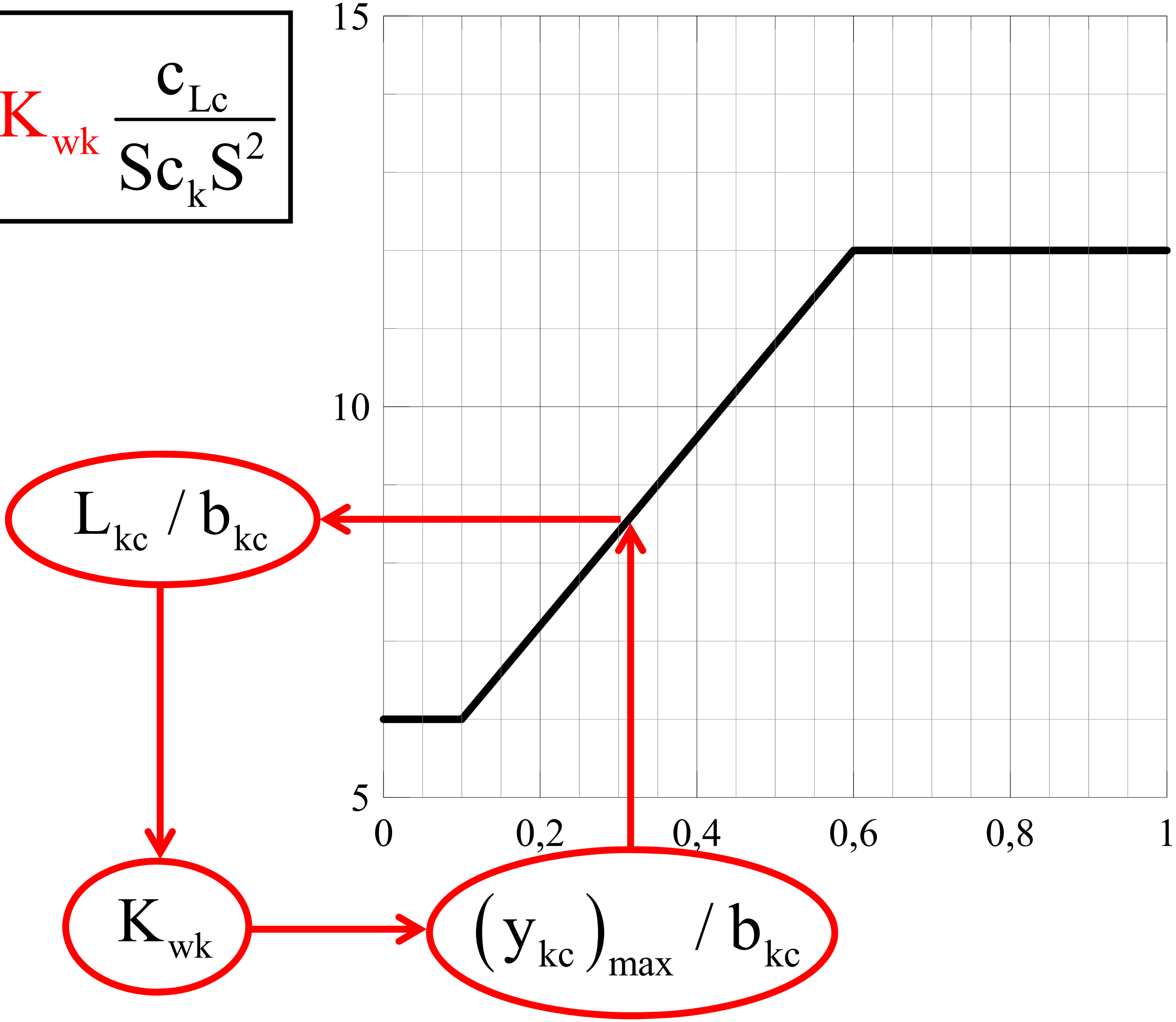
$$K_{wk} = \cos\left[\frac{\pi}{2} \cdot \left(1 - \frac{L_{kc}}{h}\right)\right] \leq 0,6$$



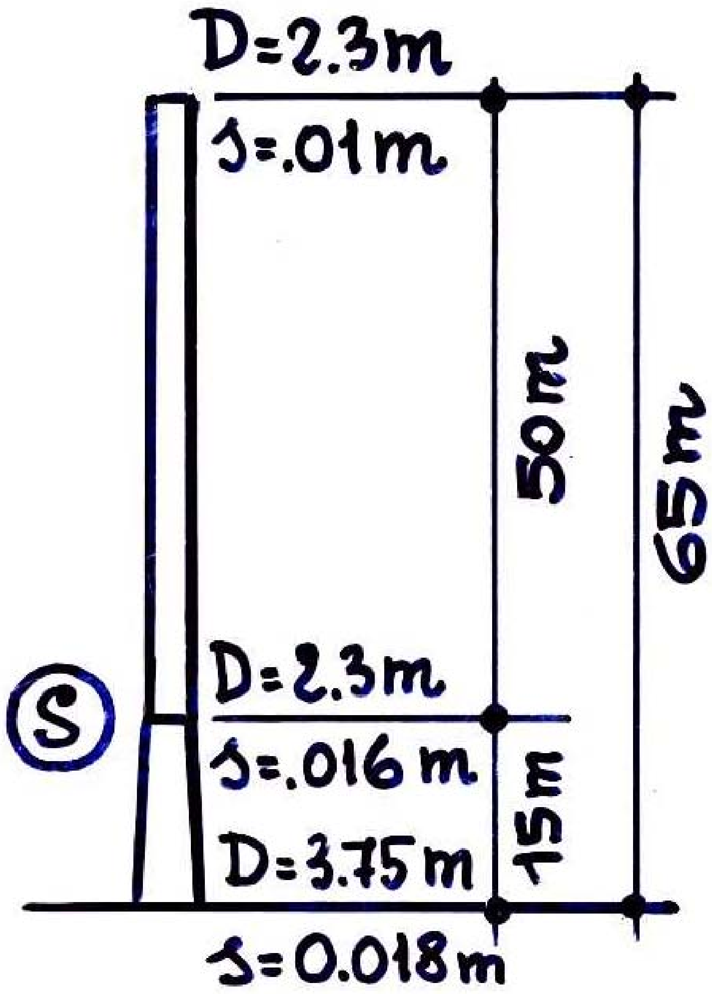
$$K_{wk} = \frac{L_{kc}}{h} + \frac{1}{\pi} \cdot \sin\left[\pi \cdot \left(1 - \frac{L_{kc}}{h}\right)\right] \leq 0,6$$

Correlation length factor

$$\boxed{\frac{(y_{kc})_{max}}{b_{kc}} = K_k K_{wk} \frac{c_{Lc}}{S c_k S^2}}$$



Correlation length method



$n_0 = 1$ Hz; $\xi_s = 0.002$

$\bar{u}_{crit} = 11.5$ m/s

$m_t = 565$ kg/m

Correlation length method

$$\frac{\bar{y}_{max}}{D} = K_w K \frac{C_{lat}}{S_c S^2}$$

$D = 2.3m$; $K = 0.13$, $Re = 1.76 \times 10^6 \Rightarrow C_{lat} = 0.2$, $S = 0.2$, $S_c = 2.57$

$L/D = 6 \Rightarrow K_w = 0.51 \Rightarrow \bar{y}_{max}/D = 0.13 \Rightarrow$

$L/D = 6.3 \Rightarrow K_w = 0.53 \Rightarrow \bar{y}_{max}/D = 0.14 \Rightarrow$

$L/D = 6.4 \Rightarrow K_w = 0.54 \Rightarrow \bar{y}_{max}/D = 0.14 \Rightarrow$

$\bar{y}_{max} = 0.32m = H/200$; $\max \sigma = 65\ N/mm^2$ S

Correlation length method

Number of stress cycles N
caused by the vortex excited oscillations

$$\boxed{N = 6.3\times10^{7}\, T n_o \varepsilon_o \left(\frac{\bar{u}_{crit}}{\bar{u}_o}\right)^2 \exp\left\{-\left(\frac{\bar{u}_{crit}}{\bar{u}_o}\right)^2\right\}}$$

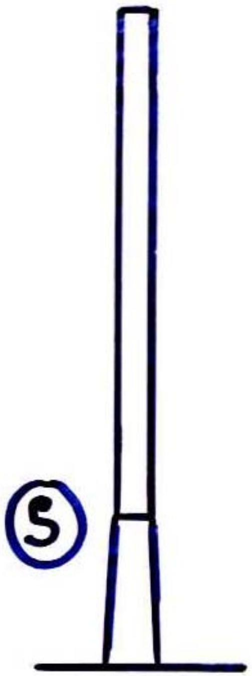
T = life time (years)

ε_o = bandwidth factor ($\varepsilon_o \sim 0.3$)

$\bar{u}_{crit} = D n_o / S$ = mean critical wind velocity

$\bar{u}_o = \sqrt{2}$ times the modal value of the probability density function of $\bar{u}$

Correlation length method

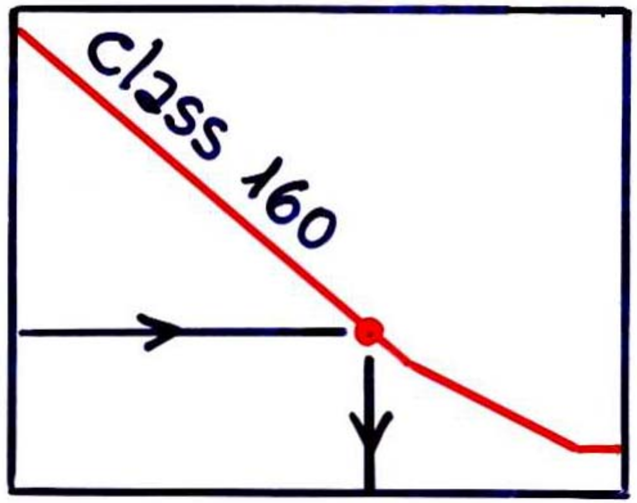


S $\max \sigma = 65\ N/mm^2 \Rightarrow$

$\Delta\sigma = 65 \times 2 = 130\ N/mm^2$

$N = 504'080$ cycles/year

Eurocode 3

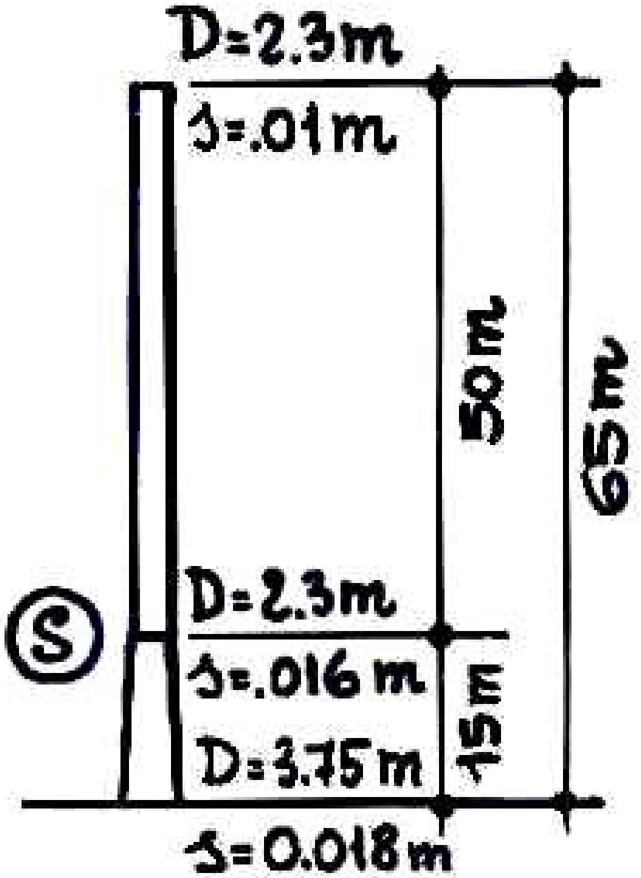


$\Delta\sigma = 130\ N/mm^2$

$N_{total} = 3 \times 10^6$

Fatigue life $= N_{total}/N = 6$ years

Correlation length method



Correlation length method

$$\bar{y}_{max} = 0,32 \text{ m}$$

Spectral method

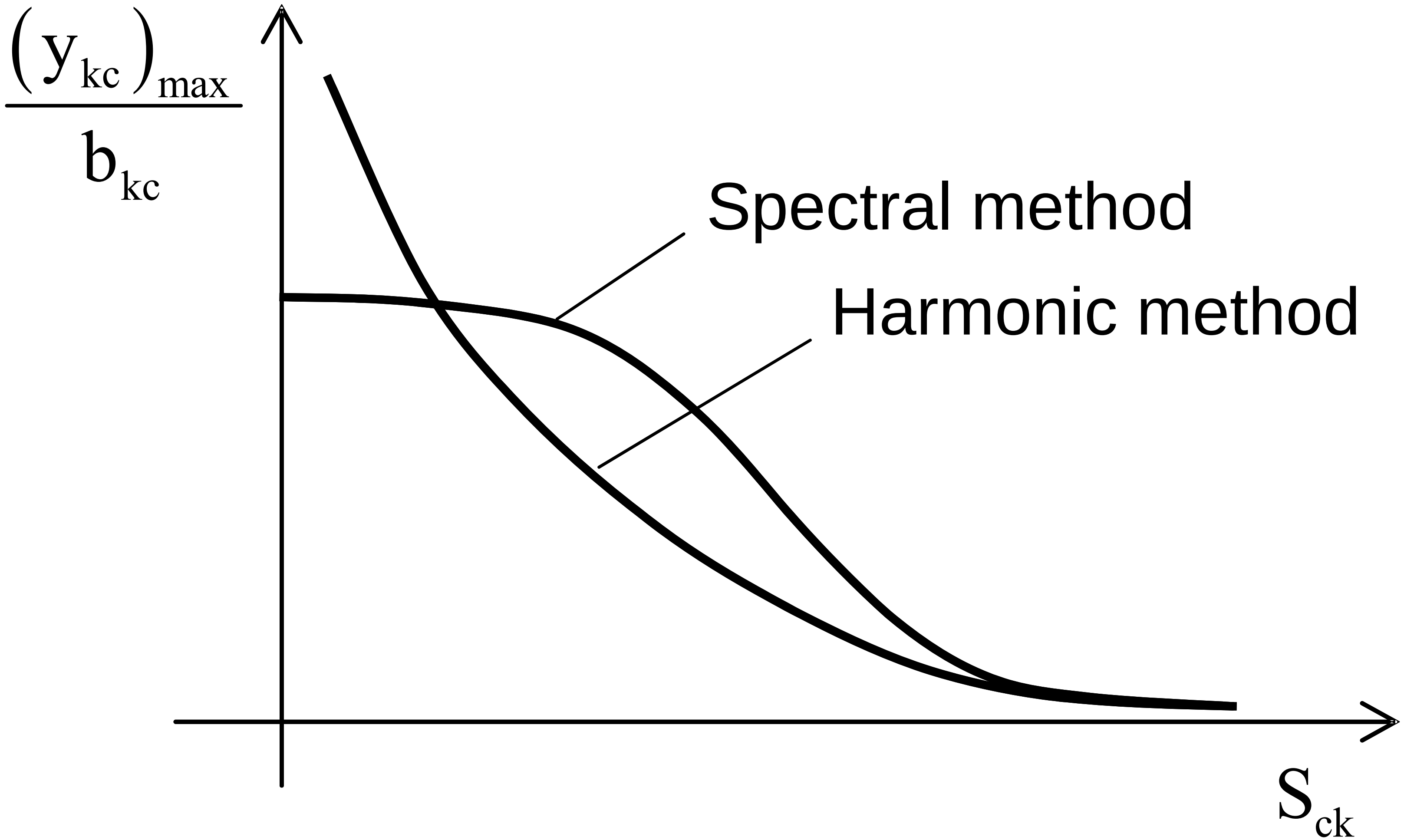
$$\text{Re} = 1,76 \times 10^6 \Rightarrow$$

