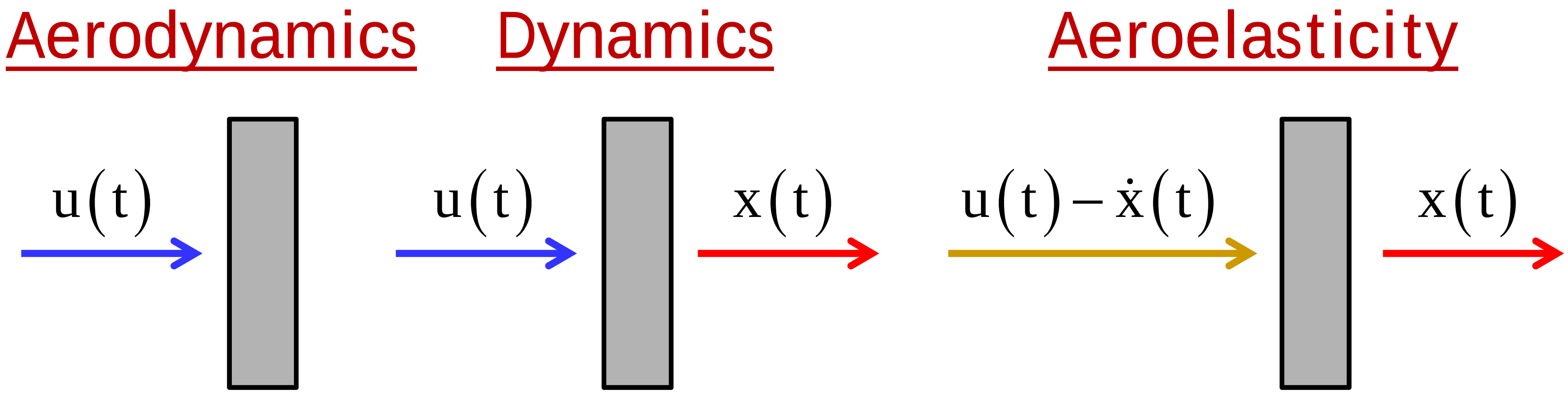


$u(t) = $ wind velocity

$x(t), \dot{x}(t) = $ structural displacement and velocity

$u(t) - \dot{x}(t) = $ relative wind velocity

Alongwind aerodynamic damping

Single Degree Of Freedom System

$$m\ddot{x}(t)+c\dot{x}(t)+kx(t)=f(t)=\frac{1}{2}\rho u_r^2(t)c_D A=\frac{1}{2}\rho\left[\bar{u}+u'(t)-\dot{x}(t)\right]^2 c_D A=$$

$$\frac{1}{2}\rho\left[\bar{u}^2+u'^2(t)+\dot{x}^2(t)+2\bar{u}u'(t)-2\bar{u}\dot{x}(t)-2u'(t)\dot{x}(t)\right]c_D A=$$

$$\frac{1}{2}\rho\bar{u}^2\left[1+\frac{u'^2(t)}{\bar{u}^2}+\frac{\dot{x}^2(t)}{\bar{u}^2}+2\frac{u'(t)}{\bar{u}}-2\frac{\dot{x}(t)}{\bar{u}}-2\frac{u'(t)\dot{x}(t)}{\bar{u}^2}\right]c_D A\cong$$

$$\cong\frac{1}{2}\rho\bar{u}^2\left[1+2\frac{u'(t)}{\bar{u}}-2\frac{\dot{x}(t)}{\bar{u}}\right]c_D A=\frac{1}{2}\rho\left[\bar{u}^2+2\bar{u}u'(t)-2\bar{u}\dot{x}(t)\right]c_D A\Rightarrow$$

$$m\ddot{x}(t)+c\dot{x}(t)+kx(t)=\frac{1}{2}\rho\left[\bar{u}^2+2\bar{u}u'(t)-2\bar{u}\dot{x}(t)\right]c_D A$$

