Power Law

Davenport (1960)

$$\bar{V}(z) = \bar{V}_G \left(\frac{z}{z_G} \right)^{\alpha}$$

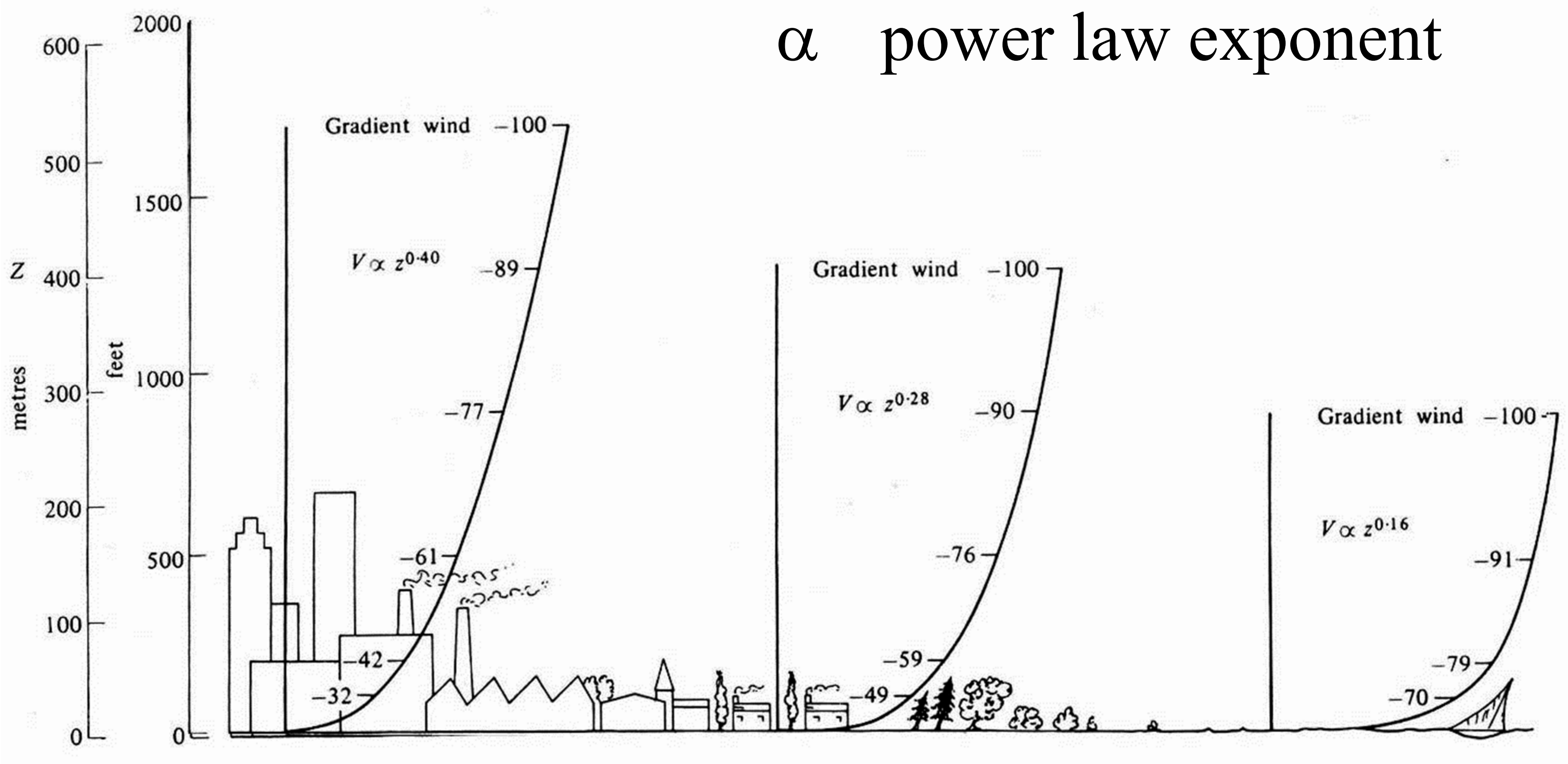
z height over ground

$\bar{V}$ mean wind velocity

$\bar{V}_G$ gradient velocity

z_G gradient height

α power law exponent



Power Law

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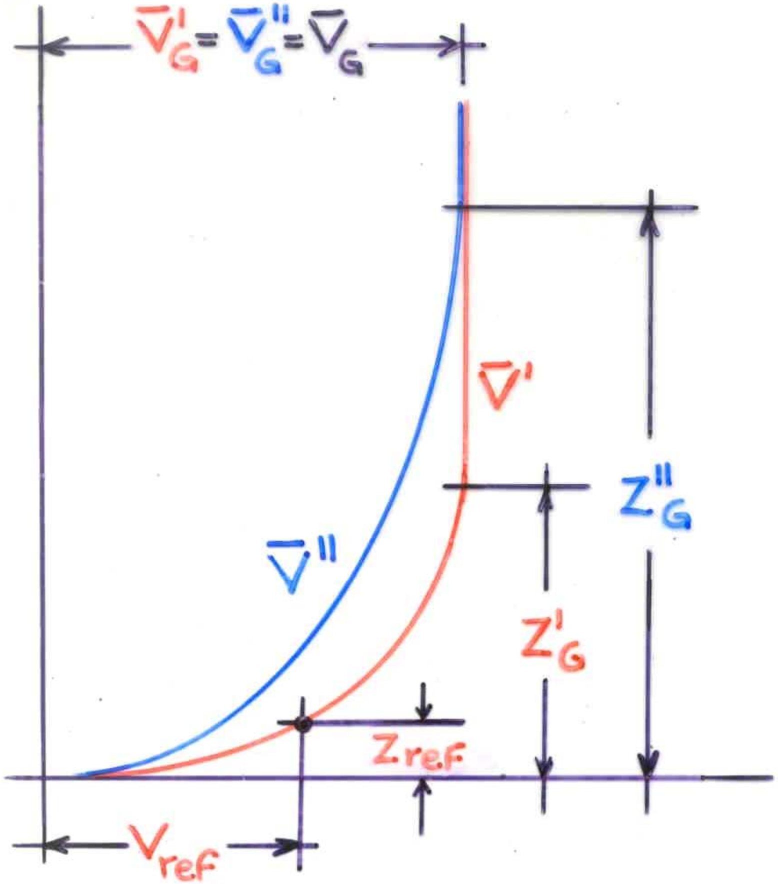
$$\bar{V}(z) = \bar{V}_G \left(\frac{z}{z_G}\right)^{\alpha}$$

z height over ground
$\bar{V}$ mean wind velocity
$\bar{V}_G$ gradient velocity
z_G gradient height
α power law exponent

Category	Description	α	z_G (ft)
1	Very smooth surfaces: e.g. large expanses of open water; low unsheltered islands; tidal flats; low-lands verging o the sea	1/8,5	800
2	Level, or surfaces with only low, surface obstructions: e.g. prairie grassland; desert; arctic tundra	1/7,5	900
3	Level, or slightly rolling surfaces, with slightly larger surface obstructions: e.g. farmland with very scattered trees and buildings, without hedgerows or other barriers; wasteland with low brush or surface vegetation; moorland	1/6,5	1.000
4	Gently rolling, or level country with low obstruction and barriers; e.g. open fields with walls and hedges scattered trees and buildings	1/5,5	1.100
5	Rolling or level surface broken by more numerous obstructions of various sizes: e.g. farmland, with small fields and dense hedges or barriers; scattered windbreaks of trees, scattered two-story buildings	1/4,5	1.200
6	Rolling or level surface, uniformly covered with numerous large obstructions: e.g. forest, scrub trees, parkland	1/3,5	1.350
7	Very broken surface with large obstructions: e.g. towns; suburbs; outskirts of large cities; farmland with numerous woods and copses and large windbreaks of tall trees	1/3	1,500
8	Surface broken by extremely large obstructions: e.g. centre of large city	1/(2,5-1,5)	1,800

Power Law



$z'_0 < z''_0 \Rightarrow \alpha' < \alpha'' \Rightarrow$

$\bar{V}'(z) > \bar{V}''(z)$

Known quantities:

$$\bar{V}_{ref} = \bar{V}'\left(z_{ref}\right), \alpha', \alpha''$$

$$\bar{V}'_G = \bar{V}_{ref}\left(z'_G / z_{ref}\right)^{\alpha'} = \bar{V}''_G$$

$$\bar{V}''(z) = \bar{V}''_G\left(z / z''_G\right)^{\alpha''} \Rightarrow$$

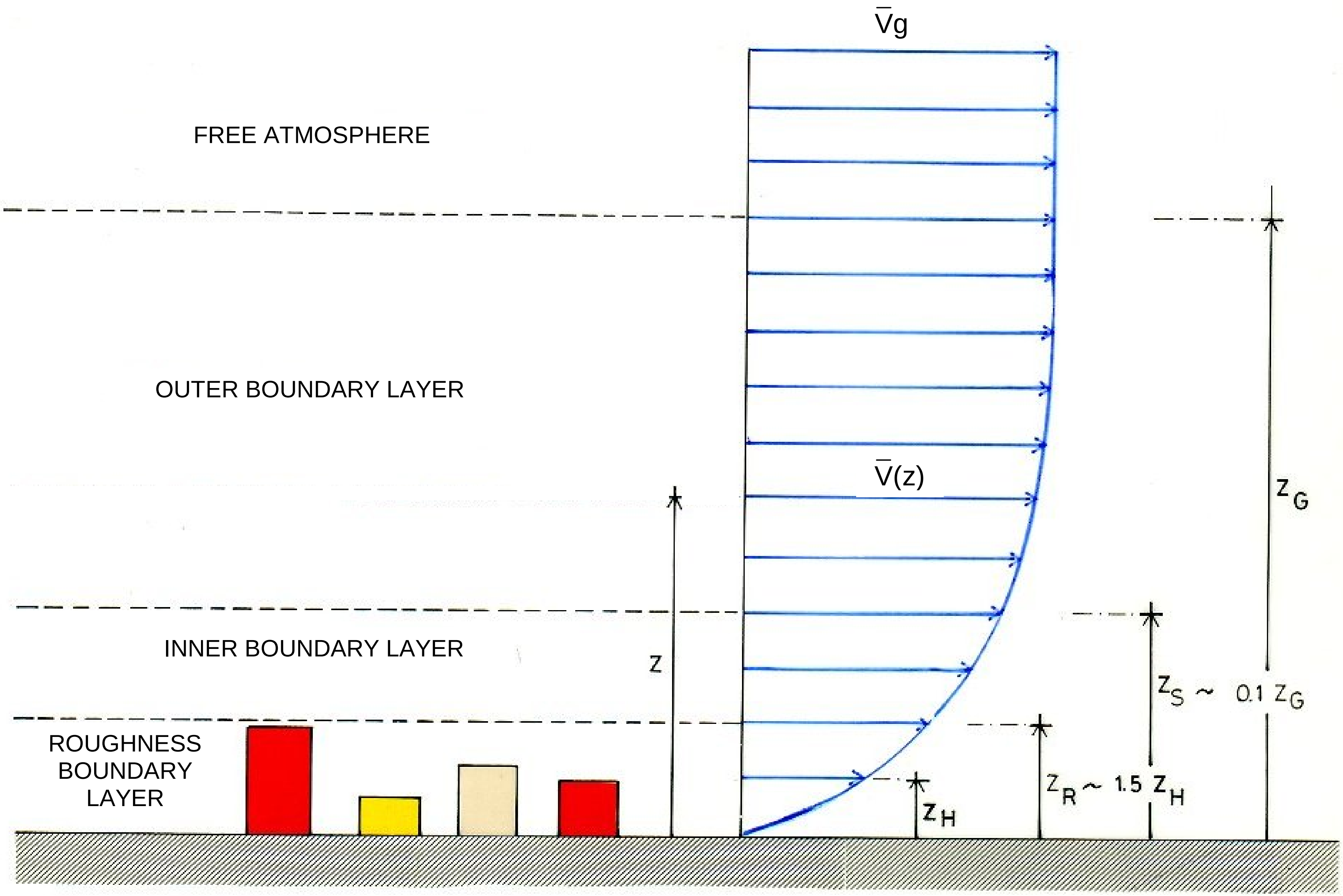
$$\bar{V}''(z) = \bar{V}_{ref}\left(z'_G / z_{ref}\right)^{\alpha'}\left(z / z''_G\right)^{\alpha''} \Rightarrow$$

$$\boxed{\bar{V}''(z) = \bar{V}_{ref} C_r(z)}$$

$\bar{V}_{ref}$ reference velocity
C_r roughness coefficient

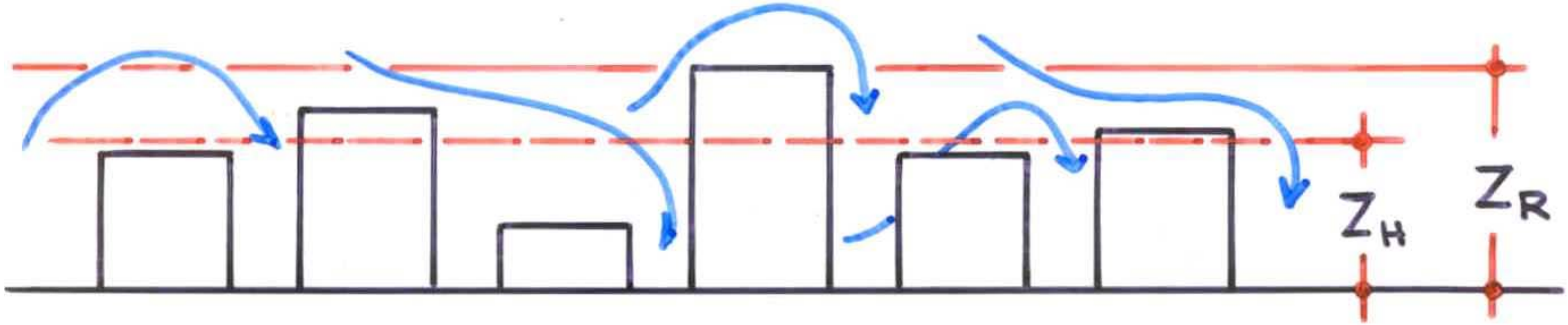
$$\boxed{C_r(z) = \left(z'_G / z_{ref}\right)^{\alpha'}\left(z / z''_G\right)^{\alpha''}}$$

Atmospheric Boundary Layer



Atmospheric Boundary Layer

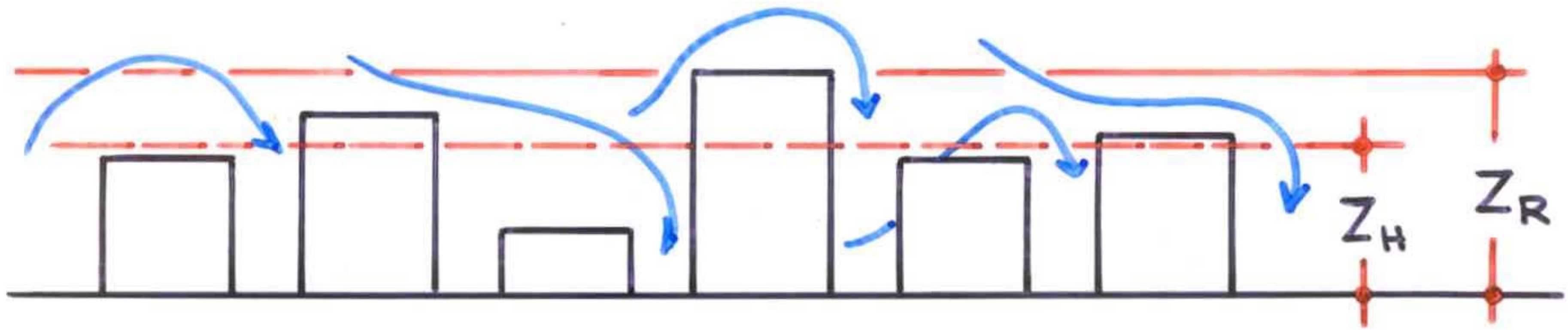
Roughness Boundary Layer



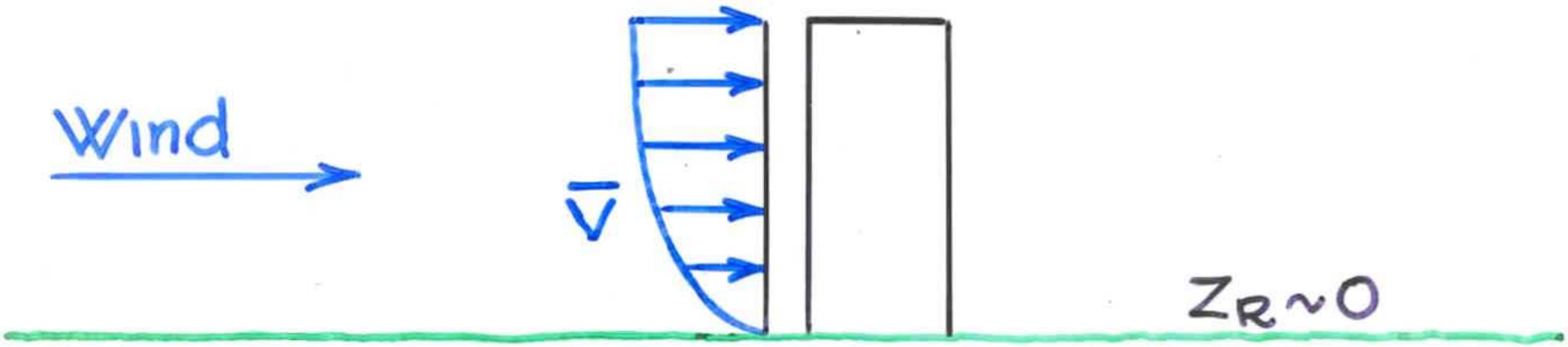
z_H	average height of obstacles
$z_R = 1{,}5 \cdot z_H$	depth of the roughness boundary layer

Atmospheric Boundary Layer

Roughness Boundary Layer

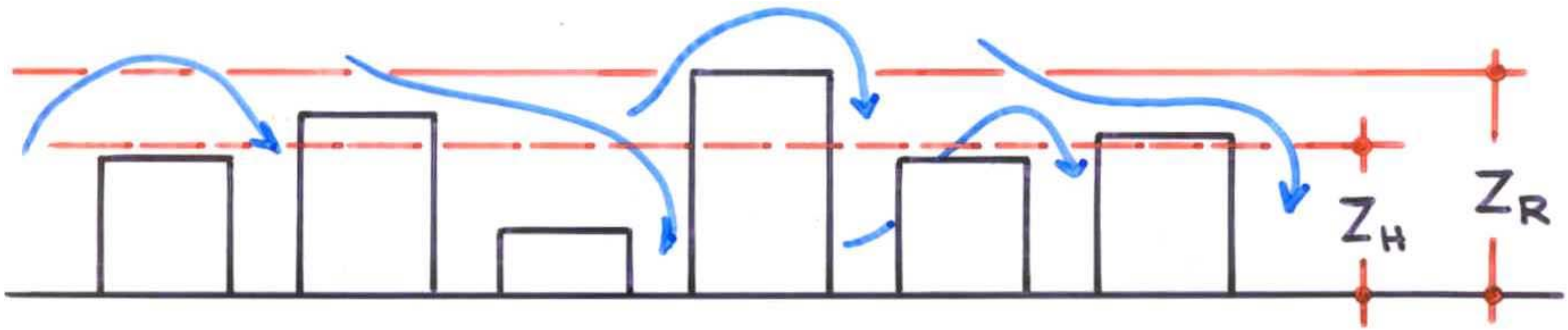


Smooth terrain

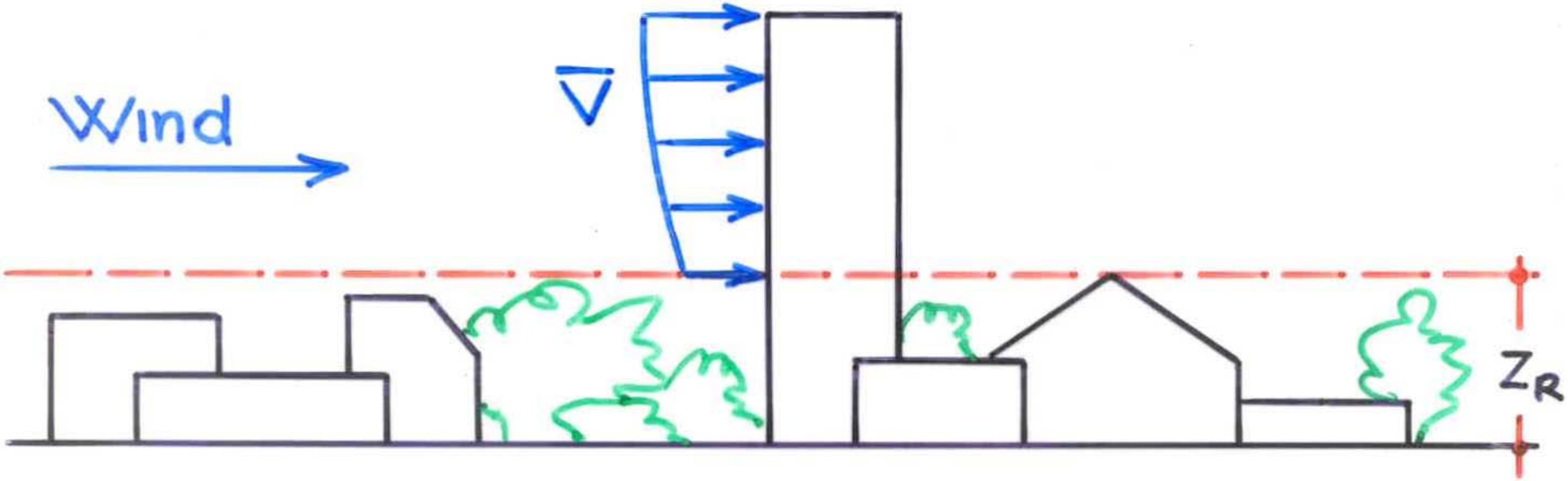


Atmospheric Boundary Layer

Roughness Boundary Layer

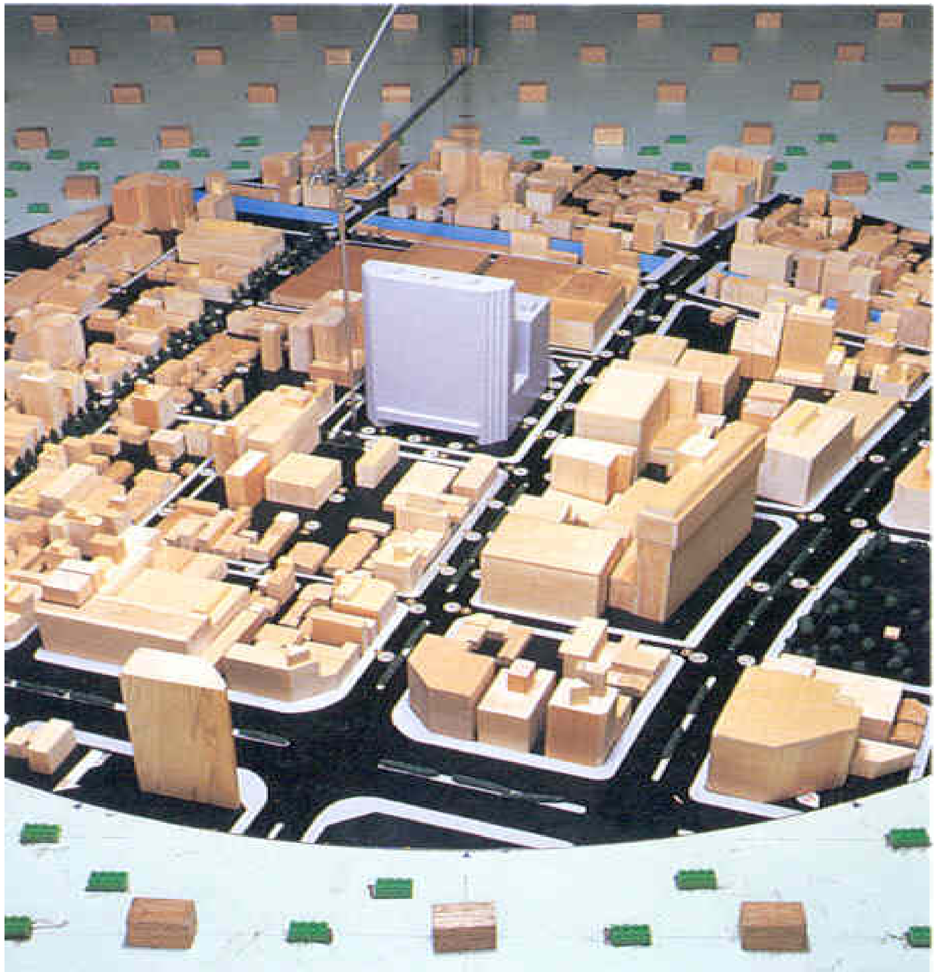


Rough terrain



Atmospheric Boundary Layer

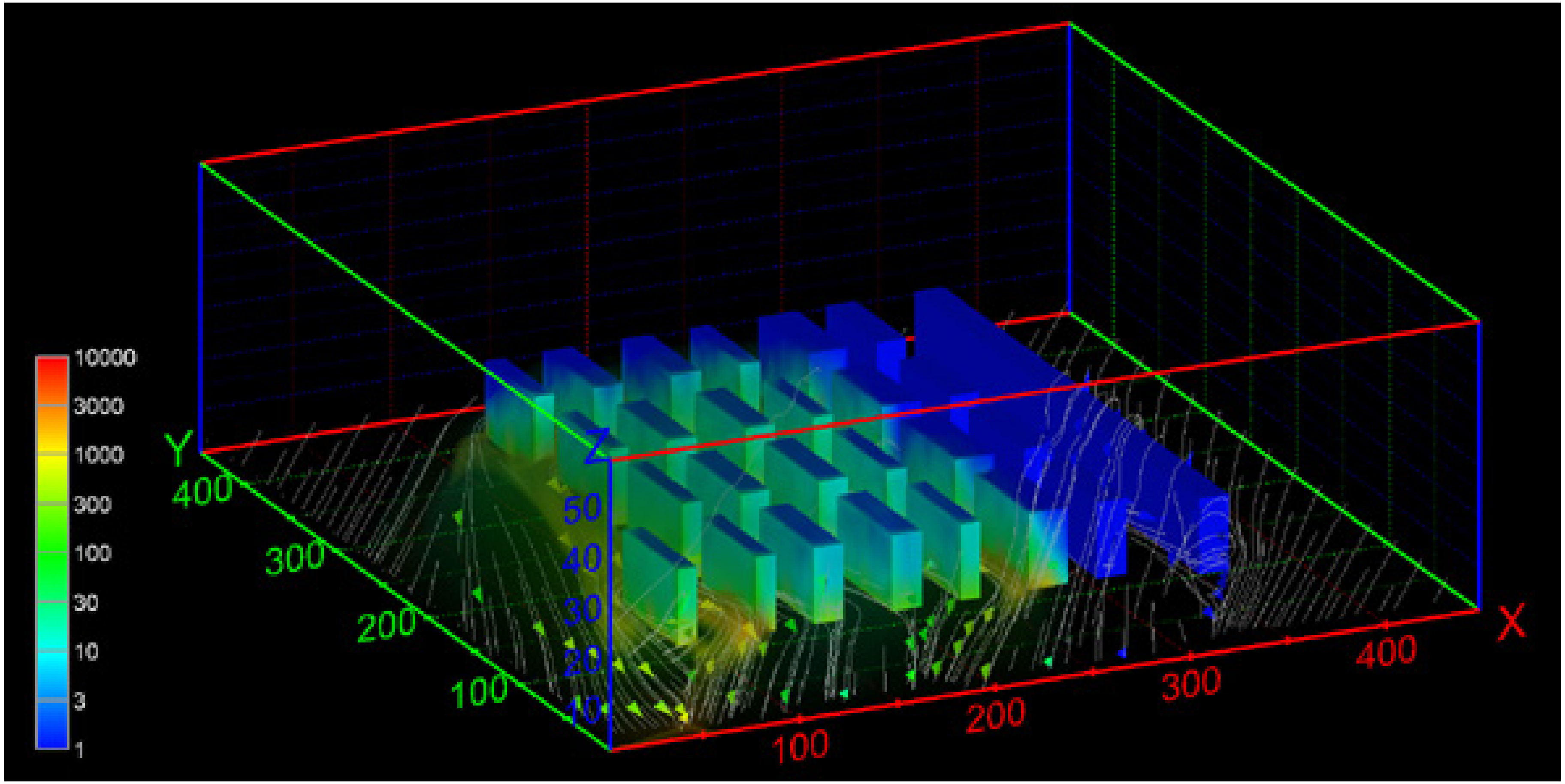
Roughness Boundary Layer



Wind tunnel tests

Atmospheric Boundary Layer

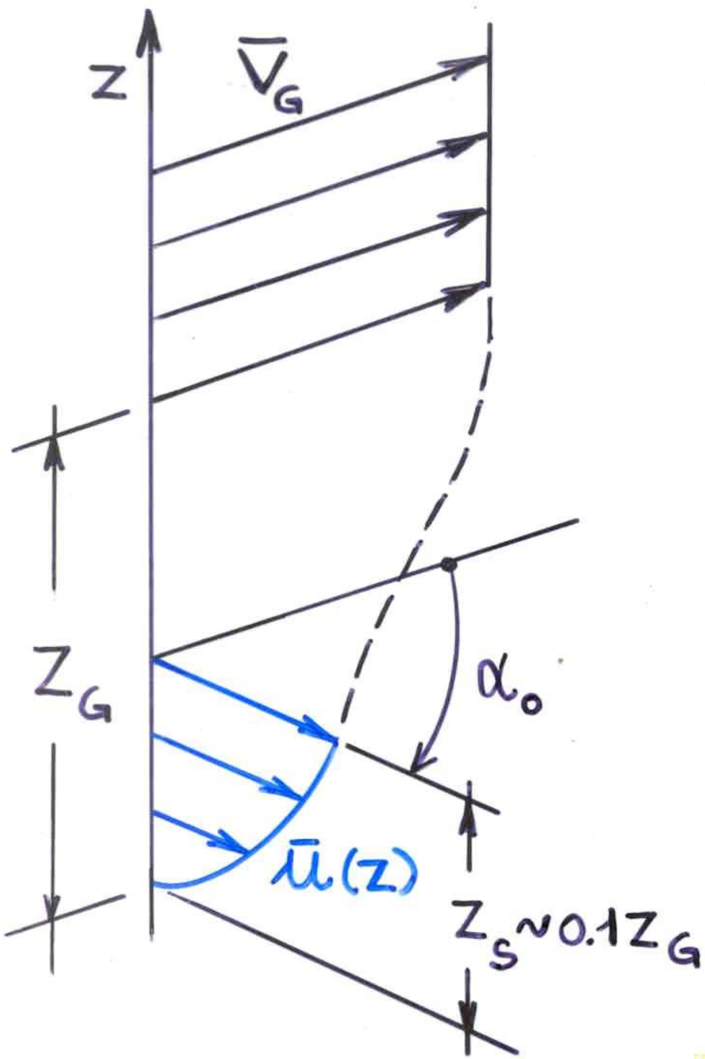
Roughness Boundary Layer



CFD numerical model

Atmospheric Boundary Layer

Inner Boundary Layer



Logarithmic profile

$$\boxed{\bar{u}(z) = \frac{1}{\kappa} u_* \ell n\left(\frac{z}{z_0}\right)}$$

κ Karman's constant $(\kappa \sim 0.4)$

u_* shear velocity $\left(u_*^2 = -\overline{u'w'}\right)$

z_0 roughness length

Marine Boundary Layer

$z_0 = \alpha u_*^2 / g$ (Charnock, 1955)