$$K_{a0} = K_{a,max} = 1;\ \alpha = 0,4 \Rightarrow$$

$$\sigma_y = 0,82 \text{ m};\ g_y = 1,416 \Rightarrow$$

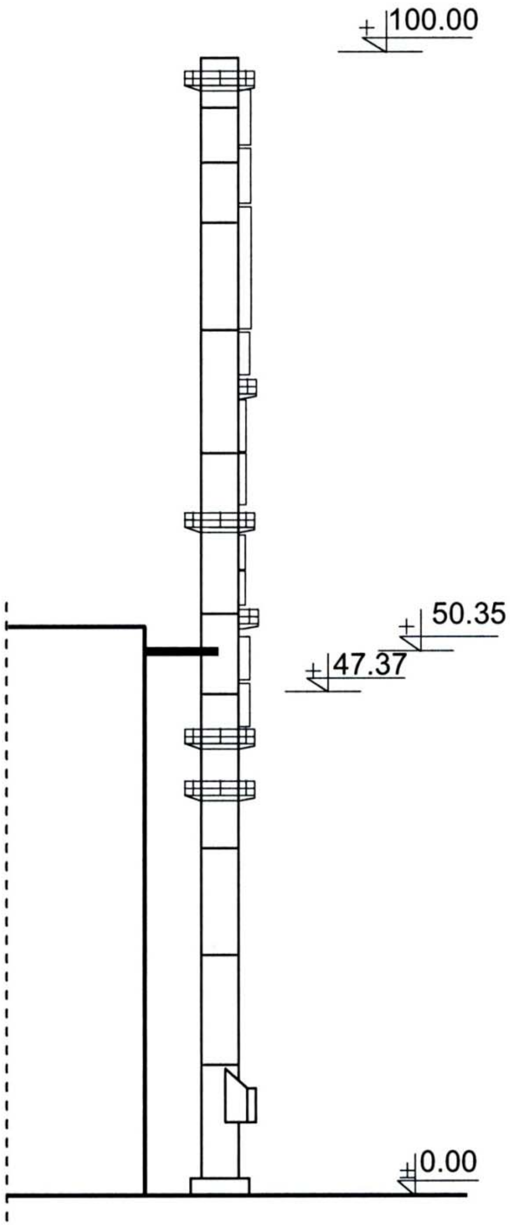
$$\bar{y}_{max} = \sigma_y \cdot g_y = 1,16 \text{ m}$$

Correlation length vs spectral method

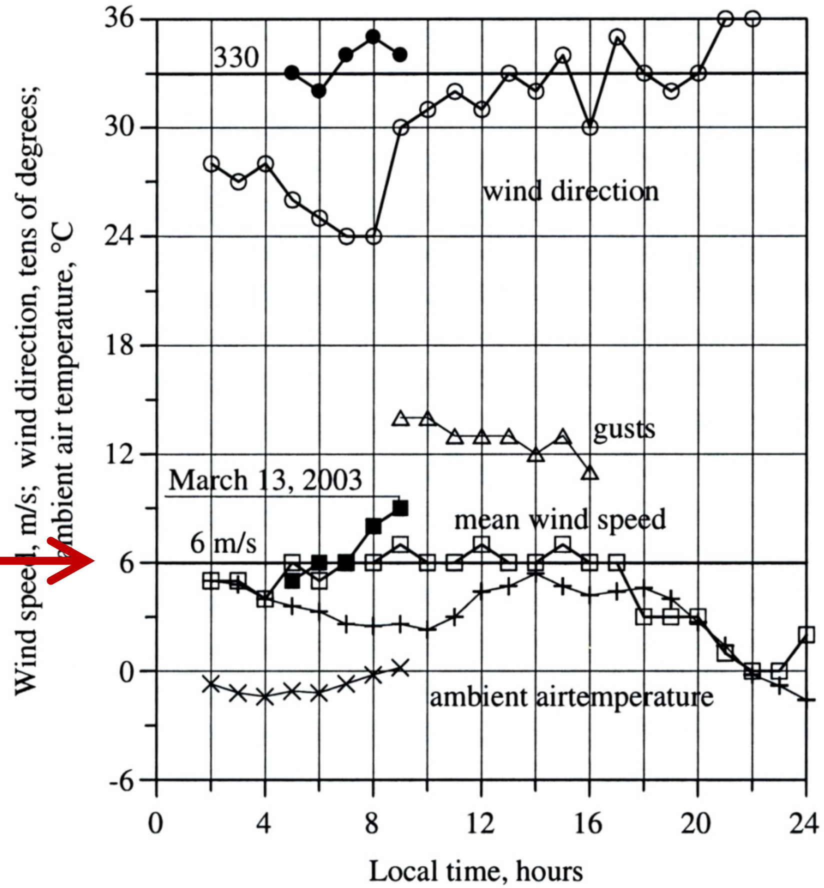


Spectral vs Harmonic method

Chimney erected in Poland in February 2003



$\bar{u}_{cr}$

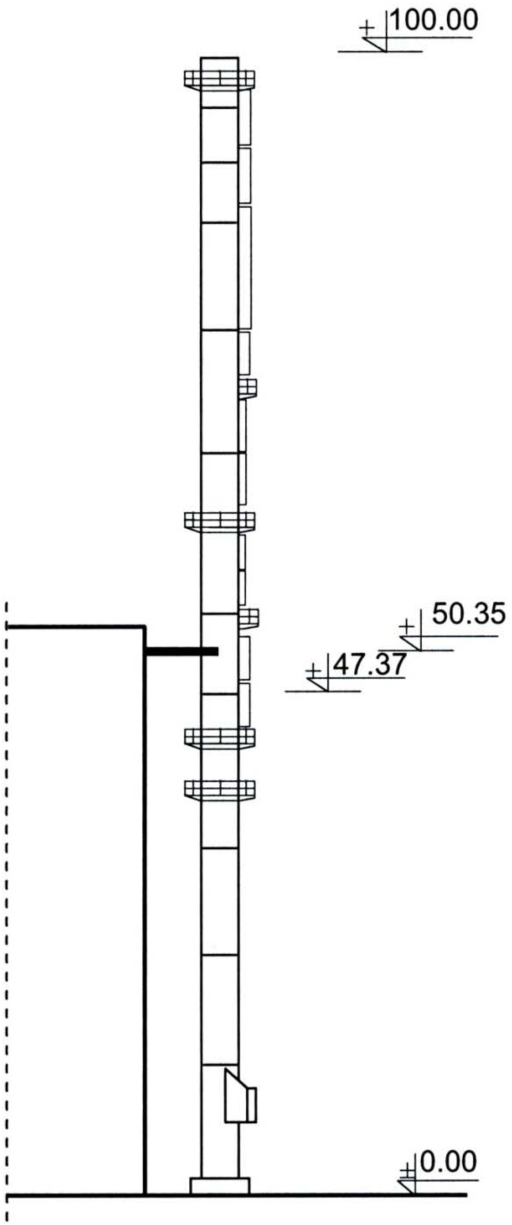


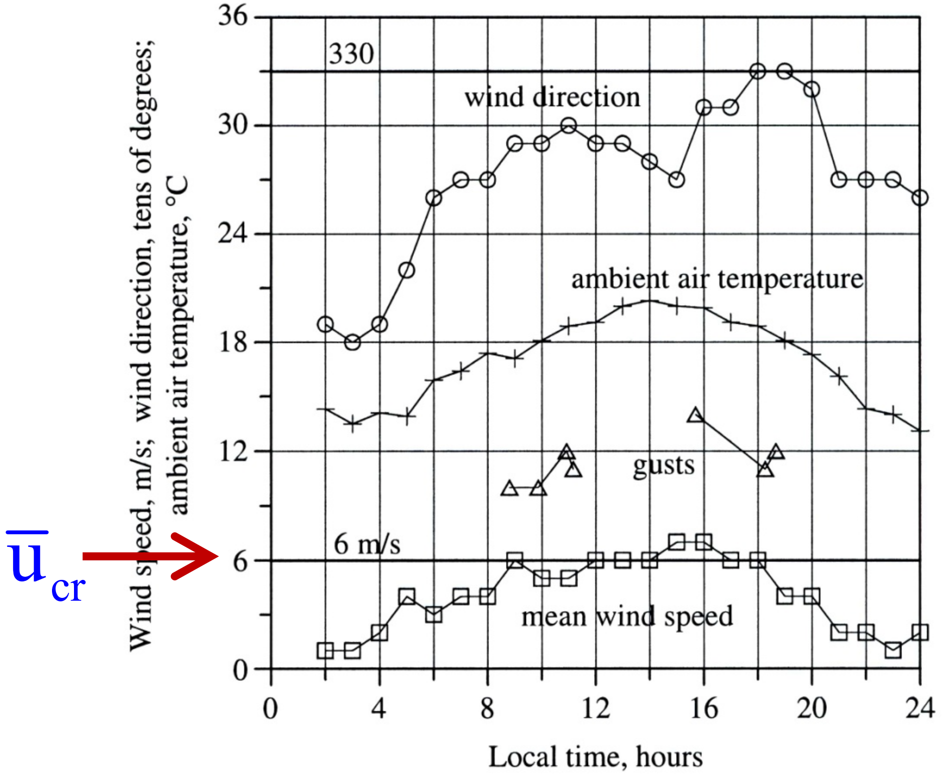
March 13, 2003. Crosswind response with 1 m amplitude. Failure of a welding, 32 bolts and a flange

Kawecki & Zuranski (2007)

Chimney erected in Poland in February 2003

Repaired in March 2003





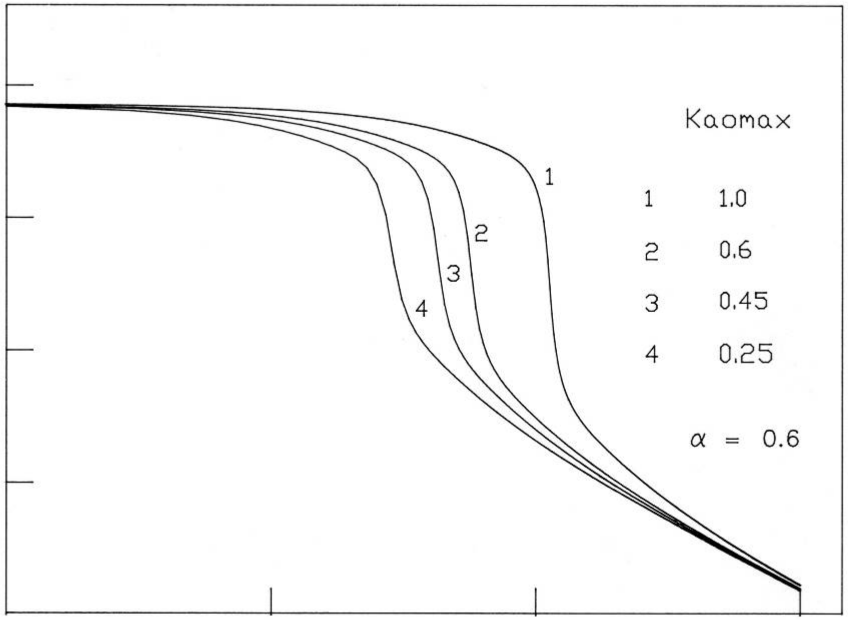
June 13, 2003. Failure of 47 bolts.

