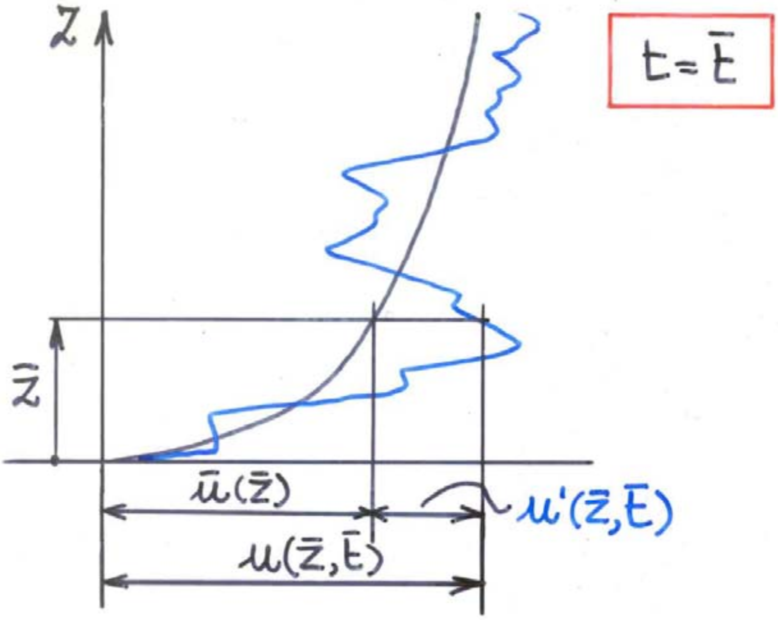
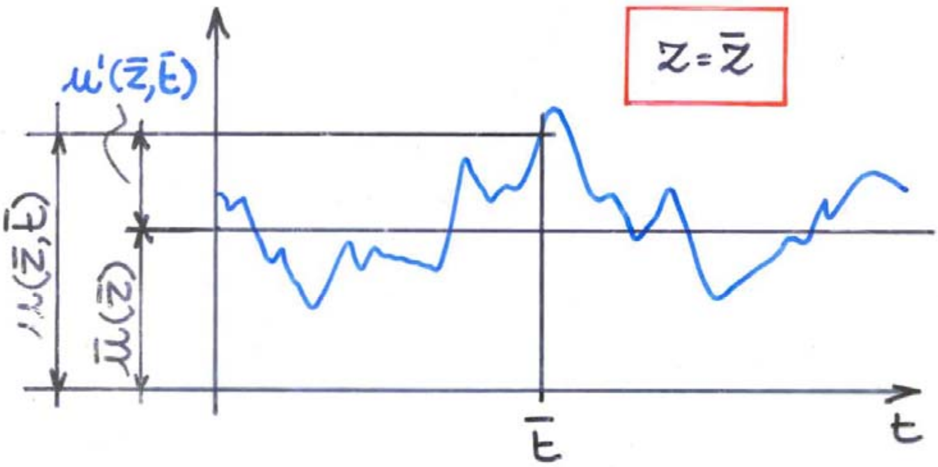
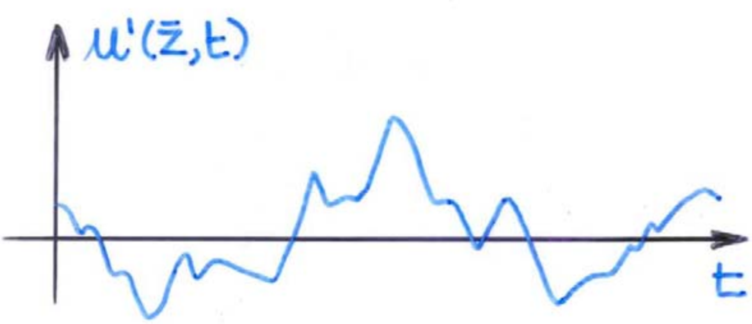
ATMOSPHERIC TURBULENCE

Wind velocity is a three-dimensional field, whose conceptual modeling may be limited to a bi-dimensional representation. The following three schemes show: 1) the instantaneous wind velocity vertical profile at time $t = \overline{t}$, $u(z, \overline{t})$; 2) the time-history of the instantaneous wind velocity at height $z = \overline{z}$, $u(\overline{z}, t)$; 3) the time-history of the instantaneous longitudinal turbulence at height $z = \overline{z}$, $u'(\overline{z}, t) = u(\overline{z}, t) - \overline{u}(\overline{z})$.



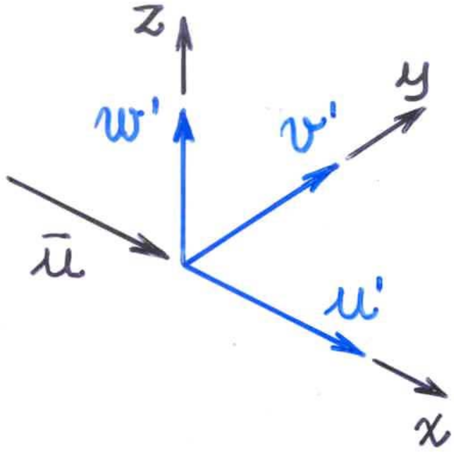




In more rigorous terms, let x,y,z be a Cartesian reference system with origin in O on the ground; z is vertical and directed upwards. The wind field is represented by the vectorial temporal law of velocity at point M of coordinates x,y,z:

$$\mathbf{V}(M;t) = \bar{\mathbf{V}}(M;t) + \mathbf{V}'(M;t)$$

where $\bar{\mathbf{V}}$ and $\mathbf{V}'$ are two vectors that denote, respectively, the mean wind velocity and the zero mean turbulent fluctuation of $\mathbf{V}$ around $\bar{\mathbf{V}}$.



Considering a flat homogeneous terrain and the inner boundary layer, they are given by:

$$\bar{\mathbf{V}}(M;t) = \mathbf{i}\,\bar{u}(z)$$

$$\mathbf{V}'(M;t) = \mathbf{i}\,u'(M;t) + \mathbf{j}\,v'(M;t) + \mathbf{k}\,w'(M;t)$$

where **i**, **j**, **k** are the unit vectors in the directions x, y, z; $\bar{u}$ is the mean wind velocity aligned with x; u', v', w' are the longitudinal (x), lateral (y) and vertical (z) turbulence components.

