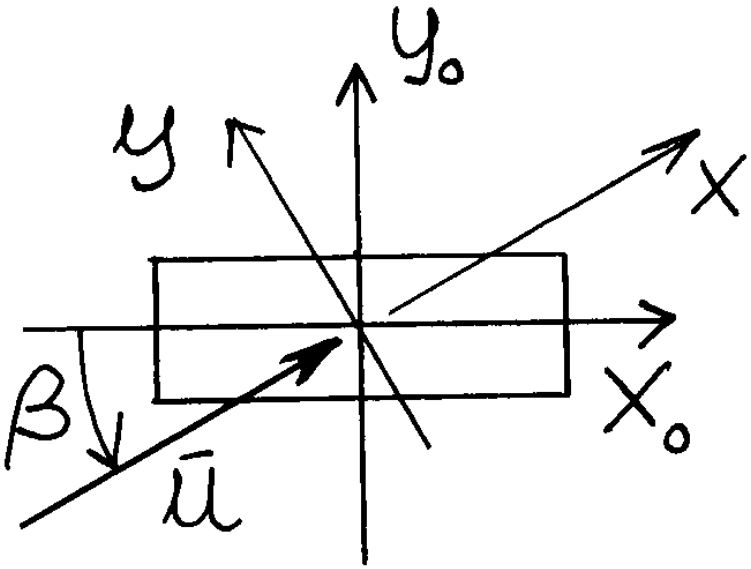
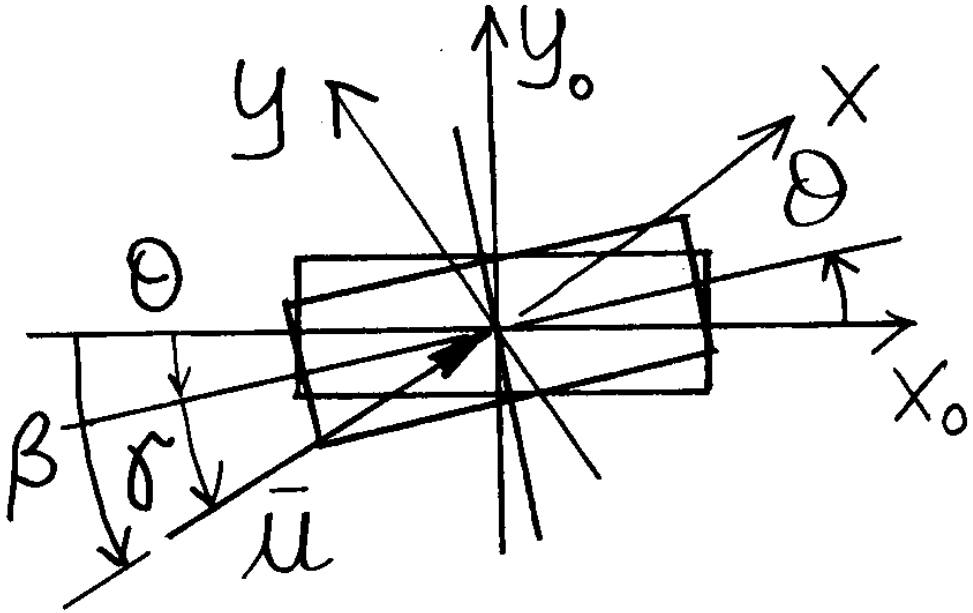