Alongwind aerodynamic damping

$$m\ddot{x}(t)+c\dot{x}(t)+kx(t)=\frac{1}{2}\rho\left[\bar{u}^2+2\bar{u}u'(t)-2\bar{u}\dot{x}(t)\right]c_D A \Rightarrow$$

$$m\ddot{x}(t)+c\dot{x}(t)+\rho\bar{u}\dot{x}(t)c_D A+kx(t)=\frac{1}{2}\rho\left[\bar{u}^2+2\bar{u}u'(t)\right]c_D A \Rightarrow$$

$$m\ddot{x}(t)+\left(c+\rho\bar{u}c_D A\right)\dot{x}(t)+kx(t)=\frac{1}{2}\rho\bar{u}^2 c_D A+\rho\bar{u}u'(t)c_D A \Rightarrow$$

$$\ddot{x}(t)+\left(\frac{c}{m}+\frac{\rho\bar{u}c_D A}{m}\right)\dot{x}(t)+\frac{k}{m}x(t)=\frac{1}{m}\left[\bar{f}_x+f'_x(t)\right] \Rightarrow$$

$$\ddot{x}(t)+\left(2\xi_s\omega_0+2\xi_a\omega_0\right)\dot{x}(t)+\omega_0{}^2 x(t)=\frac{1}{m}\left[\bar{f}_x+f'_x(t)\right] \Rightarrow$$

$$\ddot{x}(t)+2\left(\xi_s+\xi_a\right)\omega_0\dot{x}(t)+\omega_0{}^2 x(t)=\frac{1}{m}\left[\bar{f}_x+f'_x(t)\right]$$

Alongwind aerodynamic damping

$$\ddot{x}(t)+\left(\frac{c}{m}+\frac{\rho\bar{u}c_D A}{m}\right)\dot{x}(t)+\frac{k}{m}x(t)=\frac{1}{m}\left[\bar{f}_x+f'_x(t)\right]\Rightarrow$$

$$\ddot{x}(t)+2(\xi_s+\xi_a)\omega_0\dot{x}(t)+\omega_0{}^2x(t)=\frac{1}{m}\left[\bar{f}_x+f'_x(t)\right]$$

$$\xi_s=\frac{c}{2m\omega_0}=\frac{c}{2\sqrt{km}}=\text{structural damping}$$

$$\xi_a=\left(\frac{\rho c_D A}{2m\omega_0}\right)\cdot\bar{u}=\text{aerodynamic damping}$$

$$\xi_t=\xi_s+\xi_a=\text{smorzamento totale}$$

$$\xi_a\geq 0\Rightarrow\xi_t\geq 0$$

Alongwind aerodynamic damping

Continuous Mono - dimensional System

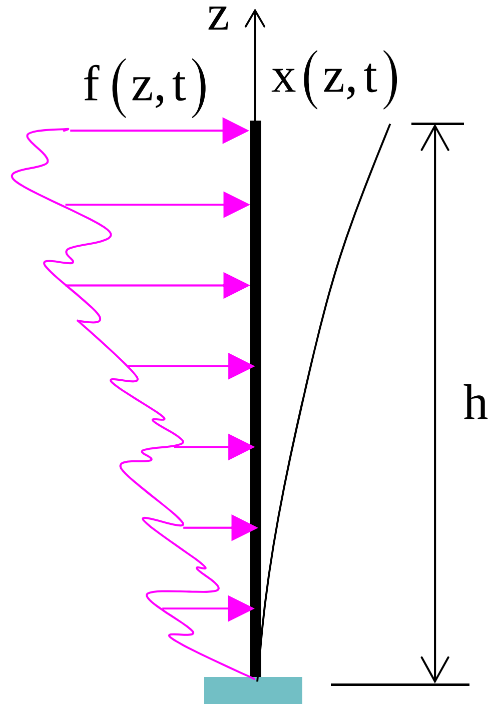
$$x(z,t) = \sum_k \psi_k(z) \cdot p_k(t)$$

$$\ddot{p}_k(t) + 2\xi_{sk}\omega_k \dot{p}_k(t) + \omega_k^2 p_k(t) = \frac{1}{m_k} f_k(t)$$

$$m_k = \int_0^h m(z)\psi_k(z)\,dz = \text{modal mass}$$

$$f_k(t) = \int_0^h f_x(z,t)\psi_k(z)\,dz = \text{modal force}$$

$$f_x(z,t) = \frac{1}{2}\rho u_r^2(z,t) c_D b = \frac{1}{2}\rho\left[\bar{u} + u'(z,t) - \dot{x}(z,t)\right]^2 c_D b$$