The mean wind velocity is given by the logarithmic profile:

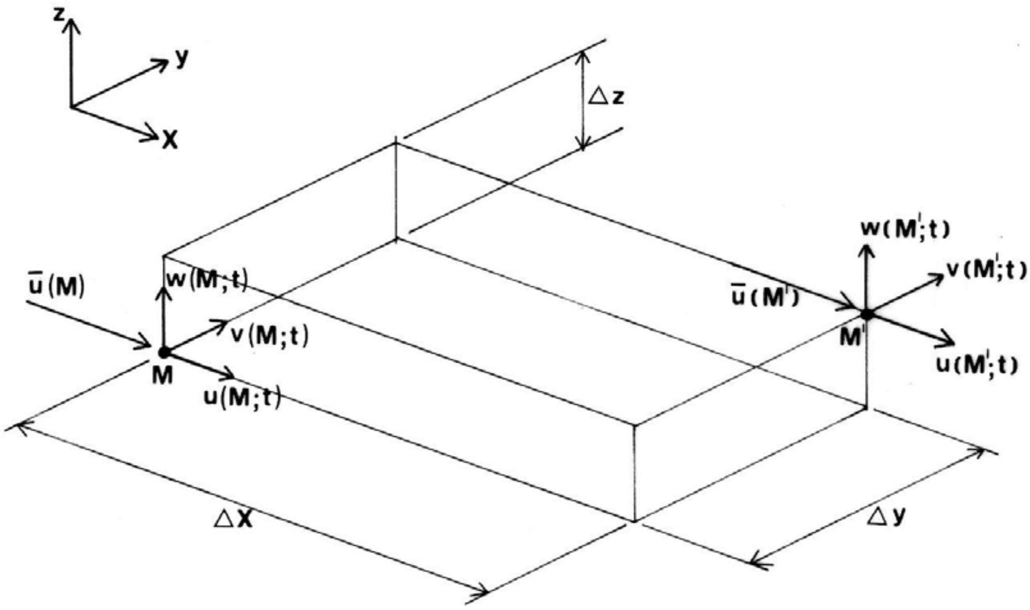
$$\bar{u}(z) = \frac{1}{\kappa} u_* ln\left(\frac{z}{z_0}\right)$$

where u_* is the shear velocity and z_0 is the roughness length.

Atmospheric turbulence is a 3-variate (3-V) 4-dimensional (4-D) random process. It is 3-V because it is described by a 3-component u', v', w'; it is 4-D because each of these components depends on 4 independent parameters, x, y, z, t. For these reason it is best described by the cross-power spectral density function (cpsdf):

$$S_{\varepsilon\eta}(M,M';n) = \sqrt{S_\varepsilon(z;n)S_\eta(z';n)}\,Coh_{\varepsilon\eta}(M,M';n)$$

where n is frequency; M' is a point of coordinates x', y', z'; $S_{\varepsilon\eta}(M,M';n)$ is the cpsdf of $\varepsilon'(M;t)$ and $\eta'(M';t)$; $S_\varepsilon(z;n) = S_{\varepsilon\varepsilon}(M,M;n)$ is the power spectral density function (psdf) of $\varepsilon'(M;t)$; $Coh_{\varepsilon\eta}(M,M';n)$ is the coherence function (cohf) of $\varepsilon'(M;t)$ and $\eta'(M';t)$. For sake of simplicity, $Coh_\varepsilon(M,M';n) = Coh_{\varepsilon\varepsilon}(M,M';n)$.



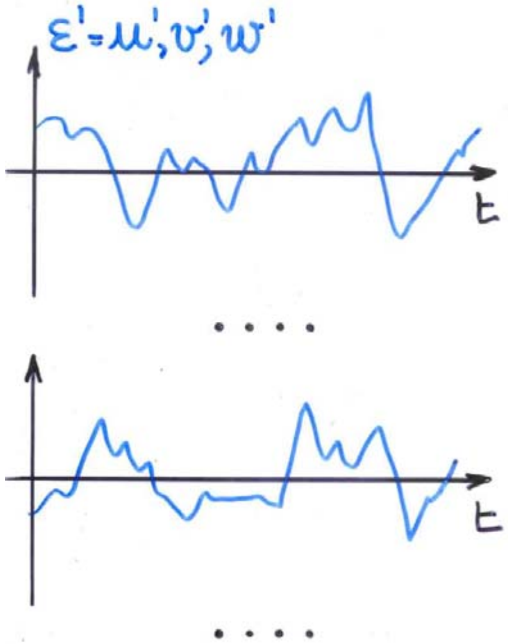
Atmospheric turbulence is thus described by four sets of functions:

1) the psdf $S_\varepsilon(z;n)$ ($\varepsilon = u,v,w$) of each turbulence component in a point M of space;
2) the cohf $Coh_{\varepsilon\eta}(z;n) = Coh_{\varepsilon\eta}(M,M;n)$ of the three turbulence components in a point M of space;
3) the cohf $Coh_\varepsilon(M,M';n) = Coh_{\varepsilon\varepsilon}(M,M';n)$ of the same turbulence component in two different points M and M' of space;
4) the cohf $Coh_{\varepsilon\eta}(M,M';n)$ of different turbulence components in two different points M and M' of space.

Wind engineering applications generally deal with cross-power spectral densities and coherence functions of turbulence as real functions. This is equivalent to assume, with a reasonable approximation, that imaginary parts may be neglected. Advanced model that take into account imaginary parts have been formulated by ESDU (1991) and by Mann (1994).

One-point / one-turbulence-component representation

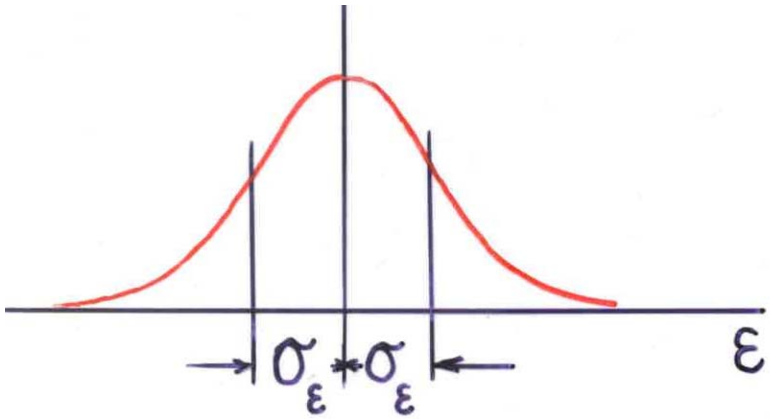
Each turbulence component ε' in a point M of space is a 1-V 1-D random stationary Gaussian process, which can be represented by a family of sample function.



The first order representation of ε' is provided by its probability density funcion (pdf):

$$f_\varepsilon(\varepsilon) = \frac{1}{\sqrt{2\pi}\sigma_\varepsilon} \exp\left\{ -\frac{\varepsilon^2}{2\sigma_\varepsilon^2} \right\}$$

where σ_ε is the standard deviation of ε'.



Turbulence standard deviation

In general terms, σ_u, σ_v and σ_w depend in a rather complicate way on: 1) the height over the ground z; 2) the roughness length z_0; 3) the shear velocity u_*; 4) the Coriolis parameter f.

In the internal boundary layer it is usual to assume, with reasonable approximation:

$$\sigma_\varepsilon^2 = \beta_\varepsilon u_*^2 \quad (\varepsilon = u, v, w)$$

where u_* is the shear velocity; β_ε is a non-dimensional turbulence factor such as: 1) it is independent of z; 2) it reduces on increasing z_0; 3) $\beta_u > \beta_v > \beta_w$ and, consequently:

$$\sigma_u^2 > \sigma_v^2 > \sigma_w^2$$

Based upon a large set of experimental measurements (Solari and Piccardo, 2001):