$\alpha \sim 0.0144$ (Garratt, 1977)

g = gravity acceleration

Atmospheric Boundary Layer

Modified logarithmic profile

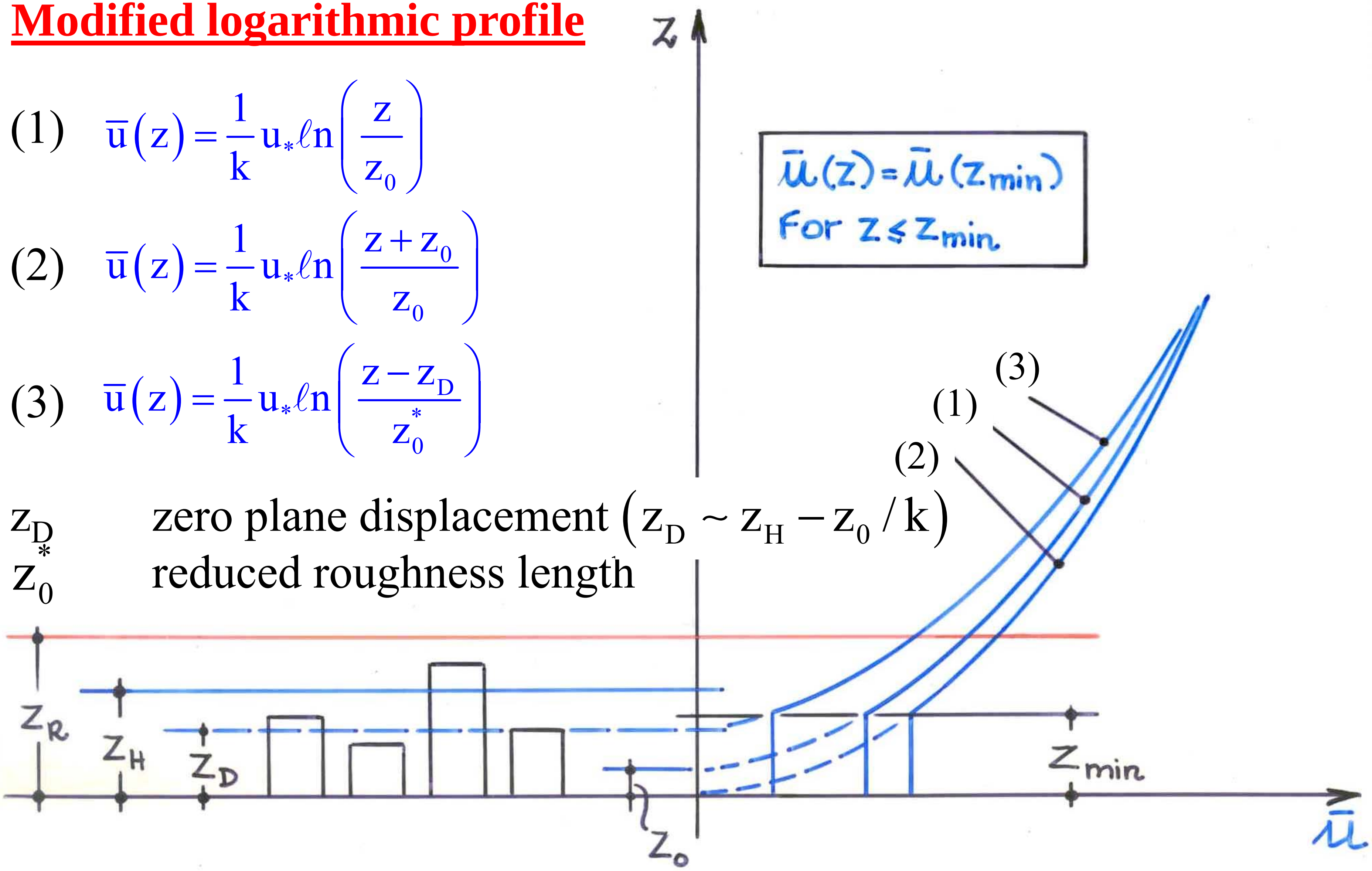
(1) $$\bar{u}(z) = \frac{1}{k} u_* \ell n\left(\frac{z}{z_0}\right)$$

(2) $$\bar{u}(z) = \frac{1}{k} u_* \ell n\left(\frac{z+z_0}{z_0}\right)$$

(3) $$\bar{u}(z) = \frac{1}{k} u_* \ell n\left(\frac{z-z_D}{z_0^*}\right)$$

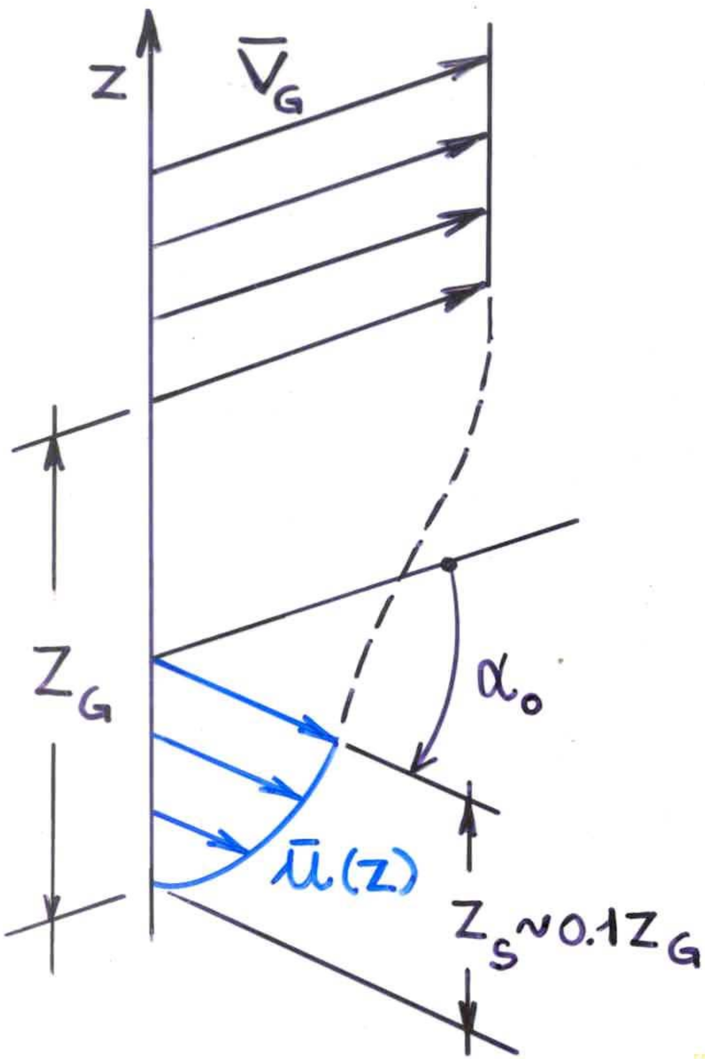
z_D zero plane displacement $\left(z_D \sim z_H - z_0 / k\right)$

z_0^* reduced roughness length



Atmospheric Boundary Layer

Free Atmosphere



Gradient velocity

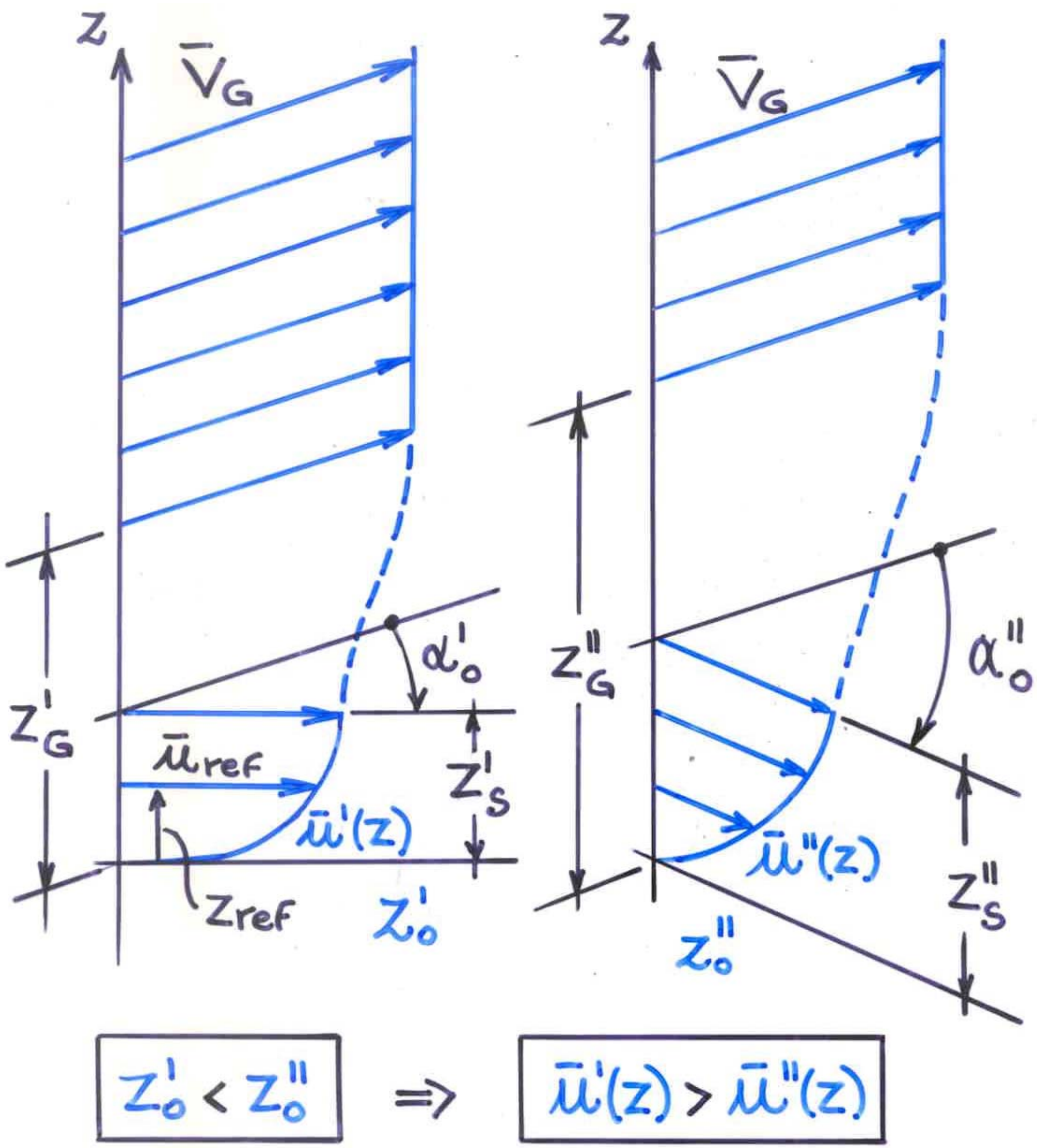
$$z_G = C\frac{u_*}{|f|}$$

$$\bar{V}_G = \frac{u_*}{k}\sqrt{\left[\ell n\left(\frac{u_*}{|f|z_0}\right) - A\right]^2 + B^2}$$

$$|\alpha_0| = \arcsin\frac{B}{\sqrt{\left[\ell n\left(\frac{u_*}{|f|z_0}\right) - A\right]^2 + B^2}}$$

$A \sim 1$	(Clarcke, 1970; Arya, 1975)
$B \sim 4.5$	(Blackadar & Tennekes, 1968)
$C \sim 1/6$	(Deaves & Harris, 1978)
$f = 2\omega \sin\varphi$	Coriolis parameter
ω	earth angular velocity
φ	latitude

Atmospheric Boundary Layer



$$\bar{u}_{ref} = \bar{u}'(z_{ref}), z'_0, z''_0 \quad \text{known}$$

$$\bar{u}_{ref} = \bar{u}'(z_{ref}) = \frac{1}{\kappa} u'_* \ell n\left(\frac{z_{ref}}{z'_0}\right)$$

$$\bar{u}''(z) = \frac{1}{\kappa} u''_* \ell n\left(\frac{z}{z''_0}\right)$$

$$u'_* / u''_* \simeq (z'_0 / z''_0)^{0,07}$$

$$\boxed{\bar{u}''(z) = \bar{u}_{ref} c_r(z)}$$

$\bar{u}_{ref}$ reference velocity

c_r roughness coefficient

$$\boxed{c_r(z) = \frac{(z''_0 / z'_0)^{0,07} \ell n(z / z''_0)}{\ell n(z_{ref} / z'_0)}}$$

Example

Reference site $z_{ref} = 10\text{ m}; z'_0 = 0{,}05\text{ m}, \bar{u}_{ref} = \bar{u}'\left(z_{ref}\right) = 25\text{ m/s}$

$$\bar{u}_{ref} = 2.5 u'_* \ell n\left(z_{ref} / z'_0\right) \Rightarrow$$

$$u'_* = 0.4 \bar{u}_{ref} / \ell n\left(z_{ref} / z'_0\right) = 0.4 \times 25 / \ell n\left(10 / 0.05\right) = 1.87\text{m/s}$$

Gradient velocity $\varphi = 45°$

$$\omega = 7.292 \times 10^{-5}\text{ rad/s}, f = 2\omega \sin\varphi = 1.0313 \times 10^{-4}\text{ s}^{-1}$$

$$z_G = Cu'_* / f = 1/6 \cdot 1.87 / 1.03313 \times 10^{-4} = 3022\text{ m}$$

$$\bar{V}_G = u'_* / k \cdot \left\{\left[\ell n\left(u'_* / f z'_0\right) - A\right]^2 + B^2\right\}^{1/2} =$$

$$= 1.87 \times 2.5 \times \left\{\left[\ell n\left(1.87 / 1.0313 \times 10^{-4} / 0.05\right) - 1\right] + \overline{4.5}^2\right\} = 59{,}05\text{ m/s}$$

Design site $z''_0 = 0.3\text{ m}$

$$c_r\left(z\right) = \left(z''_0 / z'_0\right)^{0.07} \ell n\left(z / z''_0\right) / \ell n\left(z_{ref} / z'_0\right) = \left(.3 / .05\right)^{0.07} \ell n\left(z / .3\right) / \ell n\left(10 / .05\right)$$

$$c_r\left(z\right) = 0.214 \ell n\left(z / 0.3\right) \quad \left(z\text{ in m}\right)$$

$$\bar{u}''\left(z\right) = \bar{u}_{ref} c_r\left(z\right) = 25 \times 0.214\, \ell n\left(z / 0.3\right) \quad \left(z\text{ in m}, \bar{u}\text{ in m/s}\right)$$

Example

Reference site $z_{ref} = 10\ \text{m};\ z_0' = 0,05\ \text{m},\ \bar{u}_{ref} = \bar{u}'(z_{ref}) = 25\ \text{m/s}$

$$\bar{u}_{ref} = 2.5u_*'\ell n(z_{ref}/z_0') \Rightarrow$$

$$u_*' = 0.4\bar{u}_{ref}/\ell n(z_{ref}/z_0') = 0.4 \times 25/\ell n(10/0.05) = 1.87\text{m/s}$$

Design site $z_0'' = 0.3\ \text{m}$

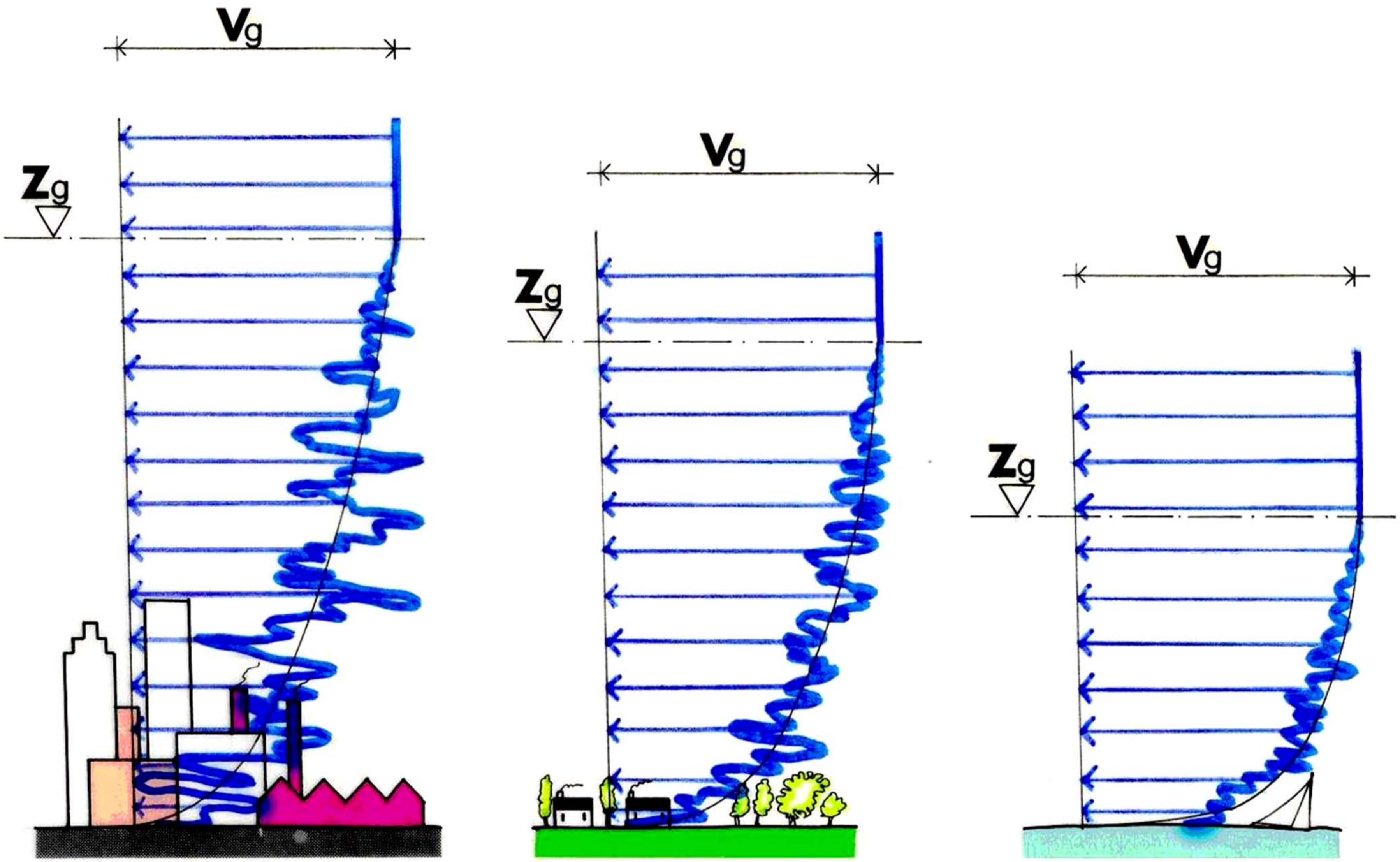
$$c_r(z) = (z_0''/z_0')^{0.07}\ \ell n(z/z_0'')/\ell n(z_{ref}/z_0') = (.3/.05)^{0.07}\ \ell n(z/.3)/\ell n(10/.05)$$

$$c_r(z) = 0.214\ell n(z/0.3) \quad (z \text{ in m})$$

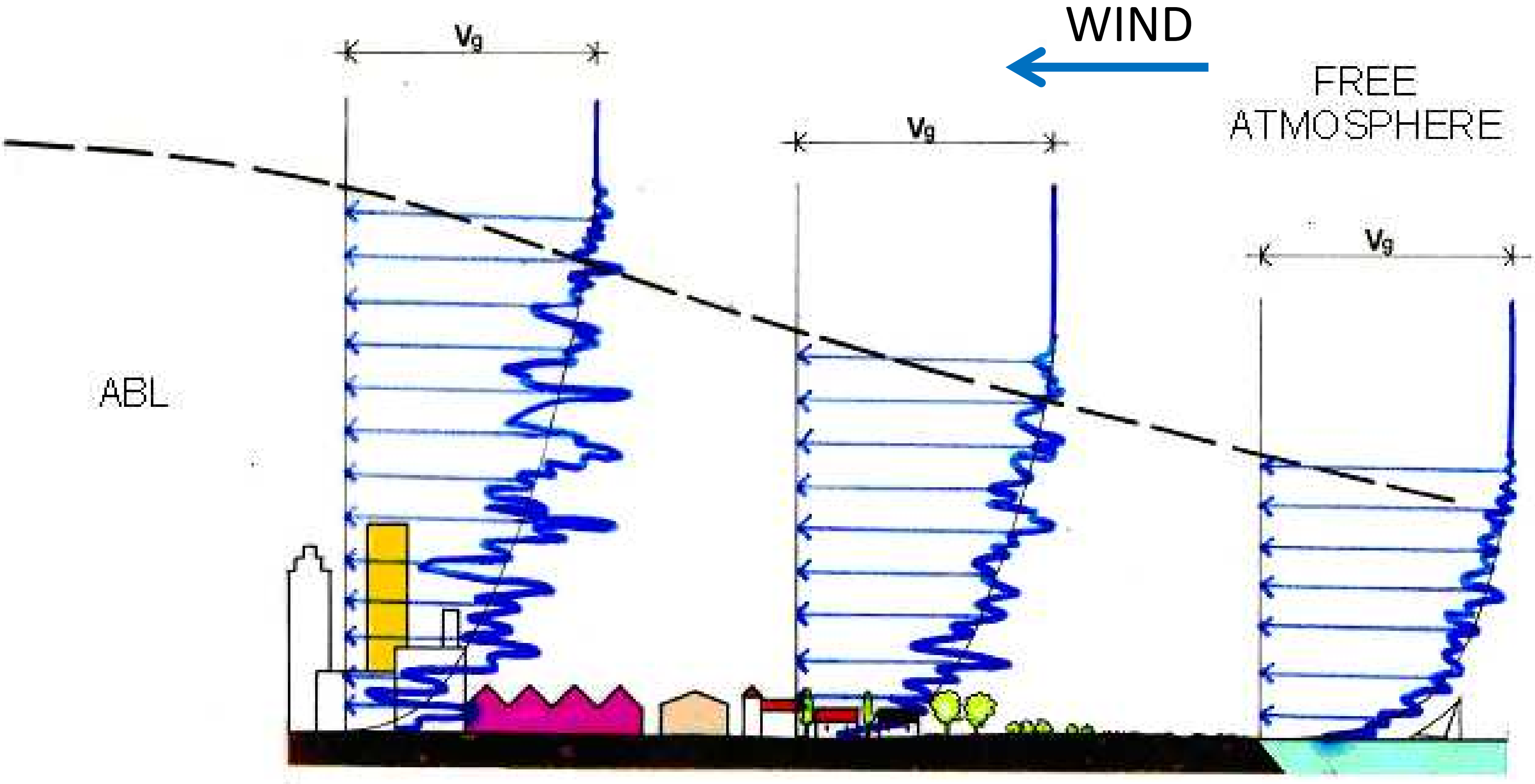
$$\bar{u}''(z) = \bar{u}_{ref}c_r(z) = 25 \times 0.214\,\ell n(z/0.3) \quad (z \text{ in m}, \bar{u} \text{ in m/s})$$

z (m)	$c_r(z)$	$\bar{u}''(z)$ (m/s)
10	0,75	18,76
20	0,90	22,47
50	1,09	27,37
100	1,24	31,08

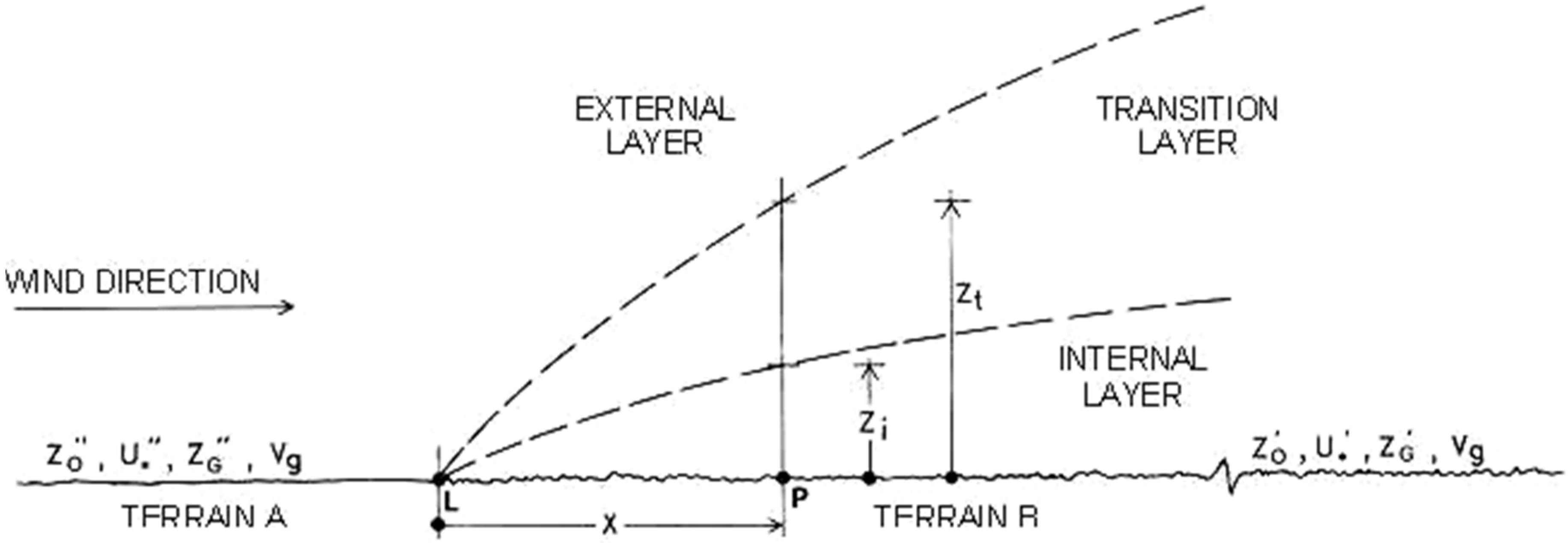
Homogeneous terrain



Transition profiles



Transition profiles



Smooth to rough $z_o'' < z_o'$

$$z_i(x) = 0,36 \cdot z_0' \cdot (x / z_0')^{0,75}$$

$$z_t(x) = 10 \cdot z_0' \cdot (x / z_0')^{0,60}$$

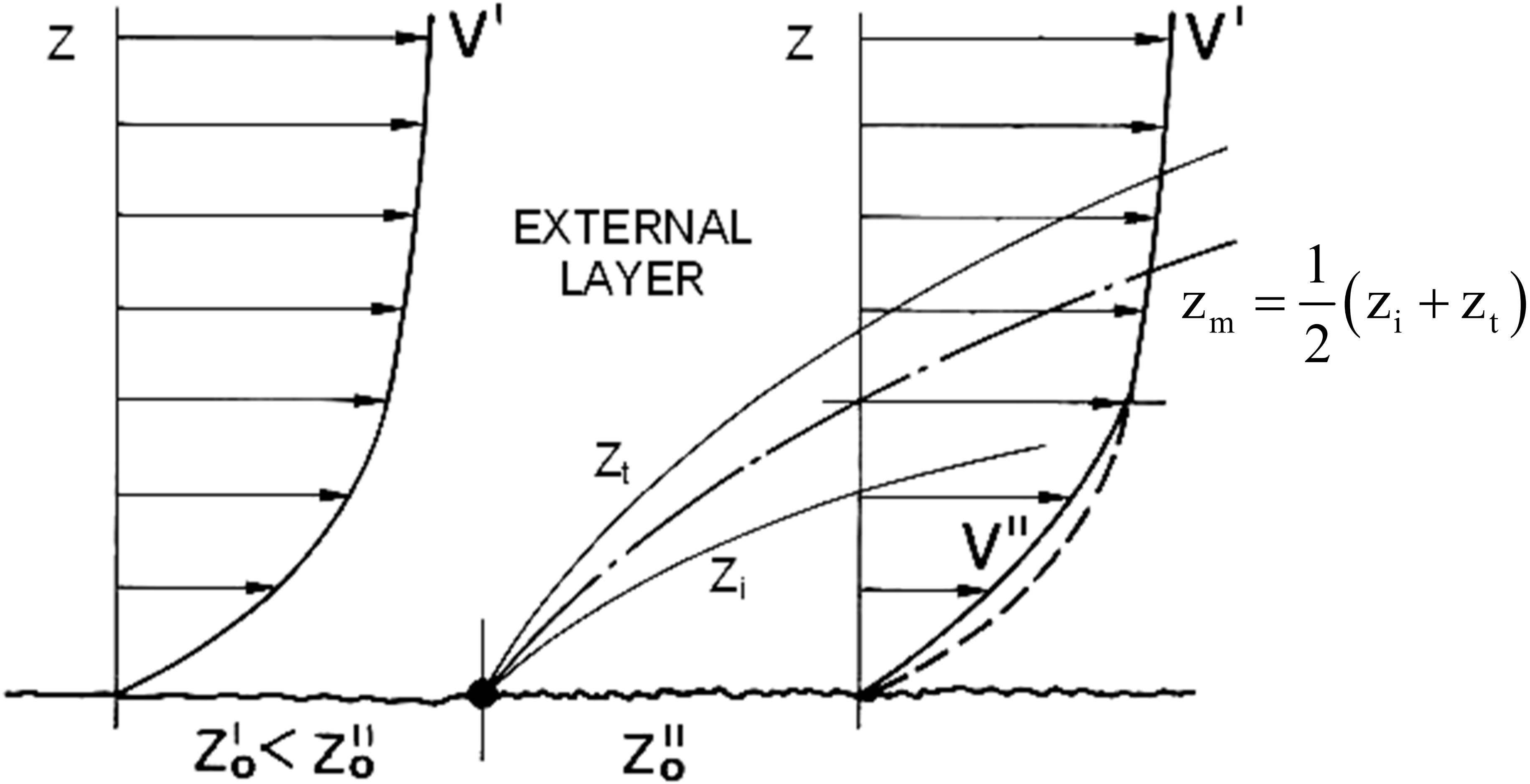
Rough to smooth $z_o'' > z_o'$

$$z_i(x) = 0,07 \cdot z_0'' \cdot (x / z_0'')^{0,50}$$

$$z_t(x) = 10 \cdot z_0'' \cdot (x / z_0'')^{0,60}$$

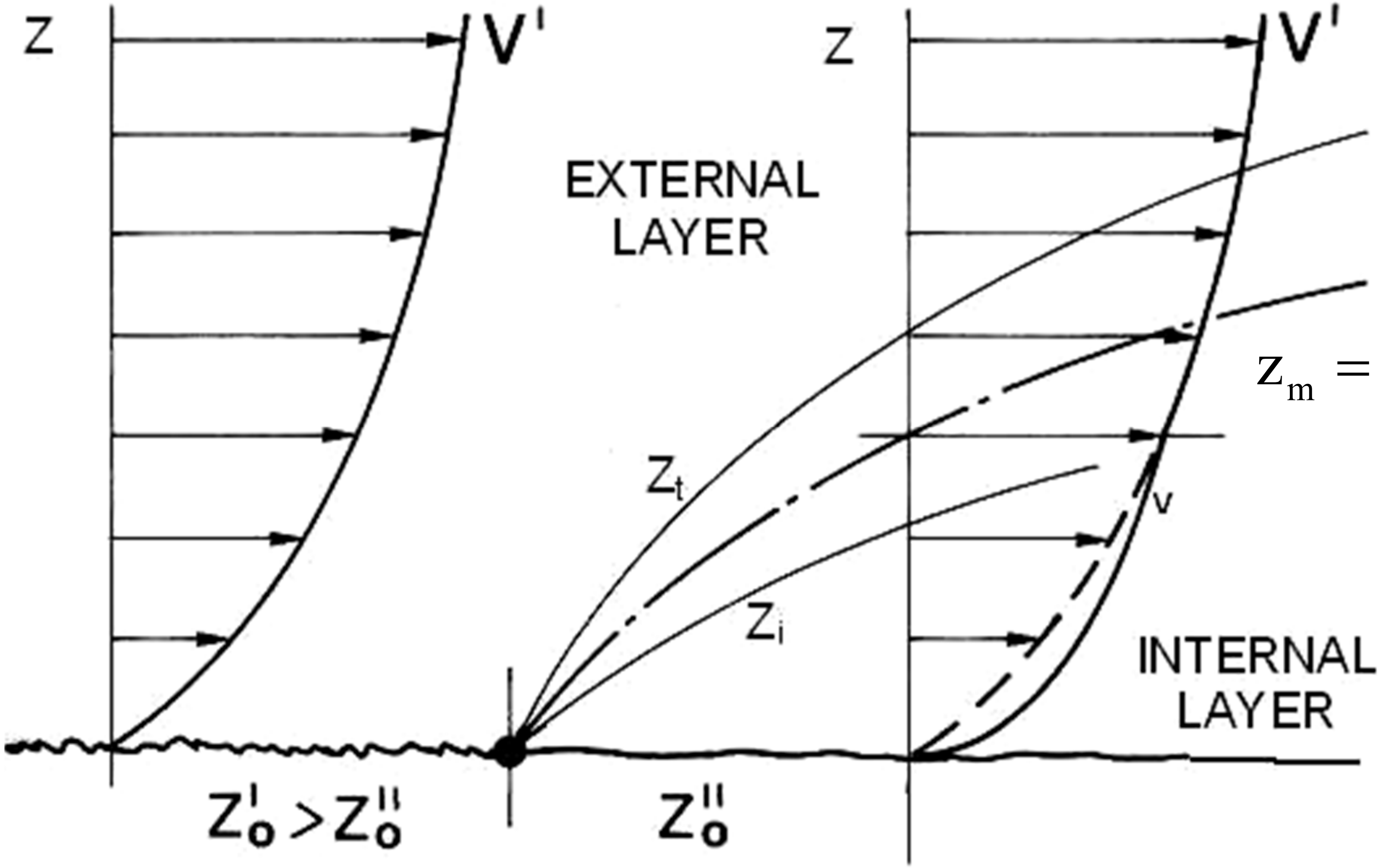
Transition profiles

Smooth to rough transition



Transition profiles

Rough to smooth transition



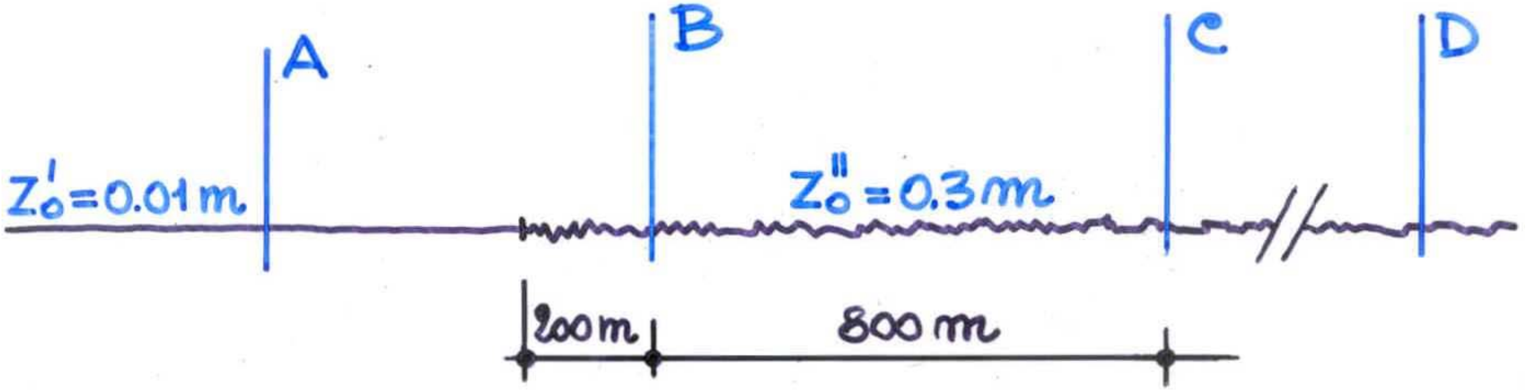
$$z_m = \frac{1}{2}\left(z_i + z_t\right)$$