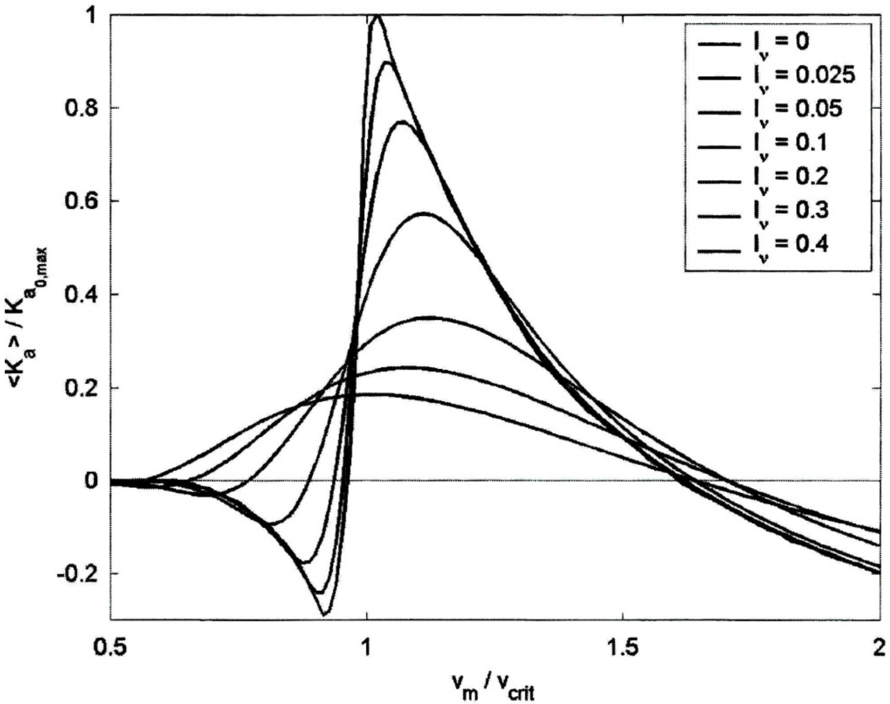
A damper is installed on October 22-23, 2003

Kawecki & Zuranski (2007)

Equivalent damping

$$\xi_{eq} \simeq \frac{\rho d^2}{4\pi m}\left\{Sc - 4\pi K_{a0}\left[1 - \frac{\sigma_y^2}{\alpha^2 d^2}\right]\right\}$$





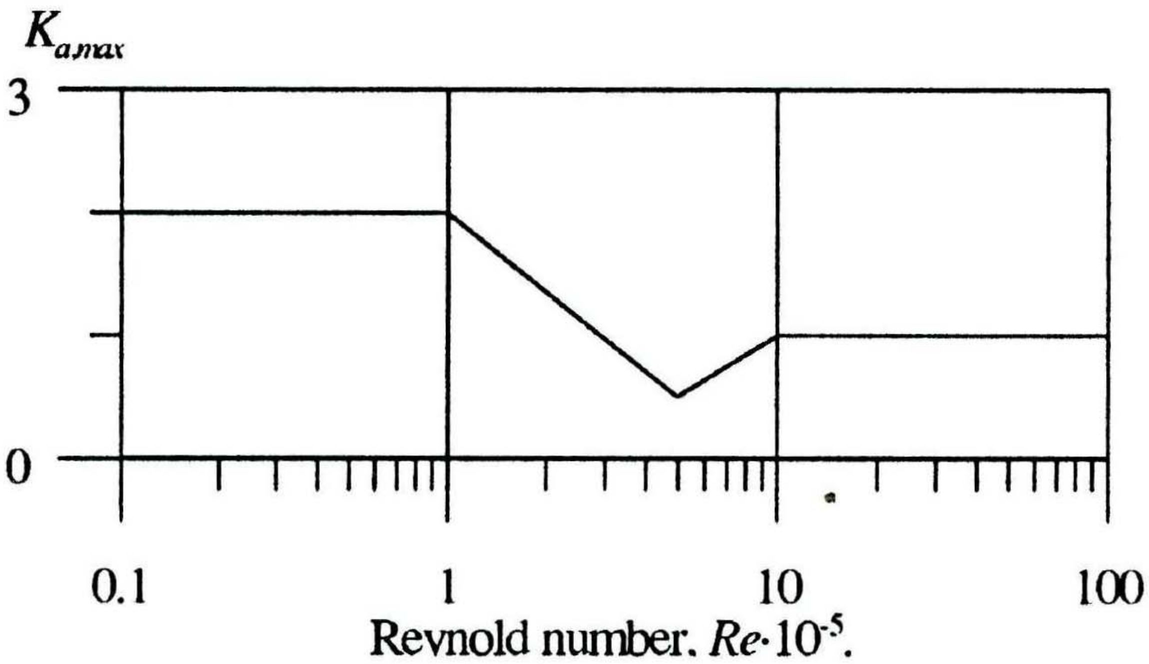
Vierboom & Van Koten (2010)

Chimney	Group	h	d	f_0	δ_s	Sc	σ_C	σ_D	σ_L
Aachen	I	28	0.914	1.71	0.015	2.6	40	29.5	16.2
Köln	I	35	0.813	0.64	0.015	7.3	40	29.5	16.2
Pirna	I	60	2.0*	0.77	0.125	16.4	40	29.5	16.2
Rechlinghausen	I	38	1.016	0.70	0.030	10.9	40	29.5	16.2
Helden	II	40	0.560*	0.65	0.013	5.8	45	33.2	18.2
Pernis	II	60	1.00*	0.57	0.012	12.4	45	33.2	18.2
Italy	II	65	4.40*	0.67	0.008	1.5	45	33.2	18.2
VEAB	II	90	2.30	0.28	0.013	3.0	35.5	26.2	14.4
0112	III	60	1.60*	0.48	0.025	5.8	45	33.2	18.2
0905	III	25.5	0.710	0.72	0.025	15.8	45	33.2	18.2
1202	III	30	0.711	0.70	0.025	12.4	45	33.2	18.2
1221	III	57	1.320*	0.44	0.025	7.0	45	33.2	18.2
1308	III	45	1.120	0.62	0.025	5.8	45	33.2	18.2

Vierboom & Van Koten (2010)

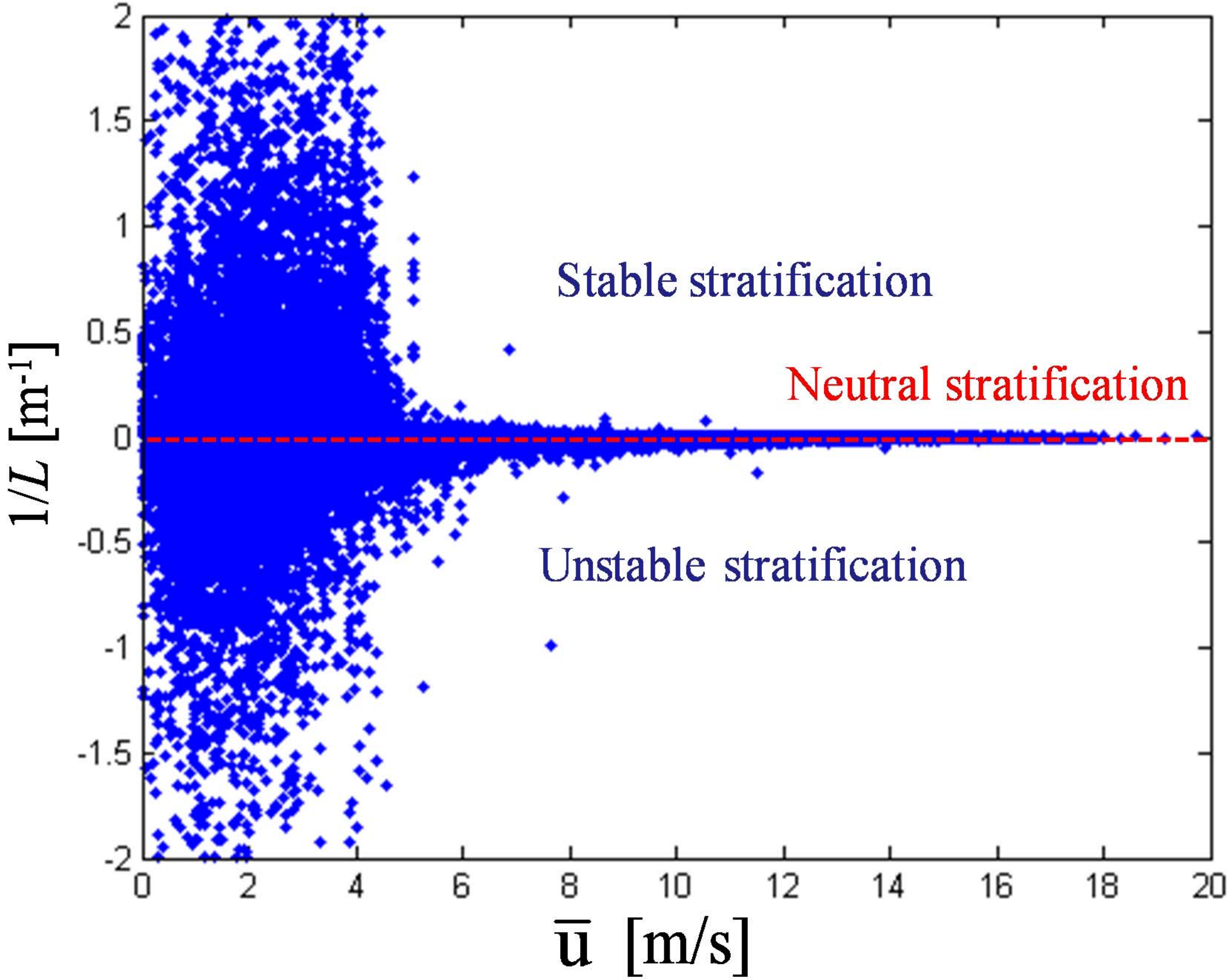
Aerodynamic damping parameter

$$\boxed{K_{a0} = K_{a0}\left(\text{Re}, I_u\right) = K_{a,\max}\left(\text{Re}\right) \cdot K_u\left(I_u\right)}$$

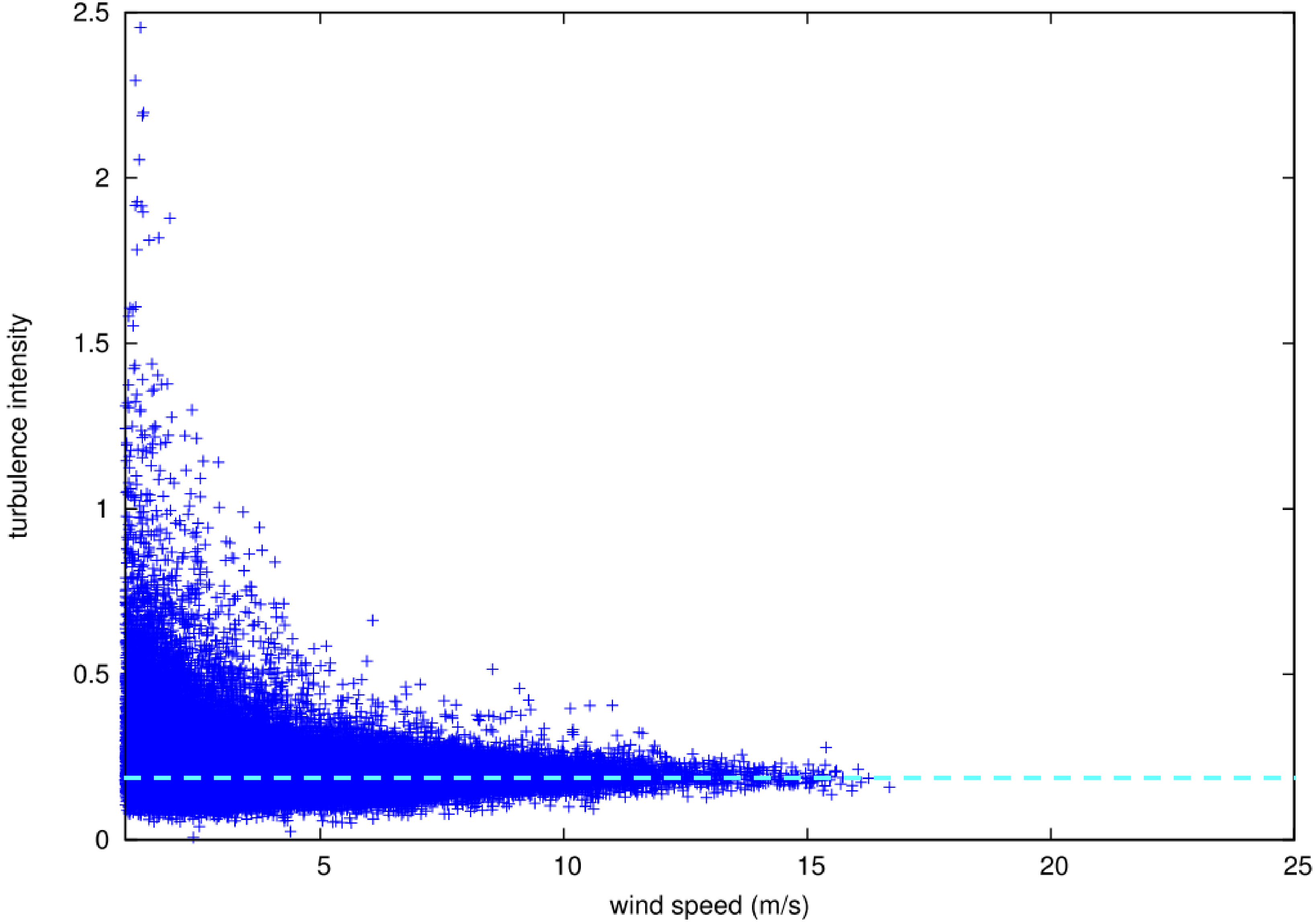


$$\boxed{K_u\left(I_u\right) = 1 - 3 \cdot I_u \geq 0,25}$$

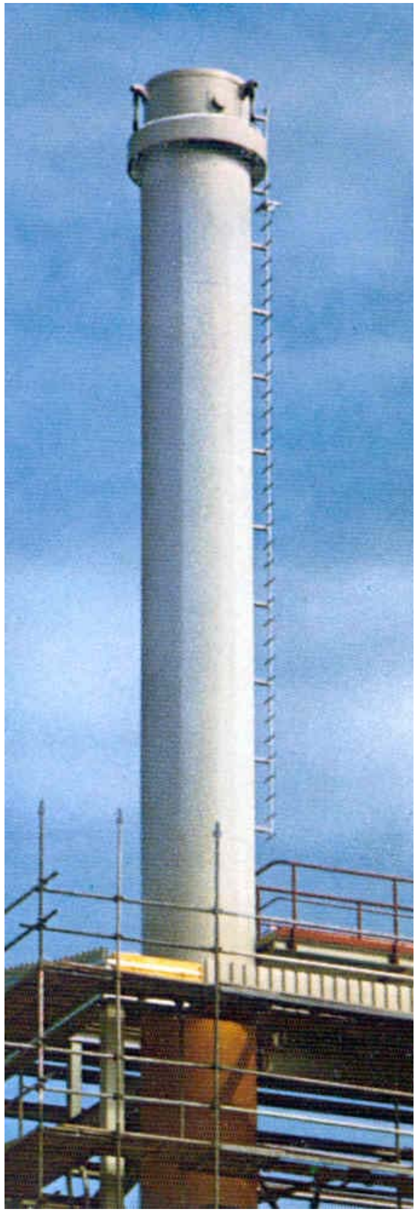
Aerodynamic damping parameter vs turbulence intensity