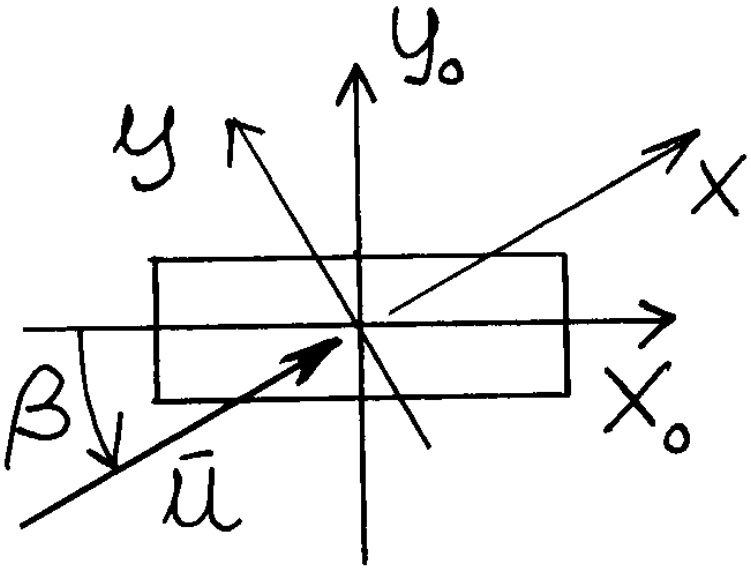


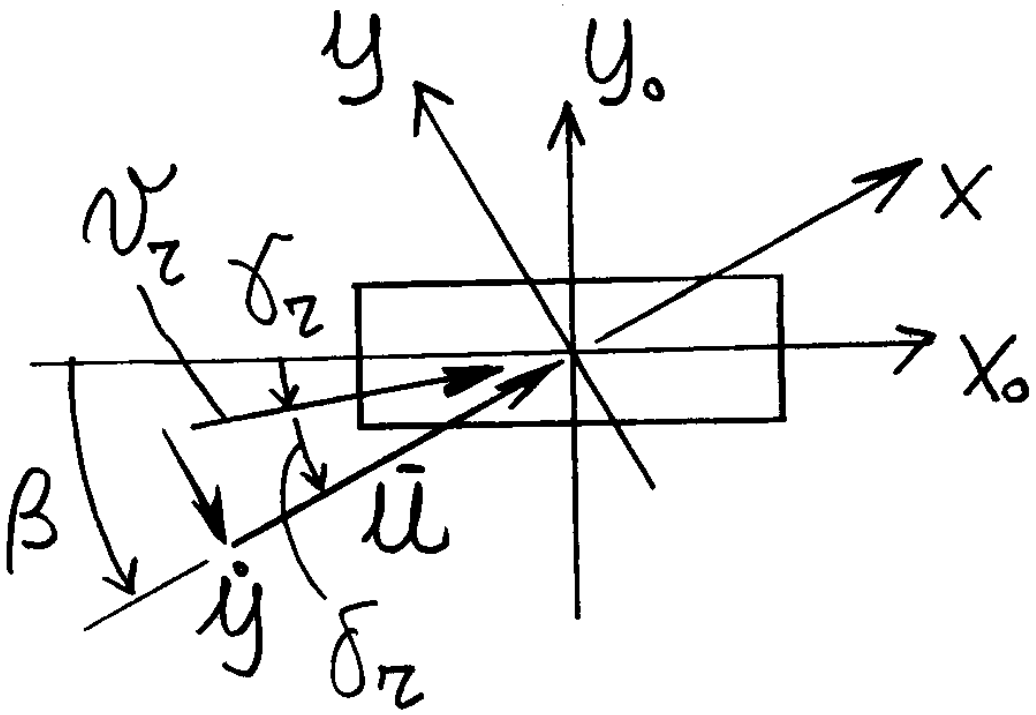
$$\bar{m}_\theta = \frac{1}{2}\rho \bar{u}^2 b^2 c_m(\beta)$$