Example

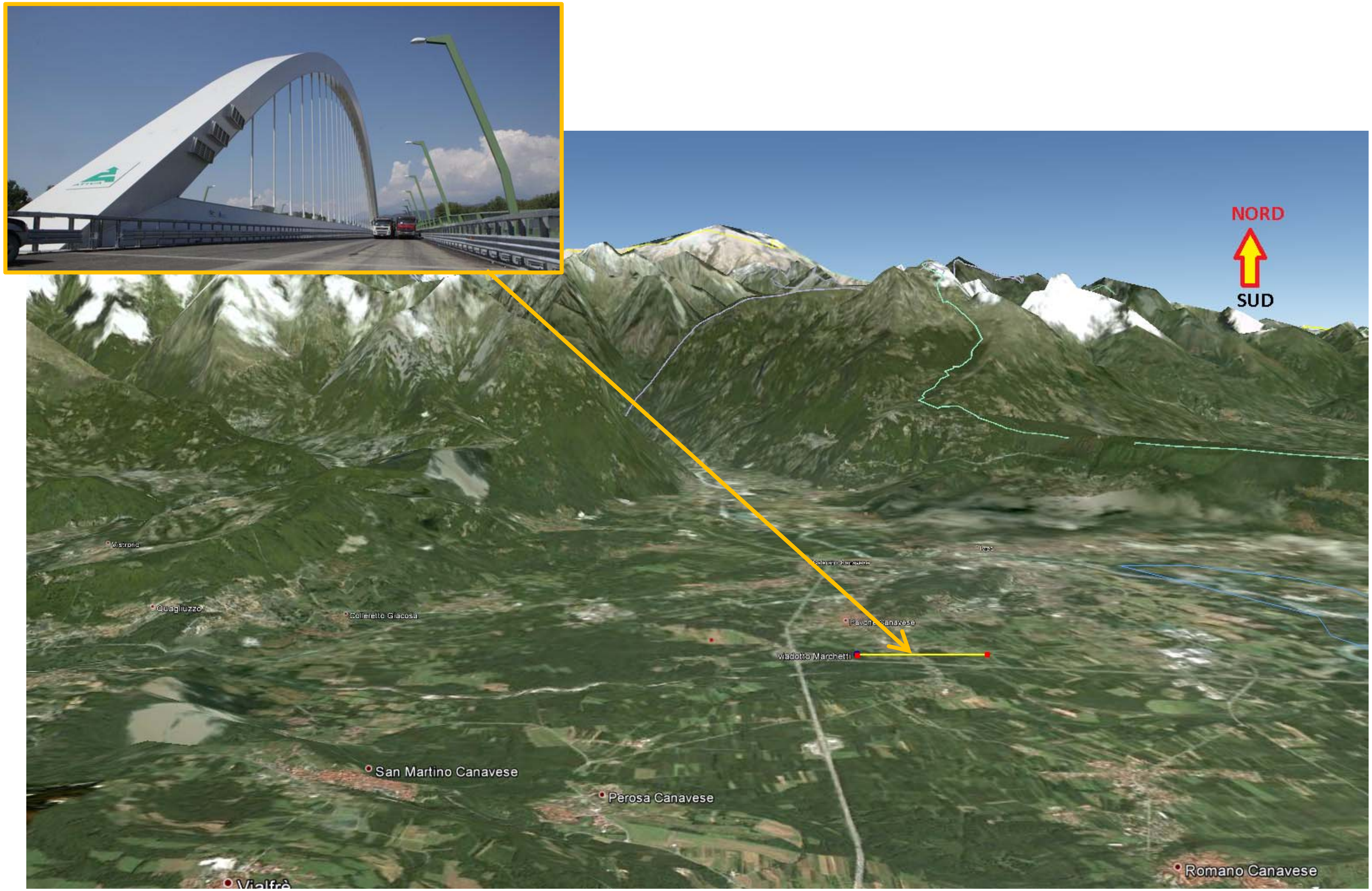
Reference site $z_{ref} = 10\text{ m};\ z'_0 = 0,05\text{ m},\ \bar{u}_{ref} = \bar{u}'(z_{ref}) = 25\text{ m/s}$

Gradient velocity $\varphi = 45°;\quad \bar{V}_G = 59,05\text{ m/s}$

Design site Engineering Science Data Unit (ESDU)

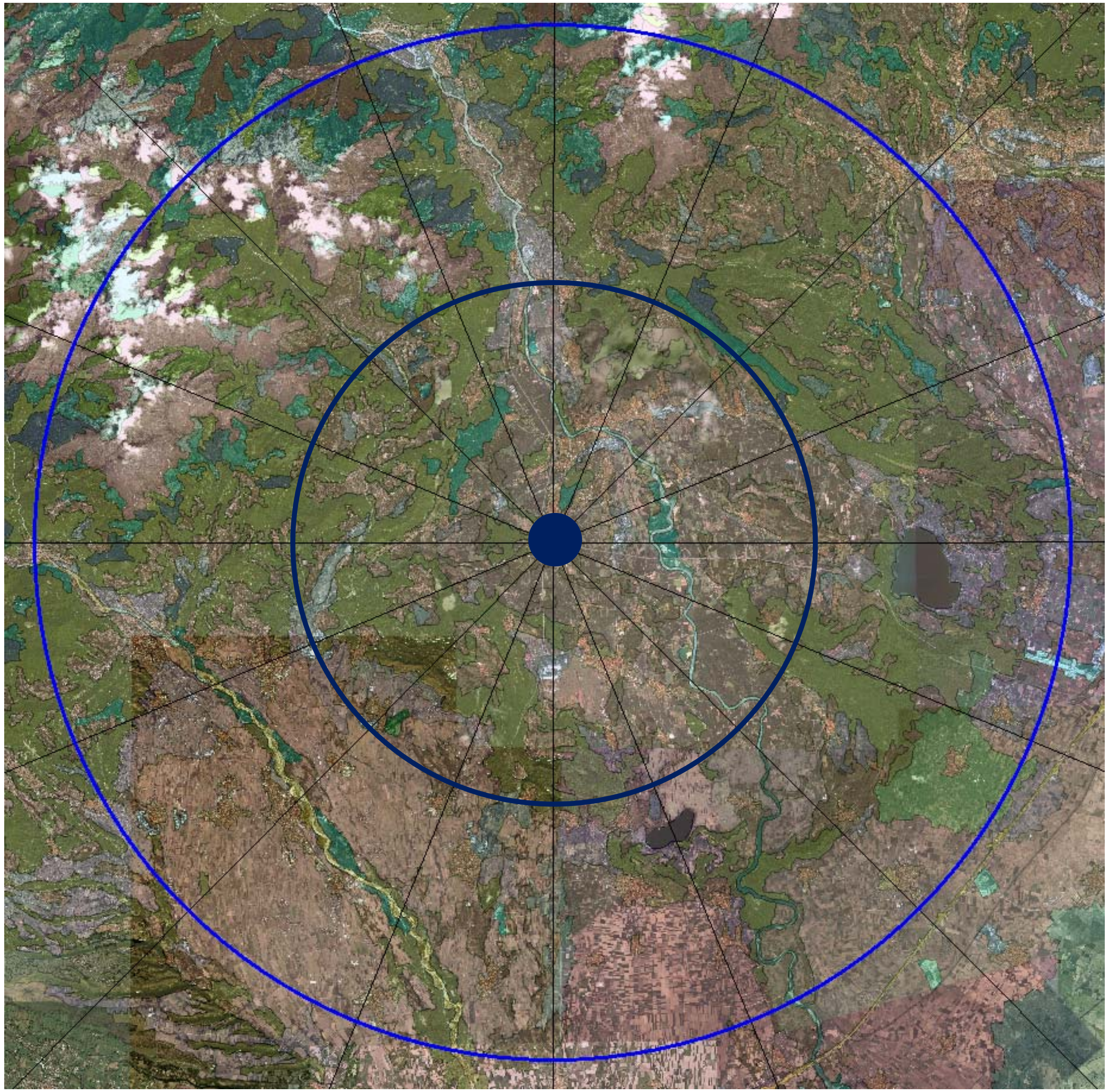


z (m)	$\bar{u}_A$ (m/s)	$\bar{u}_B$ (m/s)	$\bar{u}_C$ (m/s)	$\bar{u}_D$ (m/s)
10	29.12	24.90	23.47	18.76
20	32.04	30.10	27.55	22.47
50	35.91	35.42	33.78	27.37
100	38.83	38.06	37.78	31.08

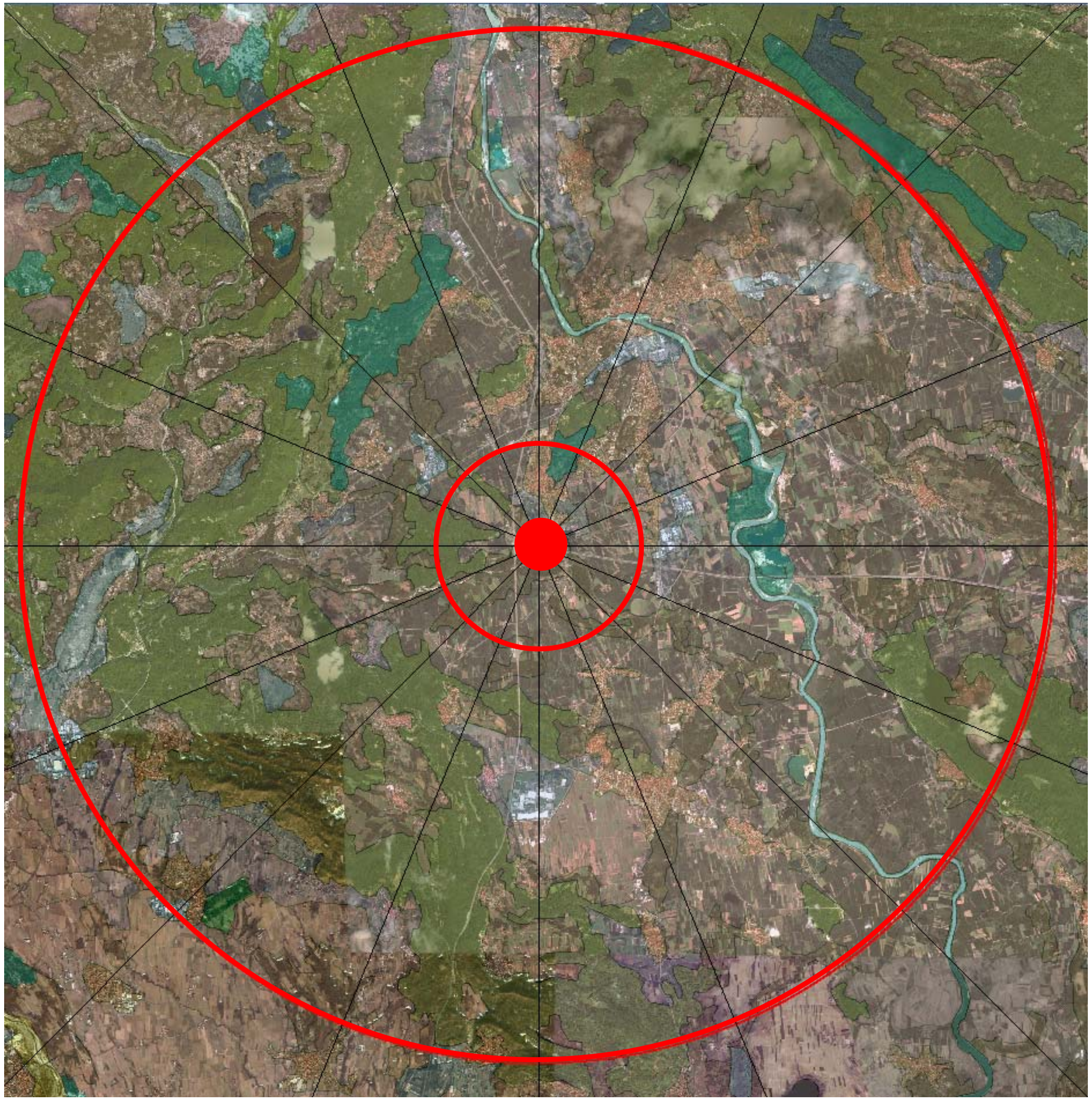


Marchetti Viaduct

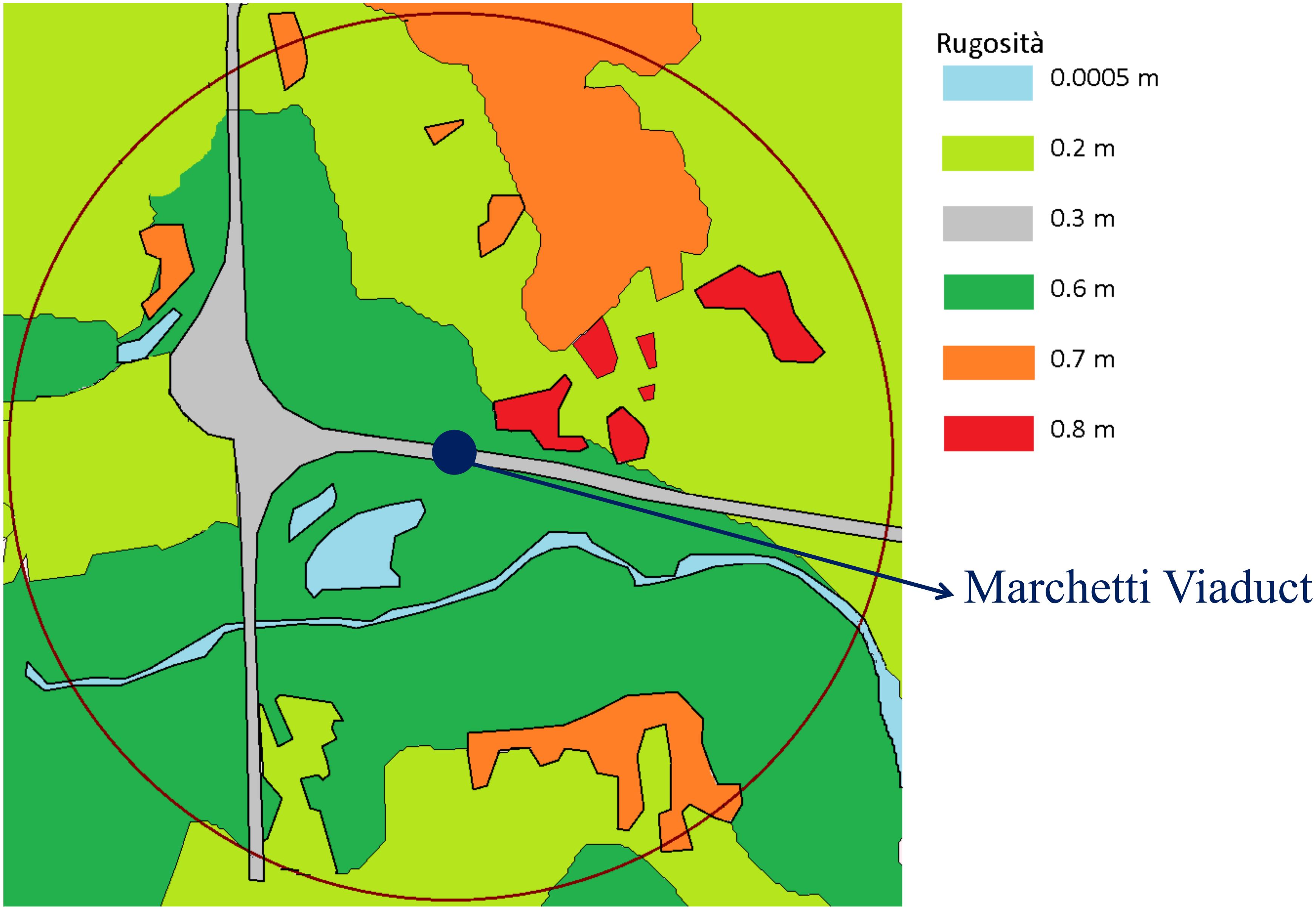
Marchetti Viaduct



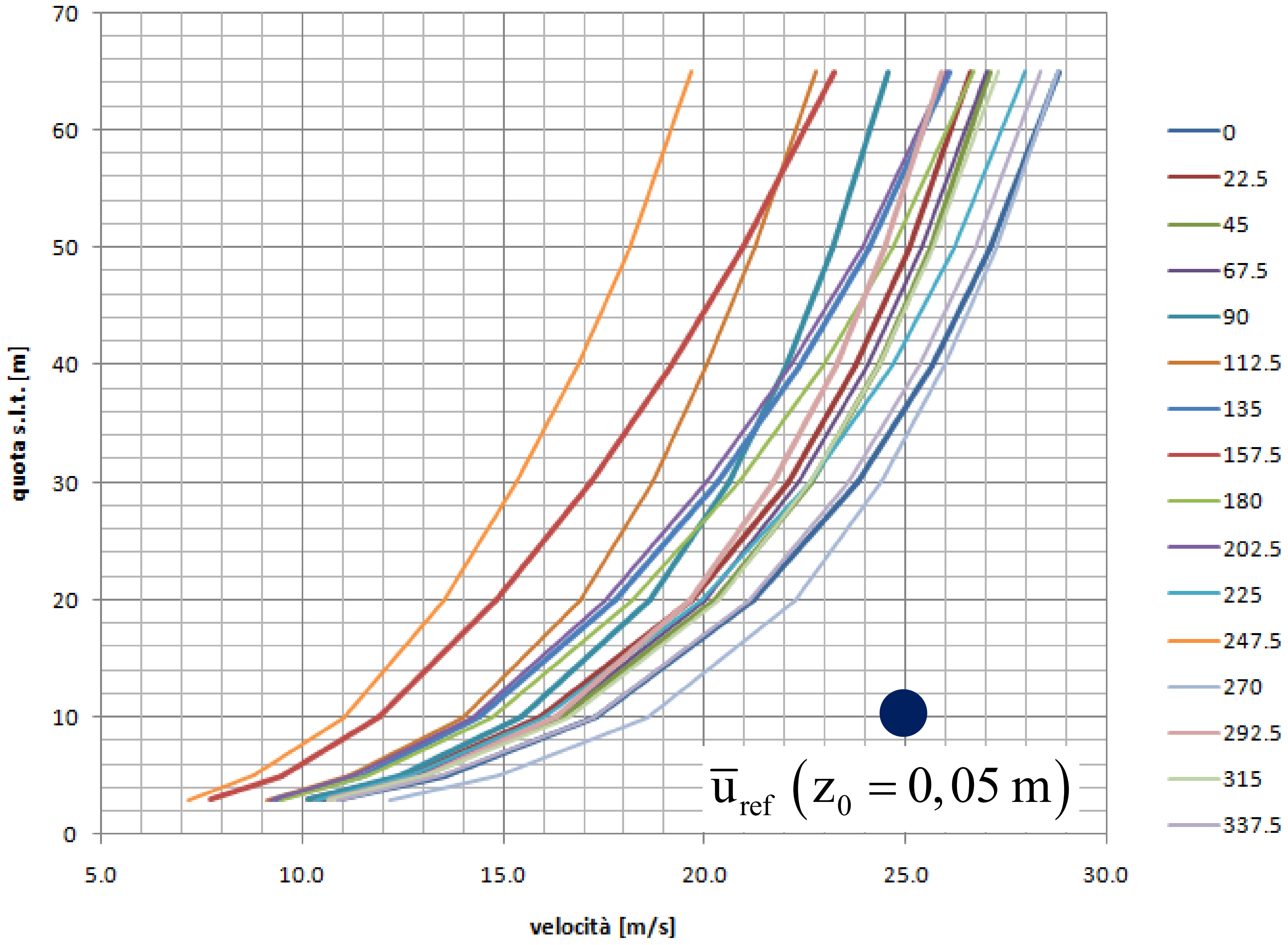
Marchetti Viaduct – 20 km radius



Marchetti Viaduct – 10 km radius

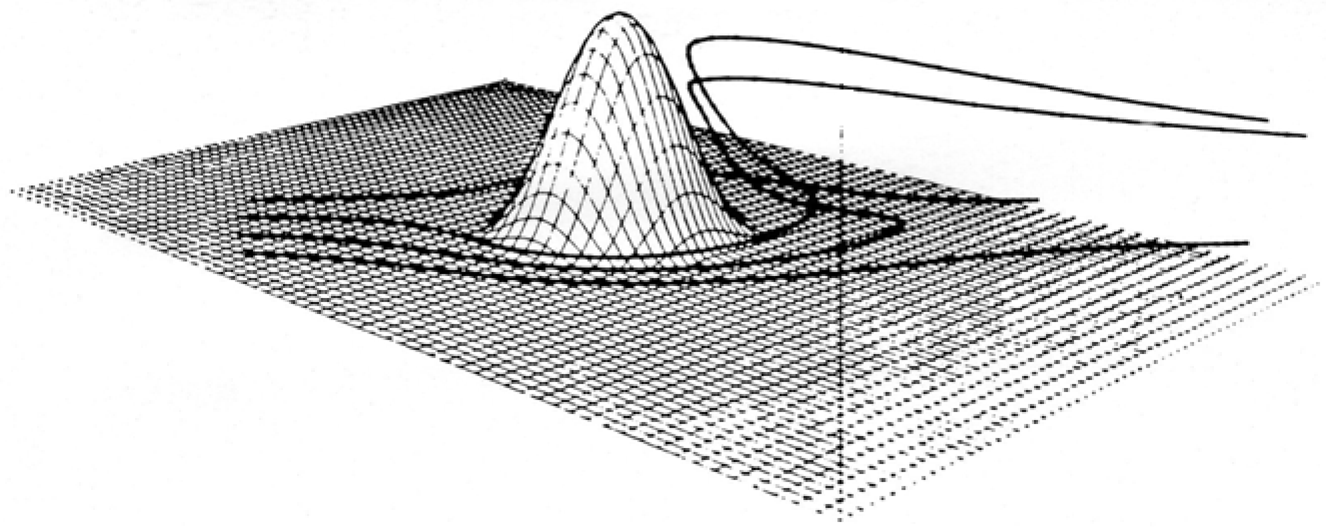


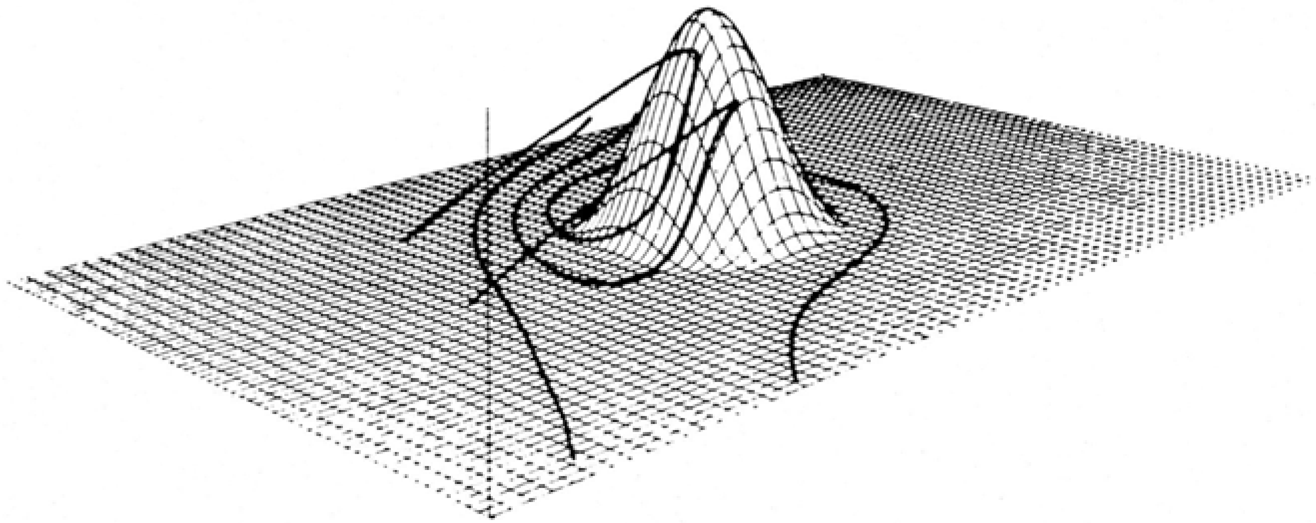
Marchetti Viaduct – Roughness map – 1 km radius



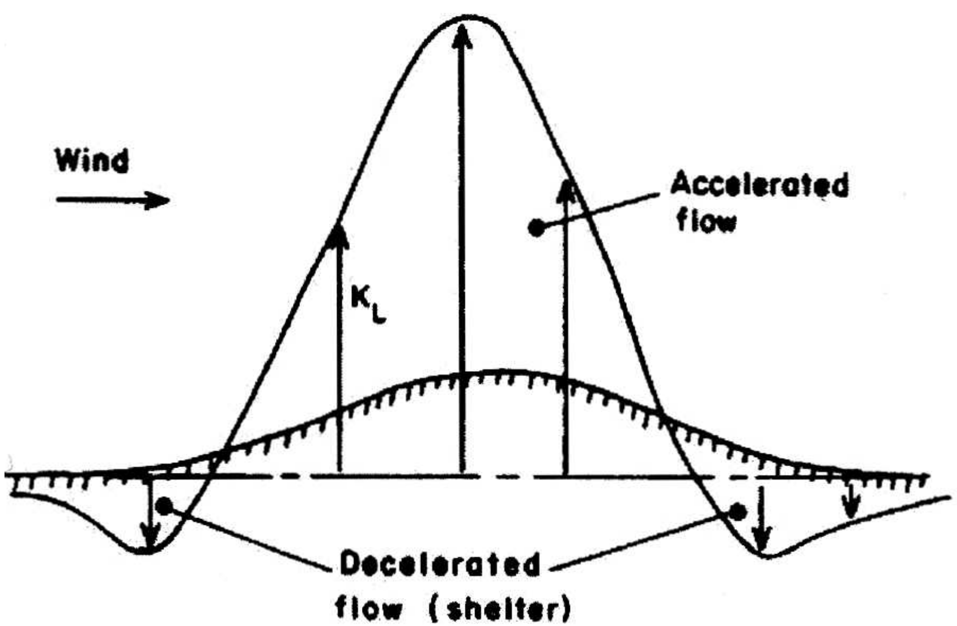
Marchetti Viaduct – Mean velocity profiles – ESDU Code