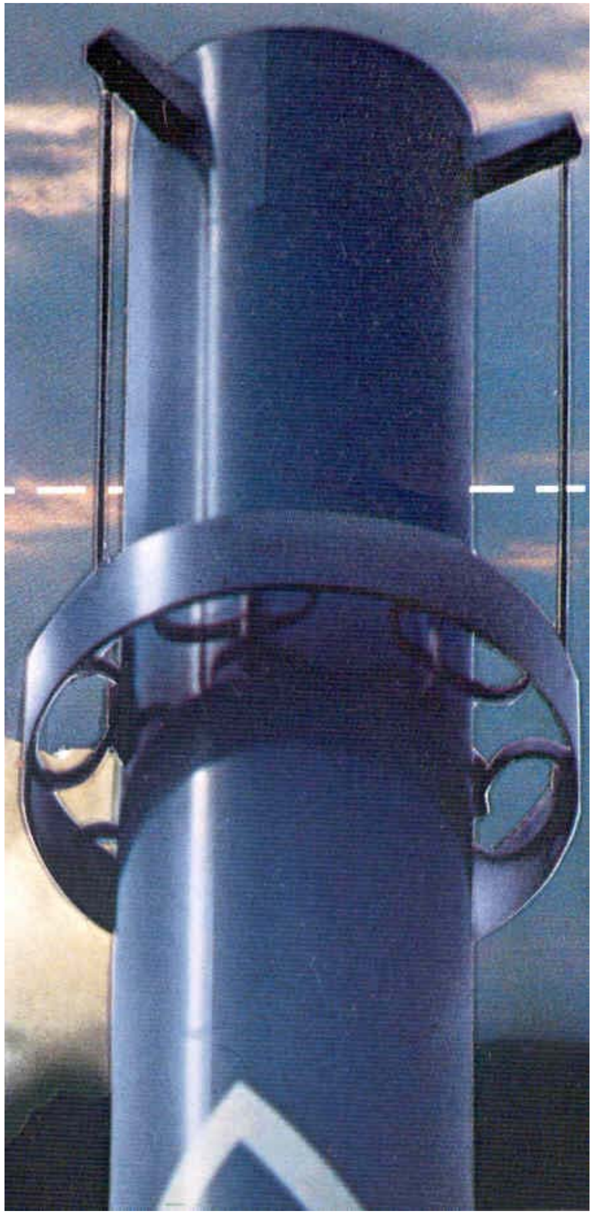


Turbulence intensity

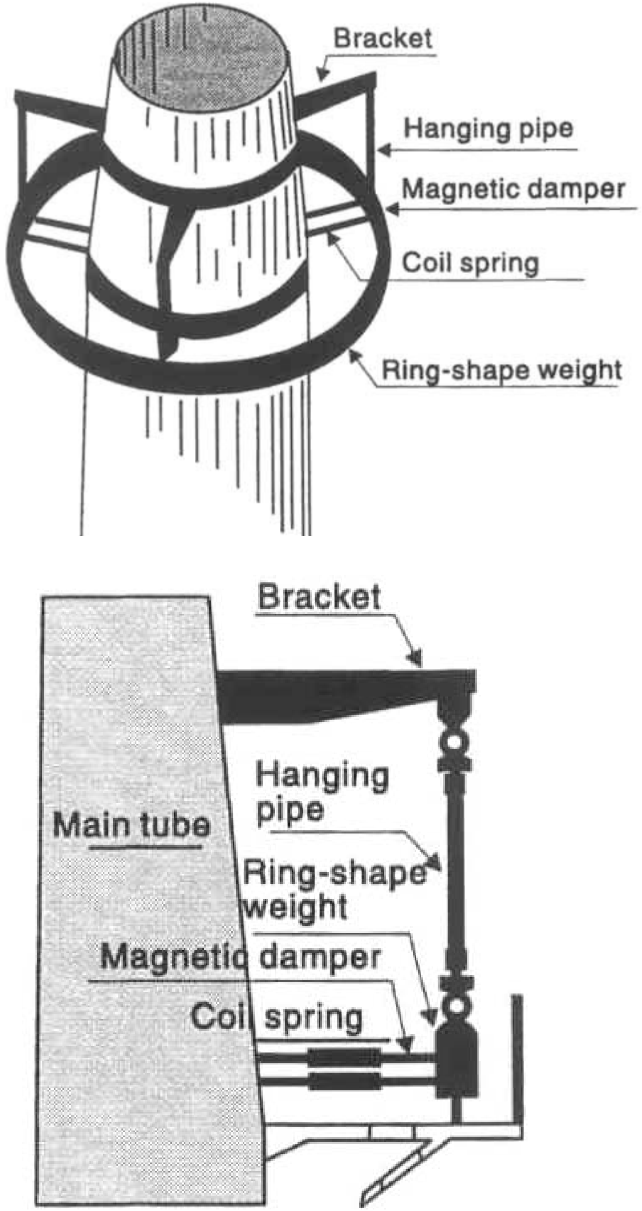


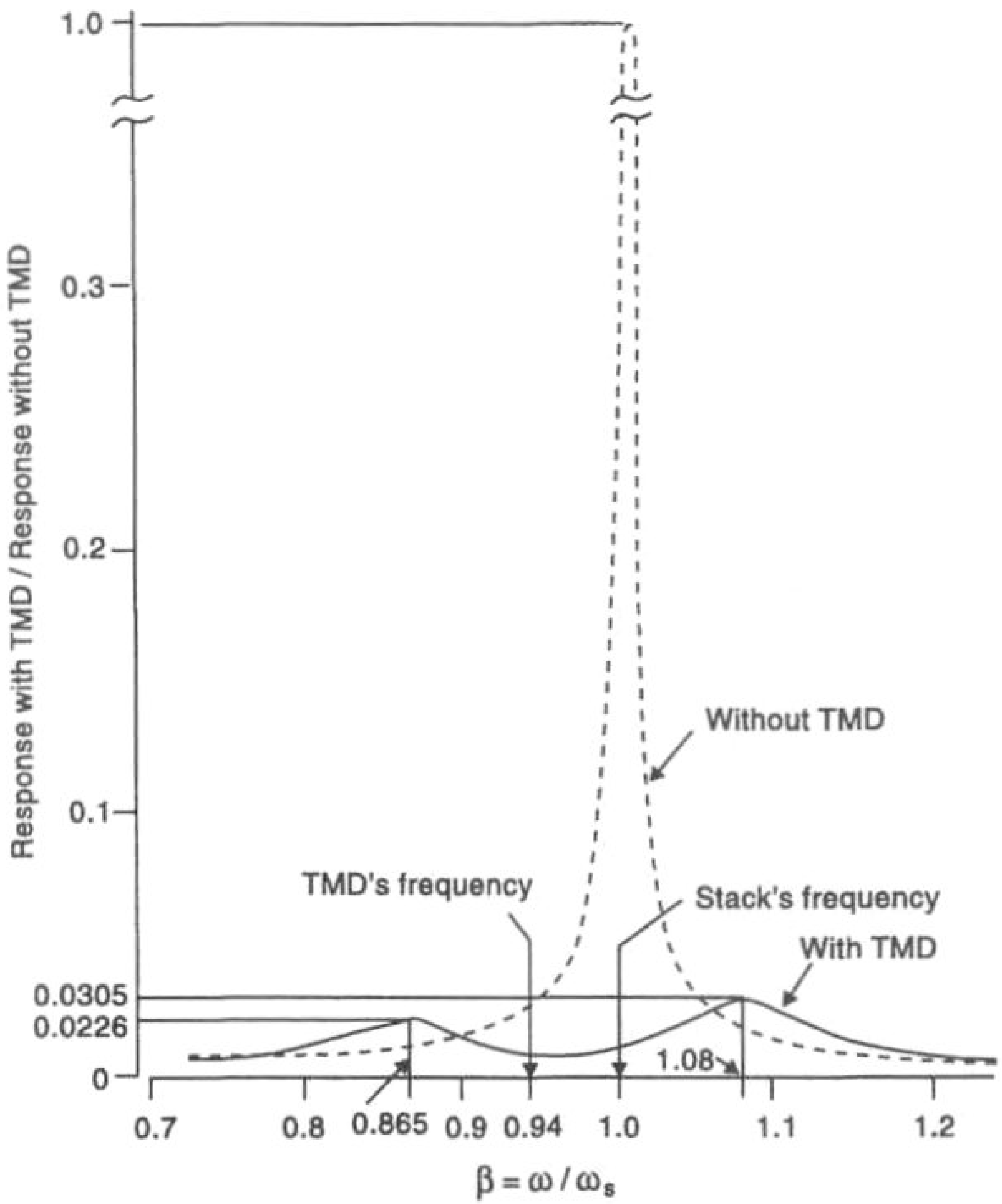
Turbulence intensity





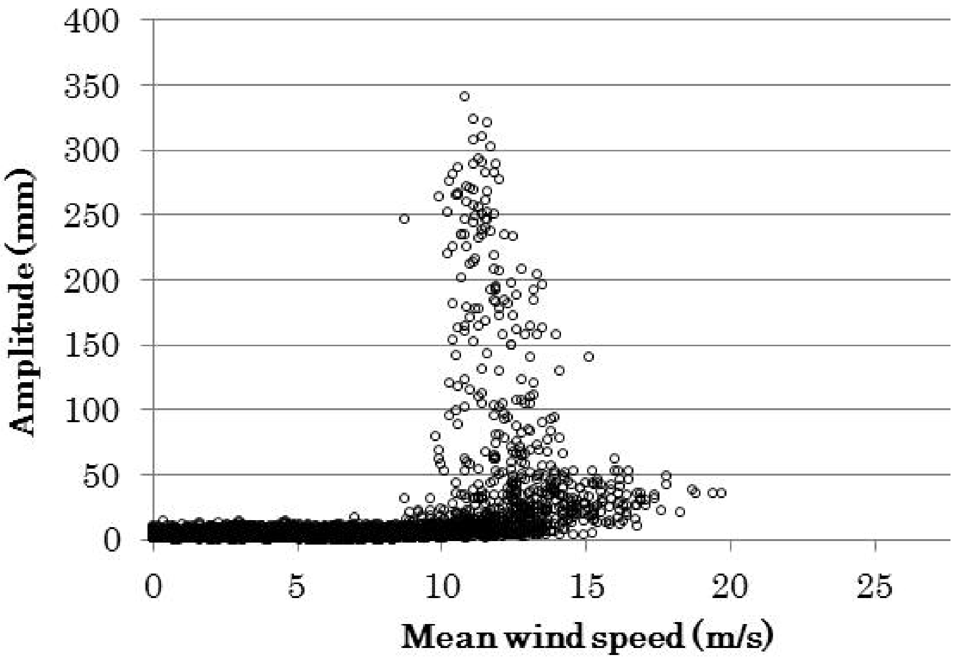
Tuned Mass Dampers for mitigating vortex-excited vibrations





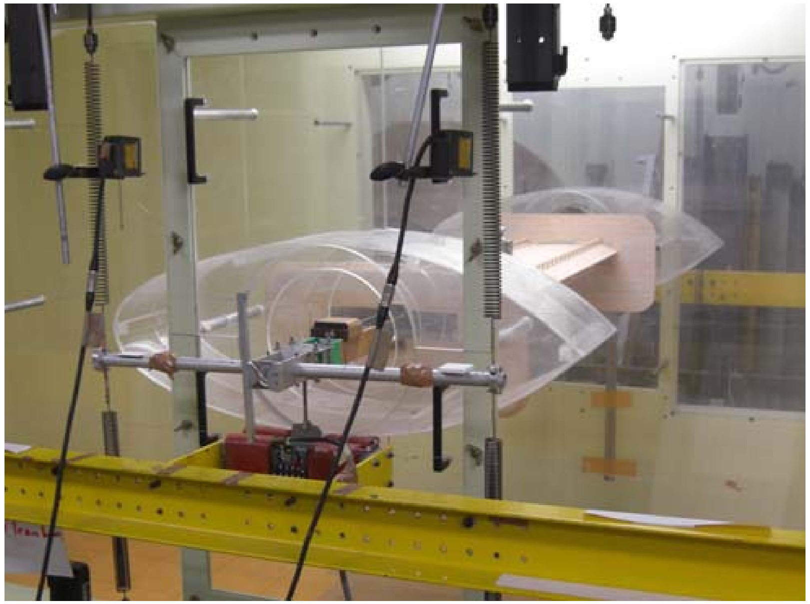
Tuned Mass Dampers for mitigating vortex-excited vibrations

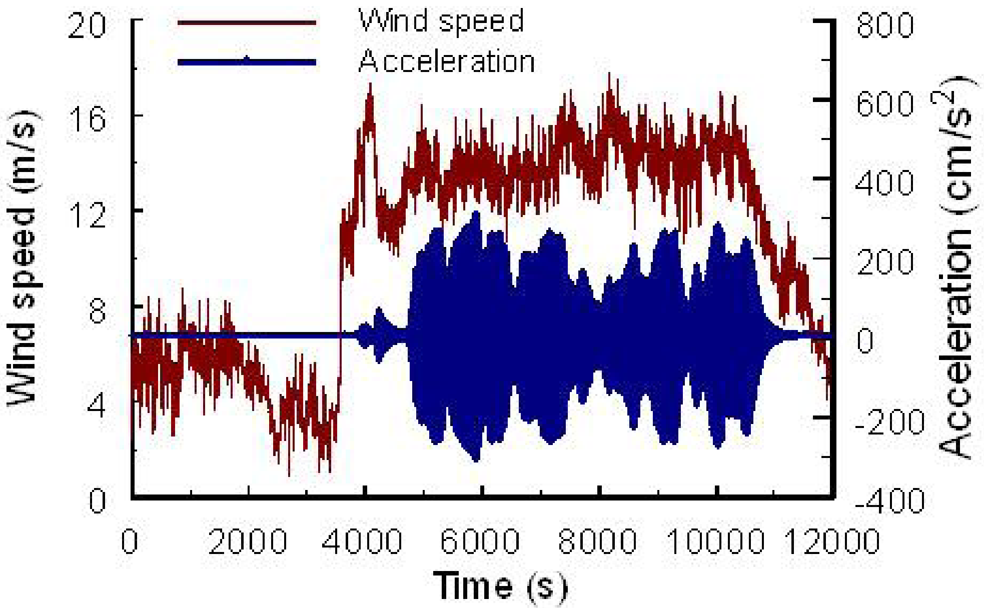
Cable-stayed bridge, Japan, central span L = 360 m