Alongwind aerodynamic damping

k – th aerodynamic damping

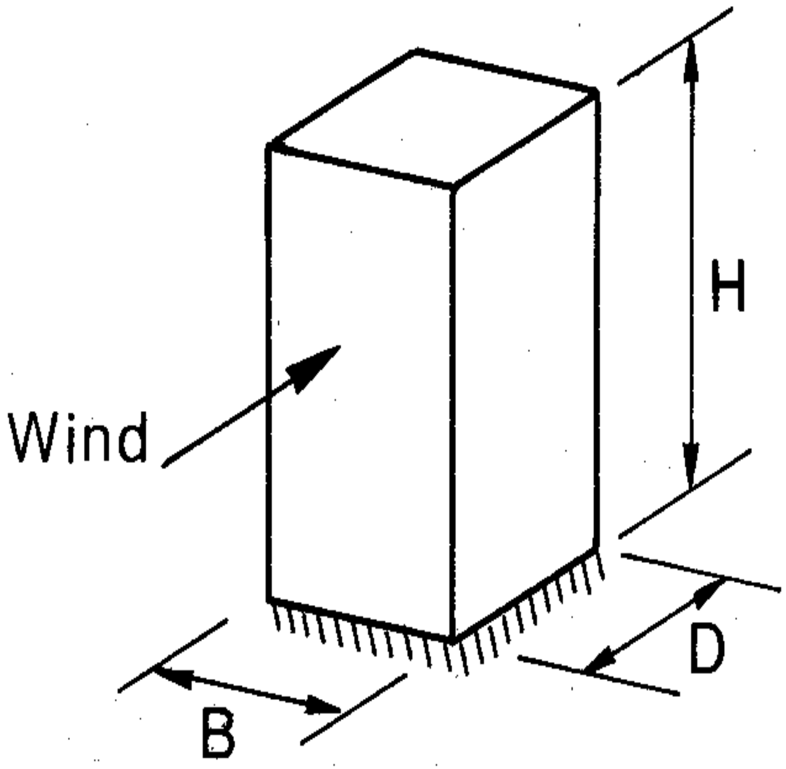
$$\xi_{ak} = \frac{\rho c_D b}{2\omega_k} \frac{\int_0^h \bar{u}(z)\psi_k^2(z)dz}{m_k} = \frac{\rho c_D b}{2\omega_k} \frac{\int_0^h \bar{u}(z)\psi_k^2(z)dz}{\int_0^h m(z)\psi_k^2(z)dz} \frac{\int_0^h \psi_k^2(z)dz}{\int_0^h \psi_k^2(z)dz}$$

$$m_{k,eq} = \frac{\int_0^h m(z)\psi_k^2(z)dz}{\int_0^h \psi_k^2(z)dz} = \text{k – th equivalent mass}$$

$$\bar{u}_{k,eq} = \frac{\int_0^h \bar{u}(z)\psi_k^2(z)dz}{\int_0^h \psi_k^2(z)dz} = \text{k – th equivalent velocity}$$

$$\xi_{ak} = \frac{\rho \bar{u}_{k,eq} c_D b}{2 m_{k,eq} \omega_k} \qquad \left(\xi_a = \frac{\rho \bar{u} c_D A}{2 m \omega_0} \;\; \text{S.D.O.F. system} \right)$$