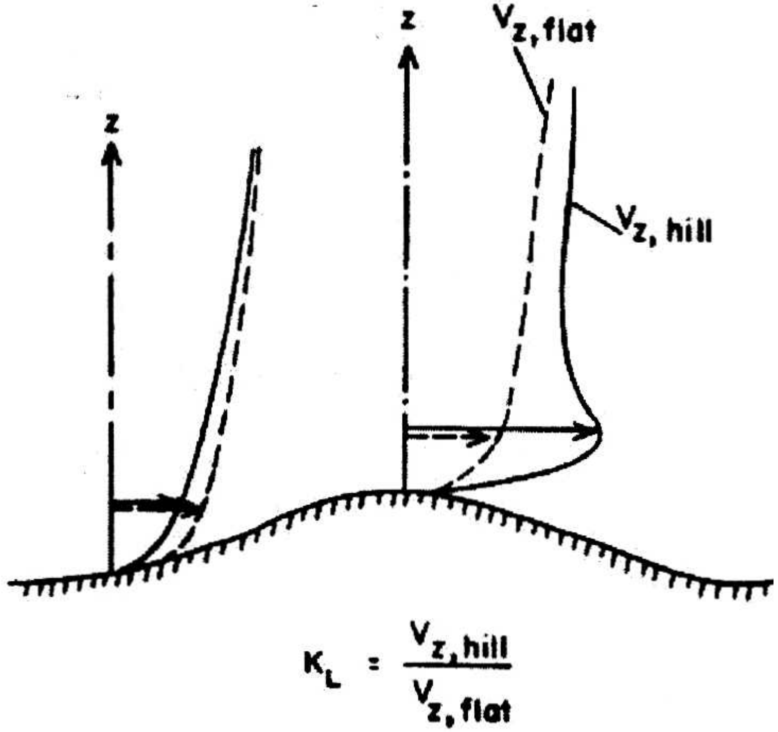
Topography effects

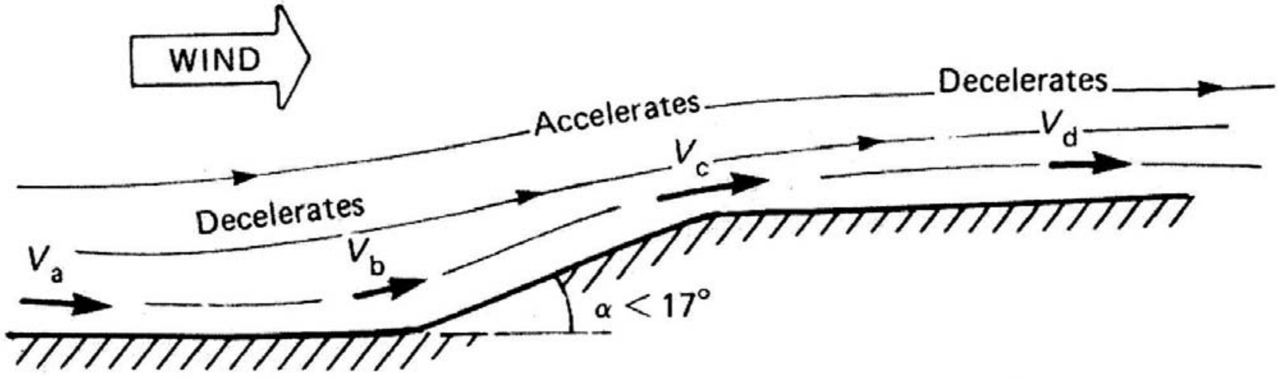




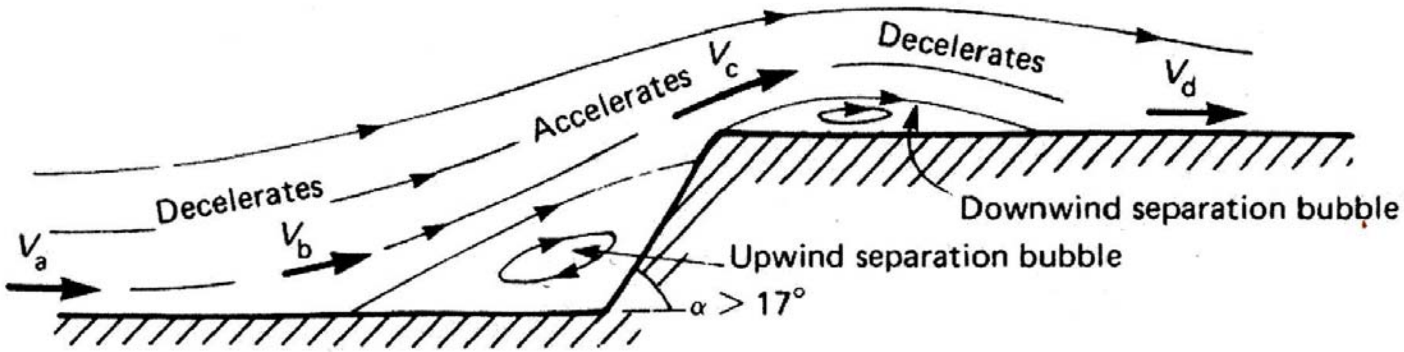
Topography effects



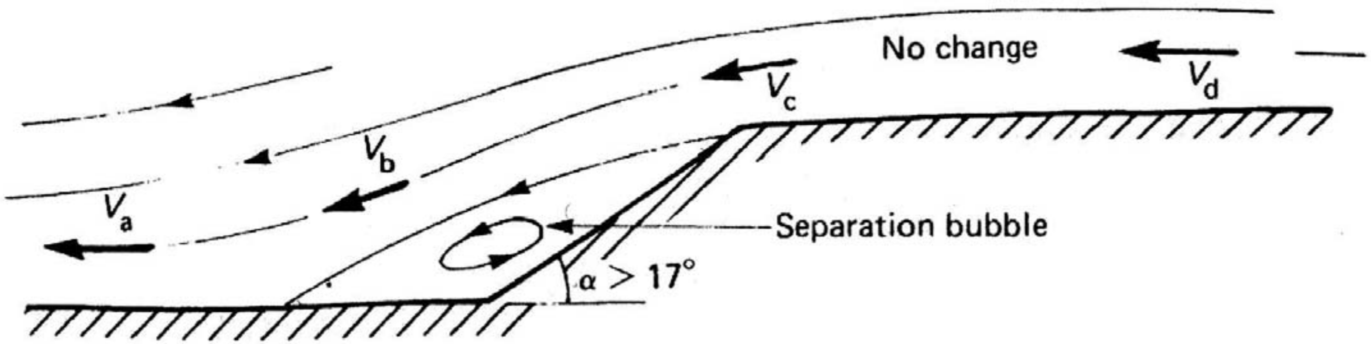




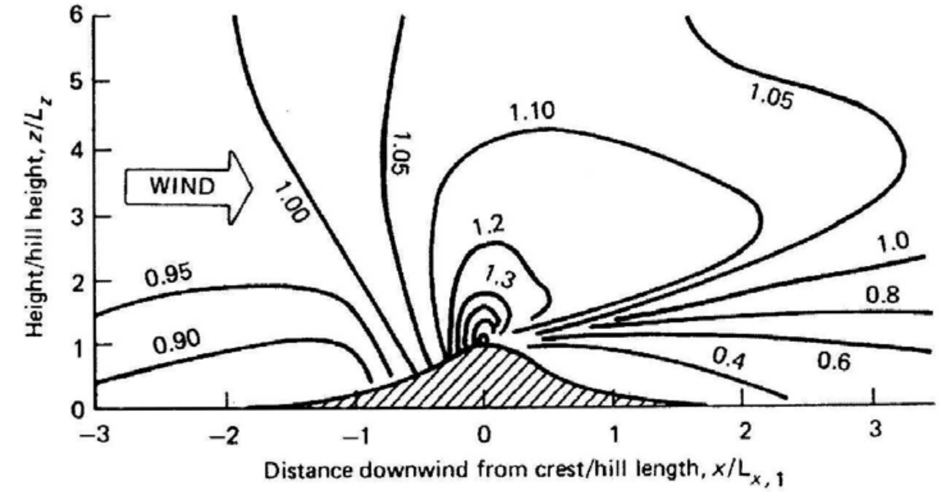
(a) Flow up shallow escarpment



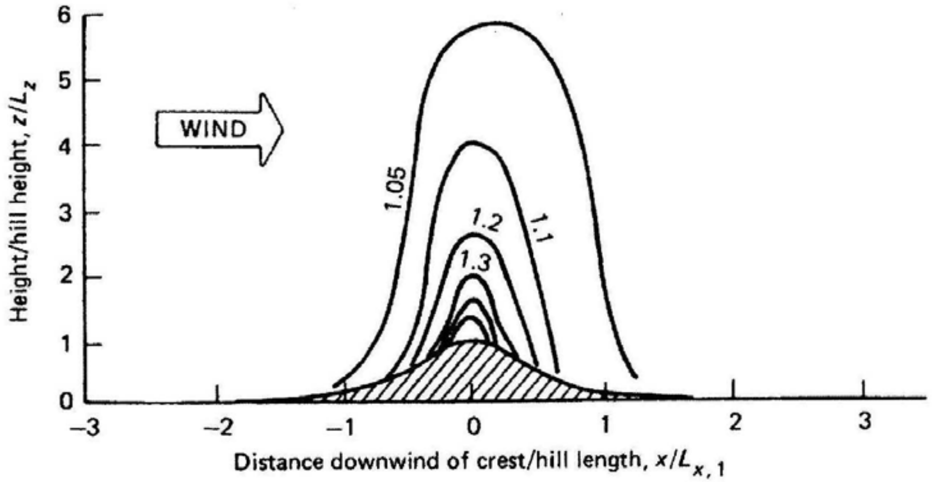
(b) Flow up steep escarpment



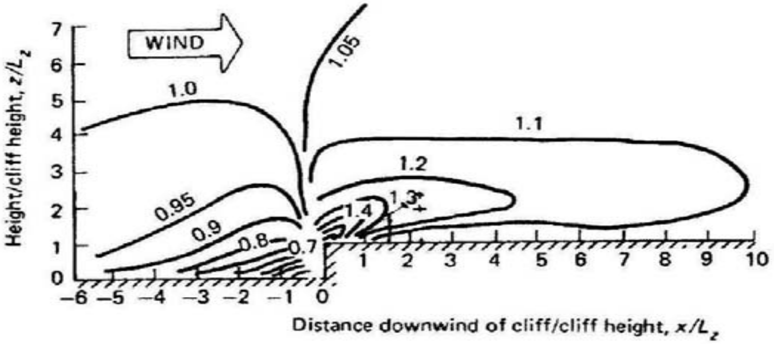
(c) Flow down steep escarpment

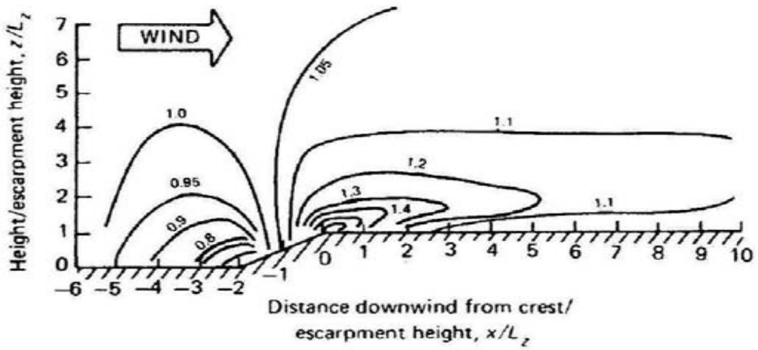


(a) Steep slope, $\psi = 0.50$

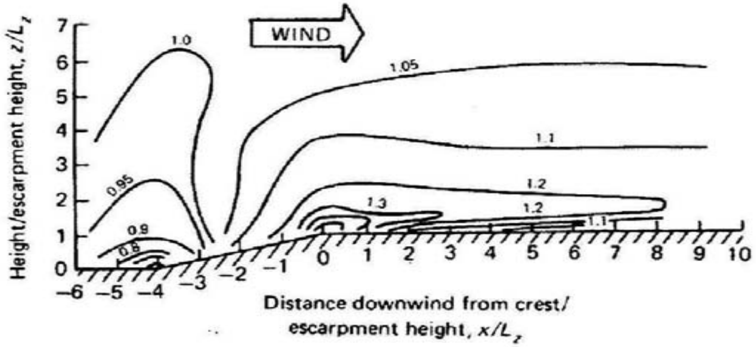


(b) Shallow slope, $\psi = 0.25$





(a) Steep slope, $\psi = 0.50$



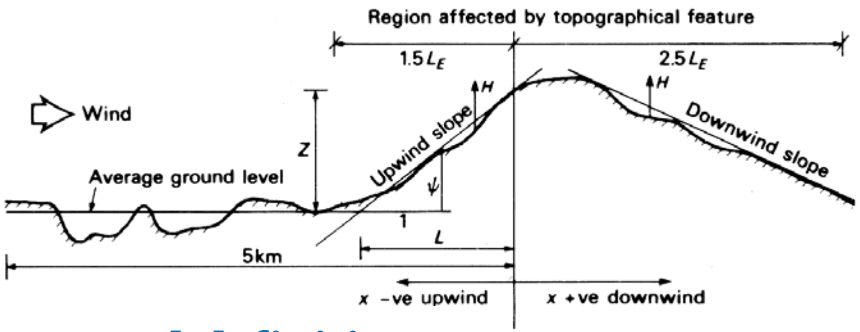
(b) Shallow slope, $\psi = 0.25$

$$\boxed{\bar{u}(z) = \bar{u}_o(z)\, c_t(x,z)}$$

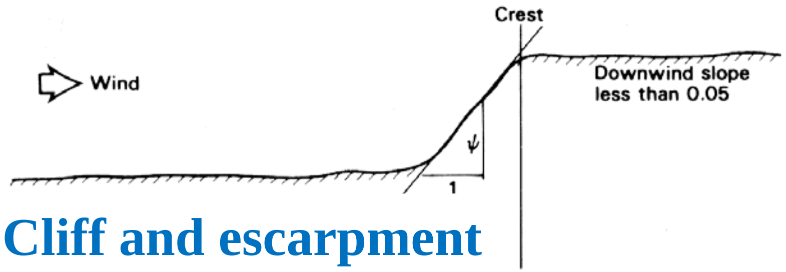
$\bar{u}_0$ mean wind velocity on flat terrain

c_t topography coefficient

$$\boxed{c_t(x,z) = 1 + 2s(x,z)\psi} \qquad \psi \le 0.3 \qquad \text{(BRE 1976; Eurocode 1)}$$

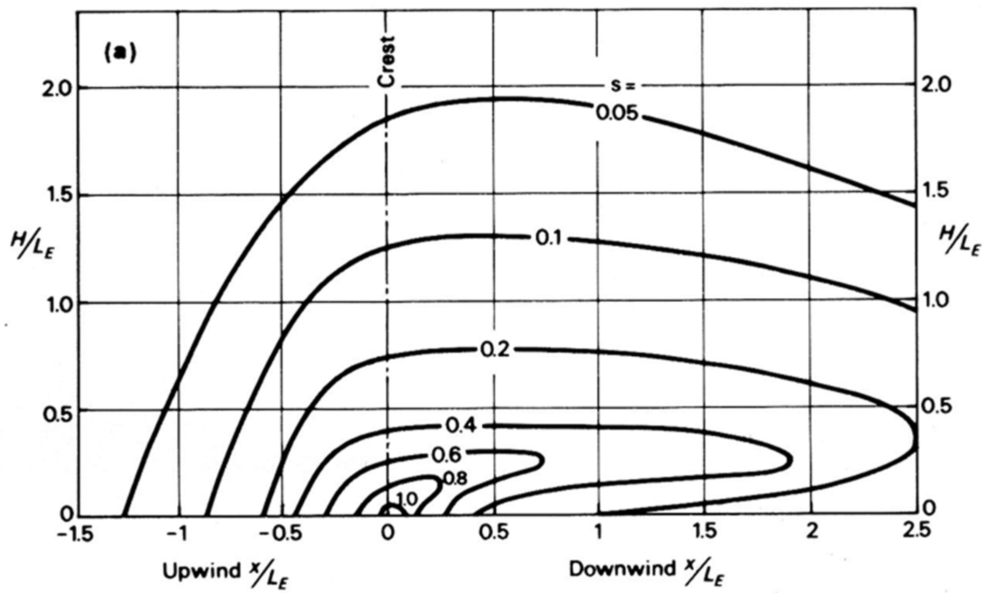


General definitions



Cliff and escarpment

Cliff and escarpment

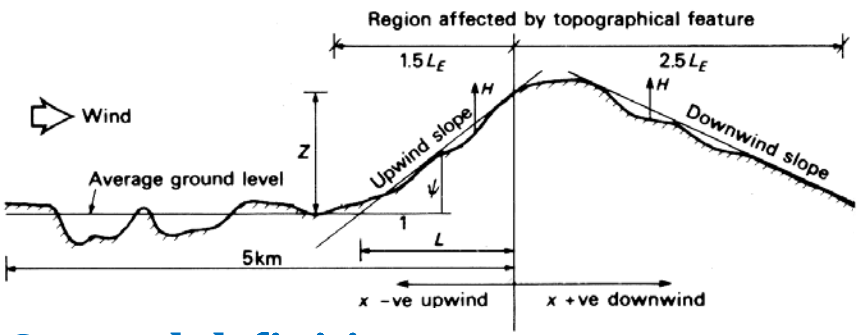


$$\boxed{\bar{u}(z) = \bar{u}_o(z)\,c_t(x,z)}$$

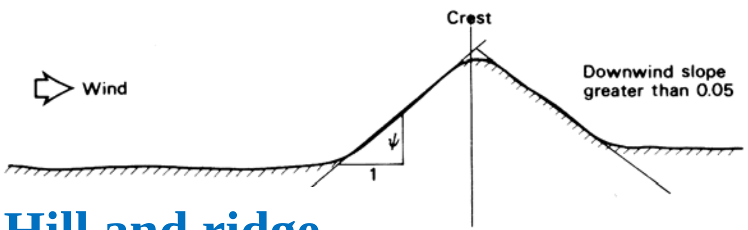
$\bar{u}_0$ mean wind velocity on flat terrain

c_t topography coefficient

$$\boxed{c_t(x,z) = 1 + 2s(x,z)\psi} \qquad \psi \leq 0.3 \qquad \text{(BRE 1976; Eurocode 1)}$$

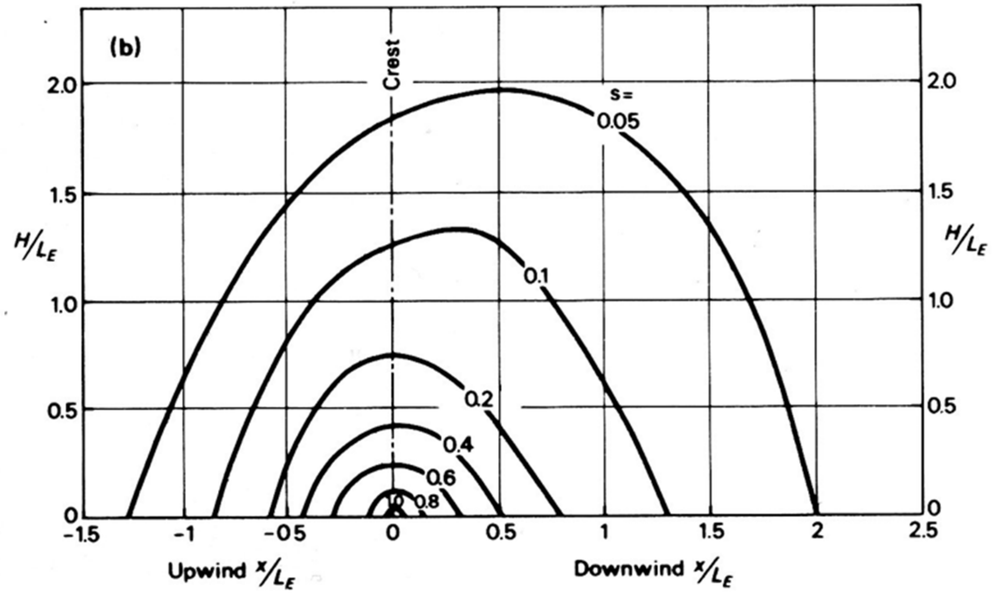


General definitions



Hill and ridge

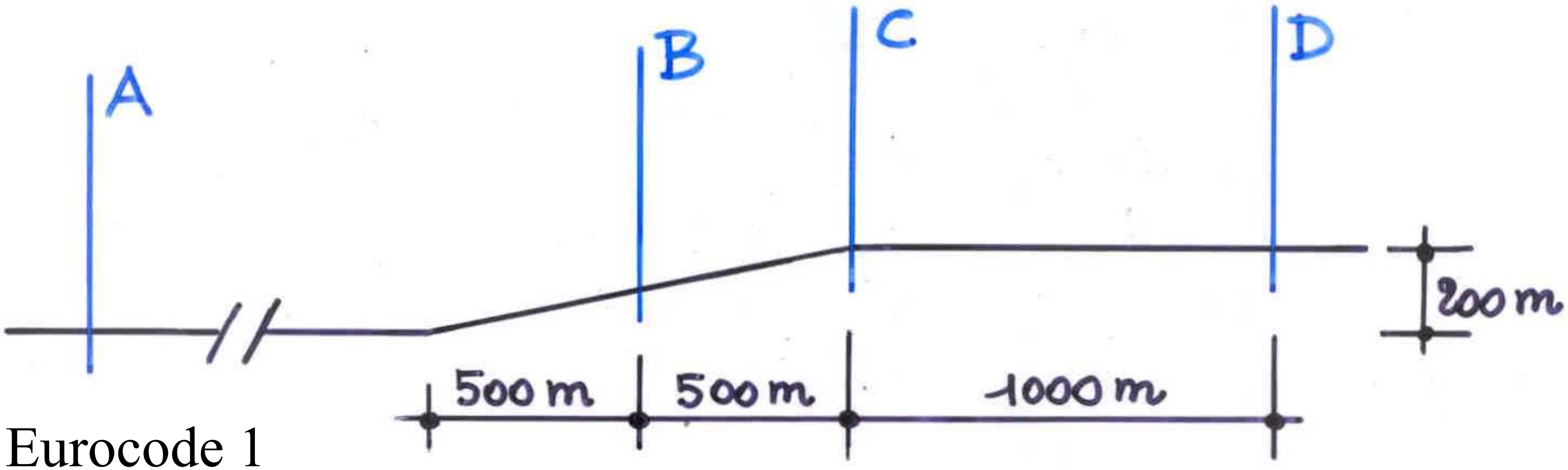
Hill and ridge



Example

Reference site $z_{ref} = 10\text{ m}; (z_0)_{ref} = 0,05\text{ m}, \bar{u}_{ref} = \bar{u}'(z_{ref}) = 25\text{ m/s}$

Design site $z_0 = 0.3\text{ m}, \ \psi = 0.2$



Eurocode 1

z (m)	$\bar{u}_A$ (m/s)	$\bar{u}_B$ (m/s)	$\bar{u}_C$ (m/s)	$\bar{u}_D$ (m/s)
10	18.76	20.62	25.77	20.25
20	22.47	24.70	30.87	24.25
50	27.37	30.08	37.60	29.55
100	31.08	34.16	42.70	33.55

Mean wind velocity in complex terrain

$$\boxed{\bar{u}(z) = \bar{u}_o(z) \cdot c_t(z)}$$

$\bar{u}_o$ mean wind velocity on flat terrain

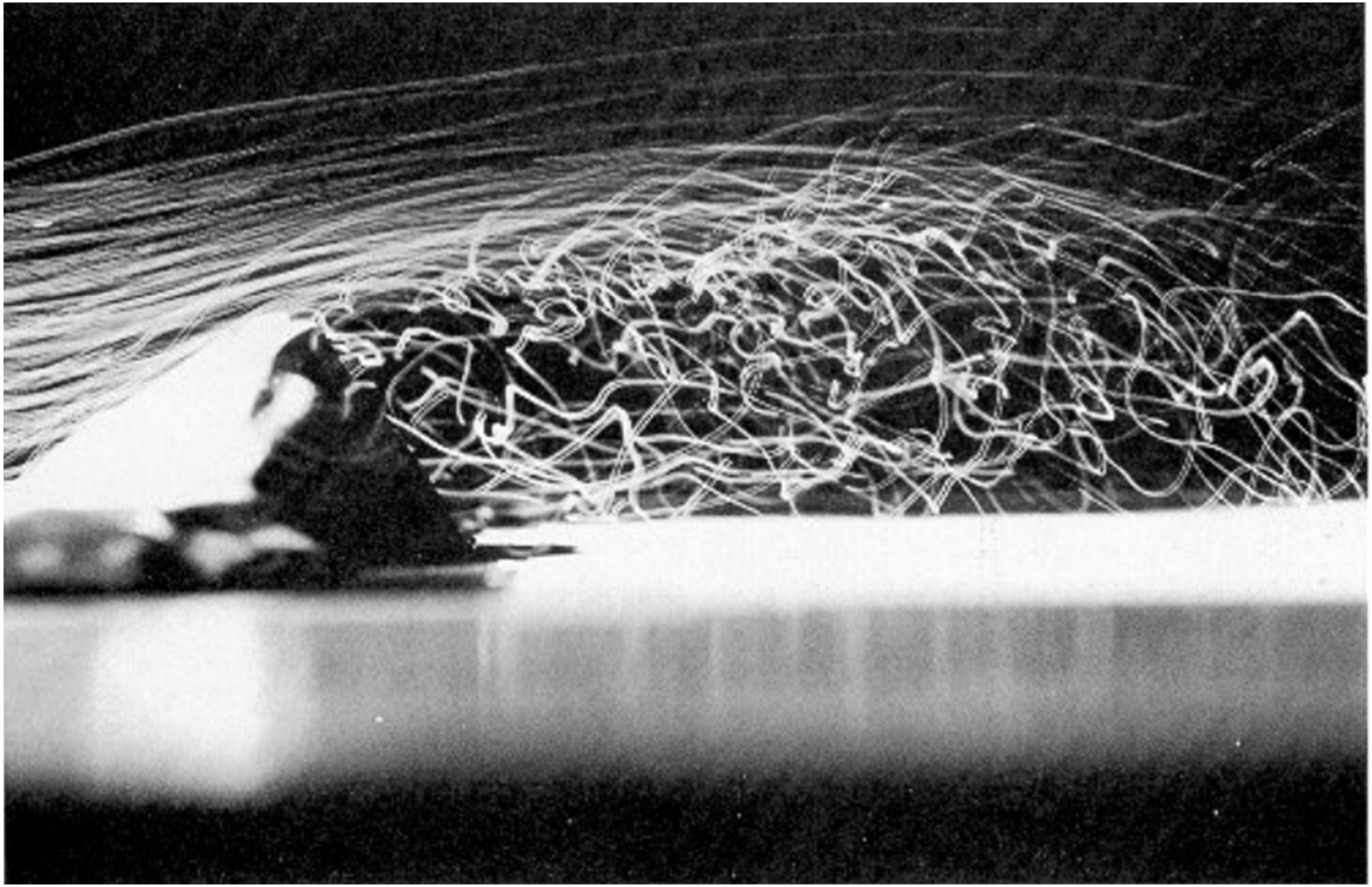
c_t topography coefficient

$$\boxed{\bar{u}_o(z) = \bar{u}_{ref} \cdot c_r(z)}$$

$\bar{u}_{ref}$ reference mean wind velocity

c_r roughness coefficient

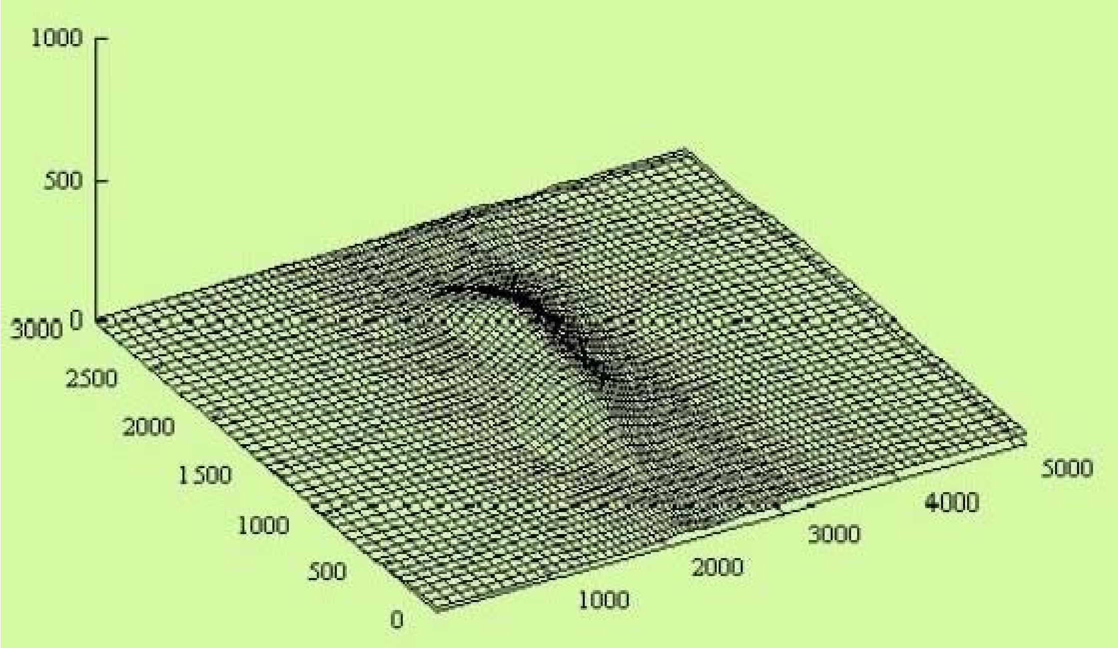
$$\boxed{\bar{u}(z) = \bar{u}_{ref} \cdot c_r(z) \cdot c_t(z)}$$



Rock of Gibraltar – Wind tunnel topographic model

Hong Kong Bay – Wind tunnel topographic model

Messina Strait – Wind tunnel topographic model



Askervein Hill (Scotland) Project

Diagnostic wind field models

Determine a time-invariant wind field by means of:

CFD models

Linearized dynamical models

Mass consistent models

Parametrized models

CFD Models

Computational Fluid Dynamics models solve the non-linear equations of conservation of mass and of motion (Navier-Sokes equations):

FLUENT

CFX

STAR-CCM+

PHOENICS

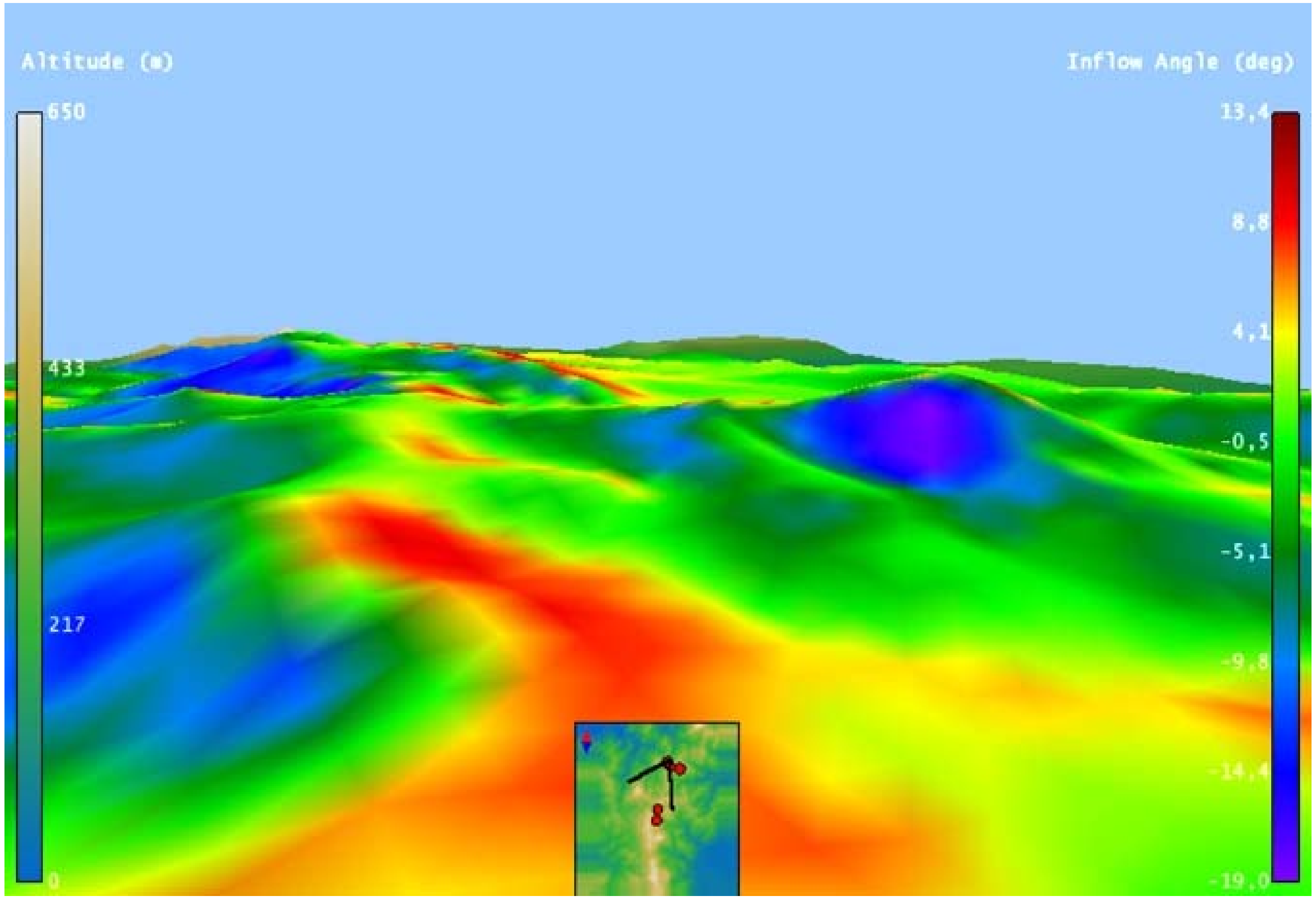
MeteodynWT

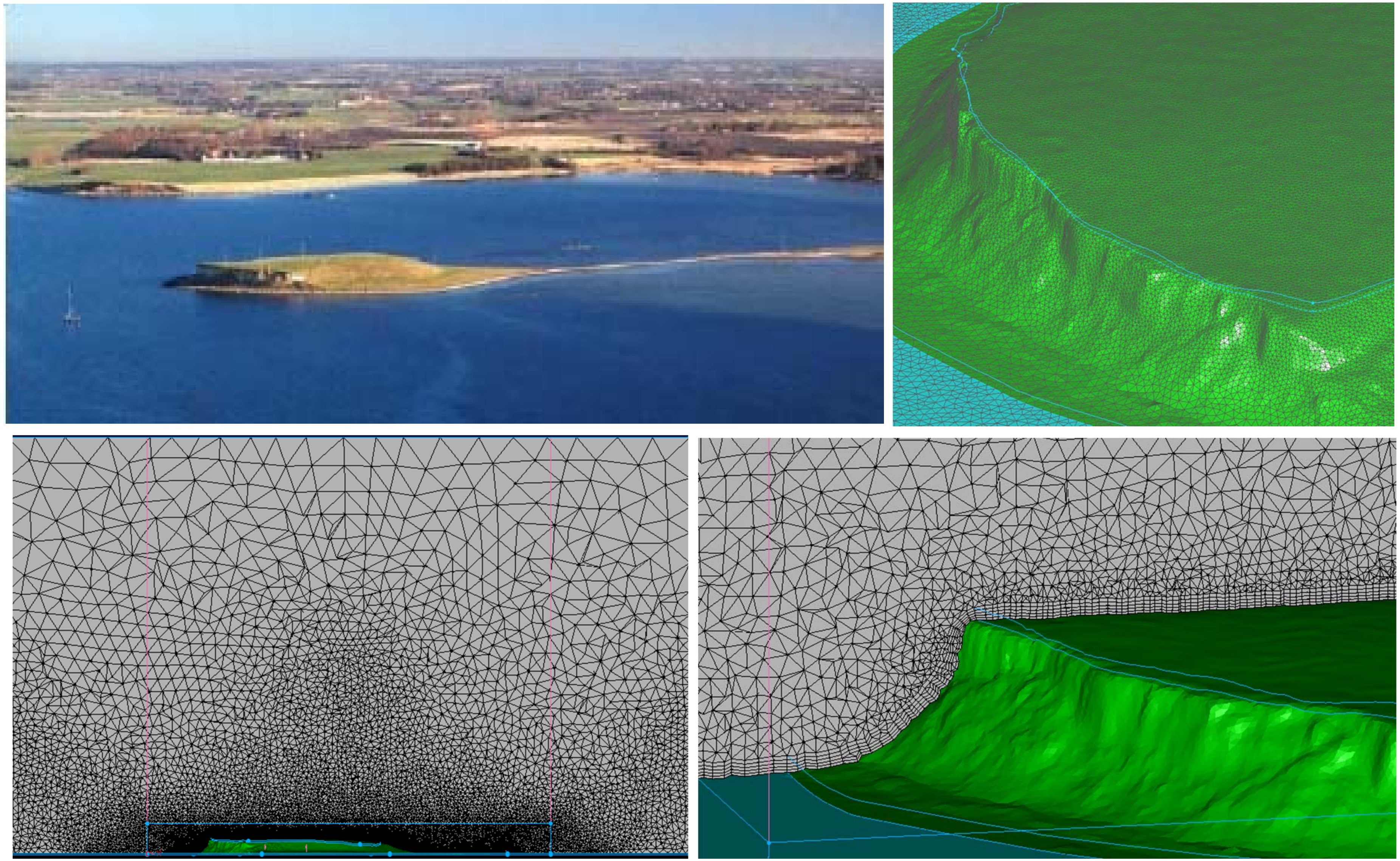
Windsym

OpenFOAM

NaSt3DGP

………



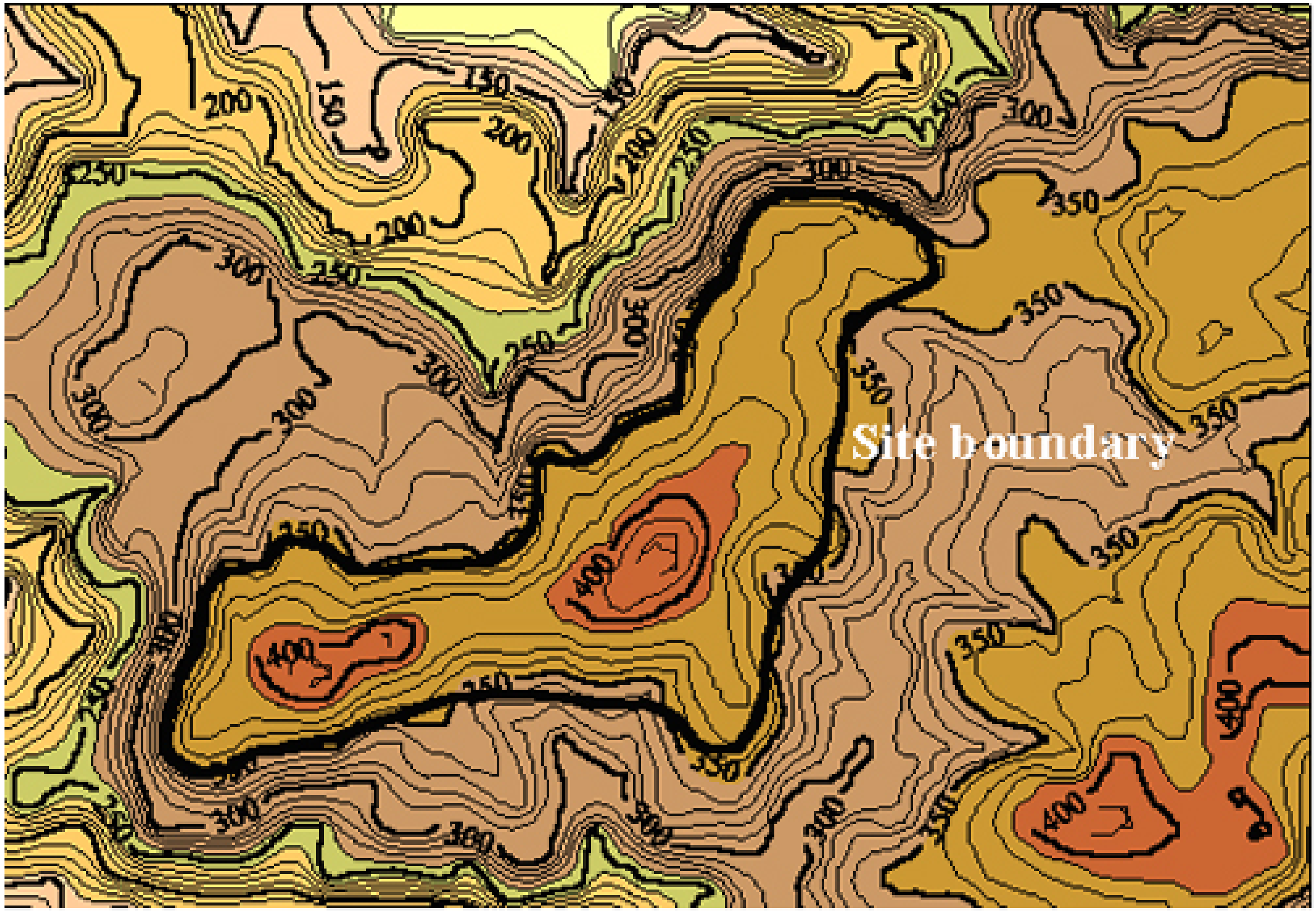


Bolund Island, Denmark

Linearized Dynamical Models

Solve the equations of conservation of mass and of motion (Navier-Sokes equations) in linearized form:

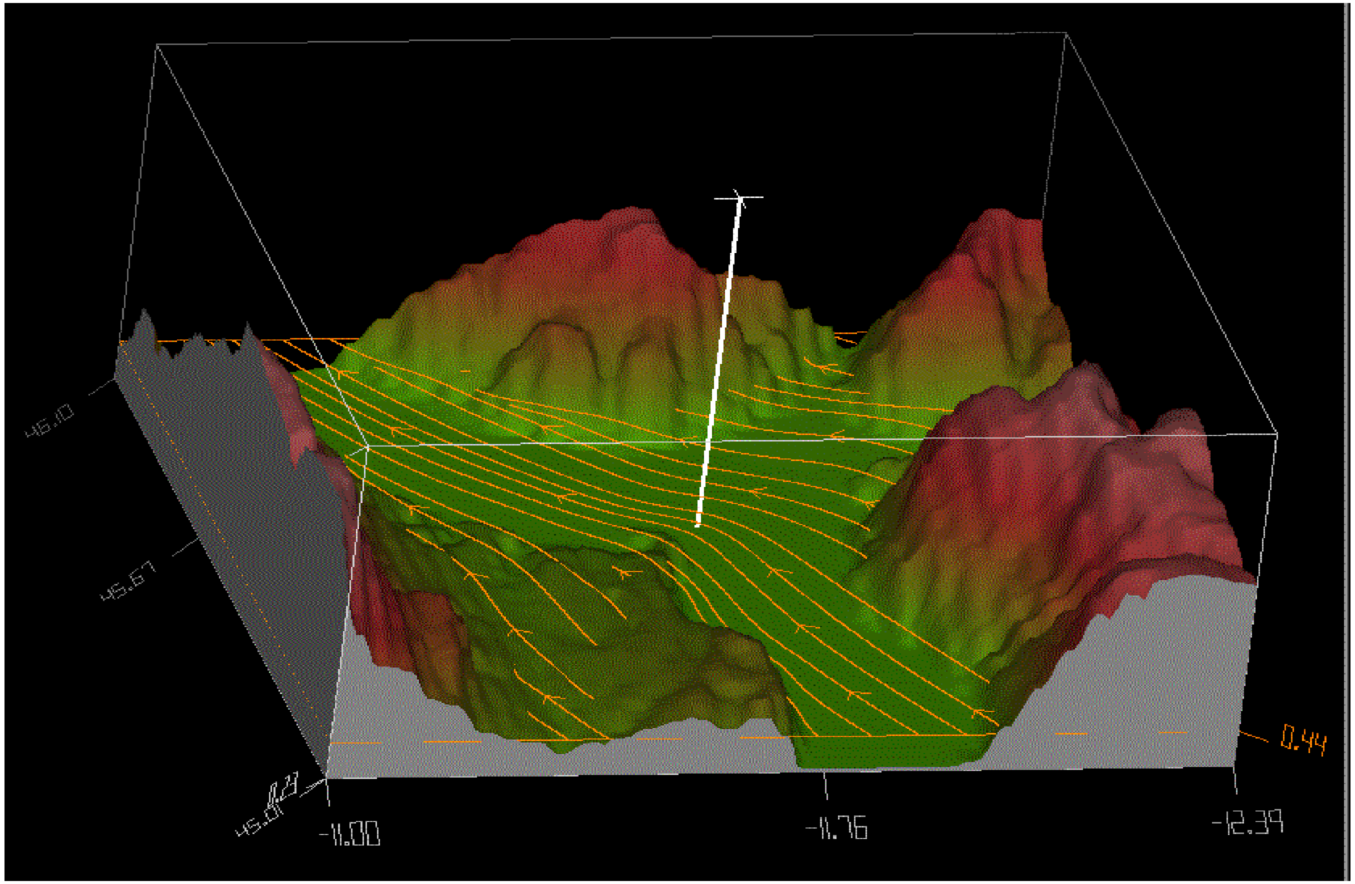
FLOWSTAR
LINCOLMN
MS3DJH/3R
MS-MICRO
MSFD
NLMSFD
WASP
……



Mass Consistent Models

Interpolate a parametrised first guess wind field by satisfying the equation of conservation of mass:

WINDS
CONDOR
MASCON
MATHEW
SHIFT
NOABL
NUATMOS
WOCSS
CALMET
....