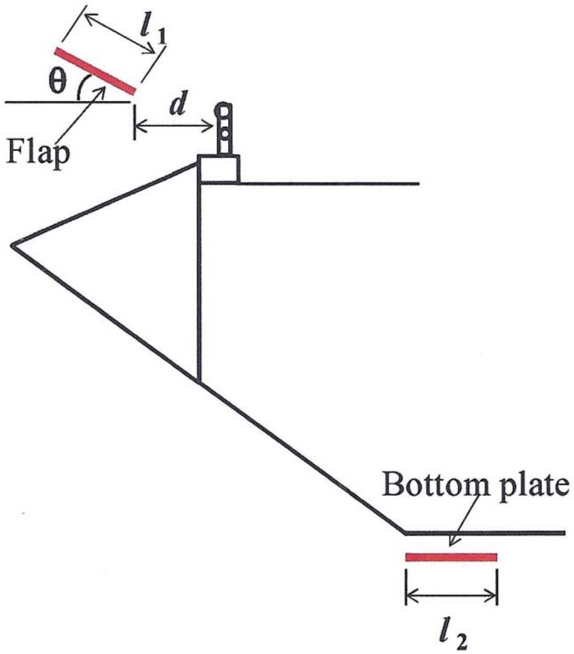
Full-scale measurements

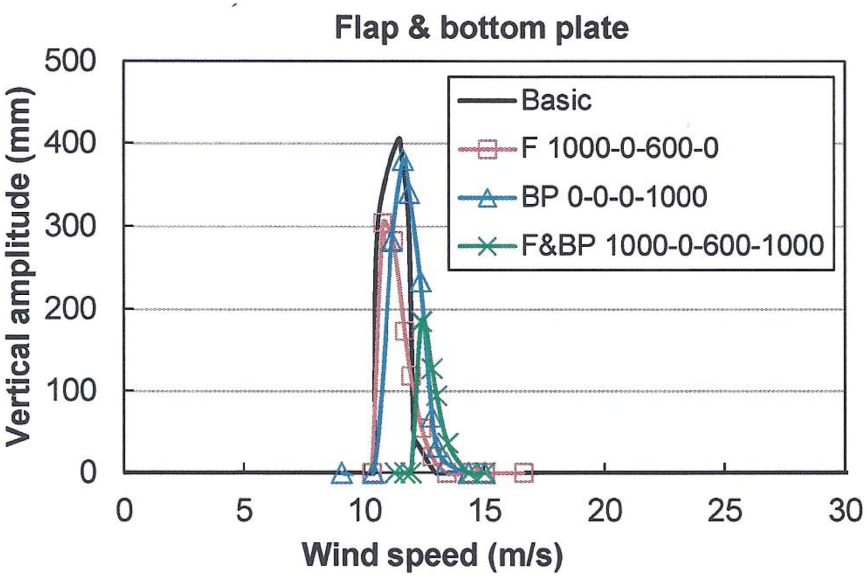
Wind tunnel tests

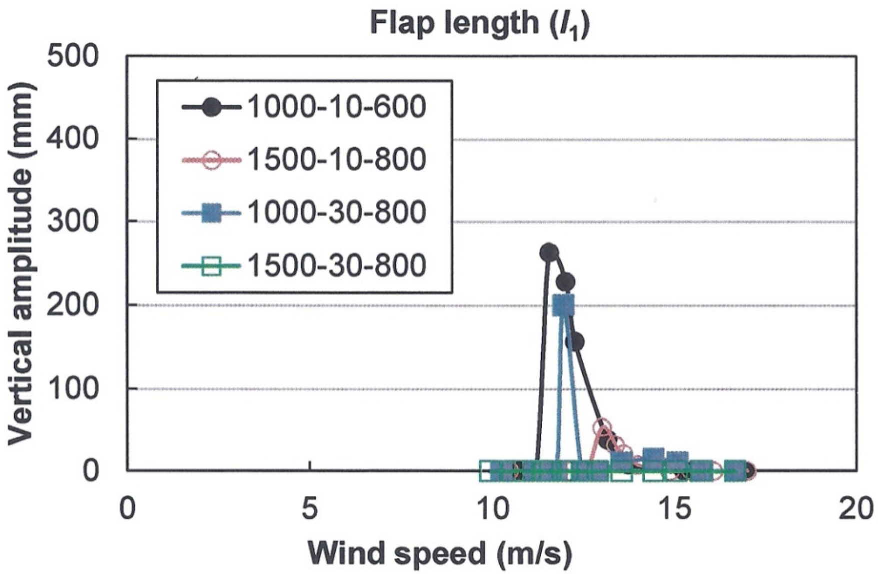




Cable-stayed bridge, Japan, central span L = 360 m







Cable-stayed bridge, Japan, equipped with flaps

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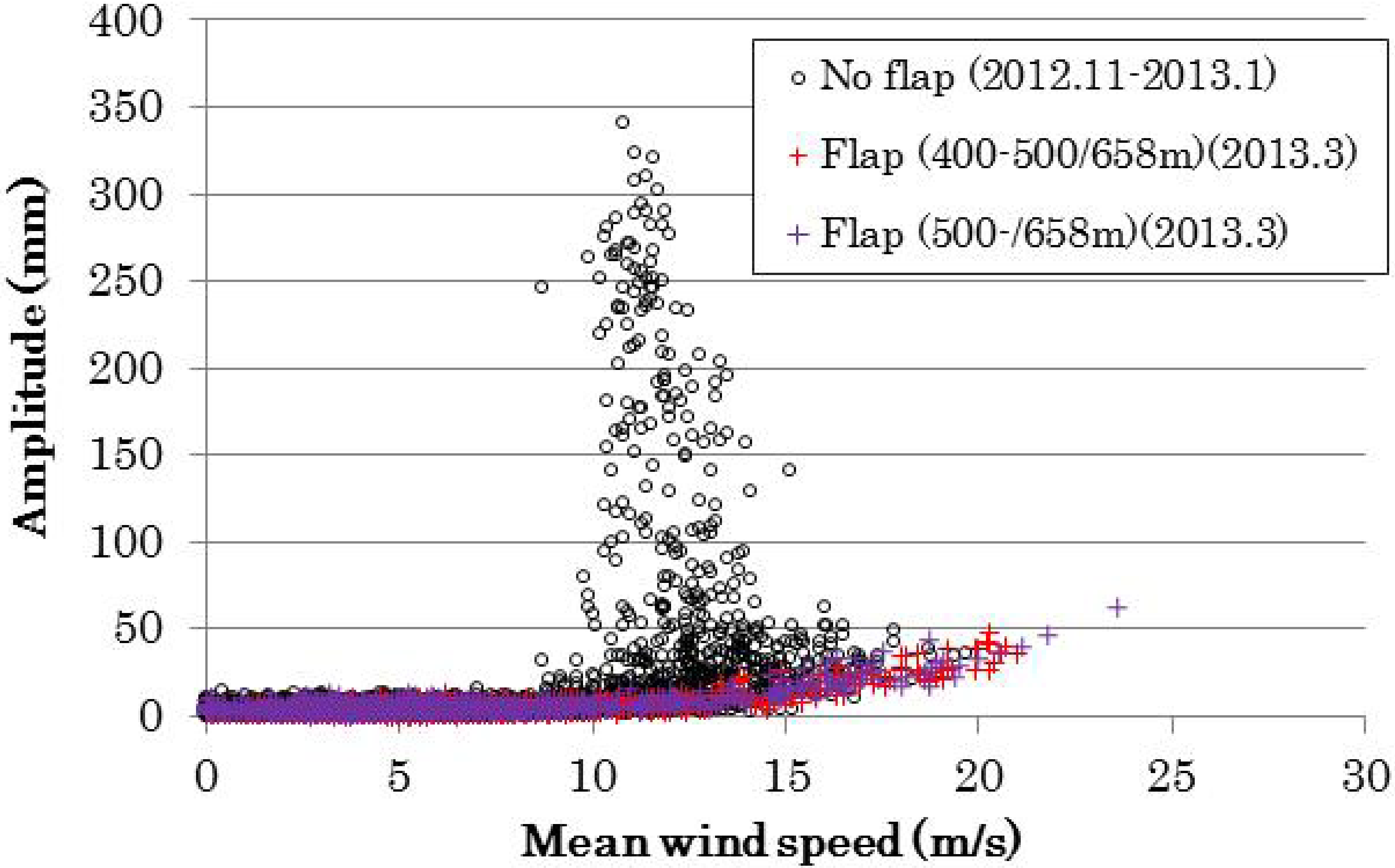
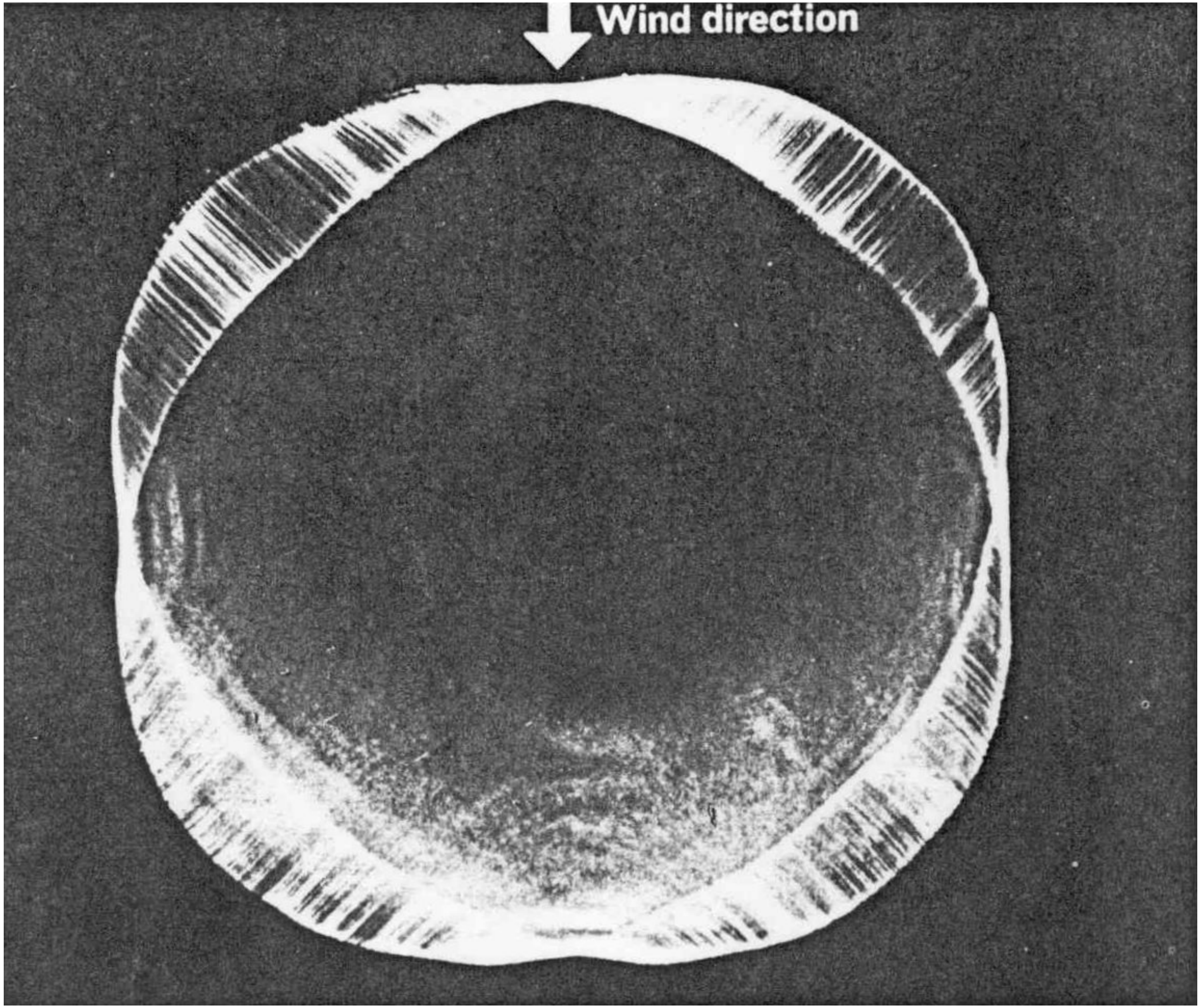
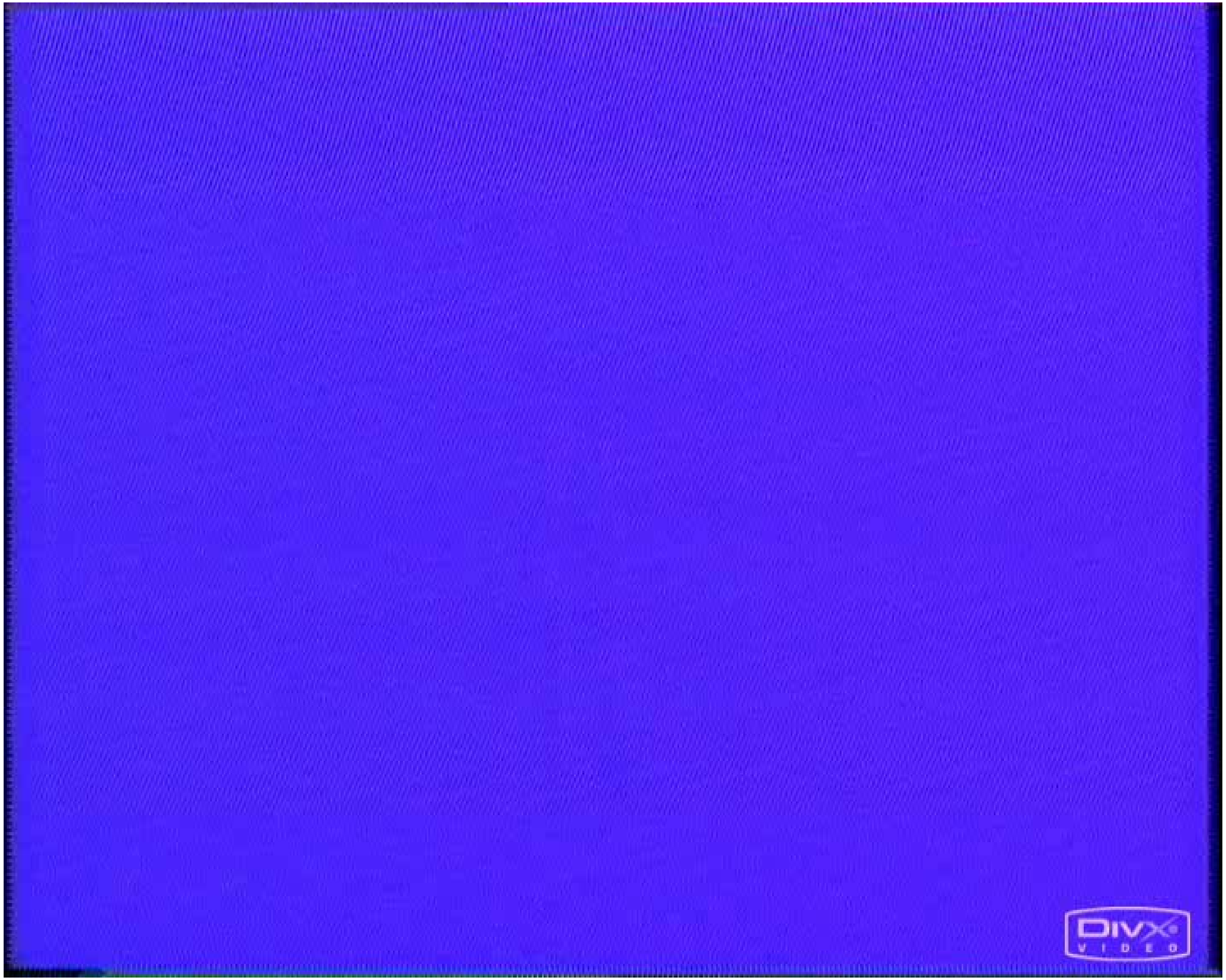