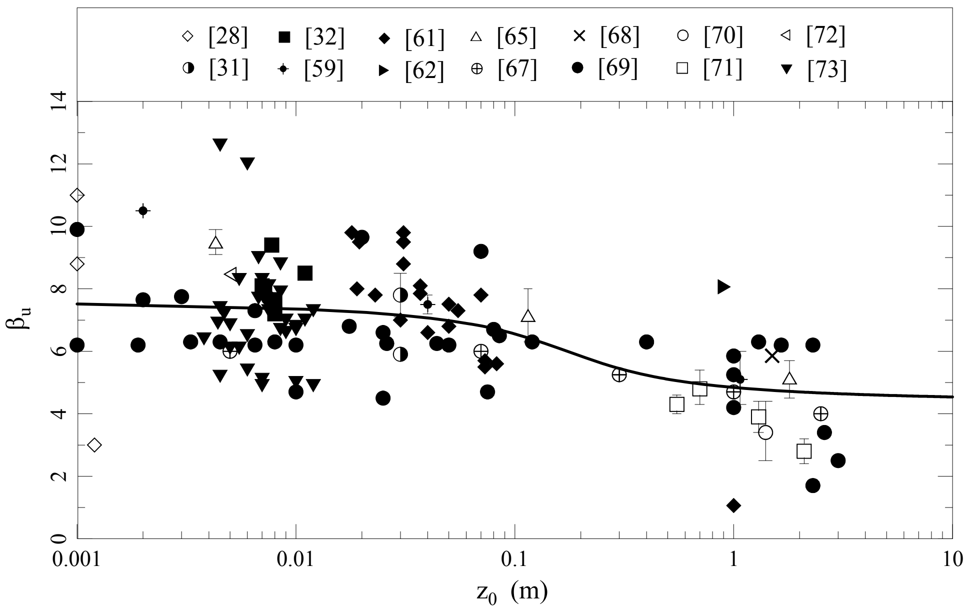
$$\beta_u = 6 - 1{,}1 \cdot \mathrm{arctg}\left[\ln(z_0) + 1{,}75\right]$$

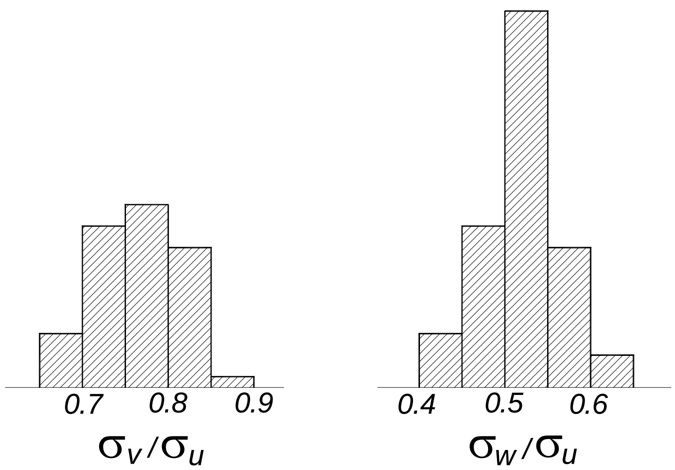
$$\beta_v = 0{,}56 \cdot \beta_u; \quad \sigma_v = 0{,}75 \cdot \sigma_u$$

$$\beta_w = 0{,}25 \cdot \beta_u; \quad \sigma_w = 0{,}5 \cdot \sigma_u$$

where z is expressed in meters.

The following pictures show the variability range of the measurements with respect to the above empirical rules.





Turbulence intensity

Wind engineering is usual to represent the first order statistical moment of turbulence through a non-dimensional parameter referred to as turbulence intensity:

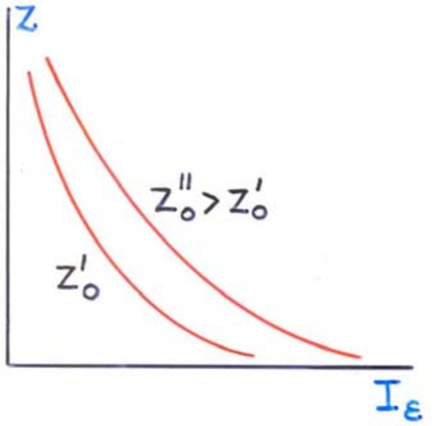
$$I_\varepsilon(z) = \frac{\sigma_\varepsilon(z)}{\bar{u}(z)} \qquad (\varepsilon = u, v, w)$$

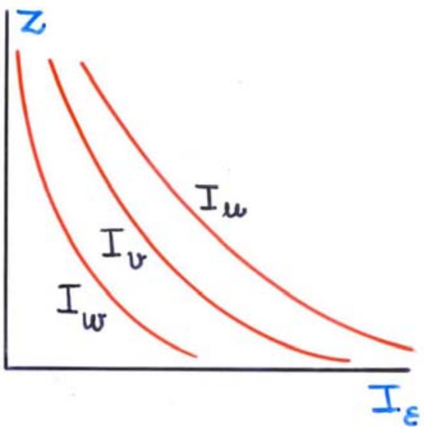
Remembering that $\bar{u}(z) = 2{,}5\,u_*\,ln(z/z_0)\cdot c_t(z)$ and $\sigma_\varepsilon^2 = \beta_\varepsilon u_*^2$, it follows that:

$$I_\varepsilon(z) = \frac{0.4\sqrt{\beta_\varepsilon}}{\ell n(z/z_0)\cdot c_t(z)}$$

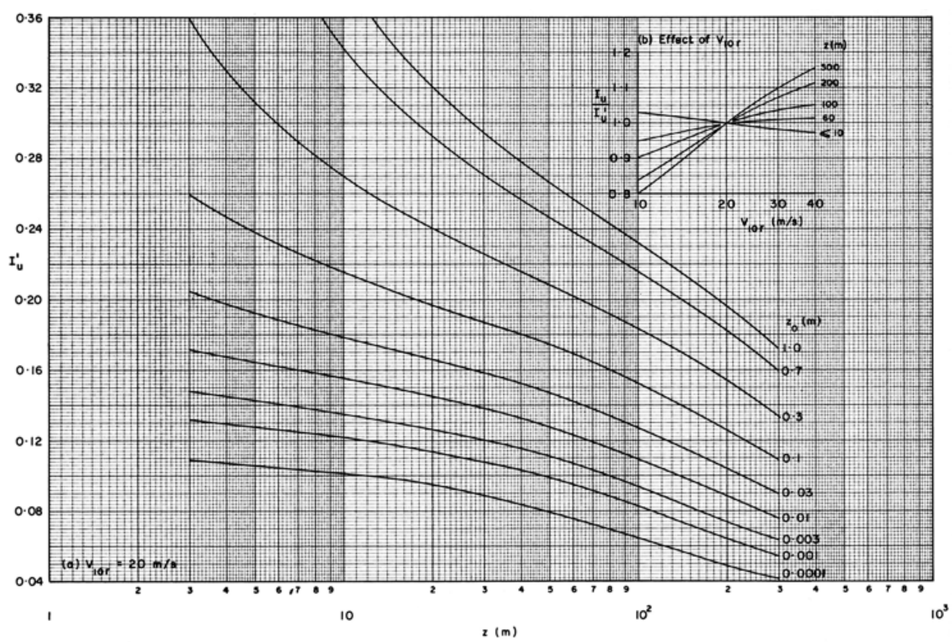
Since $0.4\sqrt{\beta_u} \sim 1$, it results:

$$I_u(z) = \frac{1}{\ell n(z/z_0)\cdot c_t(z)};\quad I_v(z) = 0.75\cdot I_u(z);\quad I_w(z) = 0.50\cdot I_u(z)$$