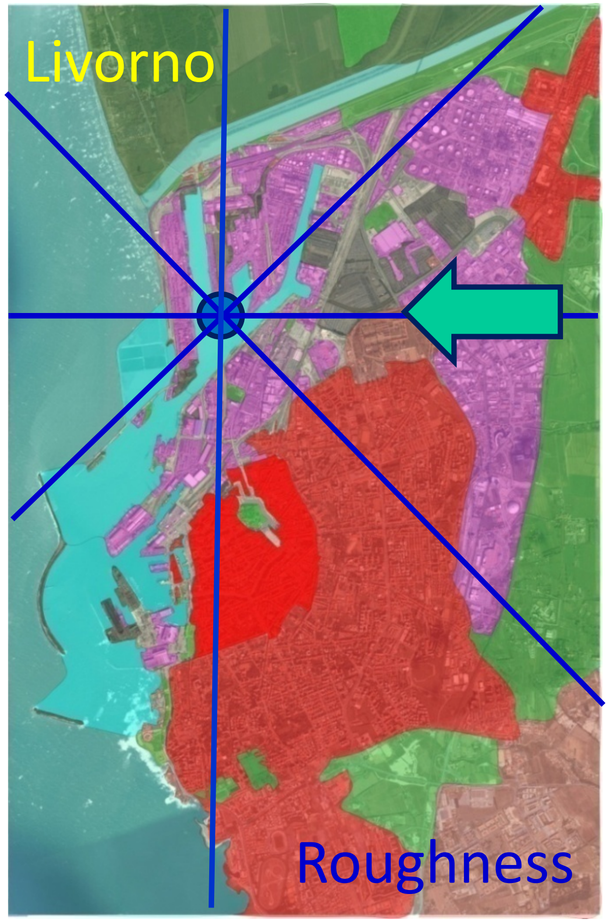


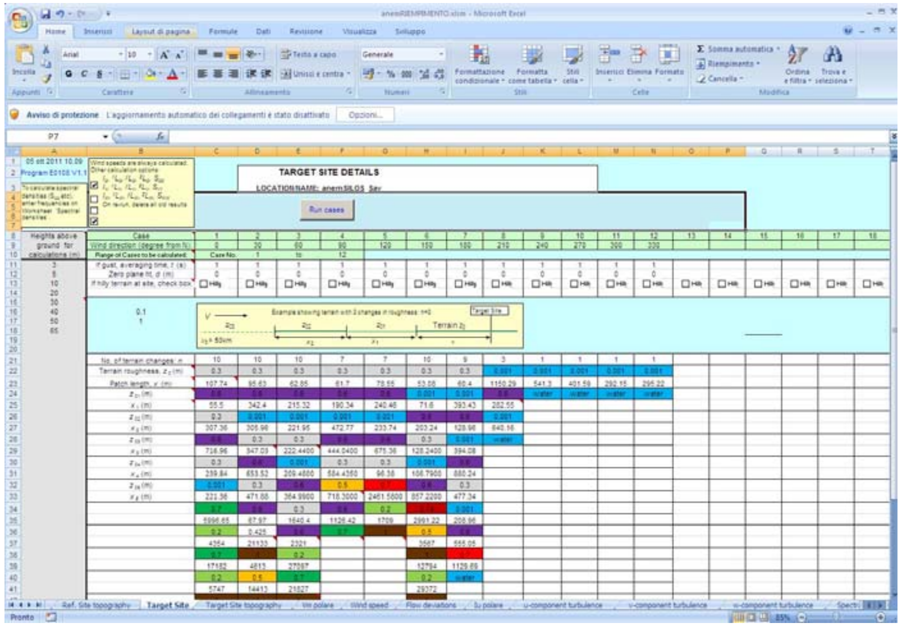
Parametrised Models

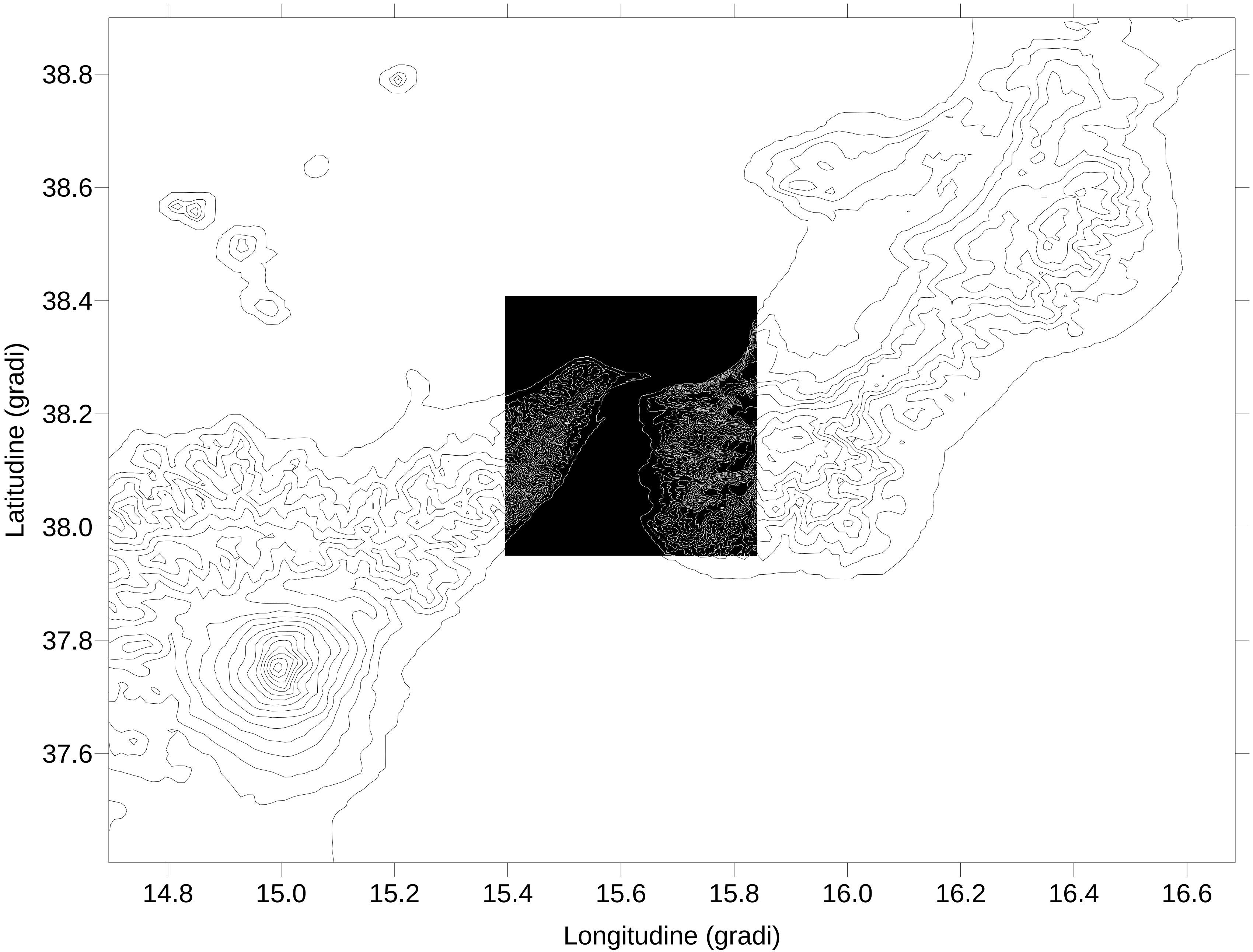
Reconstruct the wind field through parametrised equations:

ESDU

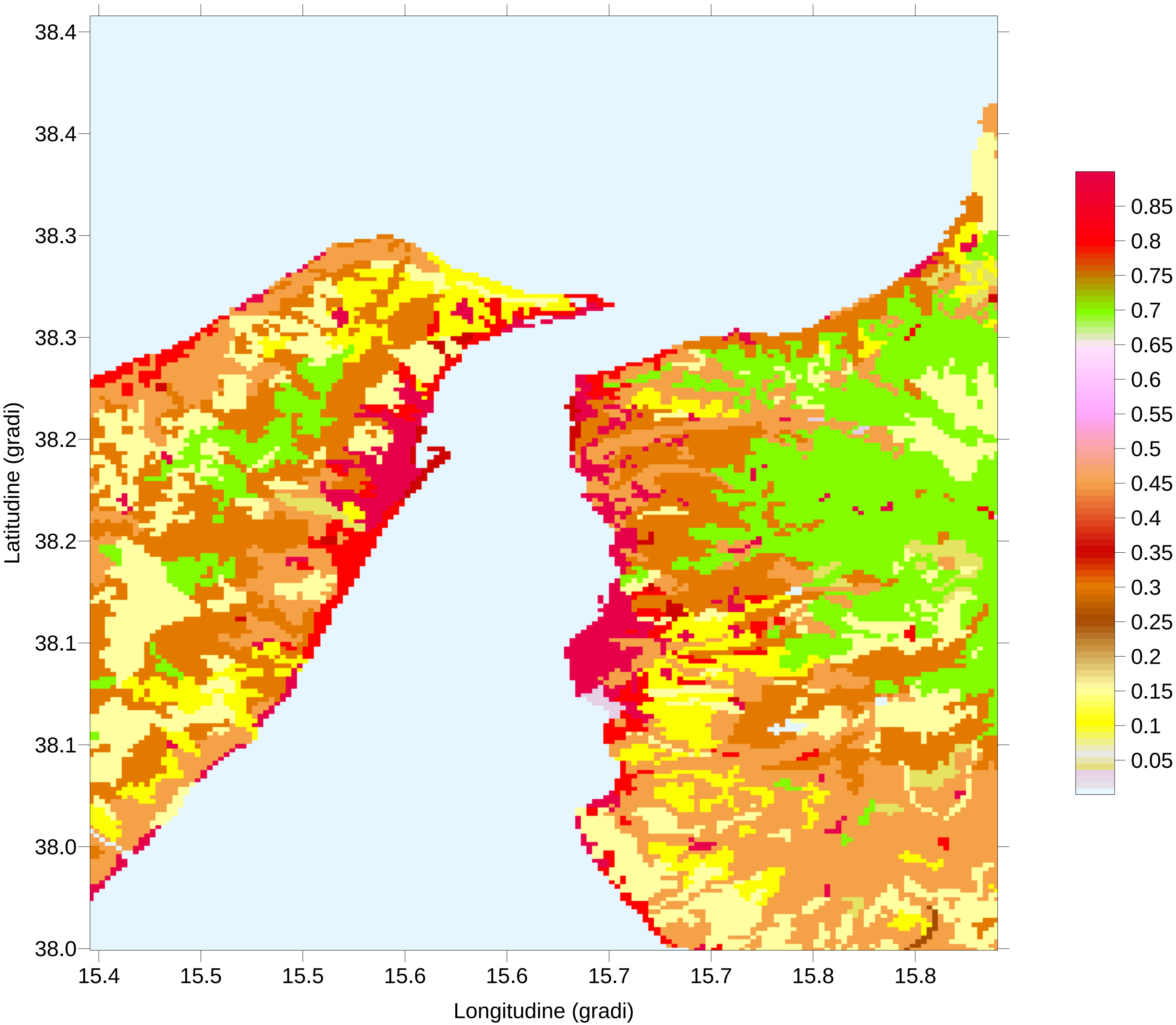
……….



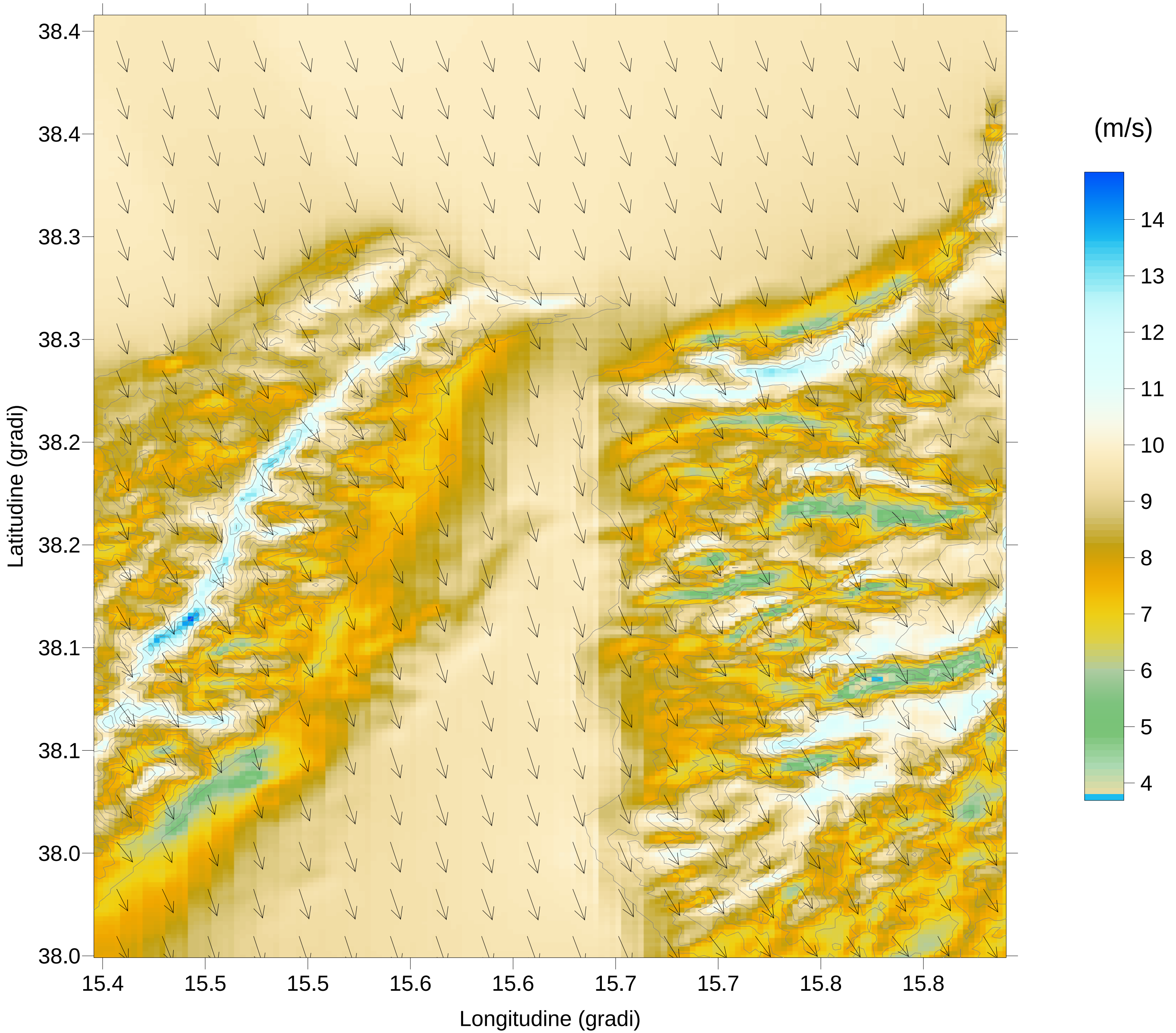




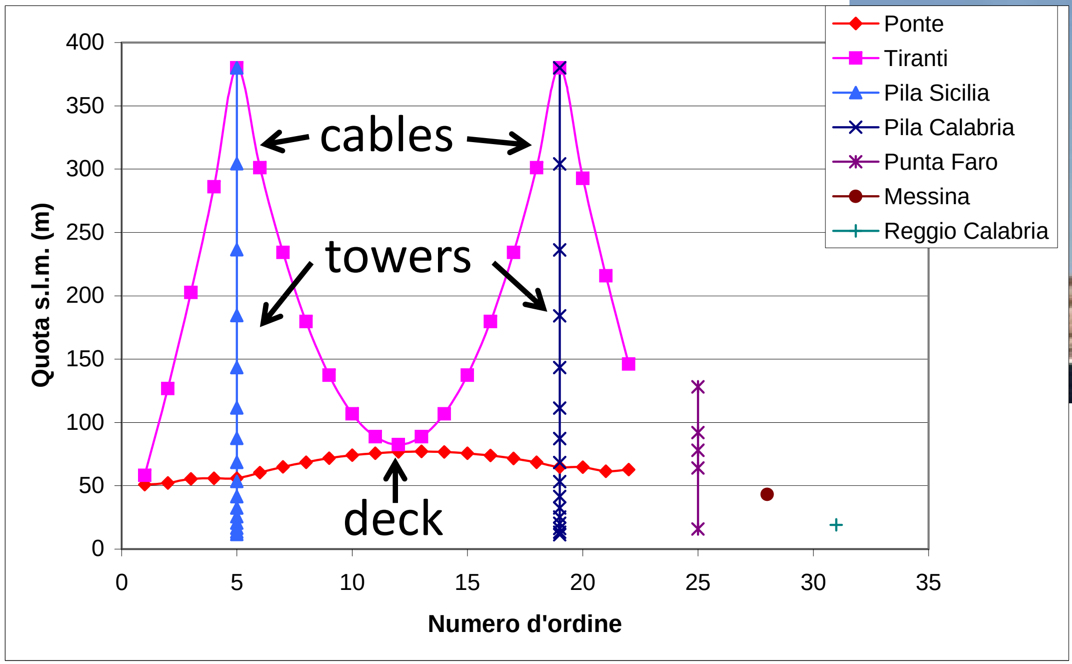
Messina Strait – Macro and micro-areas



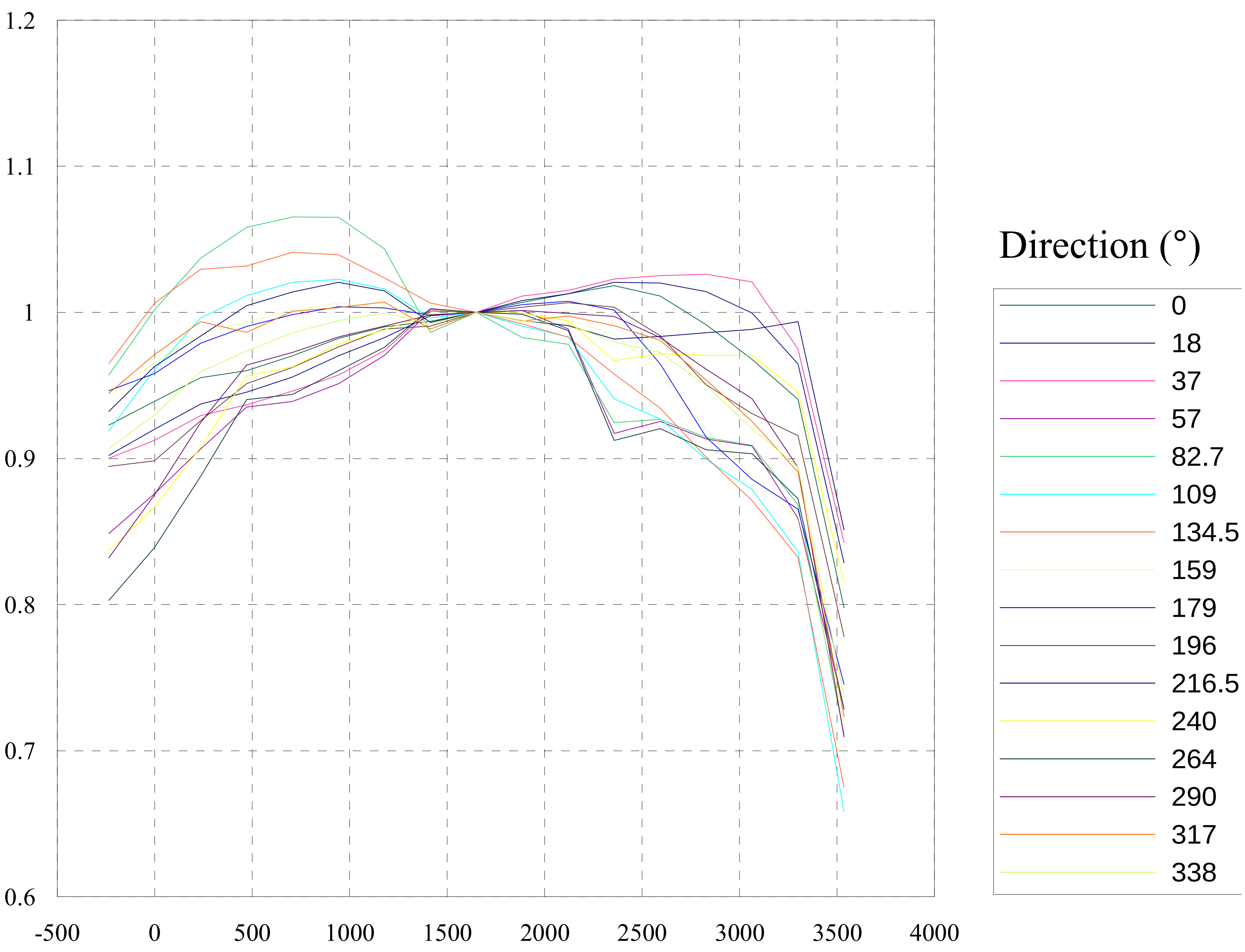
Messina Strait – Micro-area - Roughness length



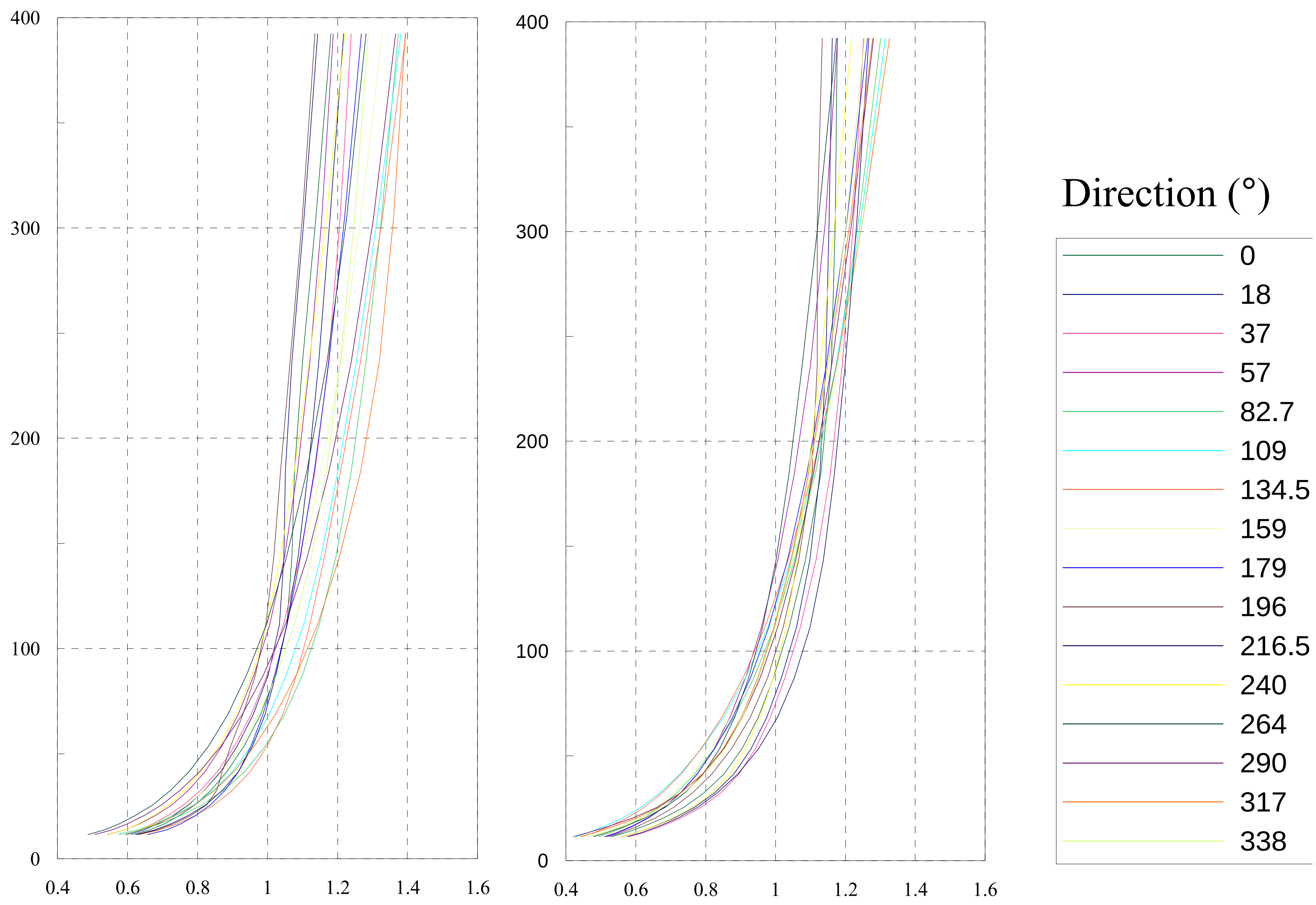
Messina Strait – Micro-area – Mean wind field