$$\bar{m}_\theta = \frac{1}{2}\rho \bar{u}^2 b^2 c_m(\gamma)$$

$$\gamma = \beta - \theta$$

Torsional instability



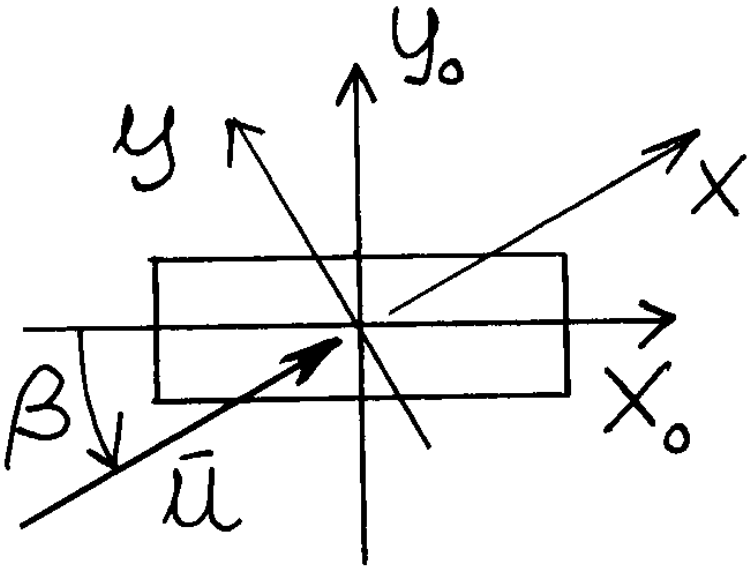


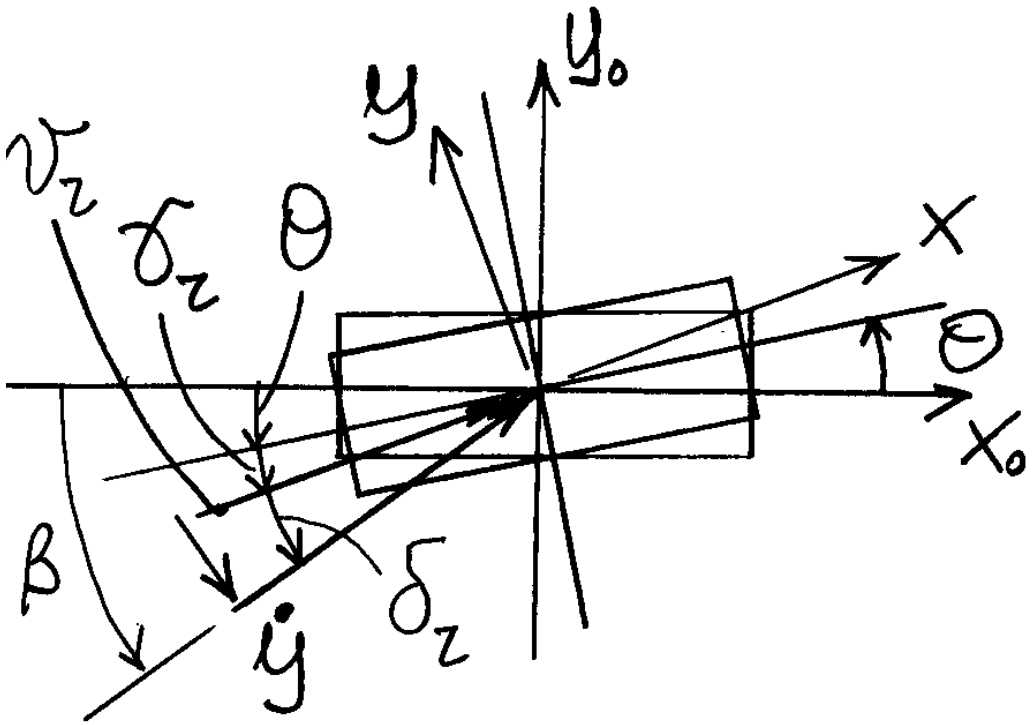
$$\bar{m}_\theta = \frac{1}{2}\rho \bar{u}^2 b^2 c_m(\beta)$$

$$m_{\theta r} = \frac{1}{2}\rho v_r^2 b^2 c_m(\gamma_r)$$

$$\gamma_r = \beta - \delta_r$$

Torsional instability





$$\bar{m}_\theta = \frac{1}{2}\rho \bar{u}^2 b^2 c_m(\beta)$$

$$m_{\theta r} = \frac{1}{2}\rho v_r^2 b^2 c_m(\gamma_r)$$

$$\gamma_r = \beta - \delta_r - \theta$$

$$v_r = \sqrt{\bar{u}^2 + \dot{y}^2}$$

$$\delta_r = \operatorname{arctg}\left(\frac{\dot{y}}{\bar{u}}\right)$$

$$m_{\theta r} = \frac{1}{2}\rho v_r^2 b^2 c_m(\gamma_r)$$

Linearized analysis $\Rightarrow$ Incipient instability

$$\gamma_r = \beta - \delta_r - \theta$$

$$v_r = \sqrt{\bar{u}^2 + \dot{y}^2} \cong \bar{u}$$

$$\delta_r = \text{arctg}\left(\frac{\dot{y}}{\bar{u}}\right) \cong \frac{\dot{y}}{\bar{u}}$$

$$c_m(\gamma_r) = \sum_k \frac{1}{k!} \left.\frac{\partial^k c_m(\gamma_r)}{\partial \gamma_r^k}\right|_{\gamma_r=\beta} \cdot \gamma_r^k \cong c_m + (\gamma_r - \beta) c'_m$$

$$c_m = c_m(\beta);\ c'_m = \left.\frac{\partial c_m(\gamma_r)}{\partial \gamma_r}\right|_{\gamma_r=\beta}$$

Torsional instability

$$m_{\theta r} = \frac{1}{2}\rho v_r^2 b^2 c_m(\gamma_r)$$

$$\gamma_r = \beta - \delta_r - \theta$$

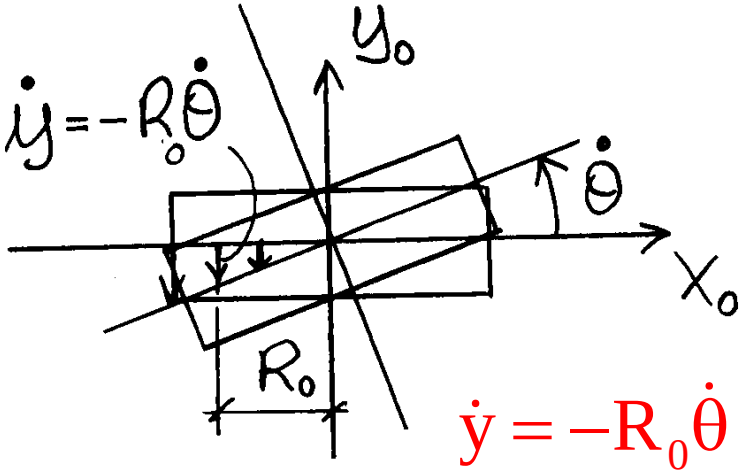
$$v_r = \sqrt{\bar{u}^2 + \dot{y}^2} \cong \bar{u}$$