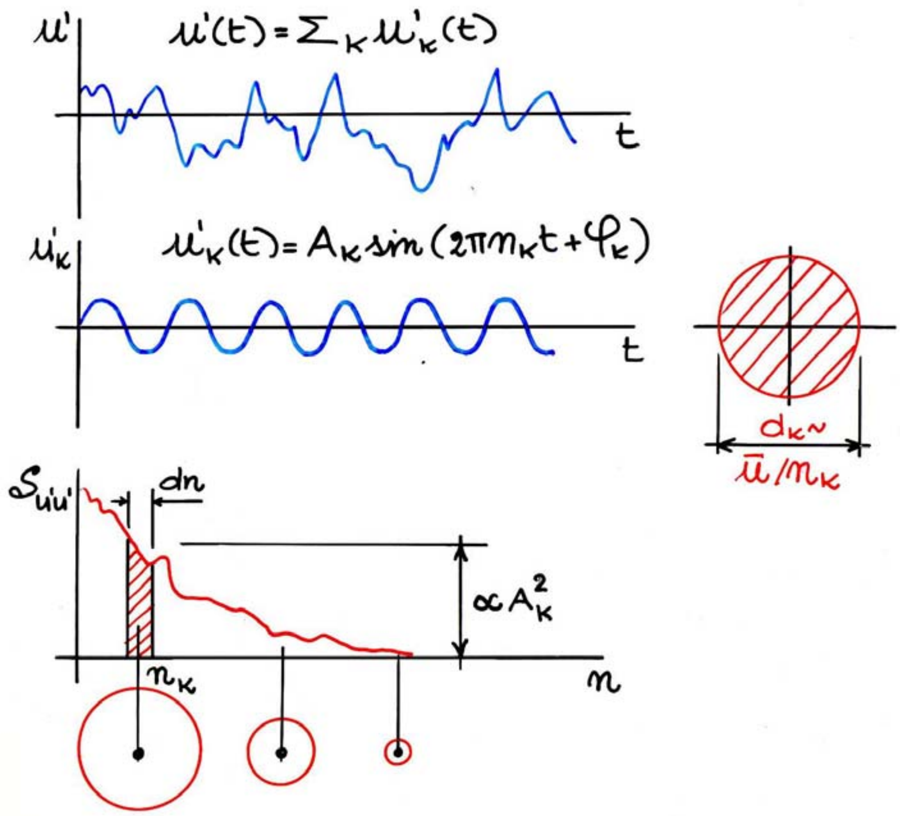


The following picture shows the diagram of the longitudinal turbulence intensity as provided by ESDU, based upon an advanced parametric model, assuming $c_t = 1$.

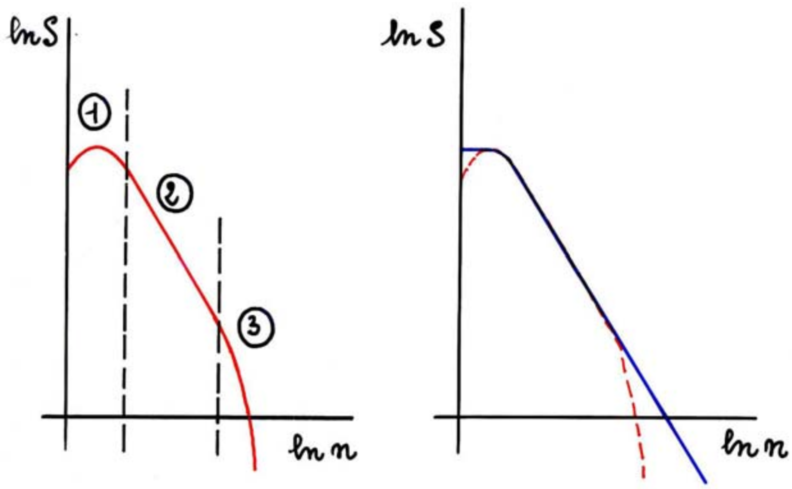


<u>Energy cascade</u>

Turbulent fluctuations may be schematized as the superposition of eddies in periodic motion with wave number $K = 2\pi n / \bar{u}$; each eddy may be idealized as sphere with diameter $d = \bar{u} / n$. The psdf of turbulence describes the distribution of the energy content associated with eddies with different wave number and diameter. Mechanical and thermal convections produce low frequency kinetic energy associated with large eddies. In the inertial sub-range there is neither production nor dissipation of energy, but an energy transfer from larger to smaller eddies. In the high frequency range small eddies dissipate viscous energy. This sequence is well known as <u>energy cascade</u>.



The following picture shows the psdf of turbulence in a bi-logarithmic scale; the first scheme corresponds to actual phenomenon of the energy cascade, the second scheme illustrates the representation usually adopted in wind engineering.



In the low frequency range:

$$S(n) \propto n^{0}$$

In the inertial sub-range:

$$S(n) \propto n^{-5/3}$$

In the high frequency range:

$$S(n) \propto n^{-k} \quad (k \approx 7)$$

Wind engineering models overestimate the energy content of turbulence in the high frequency range; this is often reasonable since in this domain energy is small.

Von Karman spectrum

In 1948 von Karman developed an extensive series of experiments during which he created a turbulent homogeneous (independent of the translation of the reference system) and isotropic (independent of the rotation of the reference system) flow in a wind tunnel. Introducing the results in the theory of turbulence, he proposed the spectral equations:

$$\frac{nS_u(n)}{\sigma_u^2} = \frac{4\frac{fL_u}{z}}{\left[1+70.8\left(\frac{fL_u}{z}\right)^2\right]^{5/6}}$$

$$\frac{nS_\varepsilon(n)}{\sigma_\varepsilon^2} = \frac{4\frac{fL_\varepsilon}{z}\left[1+755.2\left(\frac{fL_\varepsilon}{z}\right)^2\right]}{\left[1+283.2\left(\frac{fL_\varepsilon}{z}\right)^2\right]^{11/6}} \quad (\varepsilon = v, w)$$

where $f = nz/\bar{u}$ is the reduced frequency, also known as Monin coordinate; L_ε is the integral length scale of the ε turbulence component (ε = u, v, w) in the x direction. This latter quantity identifies the average size along x of turbulence eddies associated with $\varepsilon'(t)$. In mathematical terms it is defined as:

$$^{x}L_\varepsilon = \frac{1}{\sigma_\varepsilon^2}\int_0^\infty {}^{x}R_\varepsilon(r_x)\,dr_x \quad (\varepsilon = u, v, w)$$

$^{x}R_\varepsilon(r_x) = E\left[\varepsilon'(x,y,z;t)\,\varepsilon'(x',y,z;t)\right]$ being the auto-correlation function of $\varepsilon'(t)$, as a function of $r_x = |x - x'|$.

The time scale of turbulence is given by:

$$T_\varepsilon = \frac{1}{\sigma_\varepsilon^2}\int_0^\infty {}^tR_\varepsilon(\tau)\,d\tau \quad (\varepsilon = u,v,w)$$

where ${}^tR_\varepsilon(\tau) = E\left[\varepsilon'(x,y,z;t)\varepsilon'(x,y,z;t')\right]$ is the auto-correlation function of $\varepsilon'(t)$, as a function of the time lag $\tau = |t-t'|$.