Messina Strait – Mean wind field

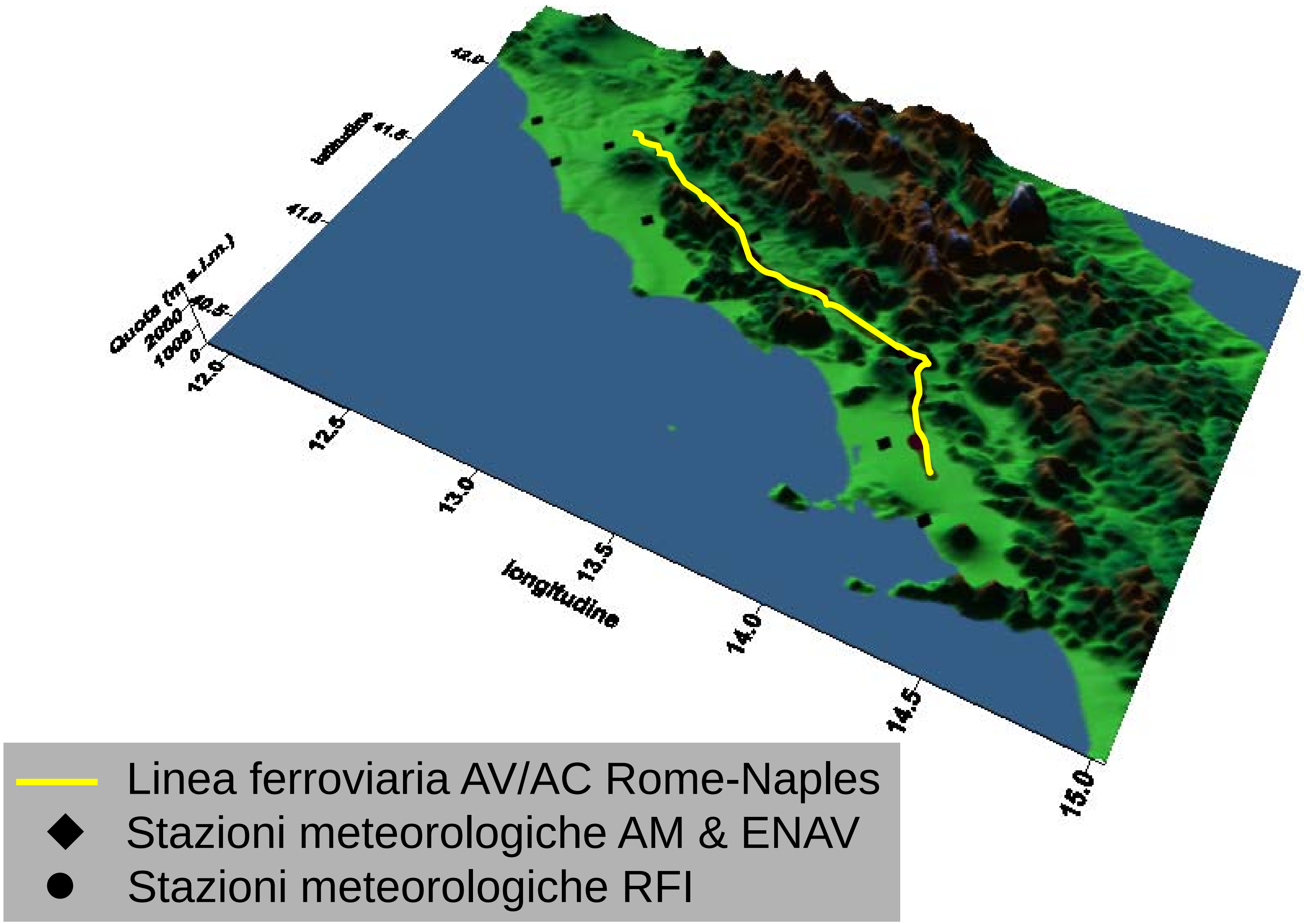


Messina Strait – Mean velocity along the bridge axis

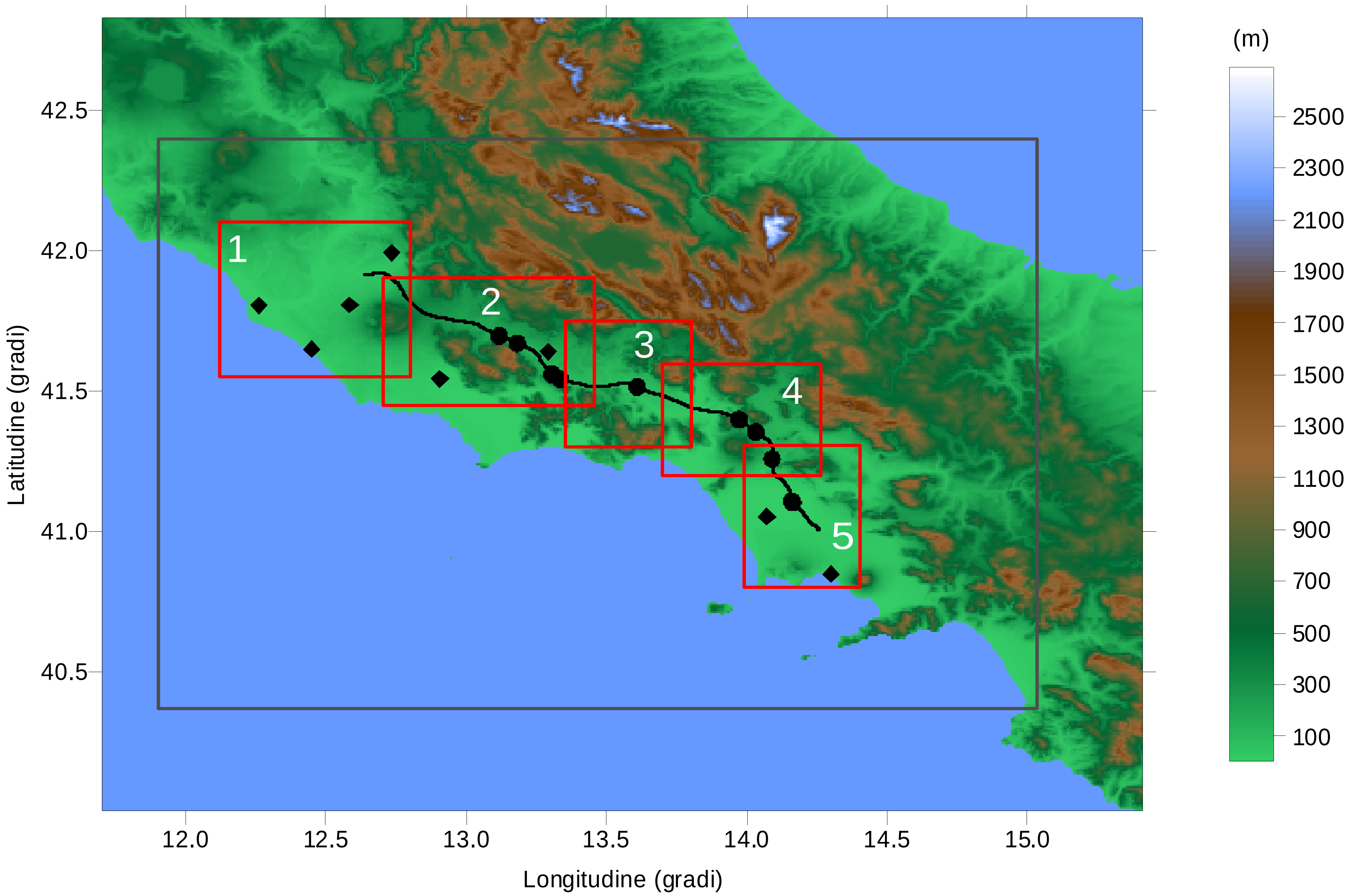


Messina Strait – Mean velocity along the Sicily and Calabria towers

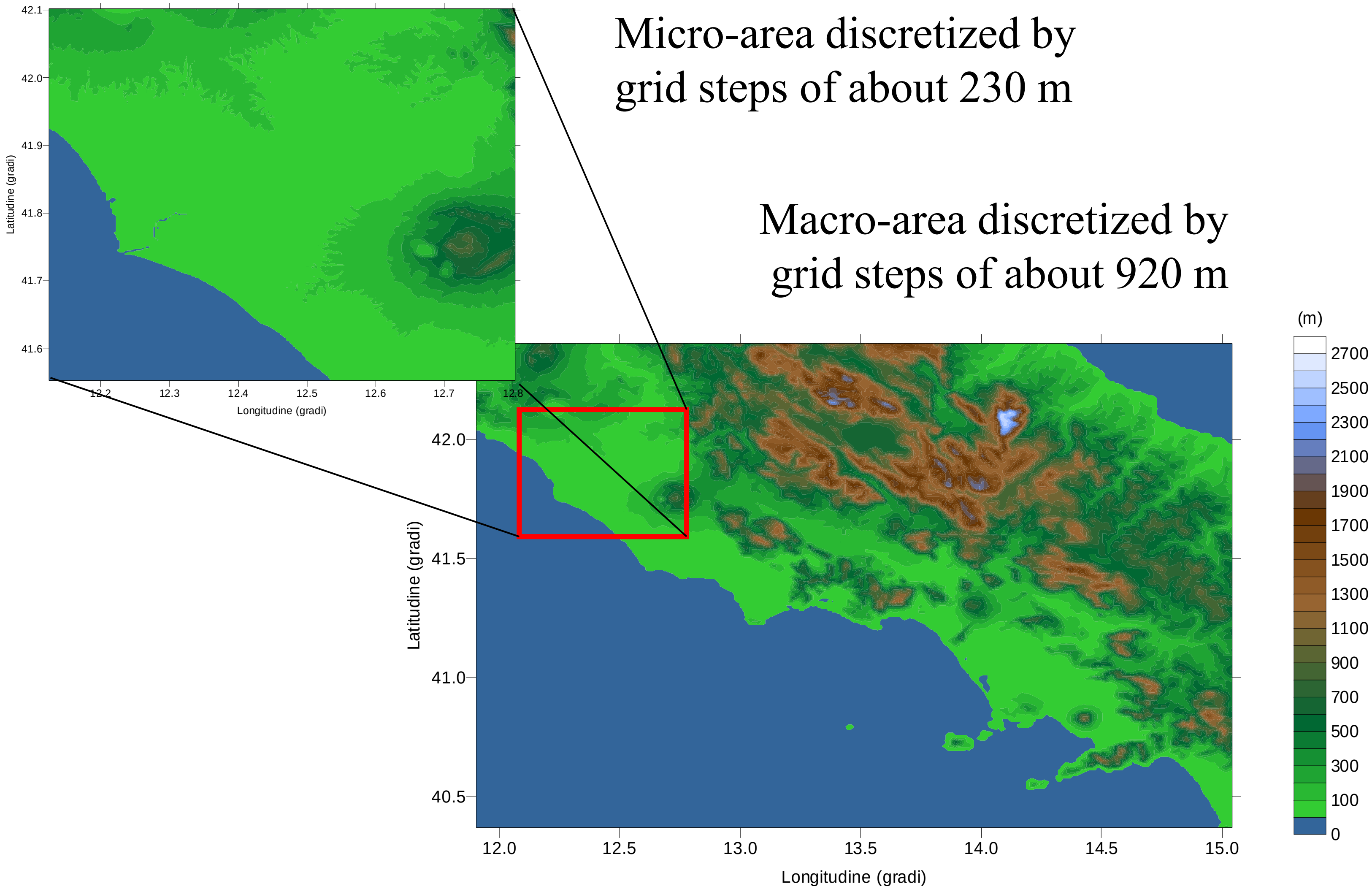
Italian High Velocity Railway Network



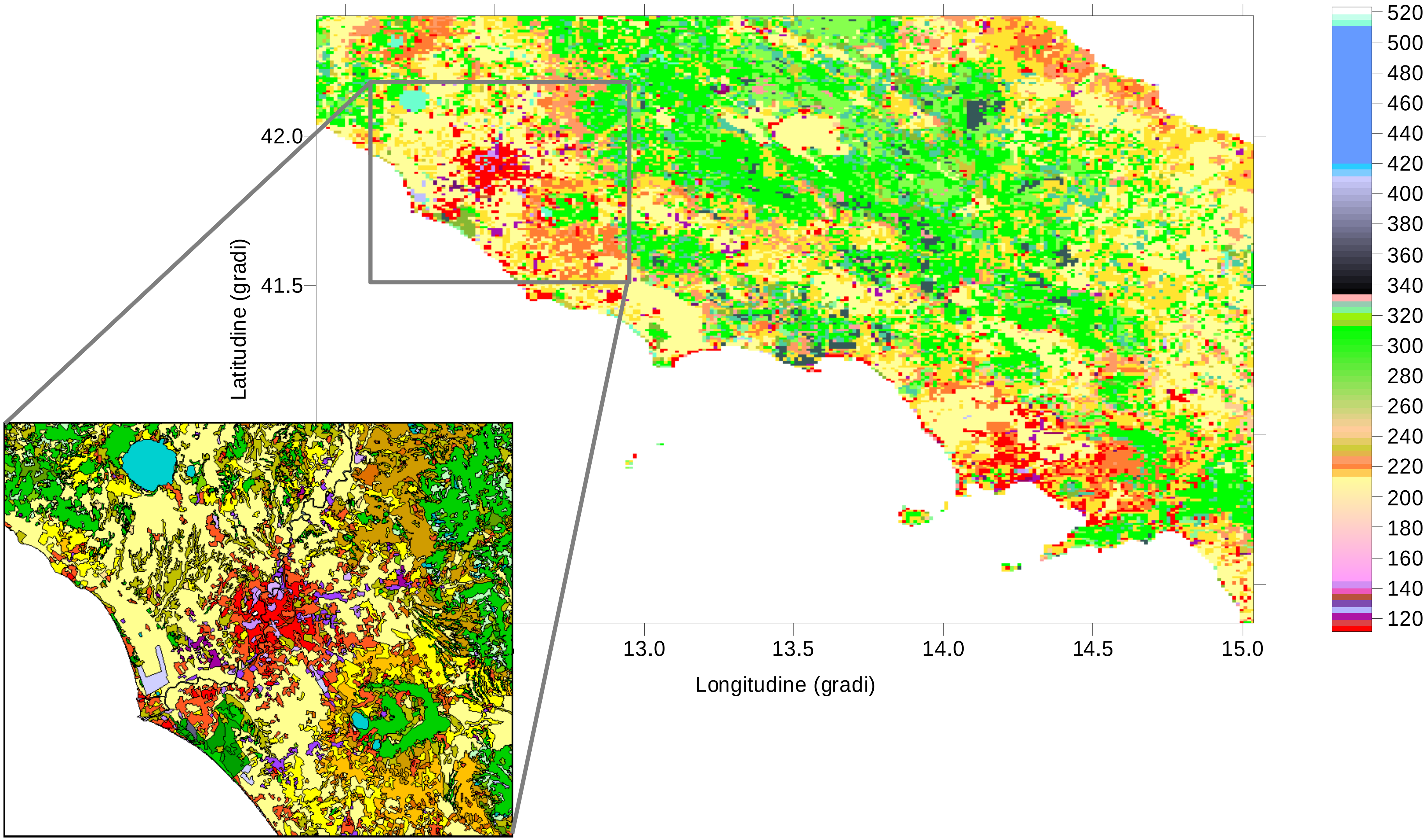
Quota (m s.l.m.)
2000
1000
0
latitudine
42.0
41.5
41.0
40.5
12.0
12.5
13.0
13.5
14.0
14.5
15.0
longitudine
Linea ferroviaria AV/AC Rome-Naples
Stazioni meteorologiche AM & ENAV
Stazioni meteorologiche RFI



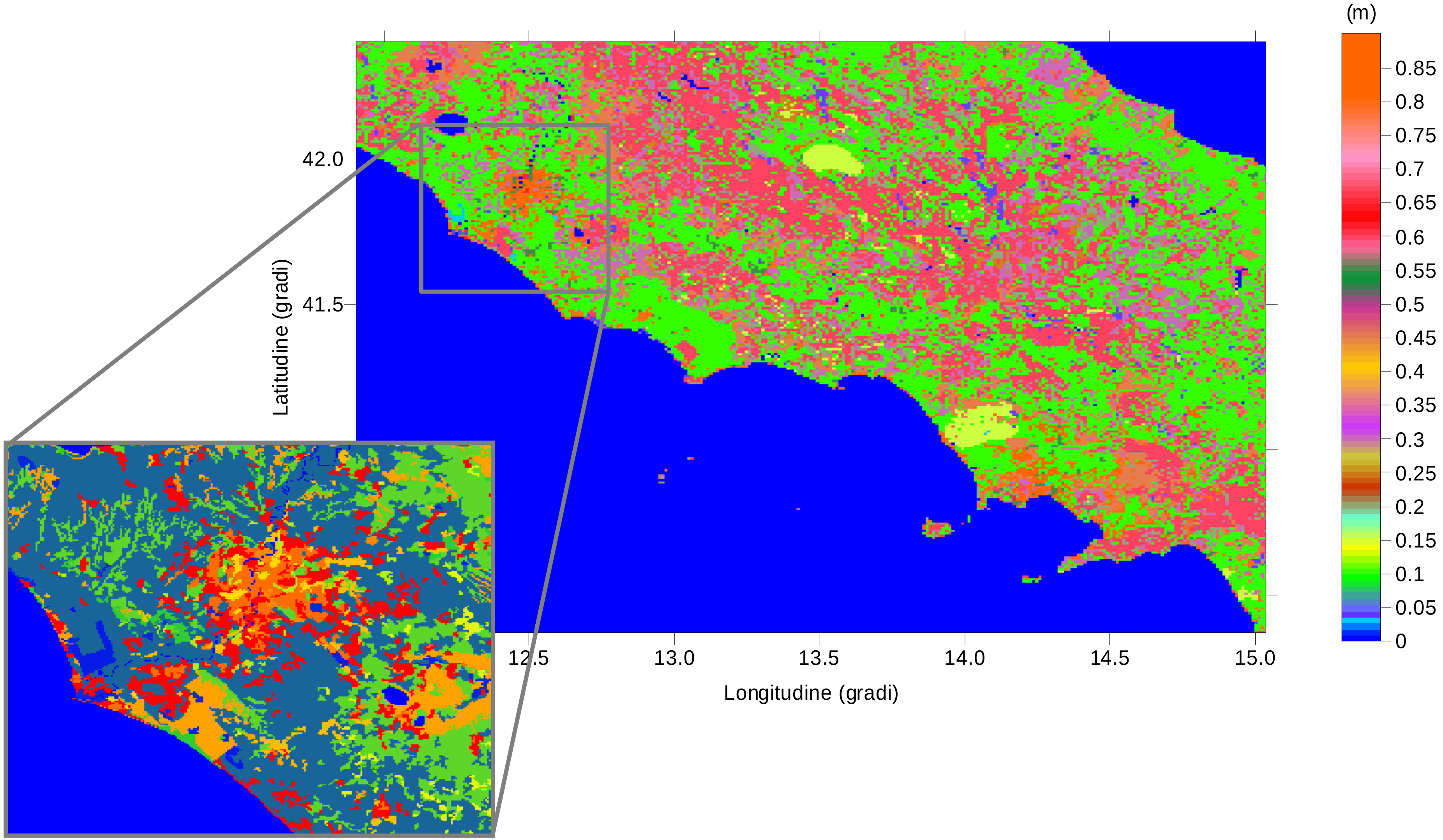
Roma-Napoli Railway Line – Topography map



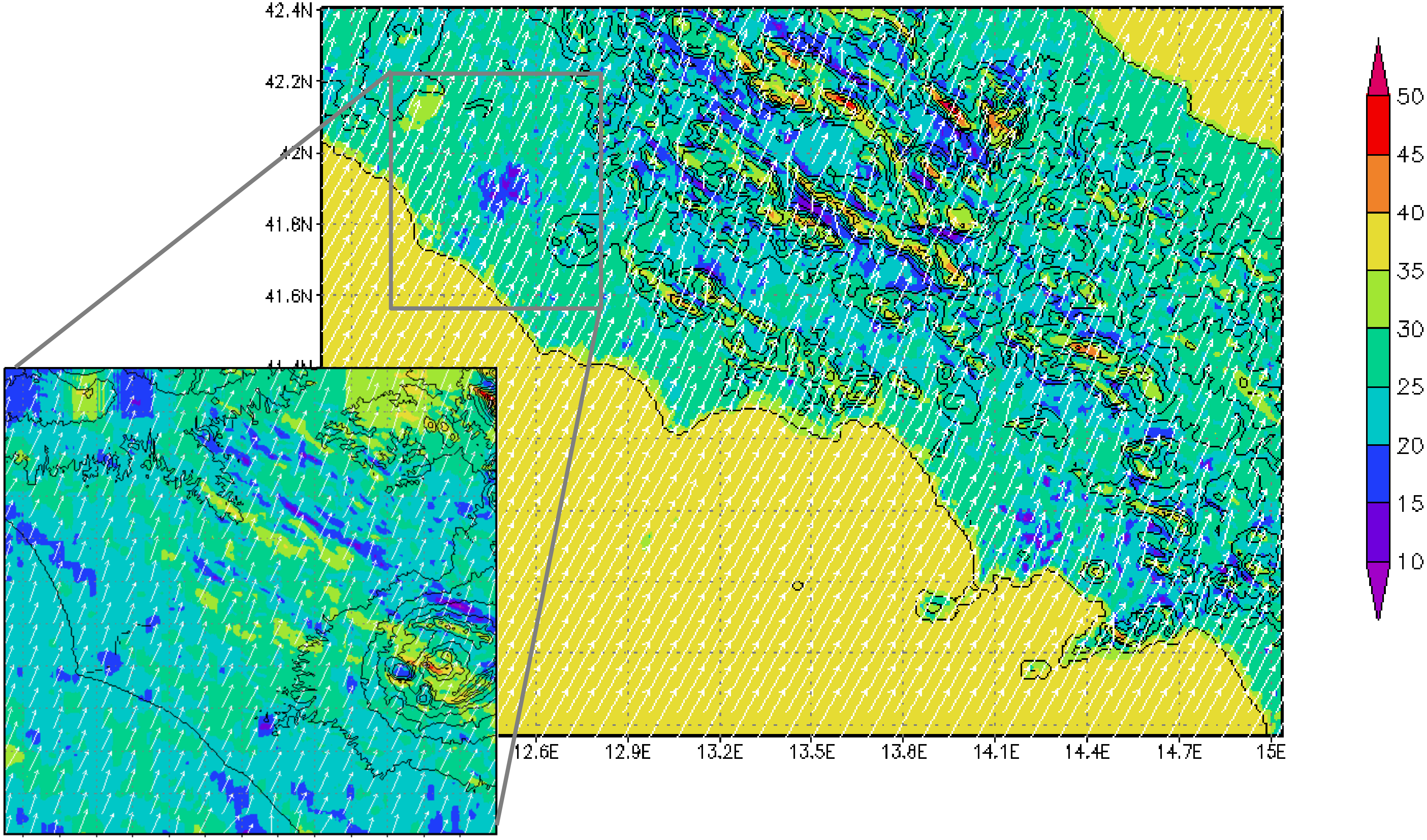
Roma-Napoli Railway Line – Topography map



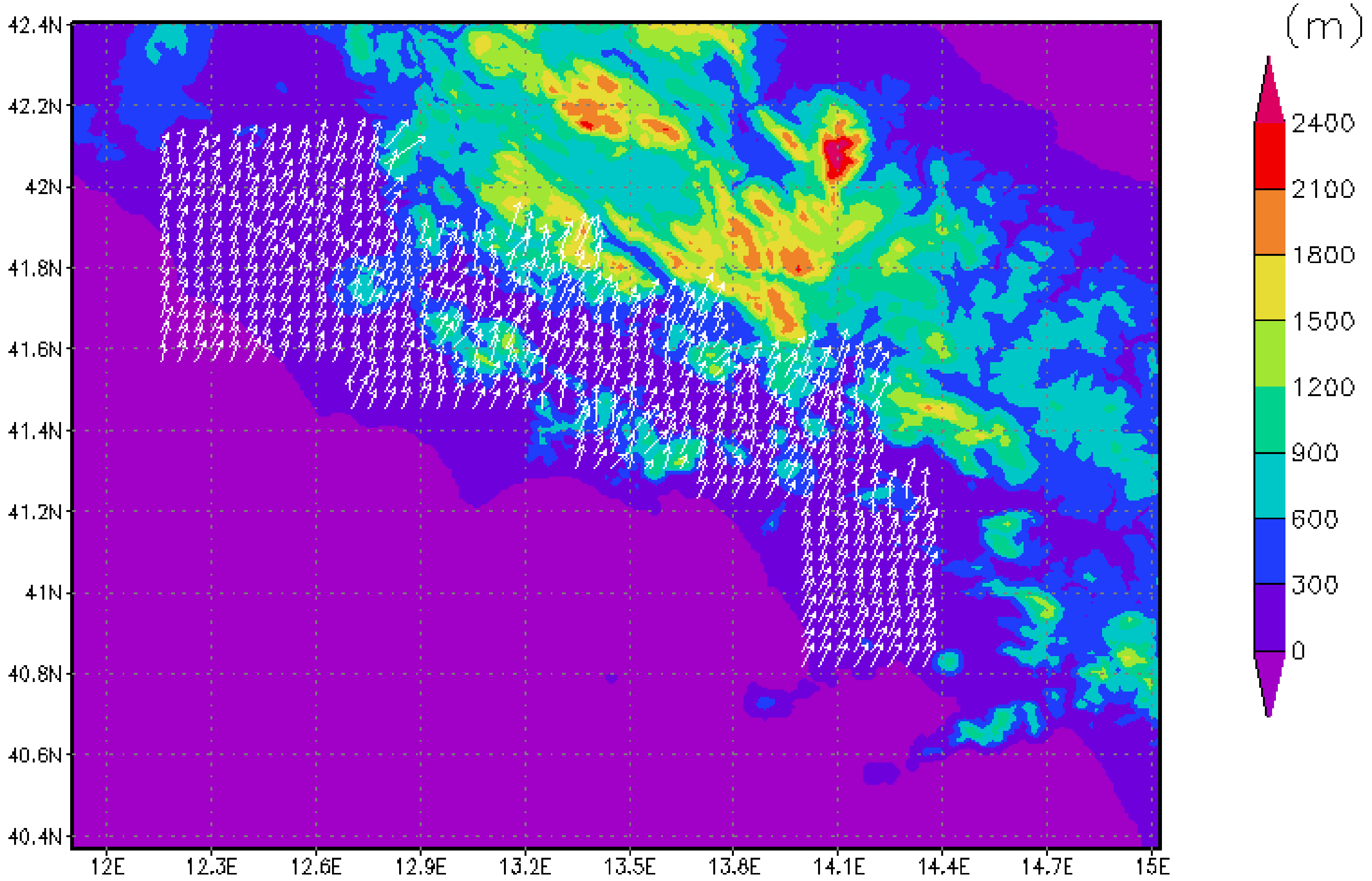
Roma-Napoli Railway Line – Cover map



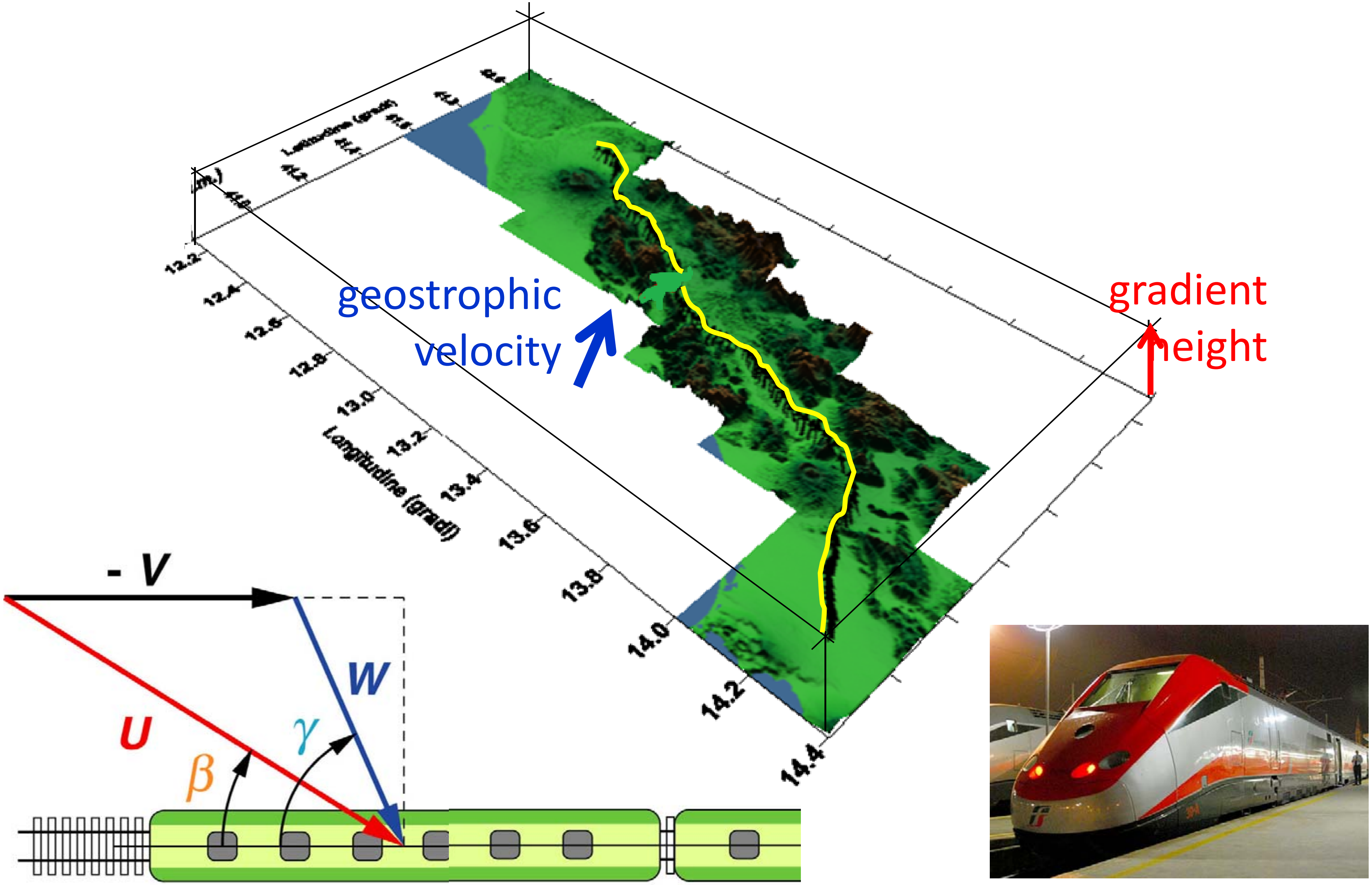
Roma-Napoli Railway Line – Roughness map



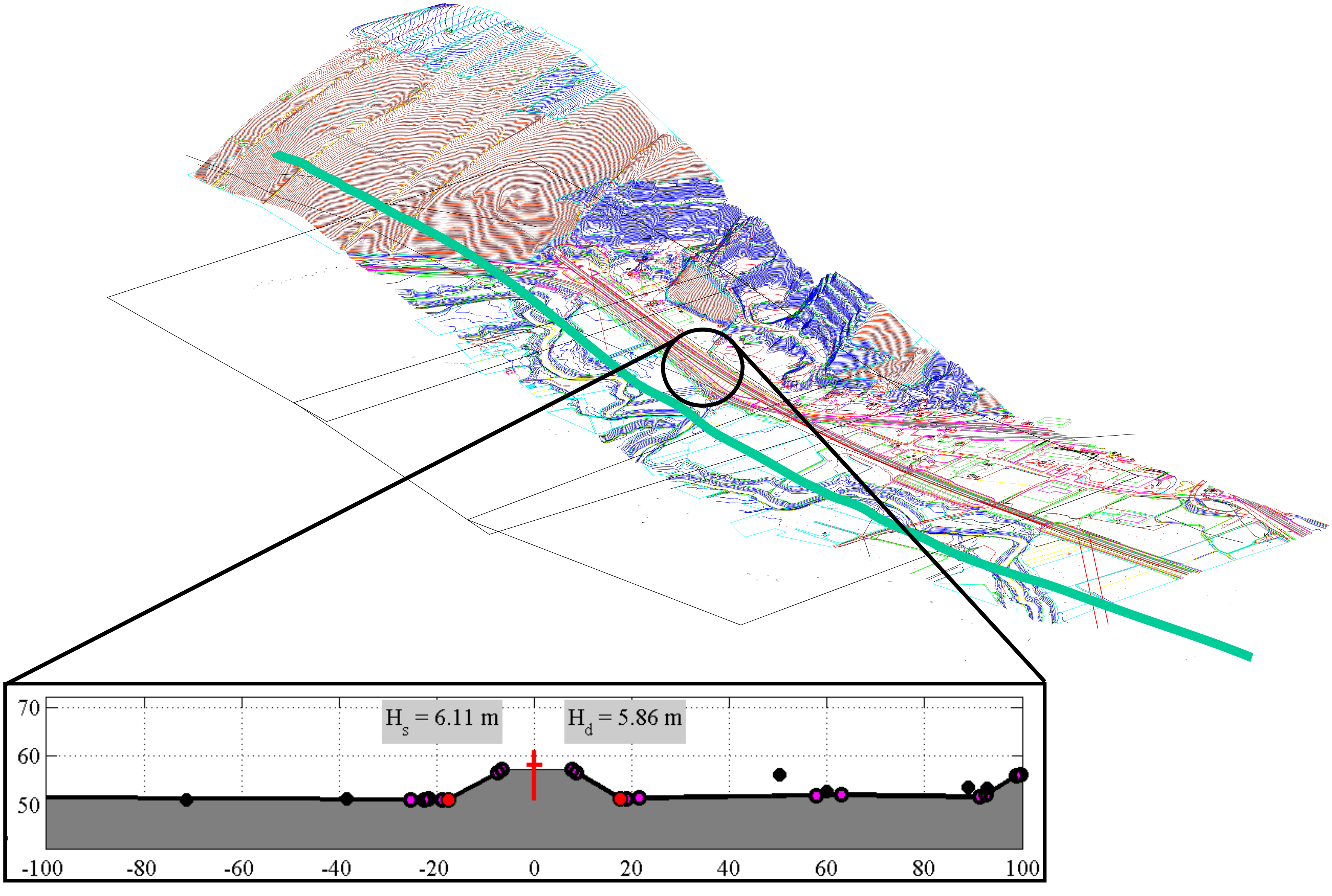
Roma-Napoli Railway Line – Mean wind field



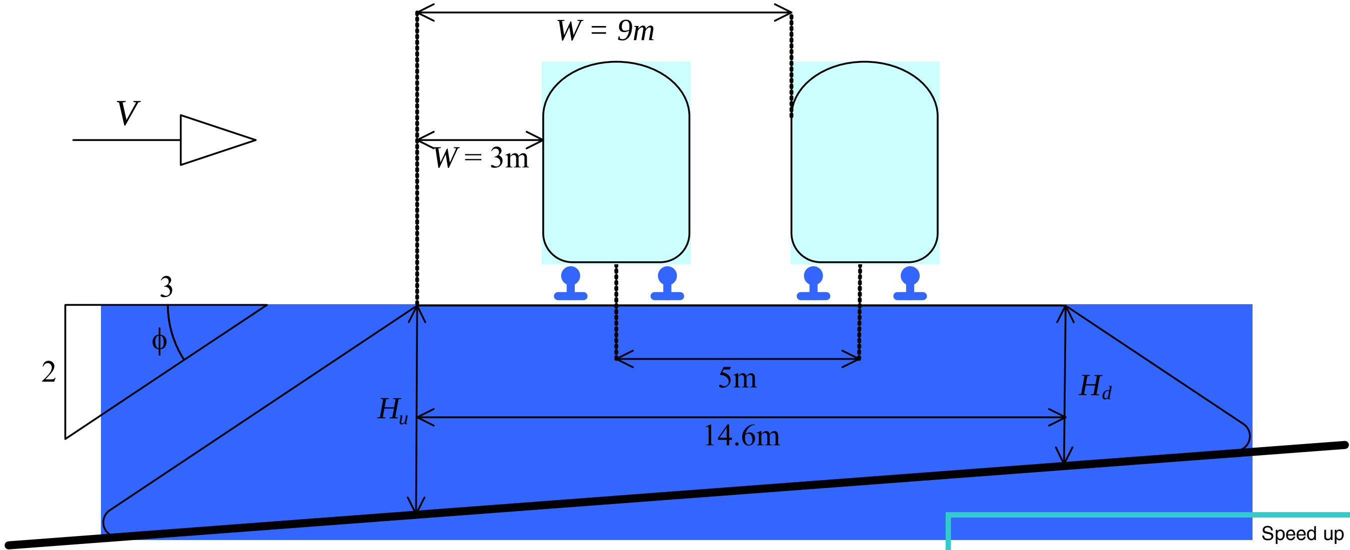
Roma-Napoli Railway Line – Mean wind field



Roma-Napoli Railway Line – Mean wind field



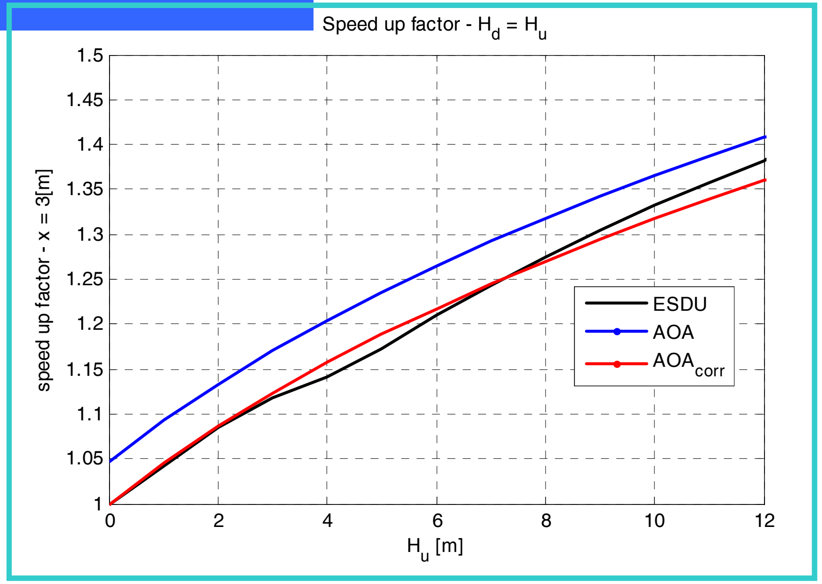
Roma-Napoli Railway Line – Mean wind field