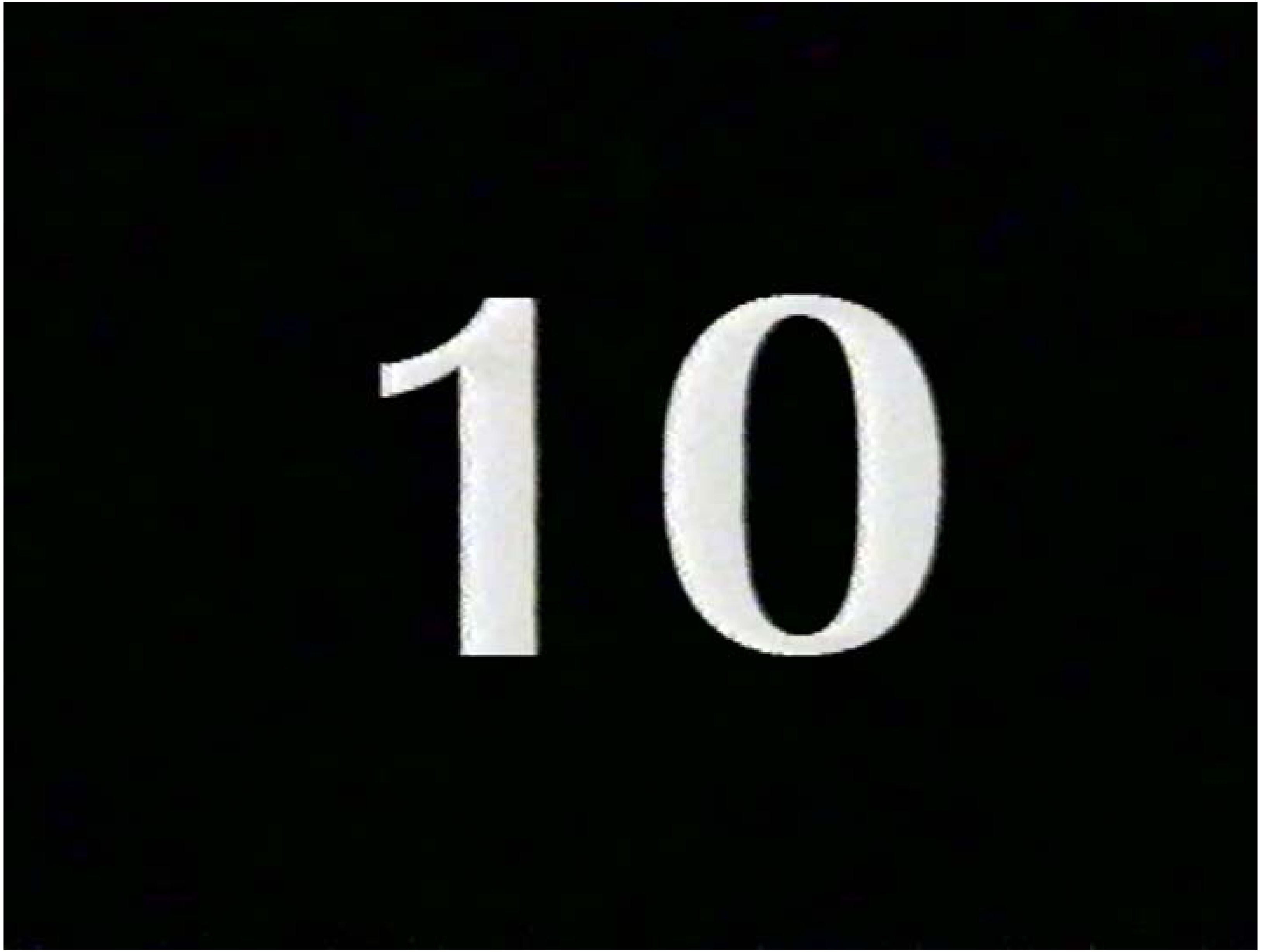
Cable-stayed bridge, Japan, equipped with flaps