This problem greatly simplifies, assuming that the turbulent field is frozen in space, and it is convected with velocity $\bar{u}$. This assumption is known as Taylor's hypothesis and provides the relationship:

$${}^tR_\varepsilon(\tau) = {}^xR_\varepsilon(r_x) = {}^xR_\varepsilon(\bar{u}\tau) \Rightarrow {}^xL_\varepsilon = \bar{u}T_\varepsilon$$

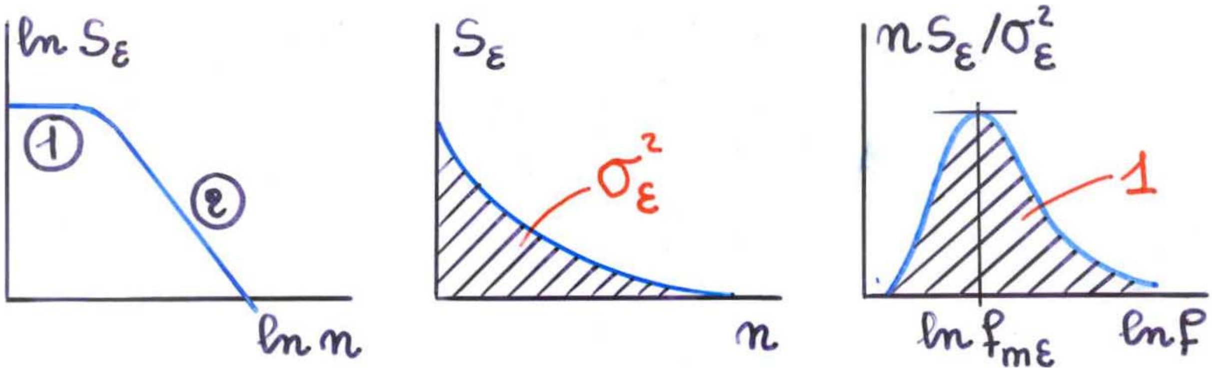
Using Wiener-Khintchine equations:

$$S_\varepsilon(n) = 4\int_0^\infty {}^tR_\varepsilon(\tau)\cos(2\pi n\tau)\,d\tau \Rightarrow$$

$$S_\varepsilon(0) = 4\int_0^\infty {}^tR_\varepsilon(\tau)\,d\tau = 4T_\varepsilon\sigma_\varepsilon^2 = \frac{4\,{}^xL_\varepsilon\sigma_\varepsilon^2}{\bar{u}} \Rightarrow$$

$${}^xL_\varepsilon = \frac{\bar{u}S_\varepsilon(0)}{4\sigma_\varepsilon^2}$$

The following figure shows three classical representations of the Karman spectrum.



The Karman spectrum has three fundamental properties:

1) for n and f tending to zero, namely in the small frequency range, $S(n) \propto n^0$:

$$S_\varepsilon(n) \cong \frac{4L_\varepsilon\sigma_\varepsilon^2}{\bar{u}}$$

2) for n and f large enough, namely in the inertial sub-range, $S(n) \propto n^{-5/3}$:

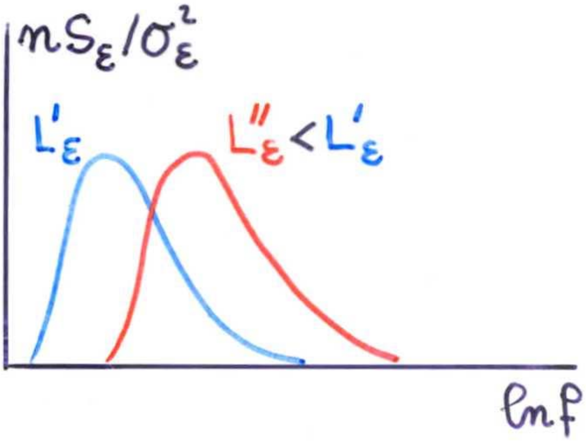
$$\frac{nS_\varepsilon(n)}{\sigma_\varepsilon^2} \cong \alpha_\varepsilon f^{-2/3}$$

where $\alpha_u = 0.115$, $\alpha_v = \alpha_w = 0.0965$; this condition perfectly matches the theories developed by Kolmogorov (1941) and Batchelor (1953).

3) the following relationship holds:

$$\frac{d}{d\ln(f)}\left[\frac{nS_\varepsilon(n)}{\sigma_\varepsilon^2}\right]=0 \Rightarrow f_{m\varepsilon}=\frac{1}{d_\varepsilon}\frac{z}{L_\varepsilon}$$

where $d_u = 6.868, d_v = d_w = 9.434$.

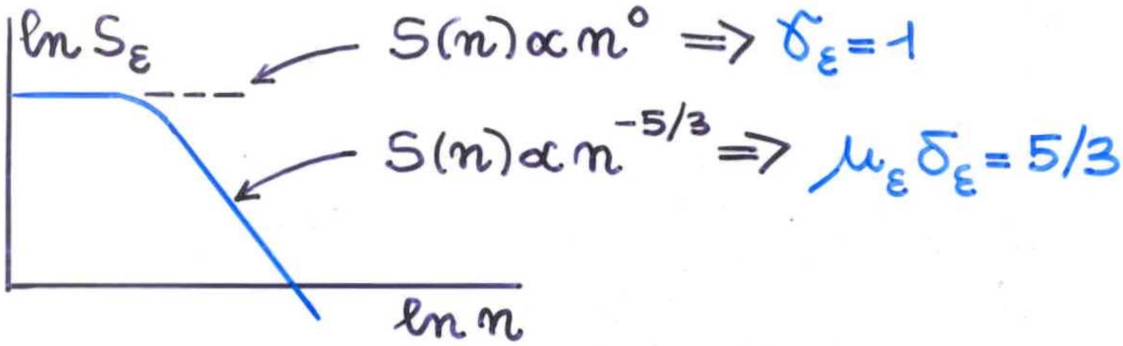


Generalized turbulence spectrum

In general terms, the spectrum of turbulence may be expressed as (Olesen et al, 1984; Tieleman, 1995):

$$\frac{nS_\varepsilon(z,n)}{u_*^2}=\frac{A_\varepsilon f^{\gamma_\varepsilon}}{\left(1+B_\varepsilon f^{\mu_\varepsilon}\right)^{\delta_\varepsilon}}$$

where $f = nz/\bar{u}(z)$ is the reduced frequency. The non-dimensional parameters A_ε, B_ε, γ_ε, μ_ε and δ_ε shall be selected in order to satisfy the main spectral properties. The following figure illustrates the criterion in accordance with which $\gamma_\varepsilon = 1$, $\mu_\varepsilon\delta_\varepsilon = 5/3$.