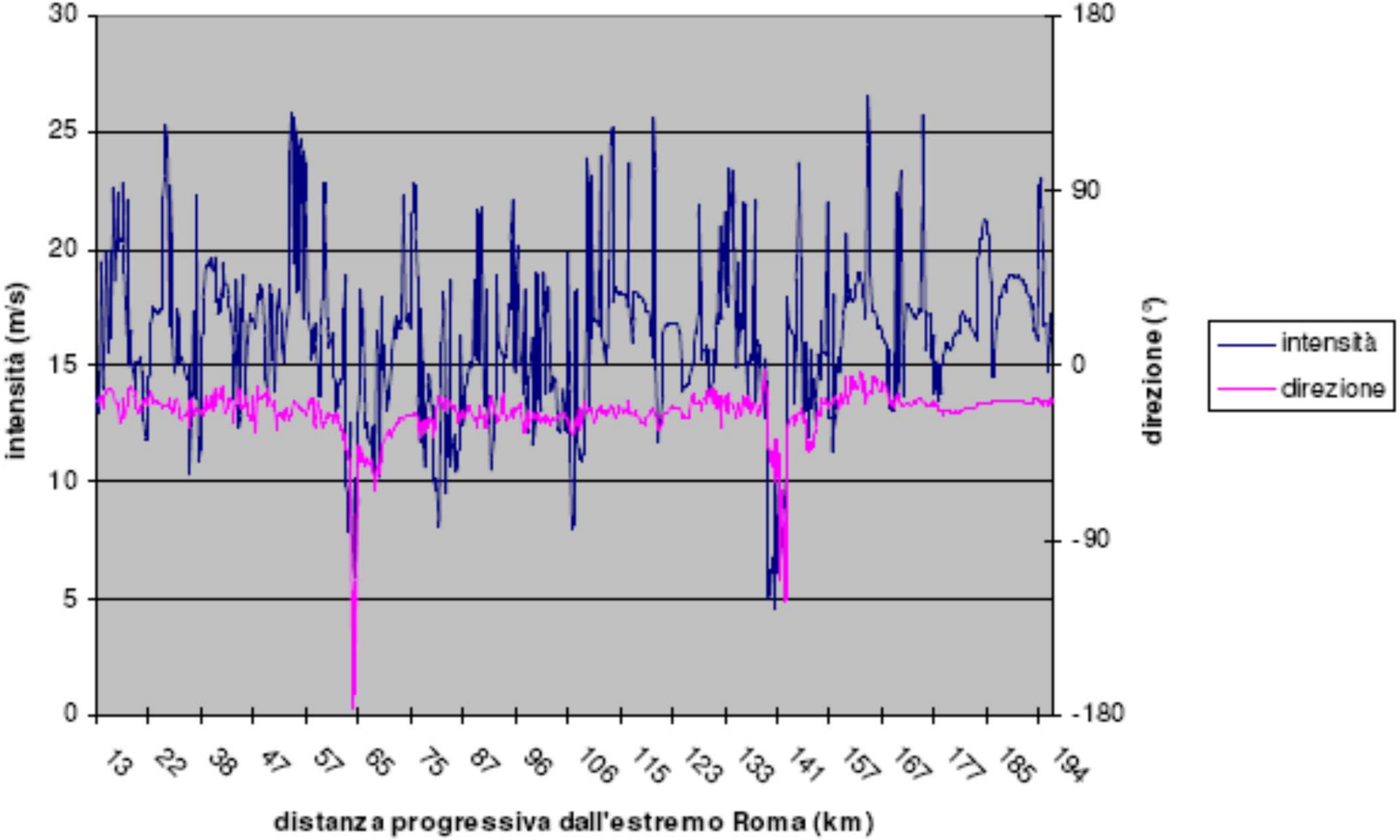
$$\bar{V}_E = \kappa_E \bar{V}_0$$

$$\kappa_E = \sqrt{(\cos\beta_0)^2 + (C_E \sin\beta_0)^2}$$

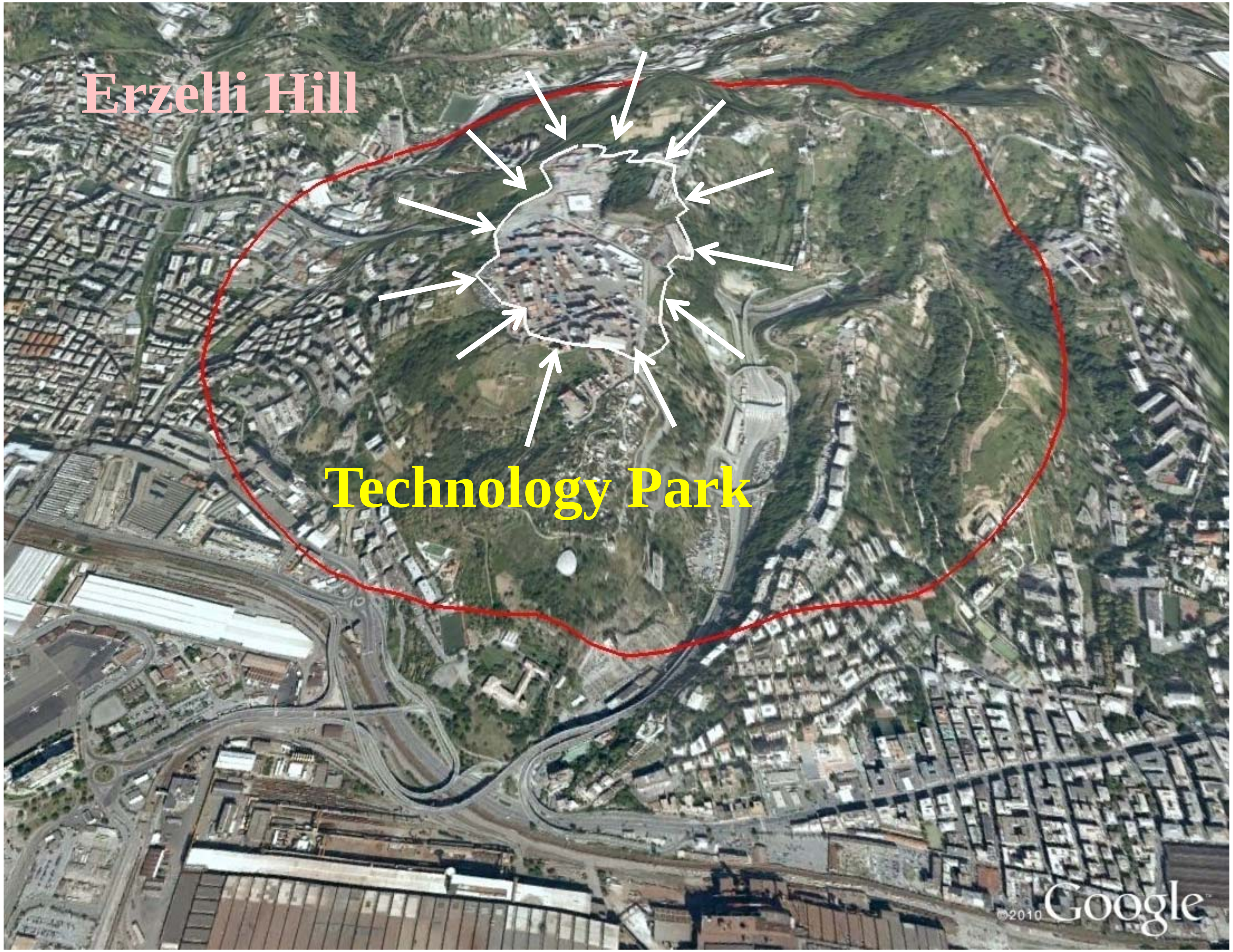
$$C_E = 0{,}94\left[1{,}63H + 10\right]^{1/5} - 0{,}49$$



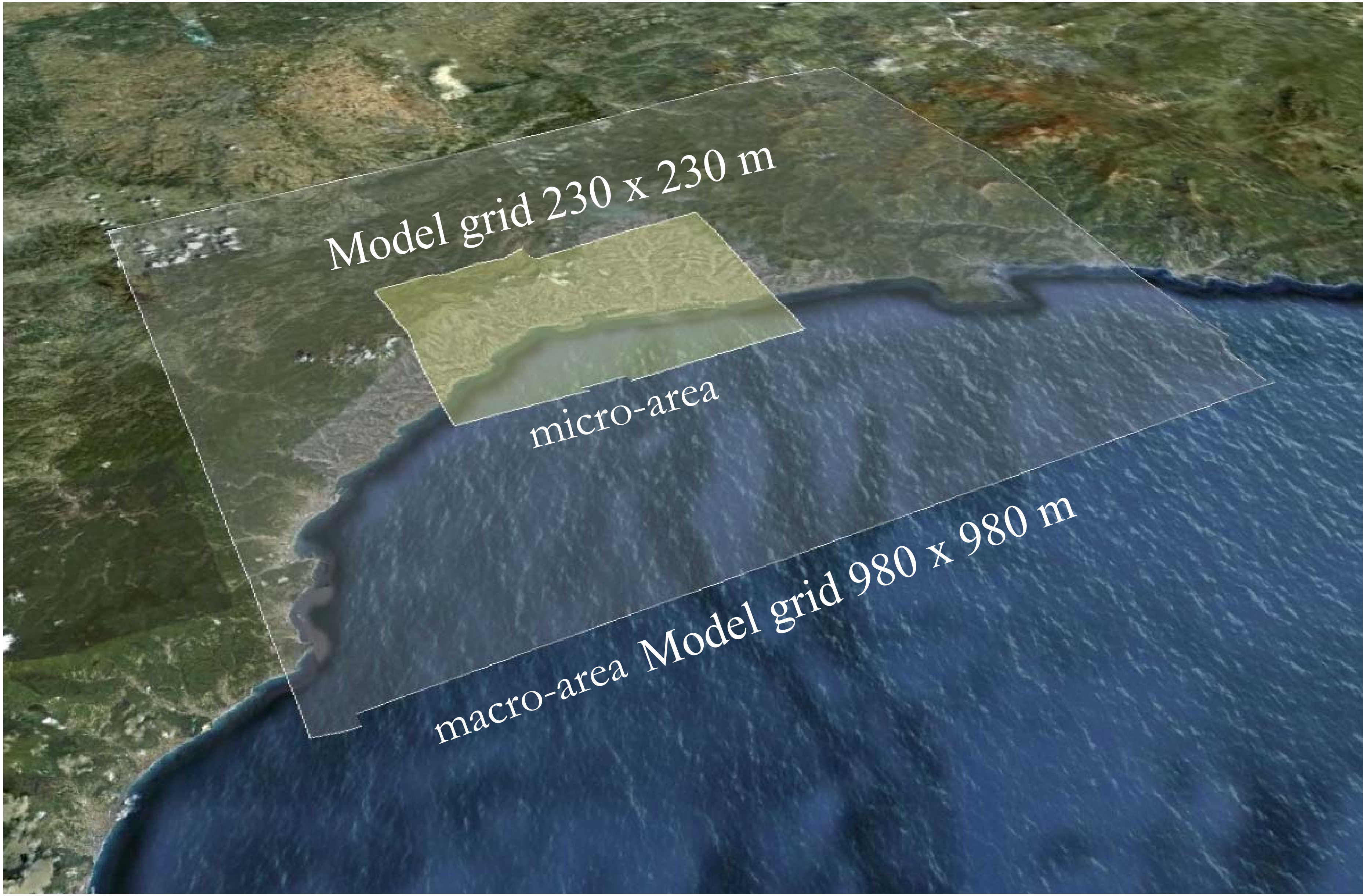
Roma-Napoli Railway Line – Mean wind field



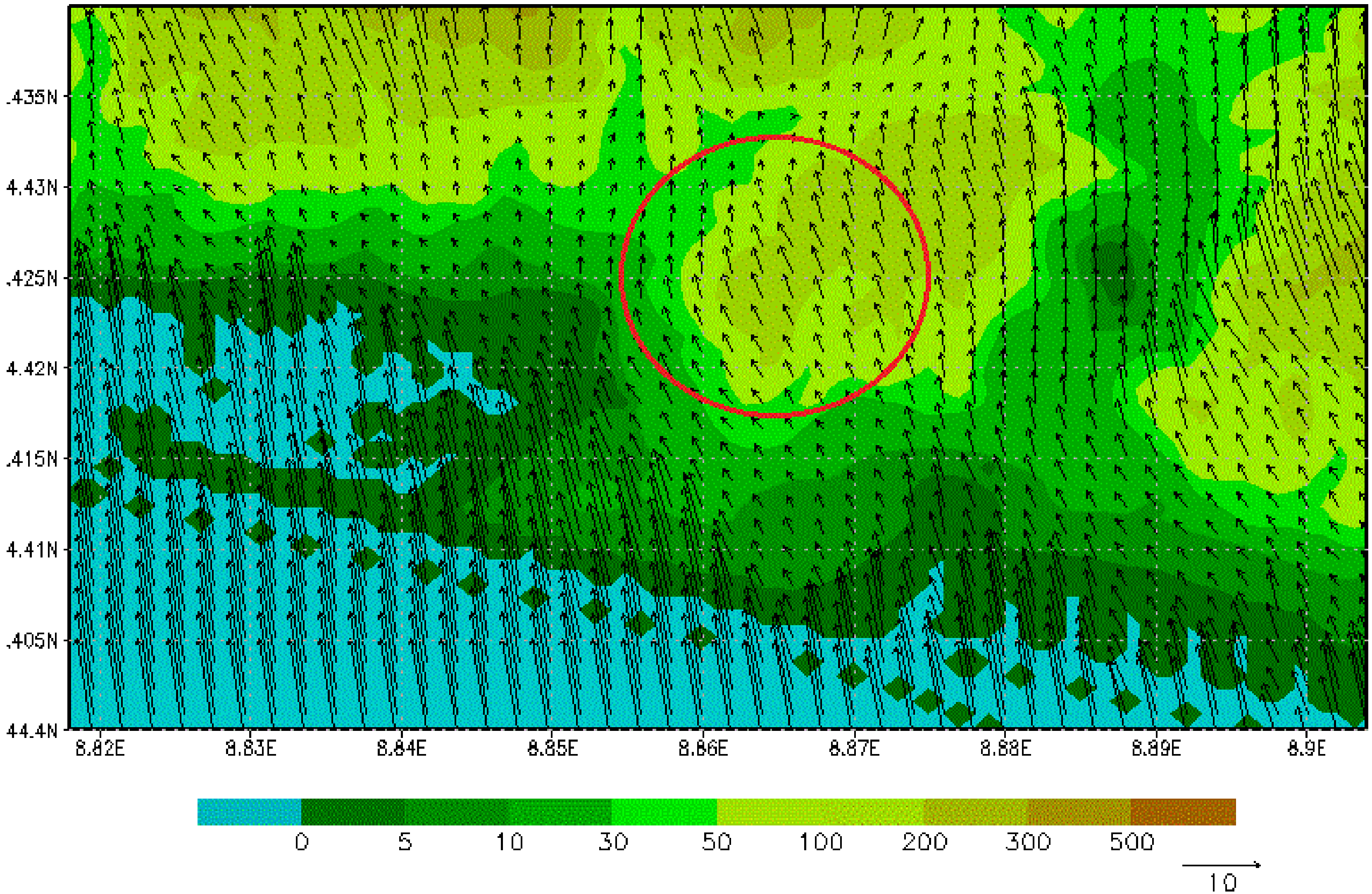
Roma-Napoli Railway Line – Topography map



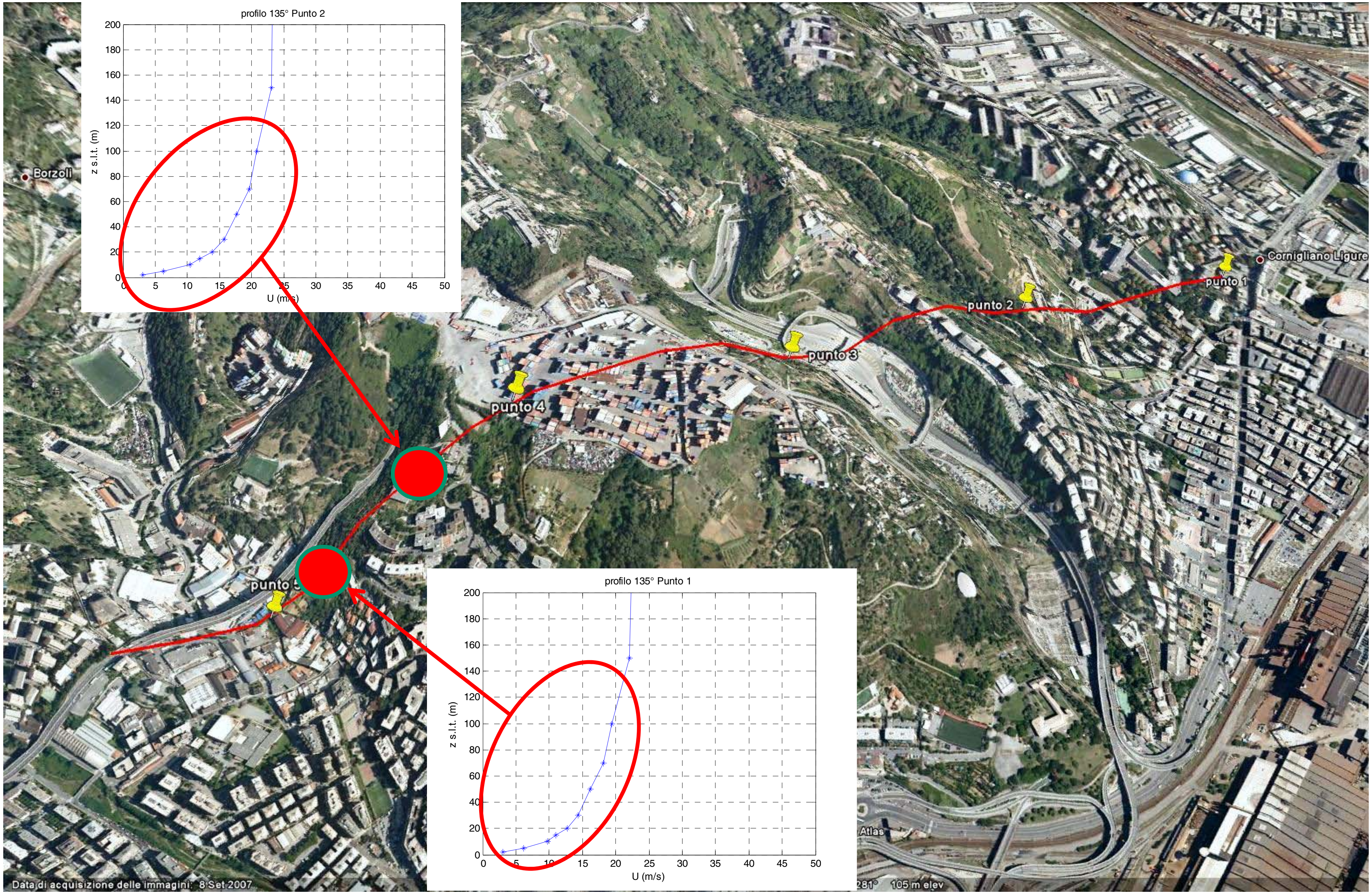
Erzelli Hill, Genova



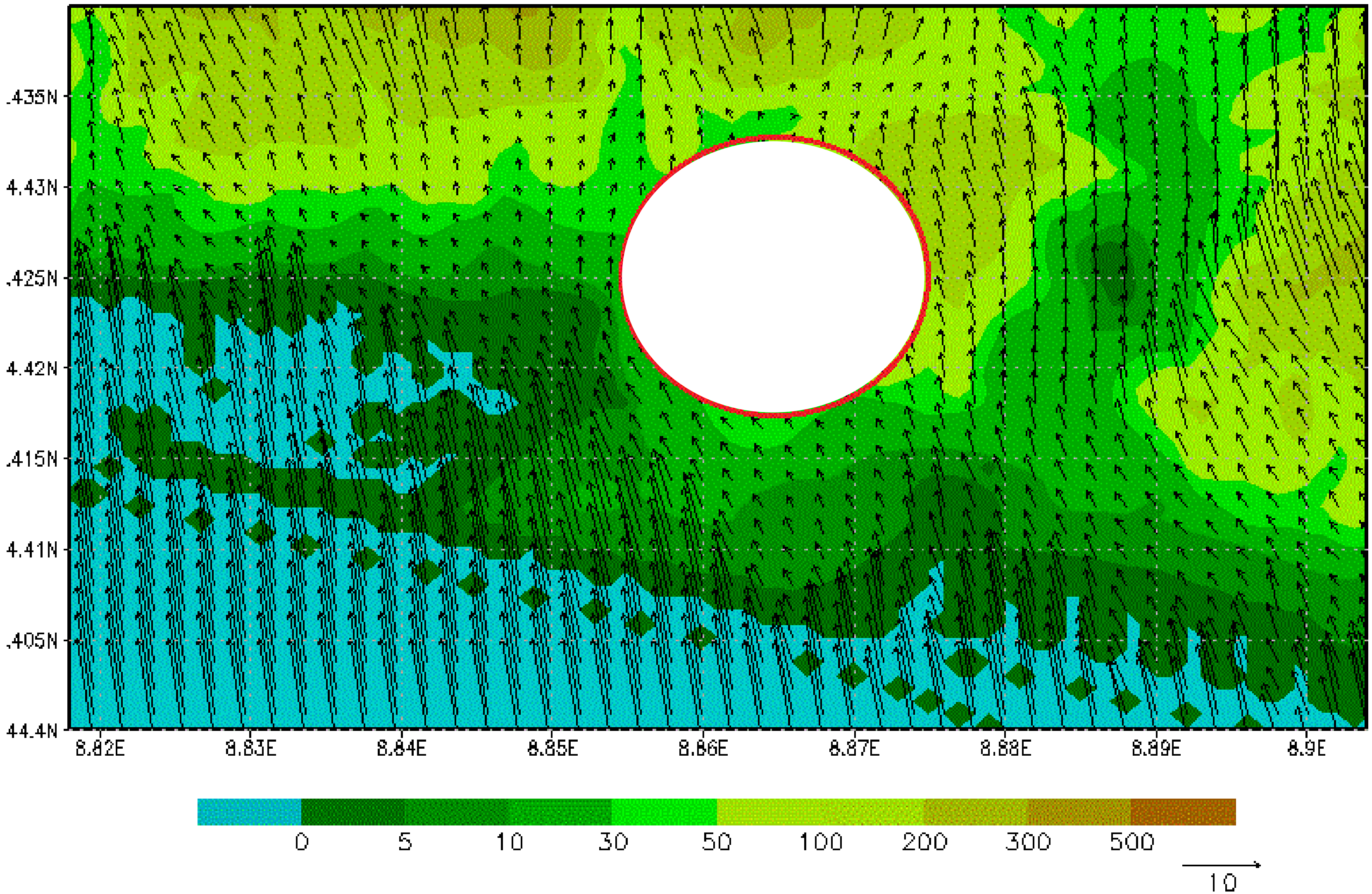
Erzelli Hill, Macro- and micro-area



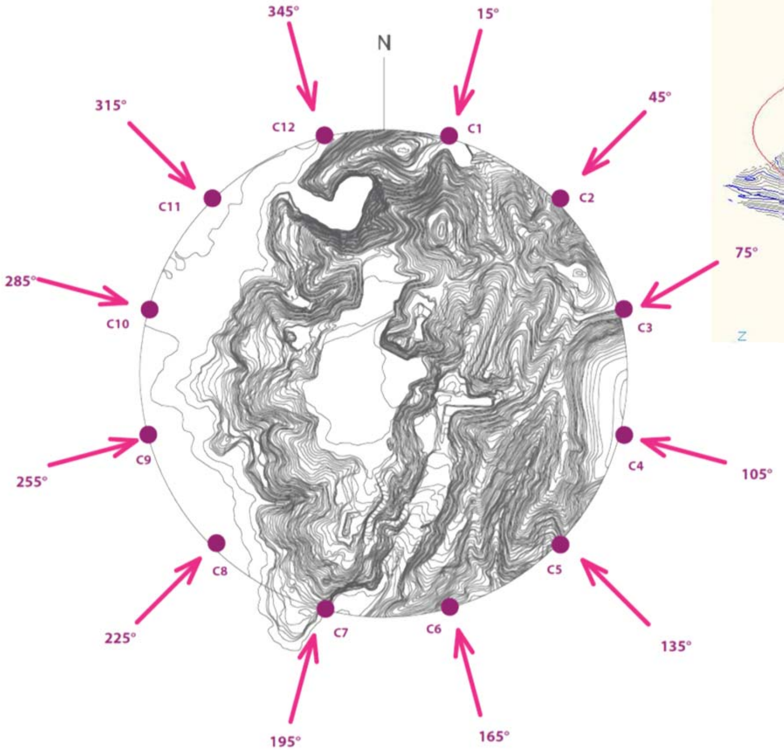
Erzelli Hill, WINDS wind field

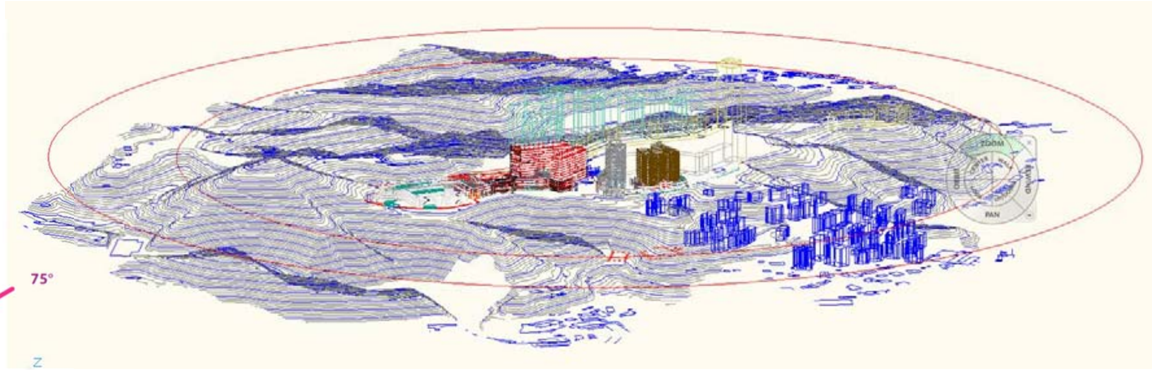


Mean wind velocity profiles along the hill



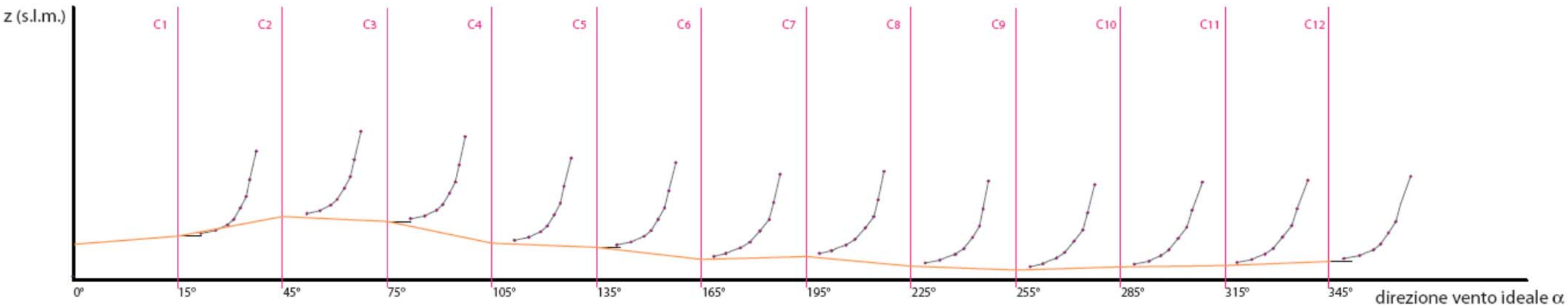
Erzelli Hill, WINDS wind field



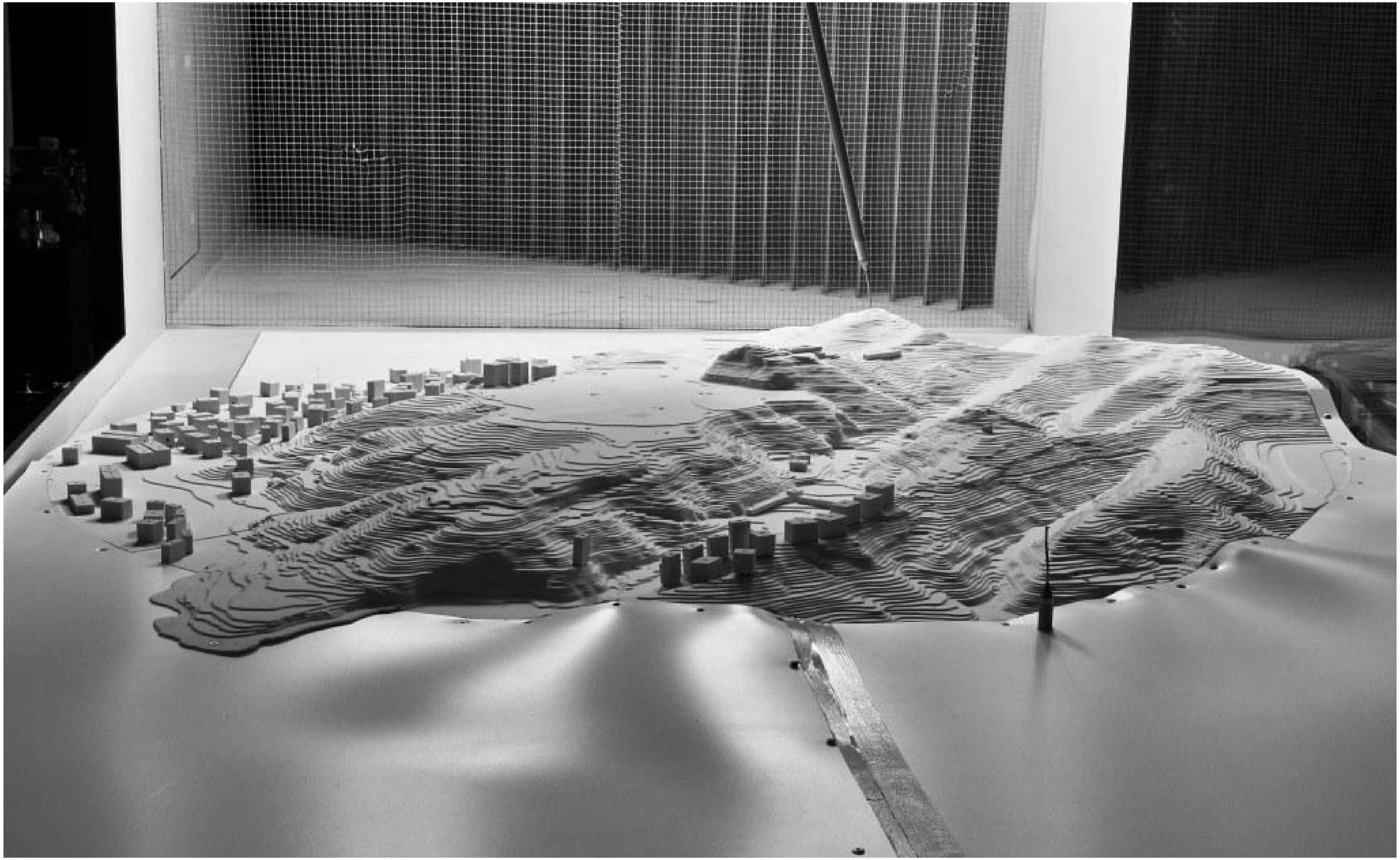


<u>Wind field downscaling</u>

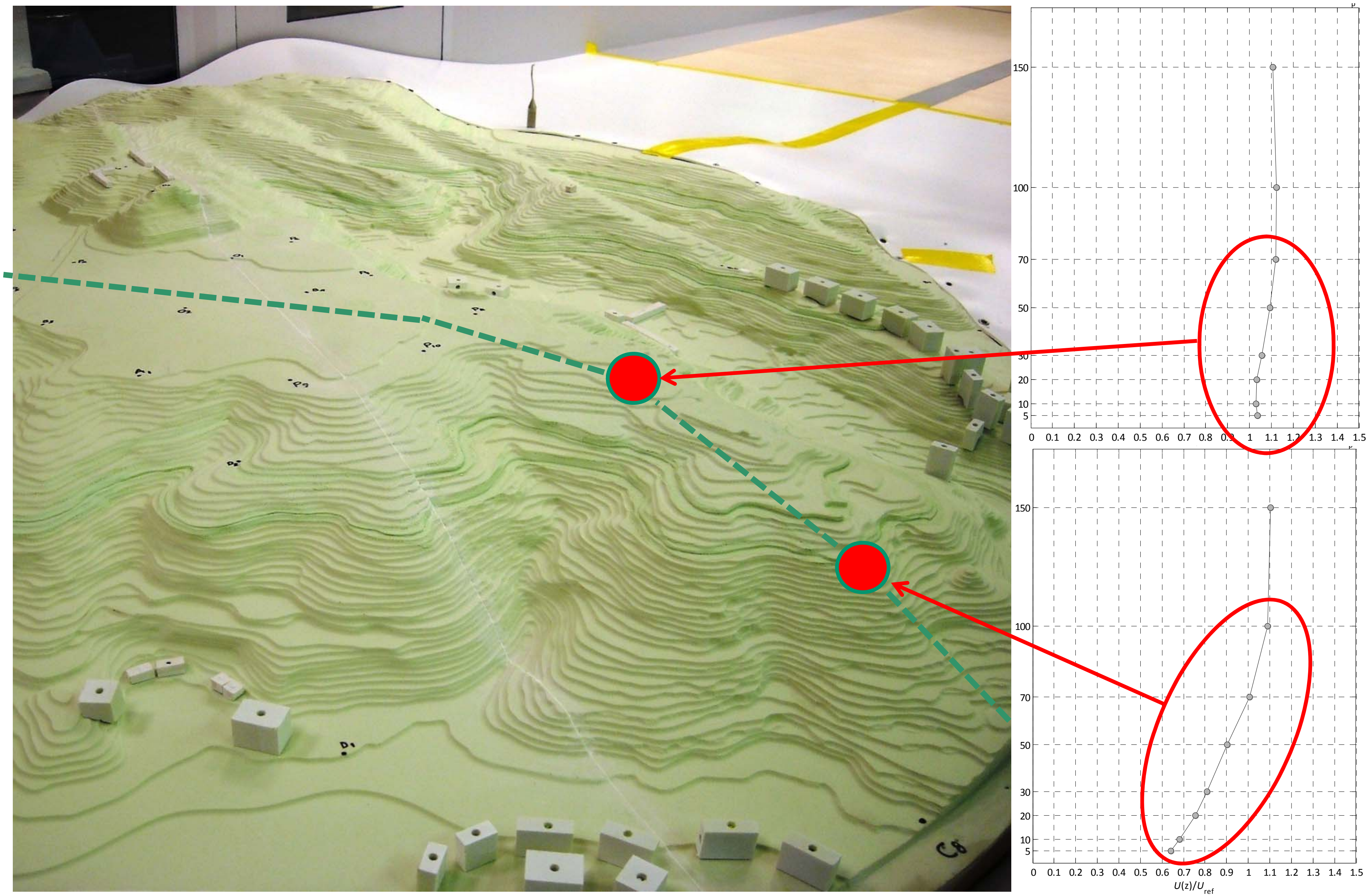
- mass-consistent model
- revised & advanced ESDU
- wind tunnel topography test
- CFD model