Ovalling

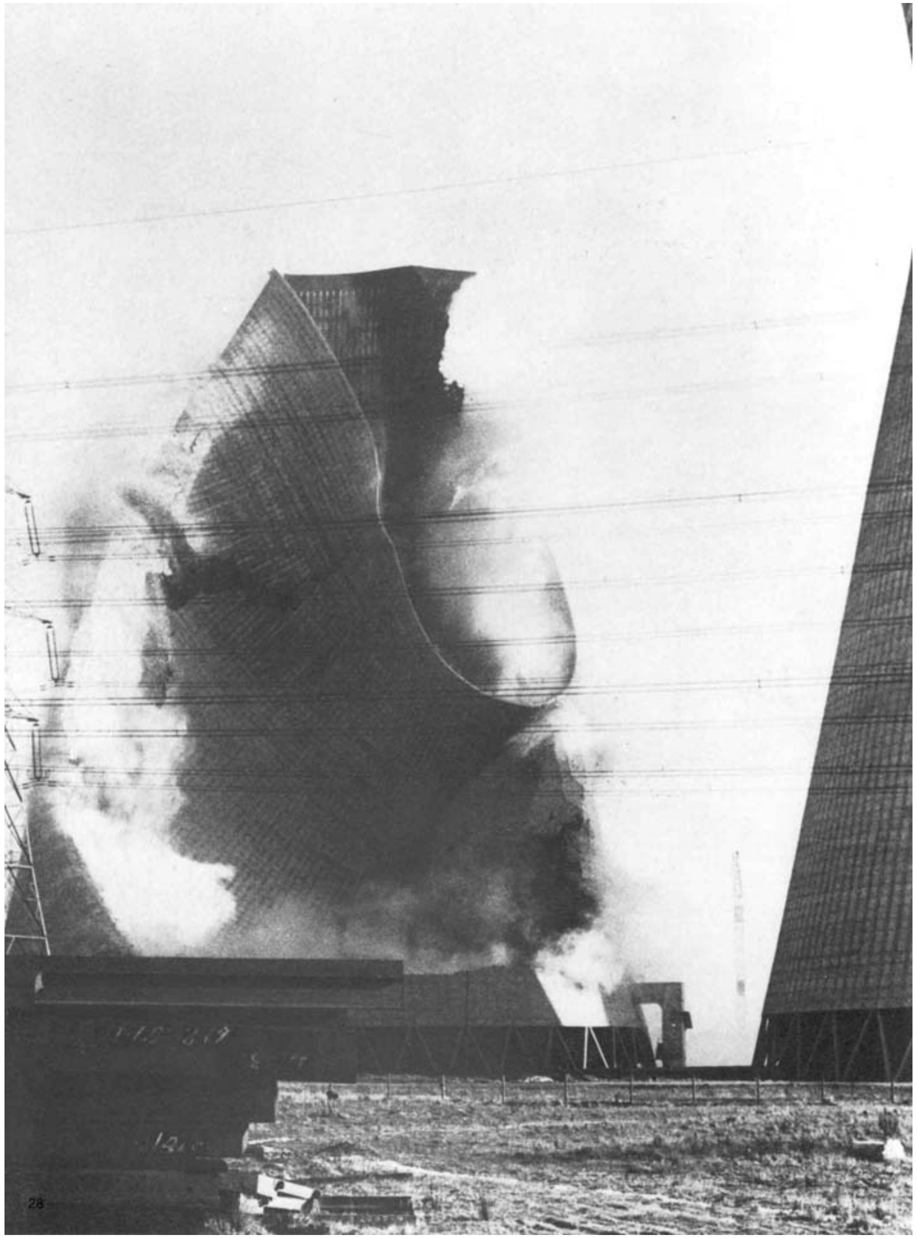


Ovalling

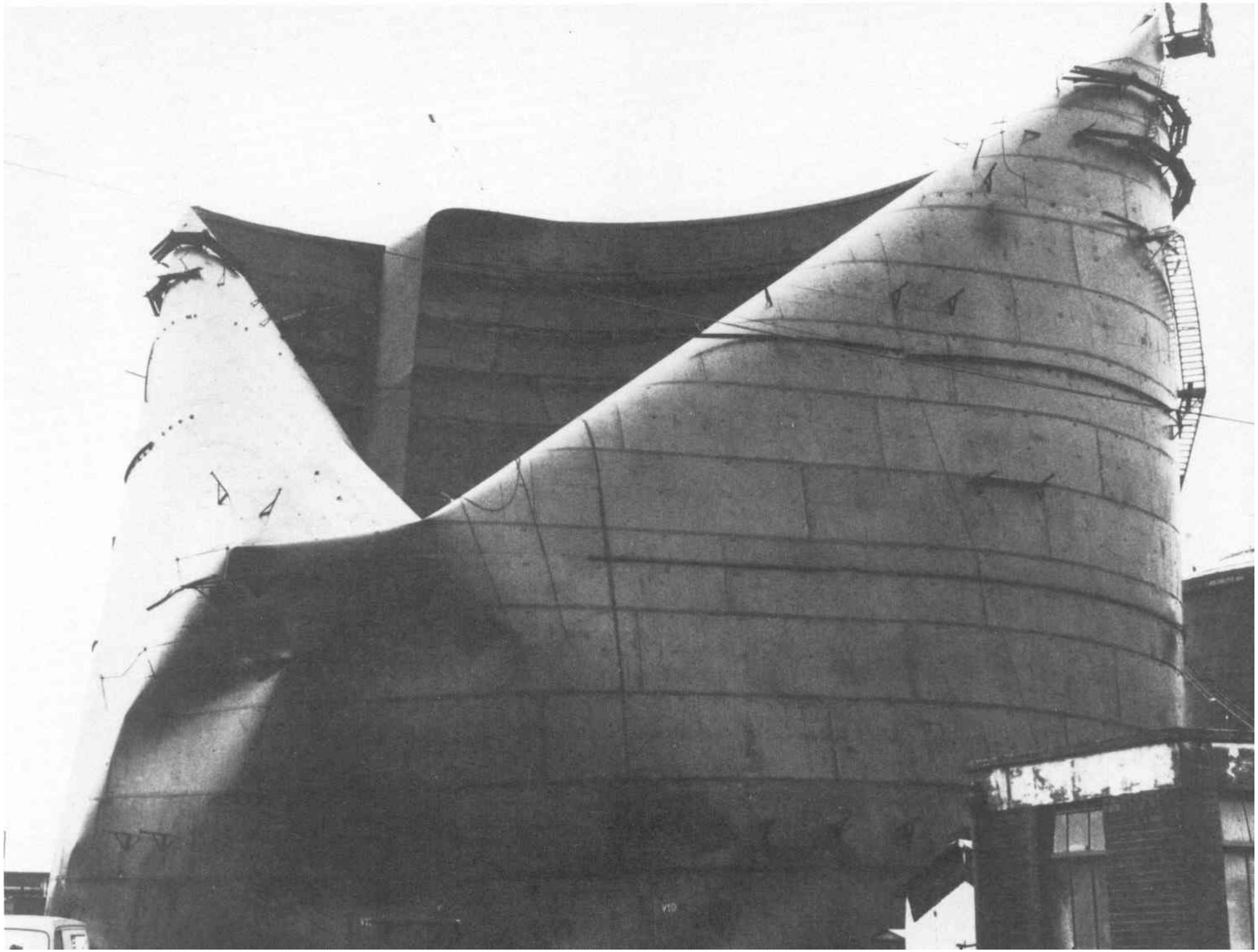


Ovalling

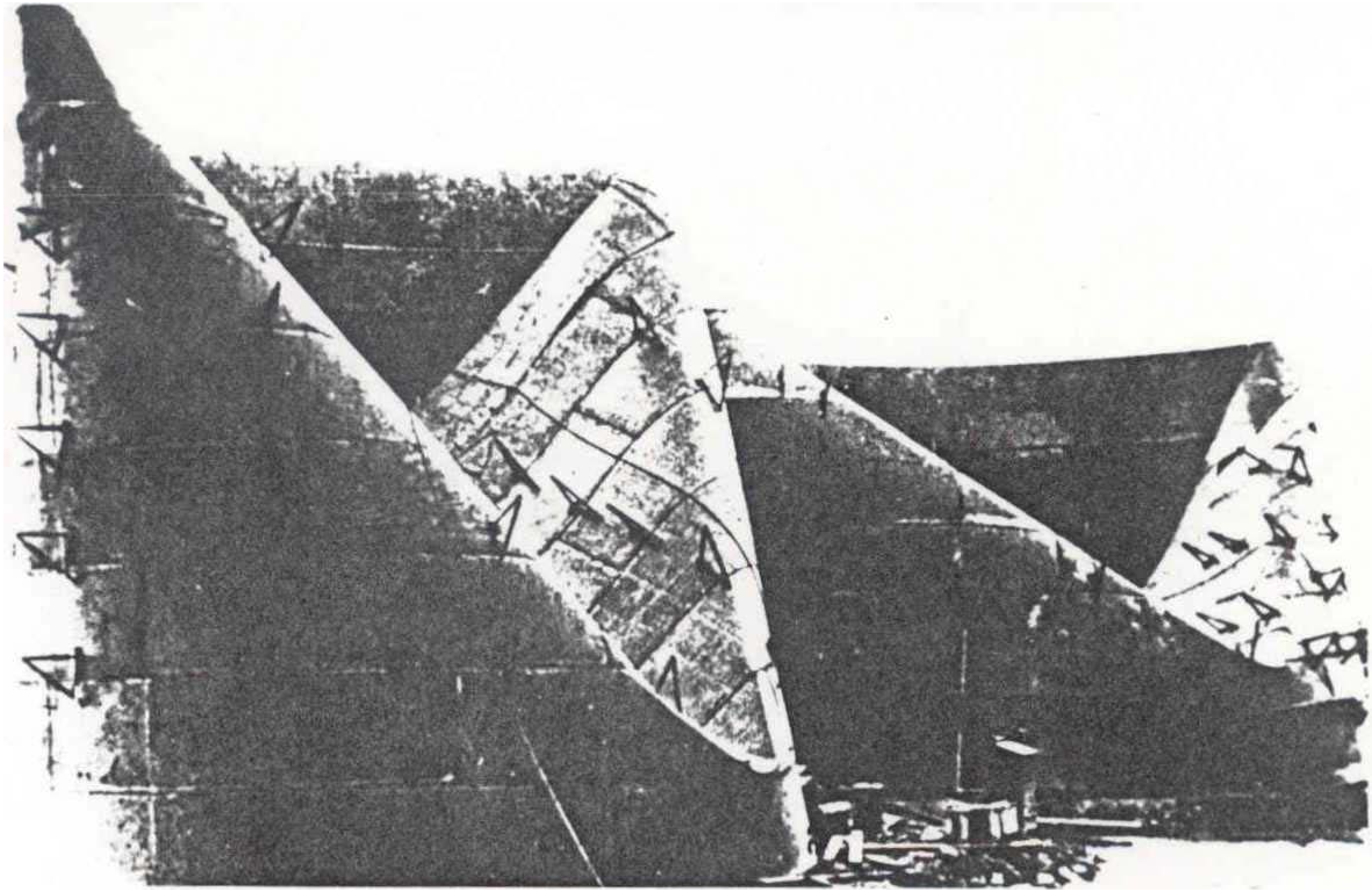
Ferrybridge Plant, UK



Ferrybridge Plant, UK

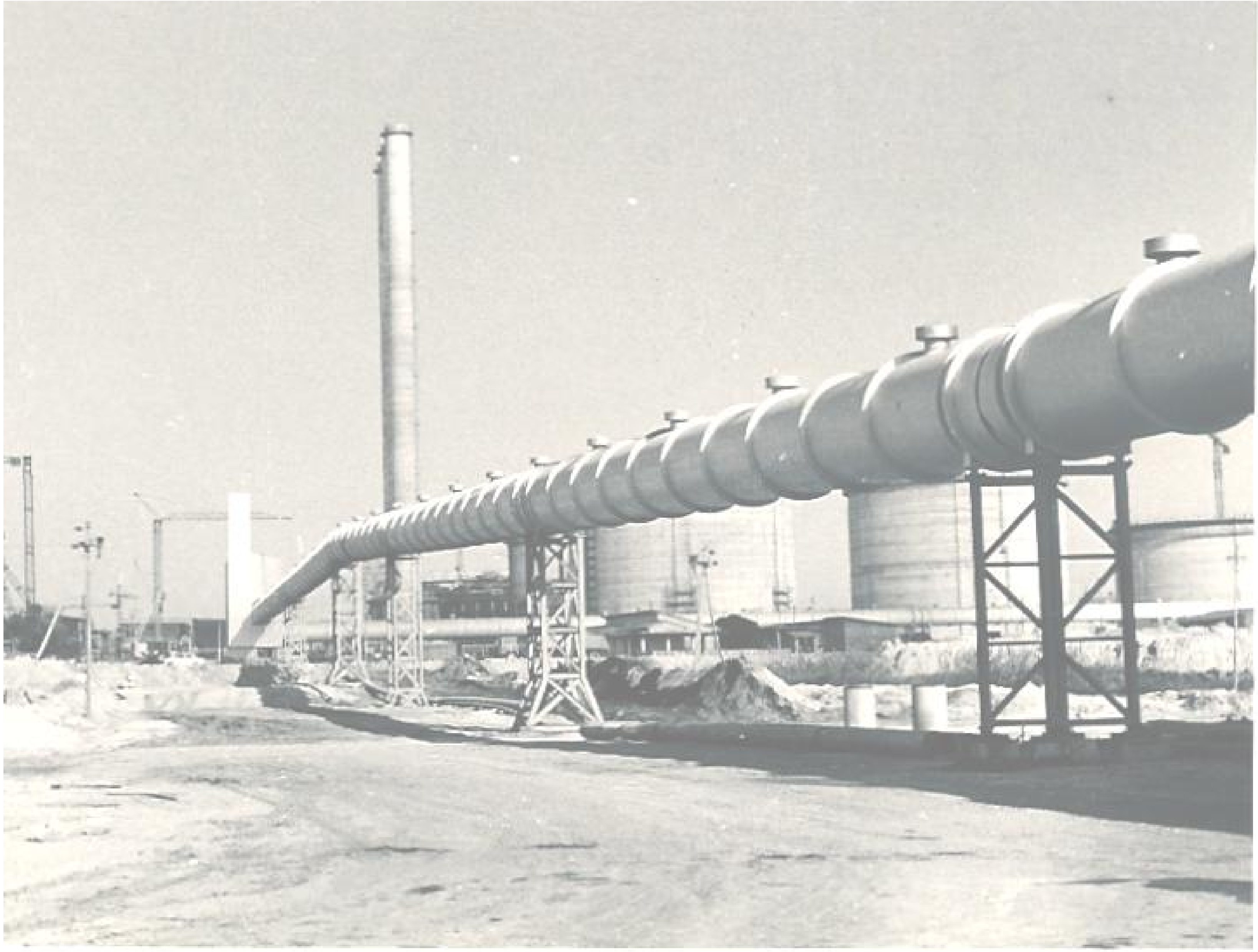


Ovalizzazione

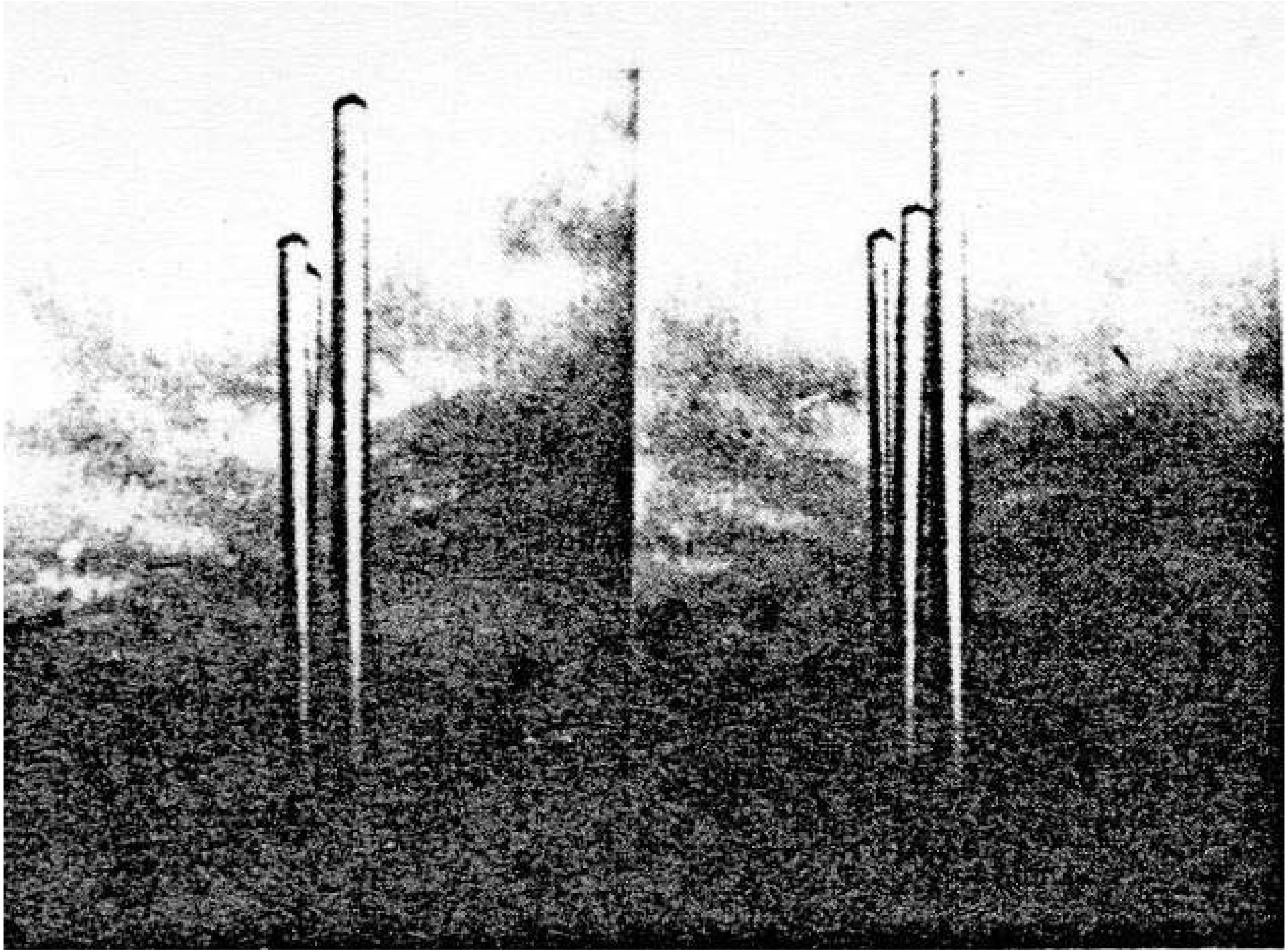


Ovalizzazione

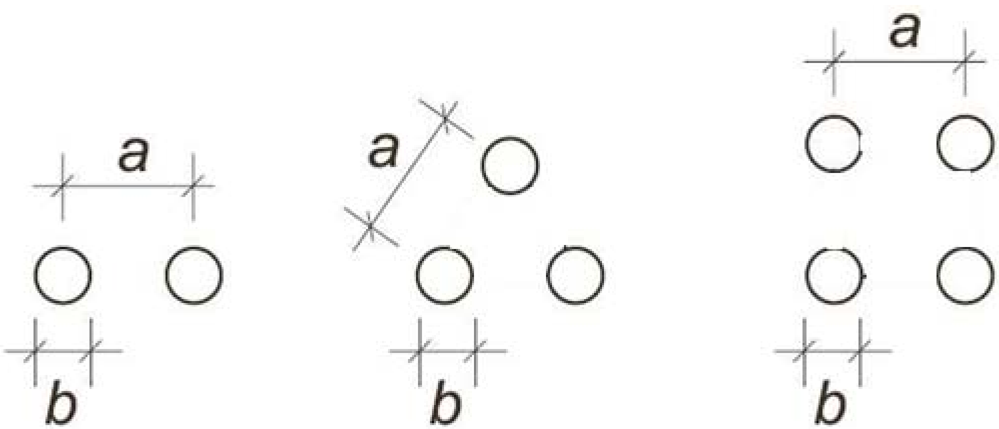
Stiffening rings for avoiding ovalling



Stiffening rings for avoiding ovalling



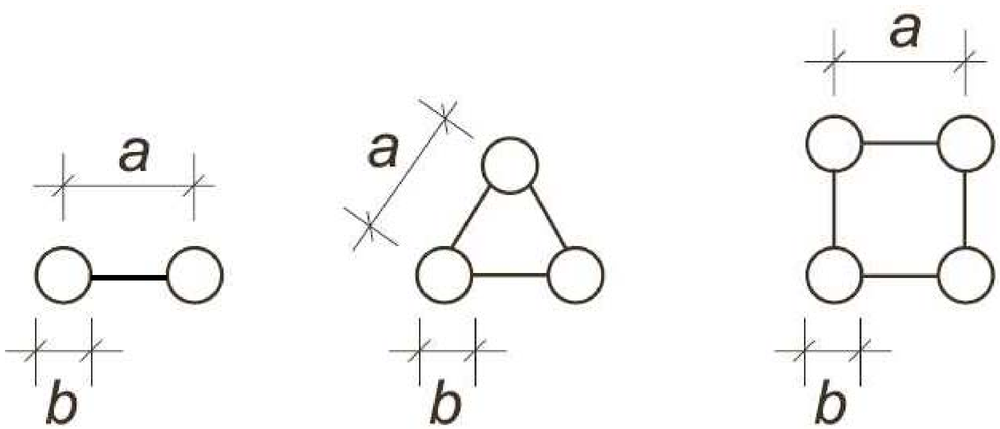
Ruscheweyh (1988)



Non structurally connected cylinders

$a > 10 \cdot b$	wake interference can be disregarded
$3 \cdot b < a < 10 \cdot b$	crosswind actions evaluated for the isolated cylinder should be increased by a factor:
	$1{,}6$ for $3 \cdot b < a < 4 \cdot b$
	$2 - 0{,}1 \cdot a/b$ for $4 \cdot b < a < 10 \cdot b$
$a < 3 \cdot b$	response should be evaluated using well documented experimental methods or seeking specialist advice

Wake interference



Structurally connected couple of cylinders

$b < a < 3 \cdot b$ crosswind actions evaluated for the single should be multiplied by a factor equal to 1,5

$a > 3 \cdot b$ wind-induced actions should be evaluated using well documented experimental methods or seeking specialist advice.