Thus, the turbulence spectrum may be rewritten as (Fichtl & Mc Vehil, 1970):

$$\frac{nS_\varepsilon(z,n)}{u_*^2}=\frac{A_\varepsilon f}{\left(1+B_\varepsilon f^{\mu_\varepsilon}\right)^{5/(3\mu_\varepsilon)}}$$

Assuming $\mu_\varepsilon = 5/3$ provides the <u>pointed model</u>:

$$\frac{nS_\varepsilon(z,n)}{u_*^2} = \frac{A_\varepsilon f}{\left(1 + B_\varepsilon f^{5/3}\right)}$$

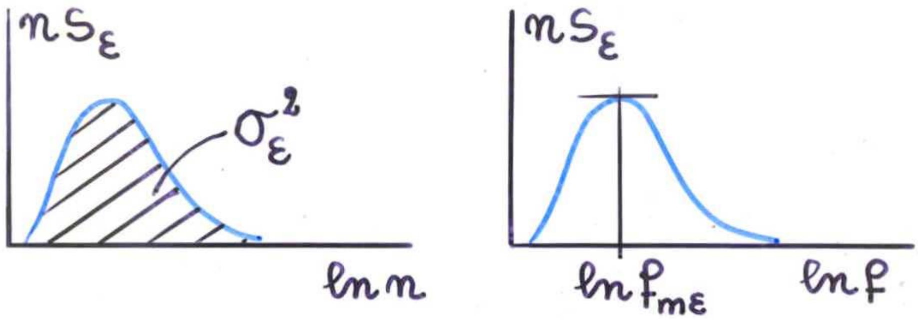
Assuming $\mu_\varepsilon = 1$ provides the <u>blunt model</u>:

$$\frac{nS_\varepsilon(z,n)}{u_*^2} = \frac{A_\varepsilon f}{\left(1 + B_\varepsilon f\right)^{5/3}}$$

Adopting the blunt model, Solari and Piccardo (2001) first imposed the condition:

$$\sigma_\varepsilon^2 = \beta_\varepsilon u_*^2 = \int_0^\infty S_\varepsilon(z,n)\,dn \Rightarrow A_\varepsilon = \frac{\beta_\varepsilon B_\varepsilon}{1.5} = \frac{\beta_\varepsilon}{f_{m\varepsilon}} \Rightarrow$$

$$\frac{nS_\varepsilon(z,n)}{\sigma_\varepsilon^2} = \frac{f / f_{m\varepsilon}}{\left(1 + 1.5 f / f_{m\varepsilon}\right)^{5/3}}$$



Matching the parameters implicit in the Karman spectrum:

$$f_{m\varepsilon} = \frac{1}{d_\varepsilon}\frac{z}{L_\varepsilon(z)} \Rightarrow$$

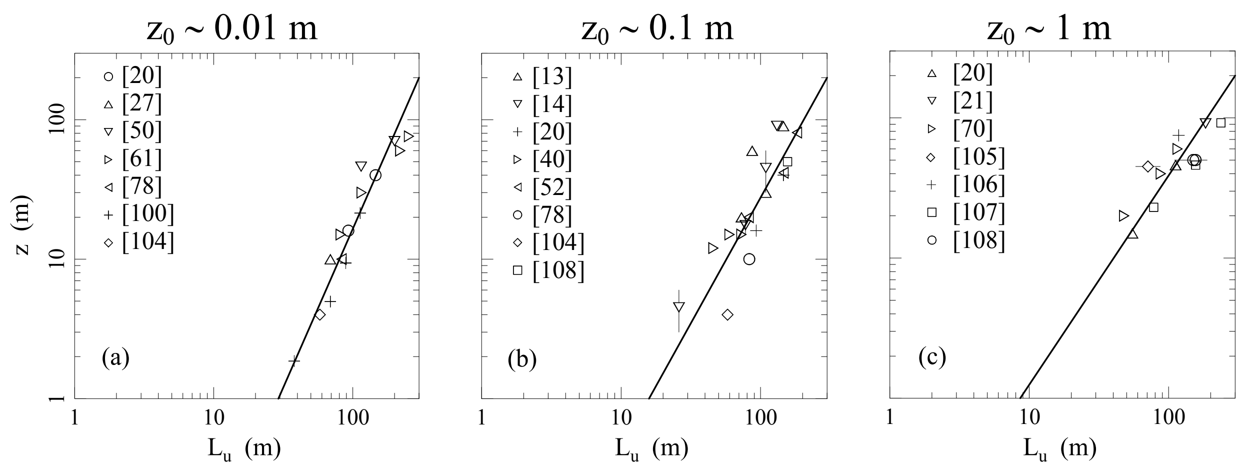
$$\frac{nS_\varepsilon(z,n)}{\sigma_\varepsilon^2} = \frac{d_\varepsilon n L_\varepsilon(z)/\bar{u}(z)}{\left[1 + 1.5 d_\varepsilon n L_\varepsilon(z)/\bar{u}(z)\right]^{5/3}} \qquad (\varepsilon = u, v, w)$$

where $d_u = 6.868, d_v = d_w = 9.434$.

Besides, selecting a set of measurements coherent with the spectrum requirements, they provided the empirical relationships:

$$L_u(z) = \bar{L}\left(\frac{z}{\bar{z}}\right)^\nu \;;\; L_v(z) = 0.25 \cdot L_u(z) \;;\; L_w(z) = 0.10 \cdot L_u(z)$$

where $\nu = 0.67 + 0.05 \cdot \ln(z_0)$, $\bar{L}$ 300 m, $\bar{z}$ = 200 m, z_0 is expressed in meters. The following diagrams, corresponding to different roughness length, show the fitting of experimental data by the above empirical model.



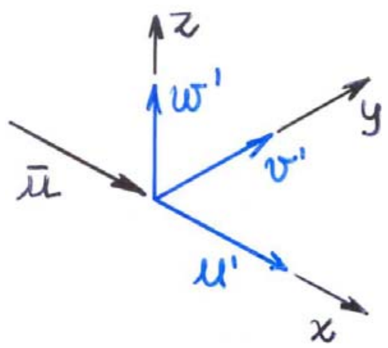
<u>One-point / three-turbulence-components representation</u>

Theory and experimental evidence show that:

u', v' statistically independent $\Rightarrow \overline{u'v'} = 0,\ S_{uv}(n) = 0$

$v'w'$ statistically independent $\Rightarrow \overline{v'w'} = 0,\ S_{vw}(n) = 0$

u', w' strongly correlated, with $\overline{u'w'} = -u_*^2$



A reasonable expression of the cross-power spectral density function (cpsdf) of u', w' is given by Teunissen (1970), ESDU (1974)

$$\frac{nS_{uw}(z,n)}{u_*^2} = -\frac{1}{A_{uw}(z)} \frac{\sqrt{nS_u(z,n)}\sqrt{nS_w(z,n)}}{\sigma_u(z)\sigma_w(z)} \cdot \frac{1}{\sqrt{1+0.4\left[nL_u(z)\bar{u}(z)\right]^2}}$$

$$A_{uw}(z) = \frac{1}{\sigma_u(z)\sigma_w(z)} \int_0^\infty \frac{\sqrt{nS_u(z,n)}\sqrt{nS_w(z,n)}}{\sqrt{1+0.4\left[nL_u(z)\bar{u}(z)\right]^2}} dn = 1.11\left[L_w(z)/L_u(z)\right]^{0.21}$$