Alongwind aerodynamic damping



$$B = D = 25\ \text{m};\ H = 100\ \text{m};\ c_D = 1{,}2$$

$$n_1 = 0{,}5\ \text{Hz};\ \psi_1(z) = \frac{z}{H};\ \xi_{s1} = 0{,}01$$

$$m(z) = 150.000\ \text{kg/m}$$

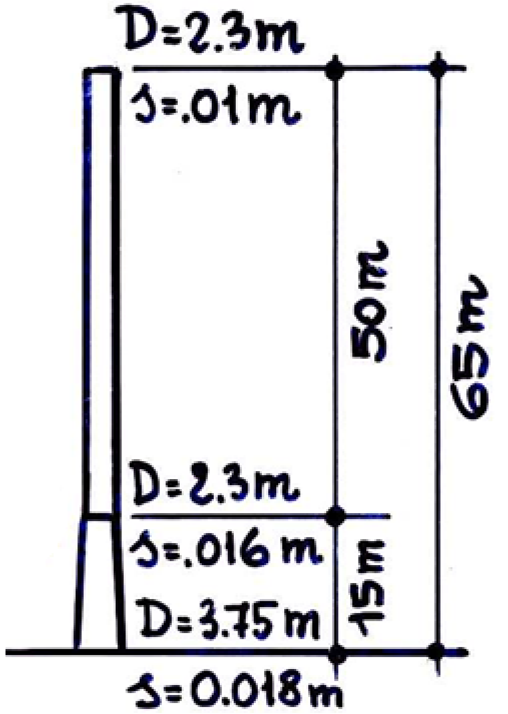
$$\bar{u}(z) = \bar{u}(H)\left(\frac{z}{H}\right)^{1/3};\ \bar{u}(H) = 25\ \text{m/s}$$

$$m_{1,eq}(z) = m(z) = 150.000\ \text{kg/m}$$

$$\bar{u}_{1,eq} = \frac{\int_0^H \bar{u}(z)\psi_1^2(z)dz}{\int_0^H \psi_1^2(z)dz} = \frac{\int_0^H \bar{u}(H)\left(\frac{z}{H}\right)^{1/3}\left(\frac{z}{H}\right)^2 dz}{\int_0^H \left(\frac{z}{H}\right)^2 dz} = \frac{\bar{u}(H)\cdot\frac{3}{10}}{\frac{1}{3}} = 25\cdot\frac{9}{10} = 22{,}5\ \text{m/s}$$

$$\xi_{a1} = \frac{\rho\bar{u}_{1,eq}c_D B}{2m_{1,eq}\omega_k} = \frac{1{,}25\times 22{,}5\times 1{,}2\times 25}{2\times 150.000\times(2\pi\times 0{,}5)} = 0{,}0009$$

Alongwind aerodynamic damping



$$D = 2{,}3\ \text{m};\ H = 65\ \text{m};\ c_D = 0{,}7$$

$$n_1 = 1\ \text{Hz};\ \psi_1(z) = \left(\frac{z}{H}\right)^2;\ \xi_{s1} = 0{,}002$$

$$m(z) = 565\ \text{kg/m}$$

$$\bar{u}(z) = \bar{u}(H)\left(\frac{z}{H}\right)^{1/3};\ \bar{u}(H) = 25\ \text{m/s}$$

$$m_{1,eq}(z) = m(z) = 565\ \text{kg/m}$$

$$\bar{u}_{1,eq} = \frac{\int_0^H \bar{u}(z)\psi_1^2(z)\,dz}{\int_0^H \psi_1^2(z)\,dz} =$$

$$= \frac{\int_0^H \bar{u}(H)\left(\frac{z}{H}\right)^{1/3}\left(\frac{z}{H}\right)^4 dz}{\int_0^H \left(\frac{z}{H}\right)^4 dz} =$$

$$= \frac{\bar{u}(H)\cdot\frac{3}{16}}{\frac{1}{5}} = 25\cdot\frac{15}{16} = 23{,}44\ \text{m/s}$$

$$\xi_{a1} = \frac{\rho\bar{u}_{1,eq}c_D D}{2m_{1,eq}\omega_k} =$$

$$= \frac{1{,}25\times 23{,}44\times 0{,}7\times 2{,}3}{2\times 565\times(2\pi\times 1)} = 0{,}0066$$

Alongwind aerodynamic damping