$$\delta_r = \operatorname{arctg}\left(\frac{\dot{y}}{\bar{u}}\right) \cong \frac{\dot{y}}{\bar{u}}$$

$$c_m(\gamma_r) \cong c_m + (\gamma_r - \beta)c'_m = c_m - (\delta_r + \theta)c'_m$$

$$m_{\theta r} = \frac{1}{2}\rho \bar{u}^2 b^2 \left[c_m - \left(\frac{\dot{y}}{\bar{u}} + \theta\right)c'_m\right] \Rightarrow$$

$$m_{\theta r} = \frac{1}{2}\rho \bar{u}^2 b^2 \left[c_m - \left(-\frac{R_0\dot{\theta}}{\bar{u}} + \theta\right)c'_m\right]$$



Torsional instability

$$m_{\theta r} = \frac{1}{2}\rho \bar{u}^2 b^2 \left[c_m + \left(\frac{R_0 \dot{\theta}}{\bar{u}} - \theta \right) c'_m \right] =$$

$$= \frac{1}{2}\rho \bar{u}^2 b^2 c_m + \frac{1}{2}\rho \bar{u} b^2 R_0 \dot{\theta} c'_m - \frac{1}{2}\rho \bar{u}^2 b^2 \theta c'_m$$

SDOF Equation of motion (per unit length)

$$I_\theta \ddot{\theta} + c_\theta \dot{\theta} + k_\theta \theta = m_{\theta r} = \frac{1}{2}\rho \bar{u}^2 b^2 c_m + \frac{1}{2}\rho \bar{u} b^2 R_0 \dot{\theta} c'_m - \frac{1}{2}\rho \bar{u}^2 b^2 \theta c'_m \Rightarrow$$

$$I_\theta \ddot{\theta} + \underbrace{\left(c_\theta - \frac{1}{2}\rho \bar{u} b^2 R_0 c'_m \right)}_{\text{dissipative moment}} \dot{\theta} + \underbrace{\left(k_\theta + \frac{1}{2}\rho \bar{u}^2 b^2 c'_m \right)}_{\text{elastic moment}} \theta = m_{\theta r} = \frac{1}{2}\rho \bar{u}^2 b^2 c_m$$

Torsional instability

Equation of motion

$$I_\theta \ddot{\theta} + \left(c_\theta - \frac{1}{2}\rho \bar{u} b^2 R_0 c'_m \right) \dot{\theta} + \left(k_\theta + \frac{1}{2}\rho \bar{u}^2 b^2 c'_m \right) \theta = m_{\theta r} = \frac{1}{2}\rho \bar{u}^2 b^2 c_m$$

Torsional galloping necessary condition

$$\boxed{c'_m > 0}$$

Torsional galloping necessary and sufficient condition

$$c_\theta - \frac{1}{2}\rho \bar{u} b^2 R_0 c'_m \le 0 \Rightarrow 2\xi_s \omega_\theta I_\theta - \frac{1}{2}\rho \bar{u} b^2 R_0 c'_m \le 0 \Rightarrow \bar{u} \ge \frac{4\xi_s \omega_\theta I_\theta}{\rho b^2 R_0 c'_m}$$

Torsional galloping critical velocity

$$\boxed{\bar{u}_{cr} = \frac{4\xi_s \omega_\theta I_\theta}{\rho b^2 R_0 c'_m}}$$

Torsional galloping is a dynamic instability condition

Torsional galloping

Equation of motion

$$I_\theta \ddot{\theta} + \left(c_\theta - \frac{1}{2}\rho \bar{u} b^2 R_0 c'_m \right) \dot{\theta} + \left(k_\theta + \frac{1}{2}\rho \bar{u}^2 b^2 c'_m \right) \theta = m_{\theta r} = \frac{1}{2}\rho \bar{u}^2 b^2 c_m$$

Torsional divergence necessary condition

$$\boxed{c'_m < 0}$$

Torsional divergence necessary and sufficient condition

$$k_\theta + \frac{1}{2}\rho \bar{u}^2 b^2 c'_m \le 0 \Rightarrow \bar{u} \ge \sqrt{-\frac{2k_\theta}{\rho b^2 c'_m}}$$

Torsional divergence critical velocity

$$\boxed{\bar{u}_{cr} = \sqrt{-\frac{2k_\theta}{\rho b^2 c'_m}} = \sqrt{-\frac{2I_\theta \omega_\theta^2}{\rho b^2 c'_m}}}$$

Torsional divergence is a static instability condition

Torsional divergence

Fokker D-8, 1920 - torsional divergence

Signboard - torsional divergence