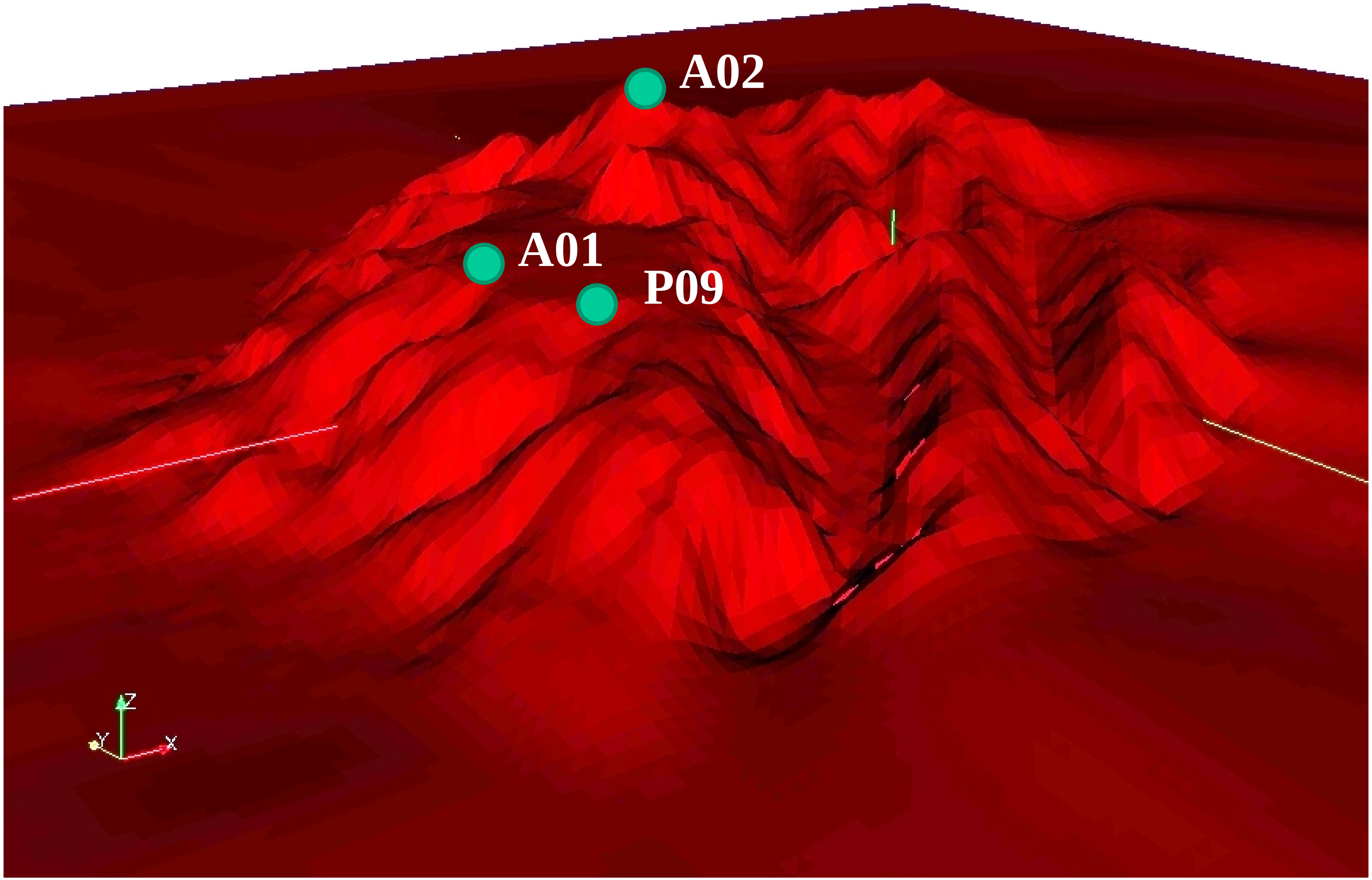
Mean wind velocity profiles at the base of the hill



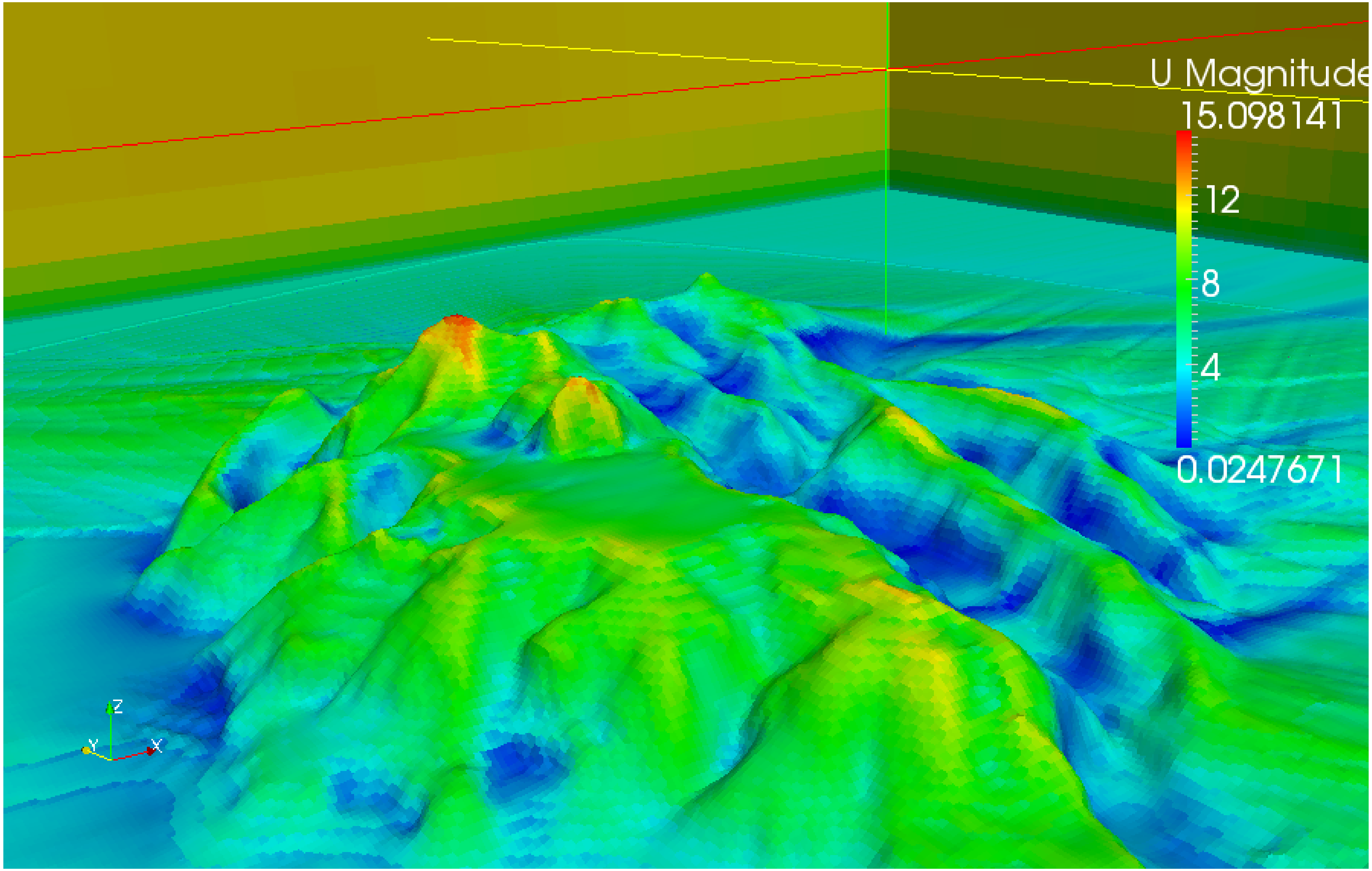
Erzelli Hill – Topography model scale 1:1000



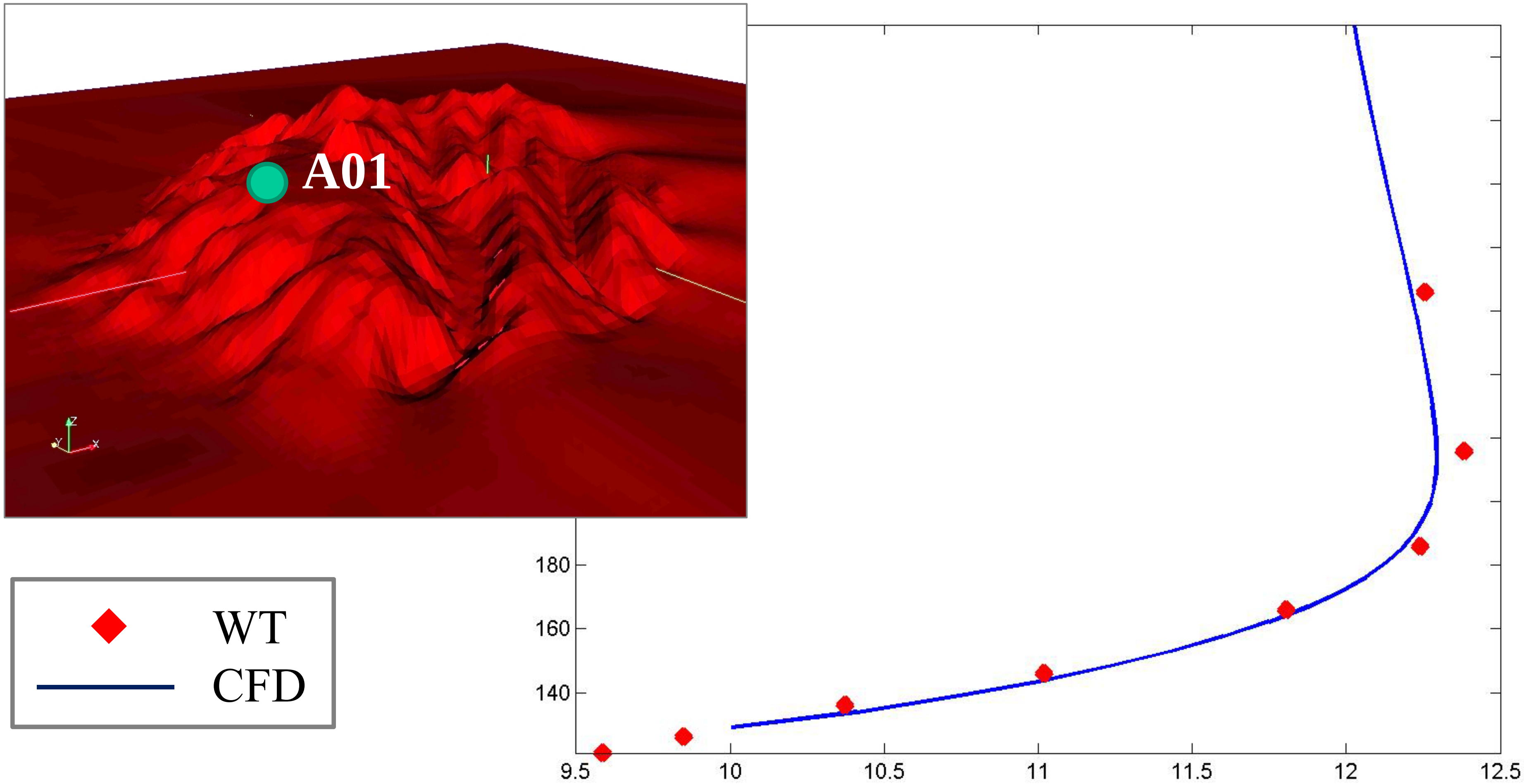
Erzelli Hill – Wind tunnel topographic model – 1:1000 scale



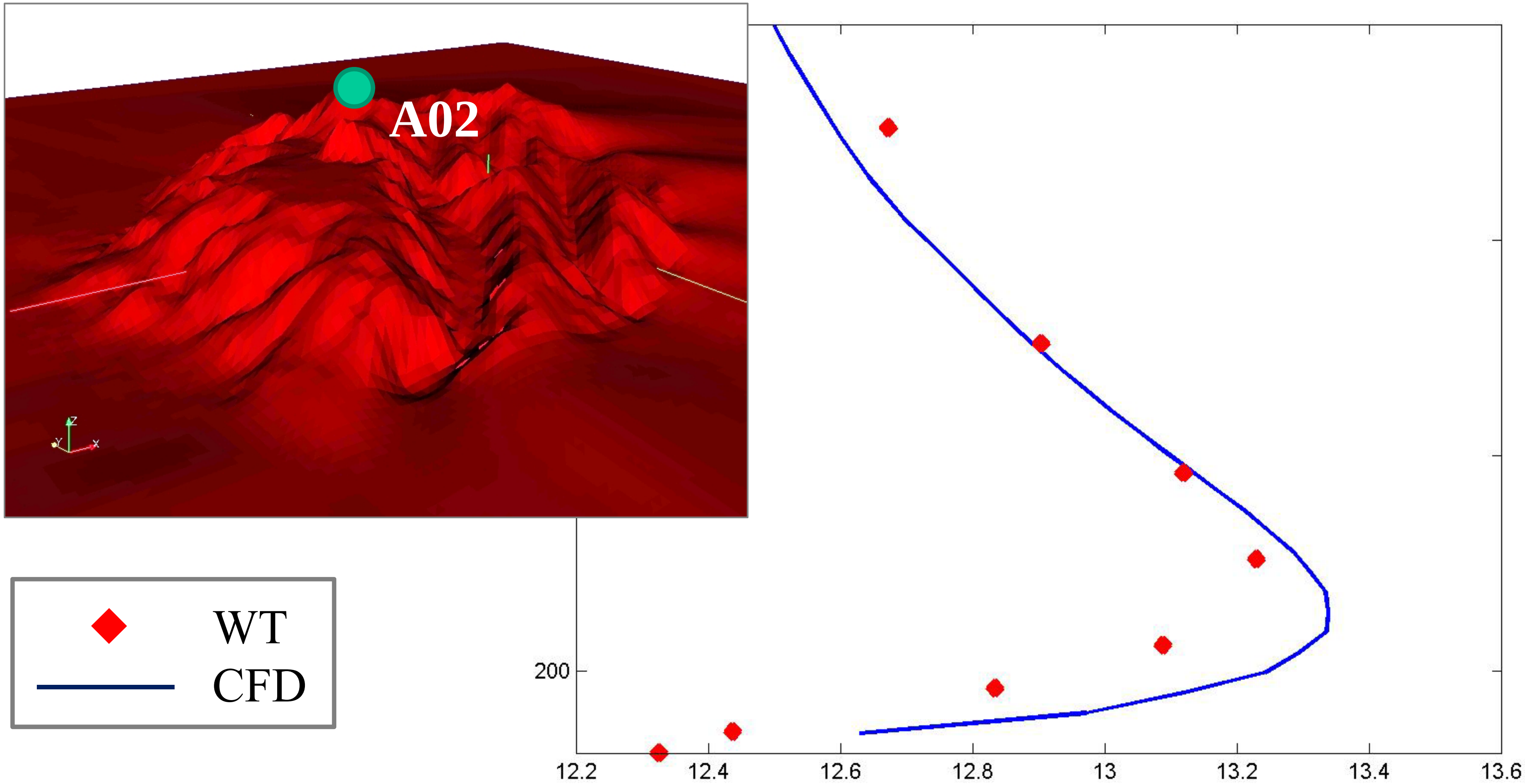
OPENFOAM CFD model



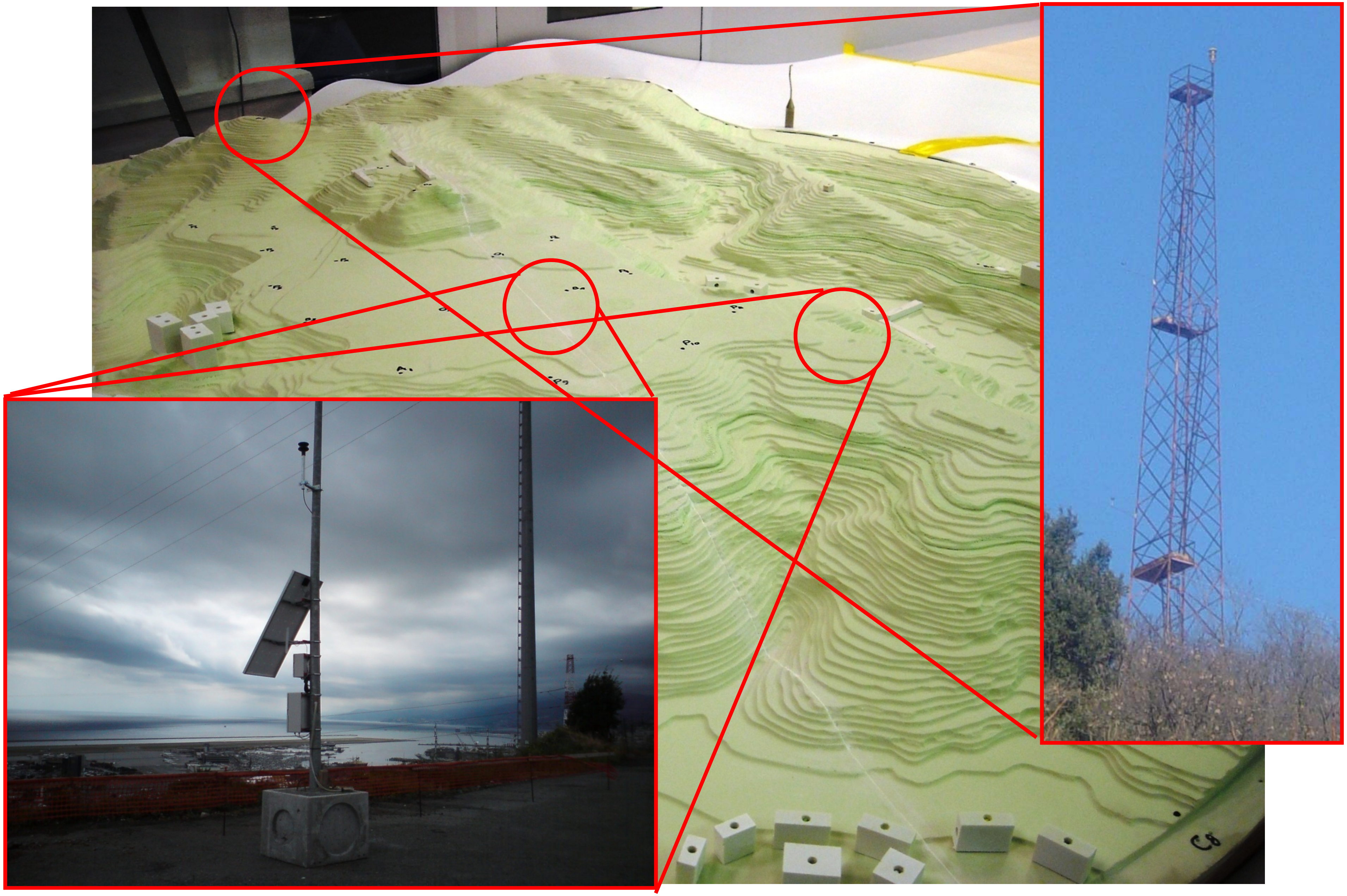
CFD model – Wind velocity at 2 m height



Comparison between Wind Tunnel and CFD results



Comparison between Wind Tunnel and CFD results



Wind monitoring at the top of the hill

Atmospheric thermal stratification

Richardson Number

$$\boxed{Ri = \frac{g}{\overline{T}} \frac{\Gamma - \Gamma_a}{(\partial \overline{u} / \partial z)^2}}$$

Obukhov Length

$$\boxed{L = \frac{u_*^3}{\kappa \frac{g}{\overline{T}} Q_0}}$$

$$\frac{z}{L} = Ri \quad \text{for } Ri \leq 0$$

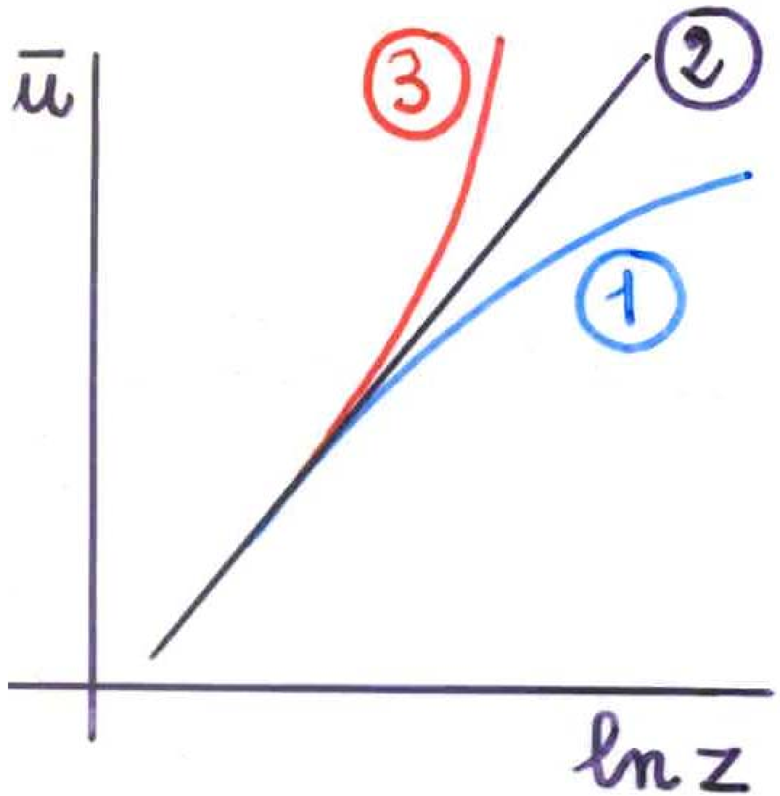
$$\frac{z}{L} = \frac{Ri}{1\text{-}5 \cdot Ri} \quad \text{for } \; 0 \leq Ri \leq 0,2$$

$\overline{T}$	mean atmospheric temperature
$\Gamma = -\partial \overline{T} / \partial z$	lapse rate
$\Gamma_a = g/c_p$	dry adiabatic lapse rate
c_p	specific heat at constant pressure
$Q_o = H_o/(\rho c_p)$	H_o being the vertical heat flux (positive upwards)

Neutral stratification	$\Gamma = \Gamma_a \;\; Ri = 0 \;\; 1/L \rightarrow 0$
Stable stratification	$\Gamma > \Gamma_a \;\; Ri > 0 \;\; 1/L > 0$
Unstable stratification	$\Gamma < \Gamma_a \;\; Ri < 0 \;\; 1/L < 0$

Mean velocity profile

$$\boxed{\bar{u}(z) = \frac{u_*}{\kappa}\left[ln\frac{z}{z_0} - \psi_m\left(\frac{z}{L}\right)\right]}$$



Neutral stratification

$$\psi_m\left(\frac{z}{L}\right) = 0 \qquad z_G = C\frac{u_*}{|f|}$$

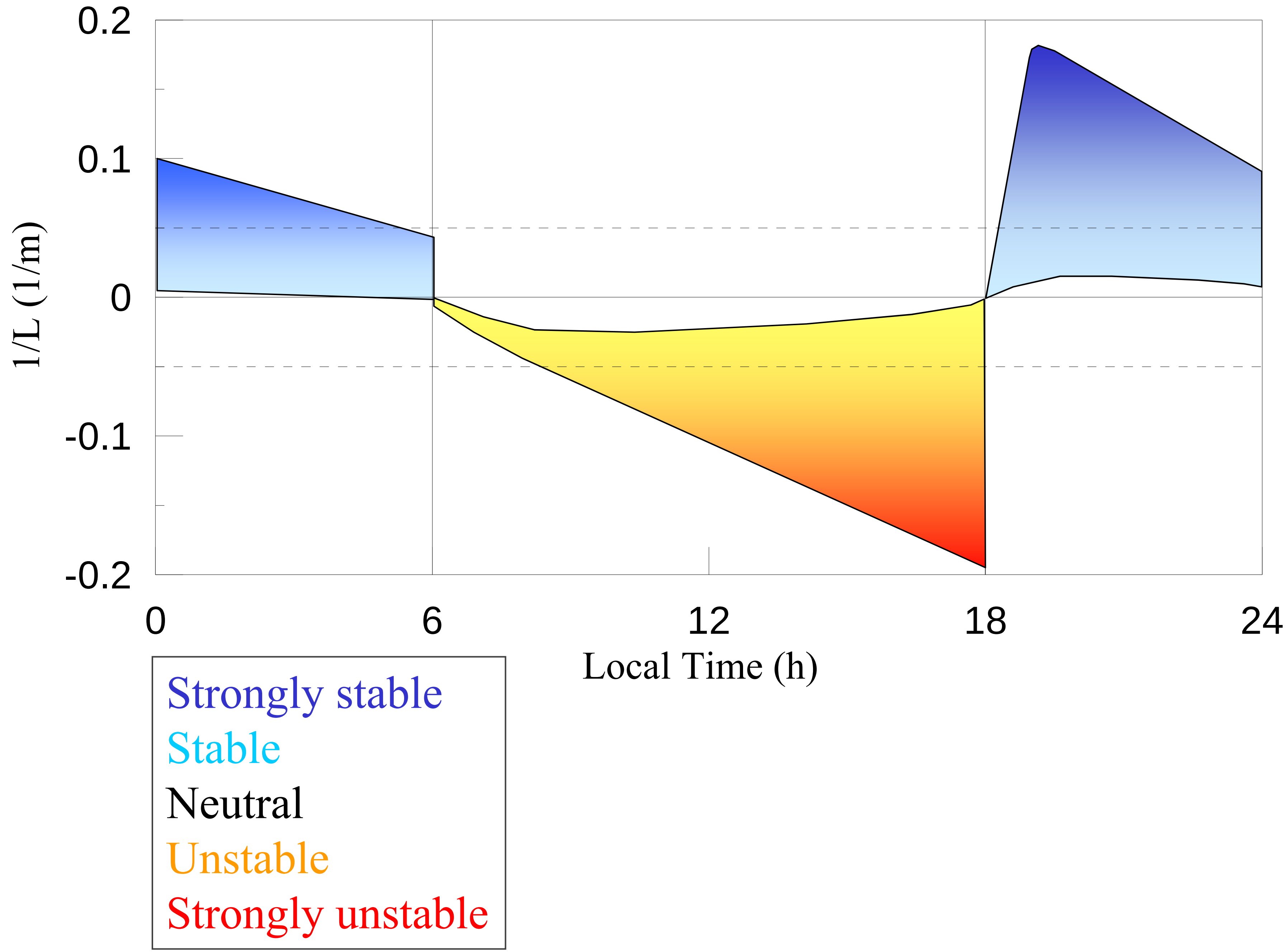
Stable stratification

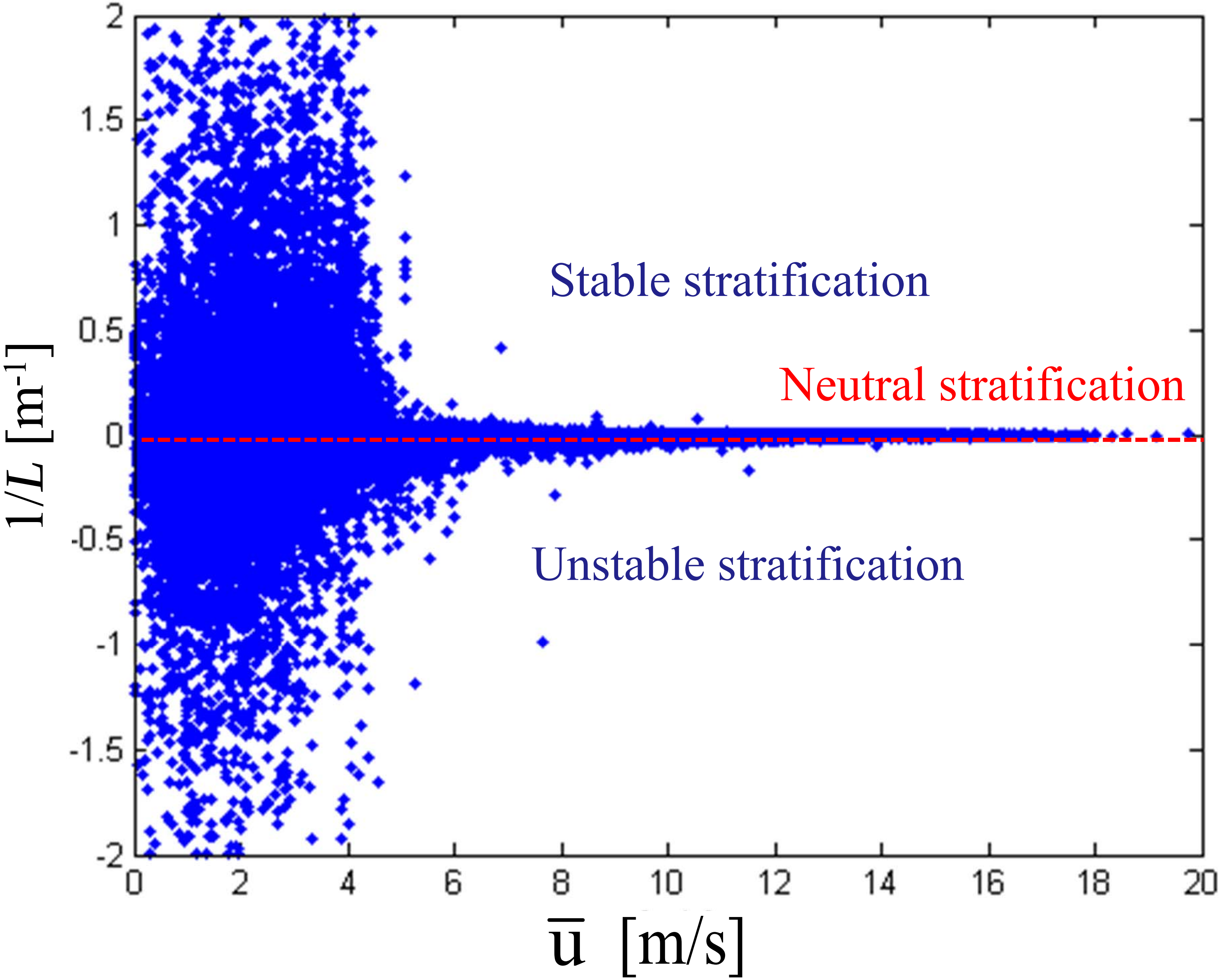
$$\psi_m\left(\frac{z}{L}\right) = -5\frac{z}{L} \qquad z_G = C\frac{u_*}{|f|}\frac{1}{1+b\sqrt{\frac{u_*}{|f|L}}}$$

Unstable stratification

$$\psi_m\left(\frac{z}{L}\right) = ln\left[\left(\frac{1+x^2}{2}\right)\left(\frac{1+x}{2}\right)^2\right] - 2atan\,x + \frac{\pi}{2} \qquad z_G = C\frac{u_*}{|f|}\left(1+d\sqrt{\frac{u_*}{|fL|}}\right)$$

$x=(1 - 16z/L)^{1/4}$


0.2
0.1
0
-0.1
-0.2
1/L (1/m)
0
6
12
18
24
Local Time (h)
Strongly stable
Stable
Neutral
Unstable
Strongly unstable



Stable stratification
Neutral stratification
Unstable stratification
1/L [m-1]
ū [m/s]
2
1.5
1
0.5
0
-0.5
-1
-1.5
-2
0
2
4
6
8
10
12
14
16
18
20