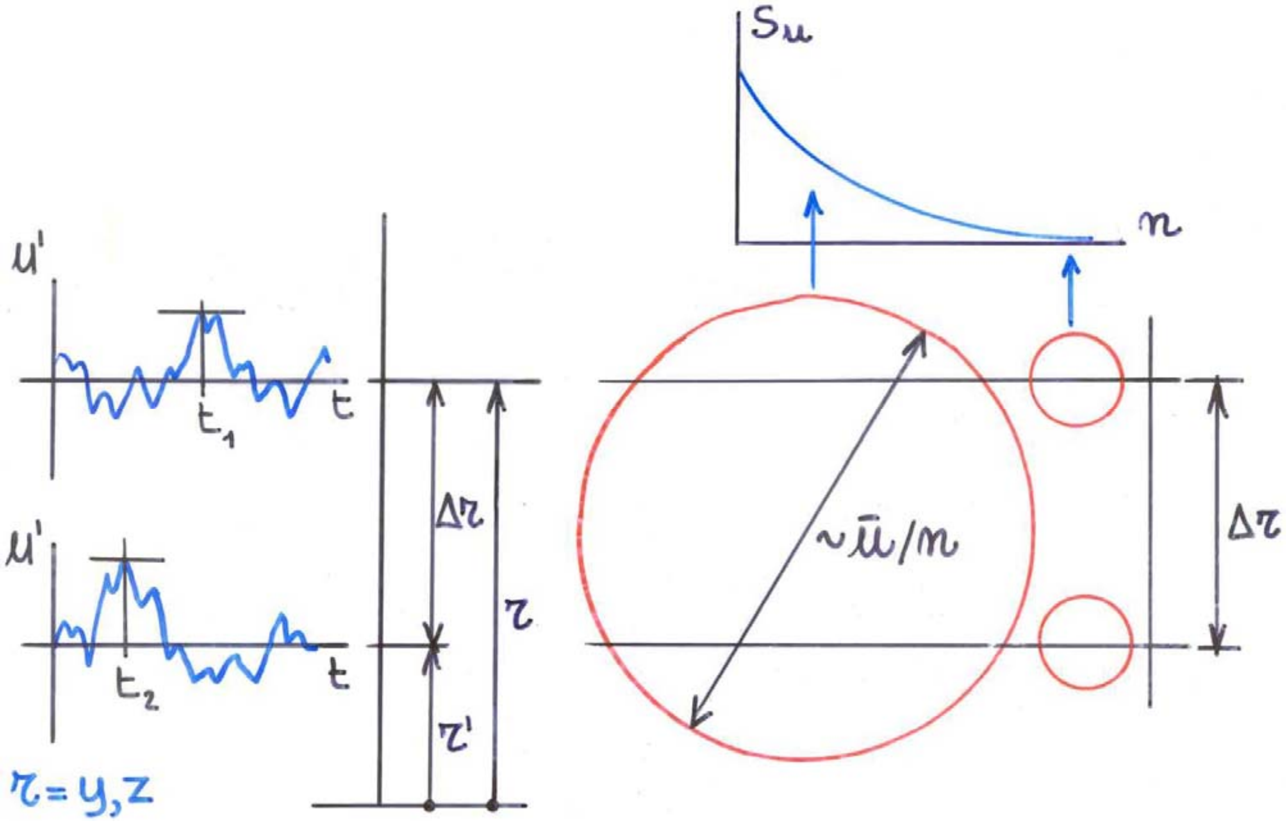
Two-points / one-turbulence-component representation

The cpsdf of the same turbulence component in two points of space is given by:

$$S_{\varepsilon}(M,M';n) = \sqrt{S_{\varepsilon}(z;n)S_{\varepsilon}(z';n)}\,Coh_{\varepsilon}(M,M';n)$$

where n is frequency; M and M' are two points with coordinates x, y, z and x', y', z'; $S_\varepsilon(z;n)$ is the psdf of ε'(M;t); $Coh_{\varepsilon\eta}$(M,M';n) is the cohf of ε'(M;t) and ε'(M';t). This latter function tends to decrease on increasing the distance between these points and the wave number $K = 2\pi n / \bar{u}$.

Line coherence function

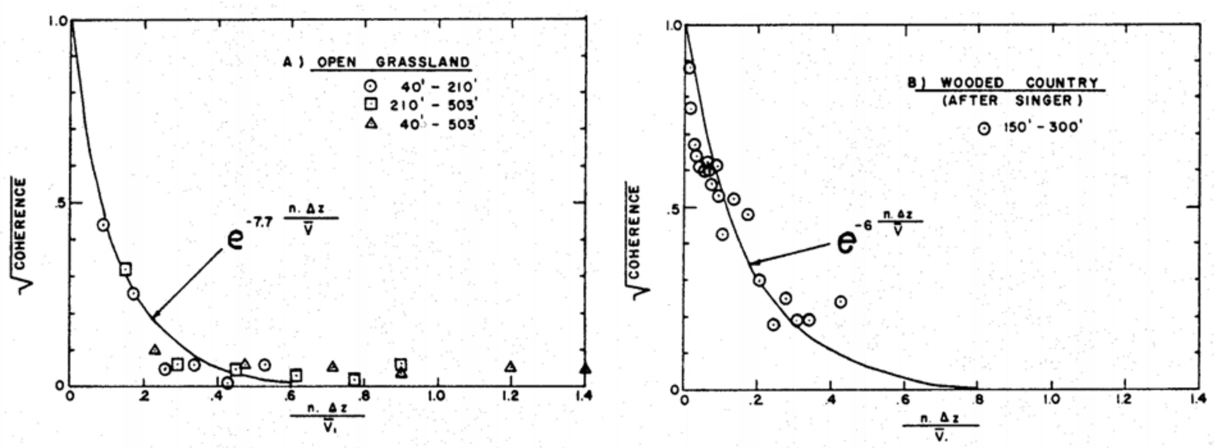


This concept was introduced by Cramer (1959) and especially by Davenport (1961), who proposed a simple empirical model to express the coherence of the longitudinal turbulence component, between any couple of points aligned in the y or z directions.

$$Coh_{u}(r,r';n) = \exp\left\{ -\frac{2nC_{ru}|r-r'|}{\bar{u}(r)+\bar{u}(r')} \right\}$$

where C_{ru} is the exponential decay coefficient of the turbulence component u' in the directions r = y, z.

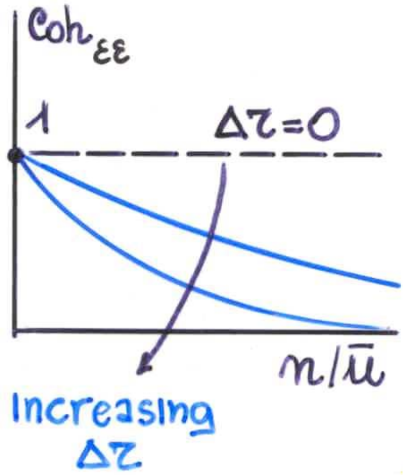
The following figure shows the first diagrams provided by Davenport in 1961.



Subsequent analyses carried out by Pielke and Panofsky (1970) extended the previous formula to any turbulence component and to any direction aligned with the Cartesian axes x, y, z:

$$\mathrm{Coh}_{\varepsilon}(r,r';n)=\exp\left\{-\frac{2nC_{r\varepsilon}|r-r'|}{\bar{u}(r)+\bar{u}(r')}\right\} \qquad \varepsilon=u,v,w;\ r=x,y,z$$

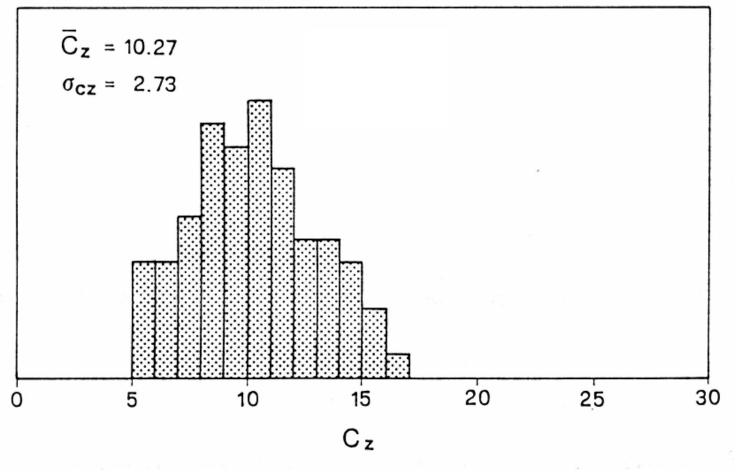
where $C_{r\varepsilon}$ is the exponential decay coefficient of the turbulence component ε' = u', v', w' in the directions r = x, y, z.