Wake interference

Thermoelectric Power Plant, Priolo Gargallo, Siracusa

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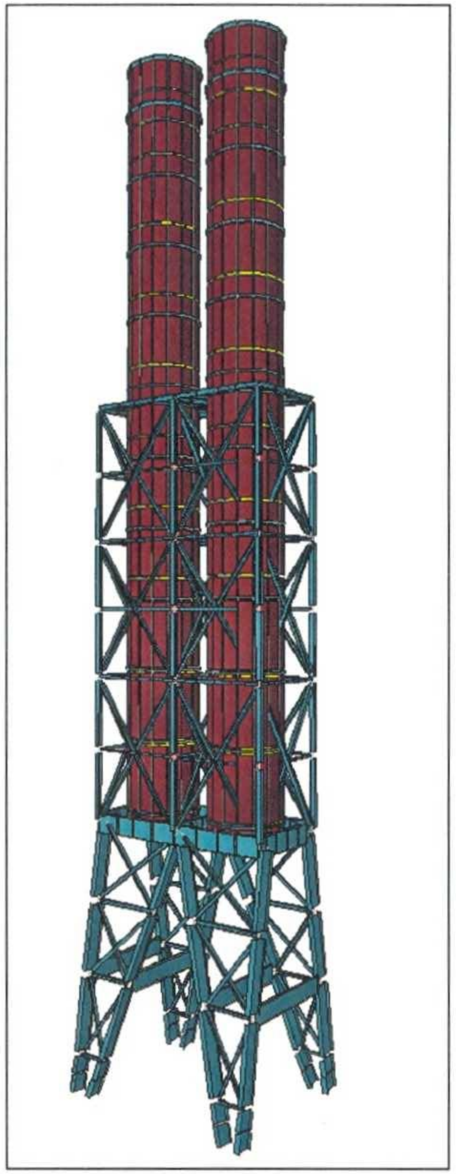
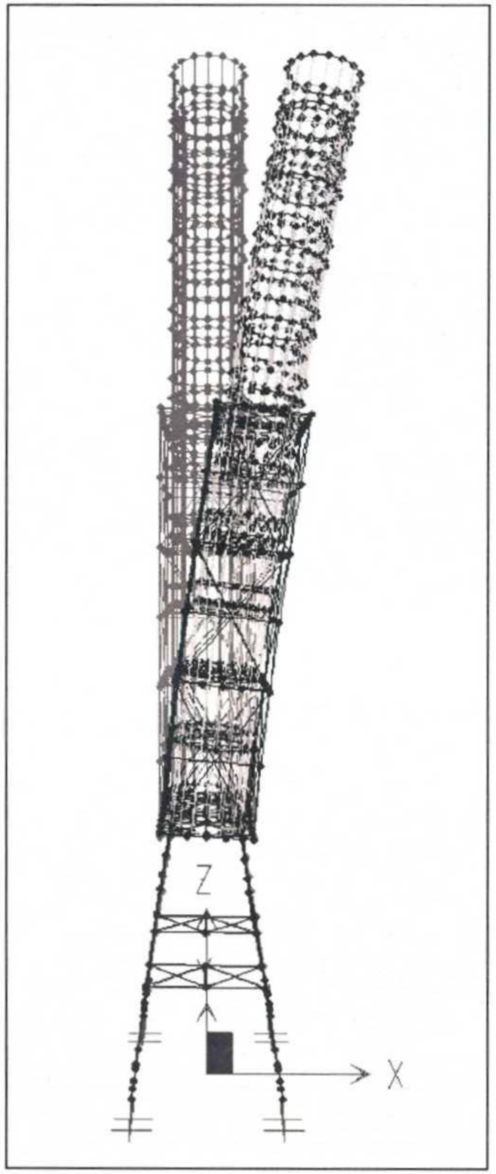
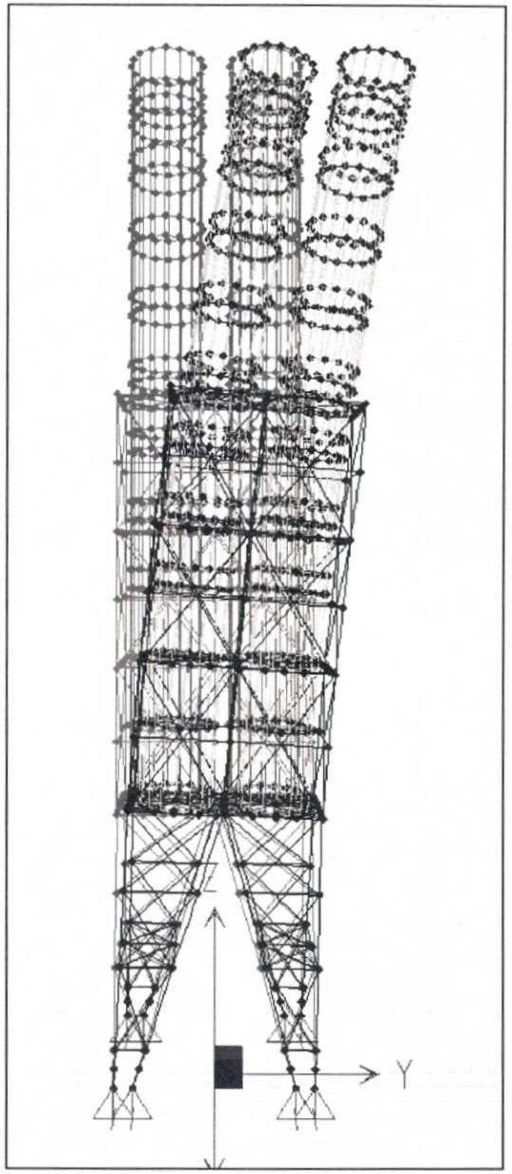


Ciminiera bicanne: render della modellazione strutturale



Ciminiera bicanne : analisi modale I° modo flessionale (1,06s- 0,93Hz)



Ciminiera bicanne : analisi modale II° modo flessionale (0,77s- 1,3Hz)

Thermoelectric Power Plant, Priolo Gargallo, Siracusa

Isolated chimney

$$h = 90\ \text{m};\ d = 6,4\ \text{m};\ n_1 = 0,93\ \text{Hz};\ \xi = 0,004;\ m = 1.683\ \text{kg/m}$$

$$\bar{u}_{cr} = \frac{n_1 \cdot d}{S} = \frac{0,93 \cdot 6,4}{0,2} = 29,76\ \text{m/s}$$

$$K = 0,13;\ L/d = 6 \Rightarrow L = 6 \times 6,4 = 38,4\ \text{m} \Rightarrow K_w = 0,6$$

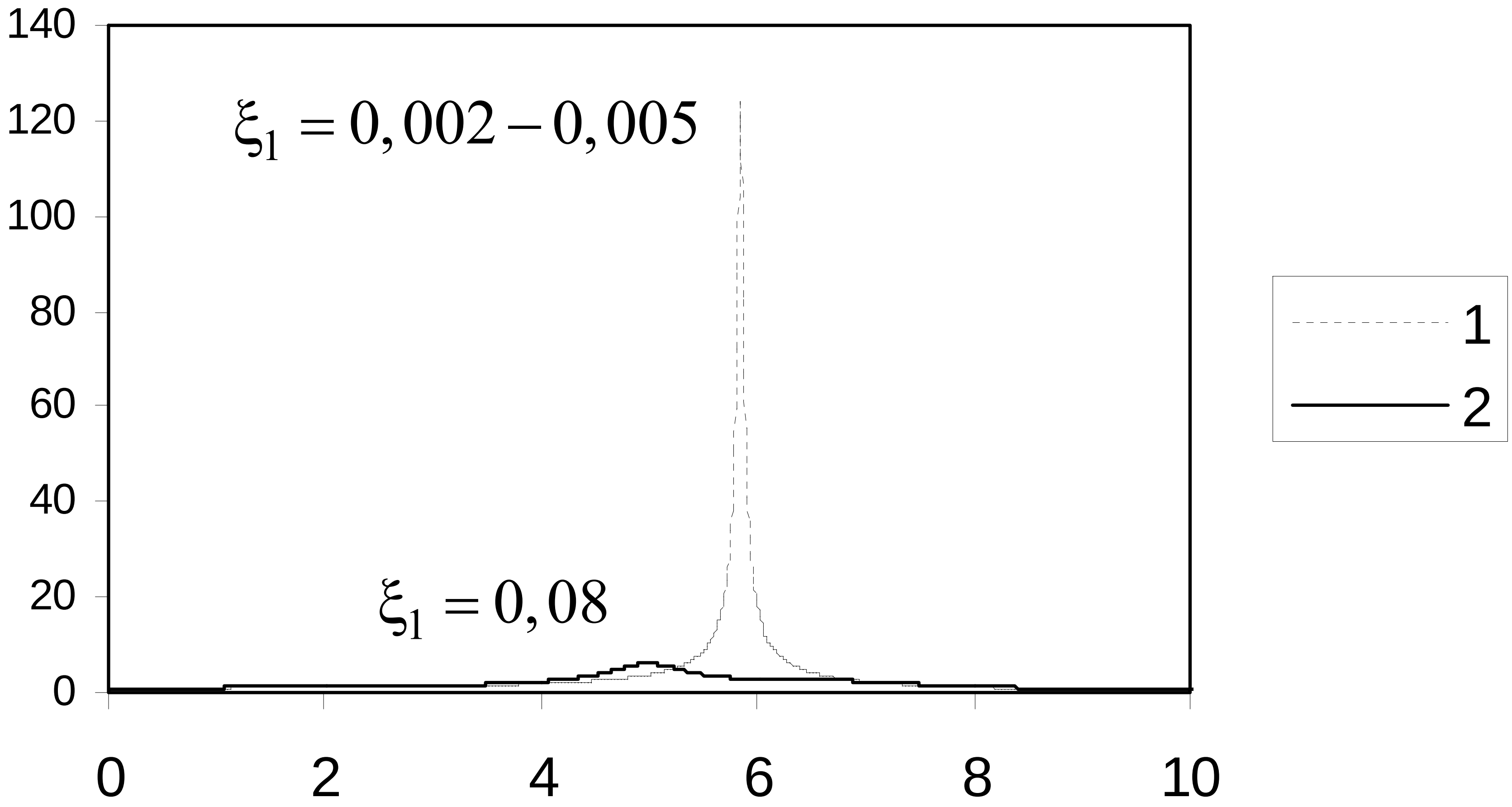
$$Re = \frac{\bar{u}_{cr} \cdot d}{\nu} = \frac{29,76 \cdot 6,4}{15 \times 10^{-6}} = 1,27 \times 10^7 \Rightarrow c_{lat} = 0,3$$

$$Sc = \frac{4\pi \cdot m \cdot \xi}{\rho \cdot d^2} = \frac{4\pi \cdot 1.683 \cdot 0,004}{1,25 \cdot 6,4^2} = 1,65\,!$$

$$\bar{y}_{max} = d \cdot K_w \cdot K \cdot \frac{c_{lat}}{Sc \cdot S^2} = 6,4 \cdot 0,6 \cdot 0,13 \cdot \frac{0,3}{1,65 \cdot 0,2^2} = 2,27\ \text{m}\,!$$

$$\bar{y}_{max}/d = 2,27/6,4 = 0,35 > 0,1 \Rightarrow \text{beginning of iteration}$$

Thermoelectric Power Plant, Priolo Gargallo, Siracusa



Thermoelectric Power Plant, Priolo Gargallo, Siracusa

Isolated chimney with Tuned Mass Damper (TMD)

$h = 90\ \text{m};\ d = 6,4\ \text{m};\ n_1 = 0,80\ \text{Hz};\ \xi = 0,08;\ m = 1.683\ \text{kg/m}$

$$\bar{u}_{cr} = \frac{n_1 \cdot d}{S} = \frac{0,80 \cdot 6,4}{0,2} = 25,60\ \text{m/s}\ (29,76\ m/s)$$

$$K = 0,13;\ L/d = 6 \Rightarrow L = 6 \times 6,4 = 38,4\ \text{m} \Rightarrow K_w = 0,6$$

$$Re = \frac{\bar{u}_{cr} \cdot d}{\nu} = \frac{25,60 \cdot 6,4}{15 \times 10^{-6}} = 1,09 \times 10^7 \Rightarrow c_{lat} = 0,3$$

$$Sc = \frac{4\pi \cdot m \cdot \xi}{\rho \cdot d^2} = \frac{4\pi \cdot 1.683 \cdot 0,08}{1,25 \cdot 6,4^2} = 33,05\ (1,65)$$

$$\bar{y}_{max} = \frac{d \cdot K_w \cdot K \cdot c_{lat}}{Sc \cdot S^2} = \frac{6,4 \cdot 0,6 \cdot 0,13 \cdot 0,3}{33,05 \cdot 0,2^2} = 0,11\ \text{m}\ (2,27\ m)$$

$\bar{y}_{max}/d = 0,11/6,4 = 0,018 < 0,1 \Rightarrow$ no iteration required

Vortex shedding – Harmonic method

Coupled chimneys with Tuned Mass Damper (TMD)

$h = 90\text{ m};\ d = 6,4\text{ m};\ a = 8,4\text{ m} \Rightarrow a/d = 1,31$

Non structurally connected chimneys

Response should be evaluated experimentally

Structurally connected chimneys

$K = 0,13;\ L/d = 6 \Rightarrow L = 6 \times 6,4 = 38,4\text{ m} \Rightarrow K_w = 0,6$

$$Re = \frac{\bar{u}_{cr} \cdot d}{\nu} = \frac{25,60 \cdot 6,4}{15 \times 10^{-6}} = 1,09 \times 10^7 \Rightarrow c_{lat} = 0,3 \times 1,5 = 0,45$$

$$Sc = \frac{4\pi \cdot m \cdot \xi}{\rho \cdot d^2} = \frac{4\pi \cdot 1.683 \cdot 0,08}{1,25 \cdot 6,4^2} = 33,05\ (1{,}65)$$

$$\bar{y}_{max} = \frac{d \cdot K_w \cdot K \cdot c_{lat}}{Sc \cdot S^2} = \frac{6,4 \cdot 0,6 \cdot 0,13 \cdot 0,45}{33,05 \cdot 0,2^2} = 0,165\text{ m}$$

$\bar{y}_{max}/d = 0,165/6,4 = 0,026 < 0,1 \Rightarrow$ no iteration required