The following table provides average values of the exponential decaying coefficients.

C_{xu}	C_{xv}	C_{xw}	C_{yu}	C_{yv}	C_{yw}	C_{zu}	C_{zv}	C_{zw}
3	3	0.5	10	6.5	6.5	10	6.5	3

The following histogram, of experimental nature, shows the typical spread of one of the above coefficients.

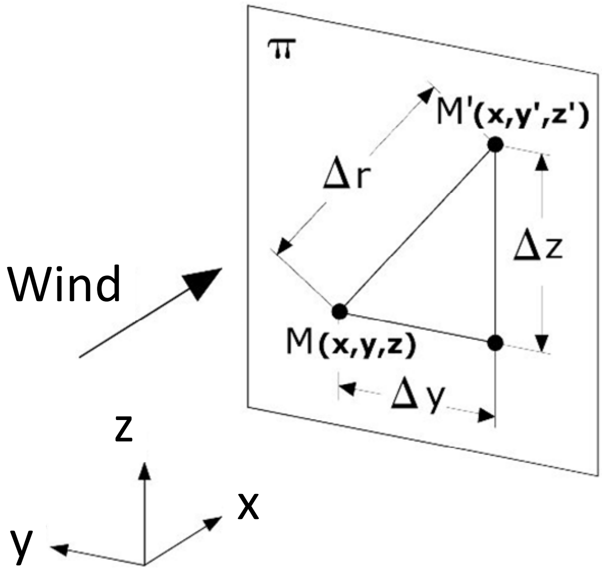


Space coherence function

The above formulation concerns the coherence function of any turbulence component in any direction aligned with the Cartesian axes x, y, z.

Vickery (1970) extended this formulation to the coherence function of the longitudinal turbulence component in a plane orthogonal to the mean wind direction x:

$$Coh_u(M,M';n) = \exp\left\{-\frac{2n\sqrt{C_{yu}^2(y-y')^2 + C_{zu}^2(z-z')^2}}{\bar{u}(z)+\bar{u}(z')}\right\}$$



This expression was extended later to any turbulence component in any point M and M' in the space:

$$Coh_\varepsilon(M,M';n) = \exp\left\{\frac{-2n\sqrt{\Sigma_r C_{r\varepsilon}^2(r-r')^2}}{\bar{u}(z)+\bar{u}(z')}\right\}$$

where the sum over r is carried out for r = x, y, z.

Two-points / three-turbulence-components representation

The cross-power spectral density matrix of the three turbulence components in two different points M and M' in the space may be written as:

$$\mathbf{S}=\begin{bmatrix}\begin{bmatrix}S_u(z) & S_{uv}(z) & S_{uw}(z)\\ S_{vu}(z) & S_v(z) & S_{vw}(z)\\ S_{wu}(z) & S_{wv}(z) & S_w(z)\end{bmatrix} & \begin{bmatrix}S_u(M,M') & S_{uv}(M,M') & S_{uw}(M,M')\\ S_{vu}(M,M') & S_v(M,M') & S_{vw}(M,M')\\ S_{wu}(M,M') & S_{wv}(M,M') & S_w(M,M')\end{bmatrix}\\ \begin{bmatrix}S_u(M',M) & S_{uv}(M',M) & S_{uw}(M',M)\\ S_{vu}(M',M) & S_v(M',M) & S_{vw}(M',M)\\ S_{wu}(M',M) & S_{wv}(M',M) & S_w(M',M)\end{bmatrix} & \begin{bmatrix}S_u(z') & S_{uv}(z') & S_{uw}(z')\\ S_{vu}(z') & S_v(z') & S_{vw}(z')\\ S_{wu}(z') & S_{wv}(z') & S_w(z')\end{bmatrix}\end{bmatrix}$$

where the dependence on frequency has been omitted for sake of simplicity.

Since $S_{uv}(z;n) = S_{vw}(z;n) = 0$, it is usually assumed that also $S_{uv}(M,M';n) = S_{vw}(M,M';n) = 0$. Thus, the cross-power spectral density matrix of the three turbulence components in two different points M and M' in the space may be rewritten as:

$$\mathbf{S}=\begin{bmatrix}\begin{bmatrix}S_u(z) & 0 & S_{uw}(z)\\ 0 & S_v(z) & 0\\ S_{wu}(z) & 0 & S_w(z)\end{bmatrix} & \begin{bmatrix}S_u(M,M') & 0 & S_{uw}(M,M')\\ 0 & S_v(M,M') & 0\\ S_{wu}(M,M') & 0 & S_w(M,M')\end{bmatrix}\\ \begin{bmatrix}S_u(M',M) & 0 & S_{uw}(M',M)\\ 0 & S_v(M',M) & 0\\ S_{wu}(M',M) & 0 & S_w(M',M)\end{bmatrix} & \begin{bmatrix}S_u(z') & 0 & S_{uw}(z')\\ 0 & S_v(z') & 0\\ S_{wu}(z') & 0 & S_w(z')\end{bmatrix}\end{bmatrix}$$

Advanced models of $S_{uw}(M,M';n)$ have been proposed by Mann (1994) and by Tubino and Solari (2005).

Atmospheric thermal stratification

Atmospheric thermal stratification may be classified in accordance with two parameters, namely the Richardson number and the Obukhov length:

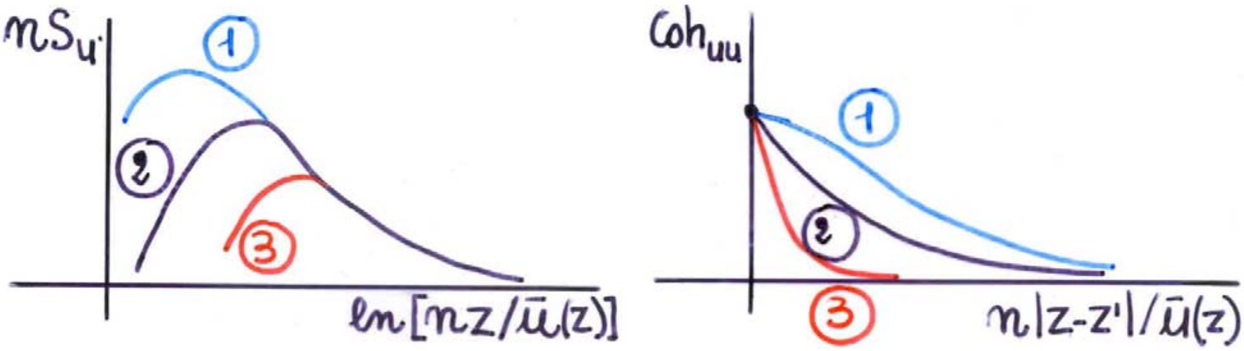
$$Ri=\frac{g}{\overline{T}}\frac{\Gamma-\Gamma_a}{\left(\partial\overline{u}/\partial z\right)^2}$$

$$L=\frac{u_*^3}{\kappa\frac{g}{\overline{T}}Q_0}$$

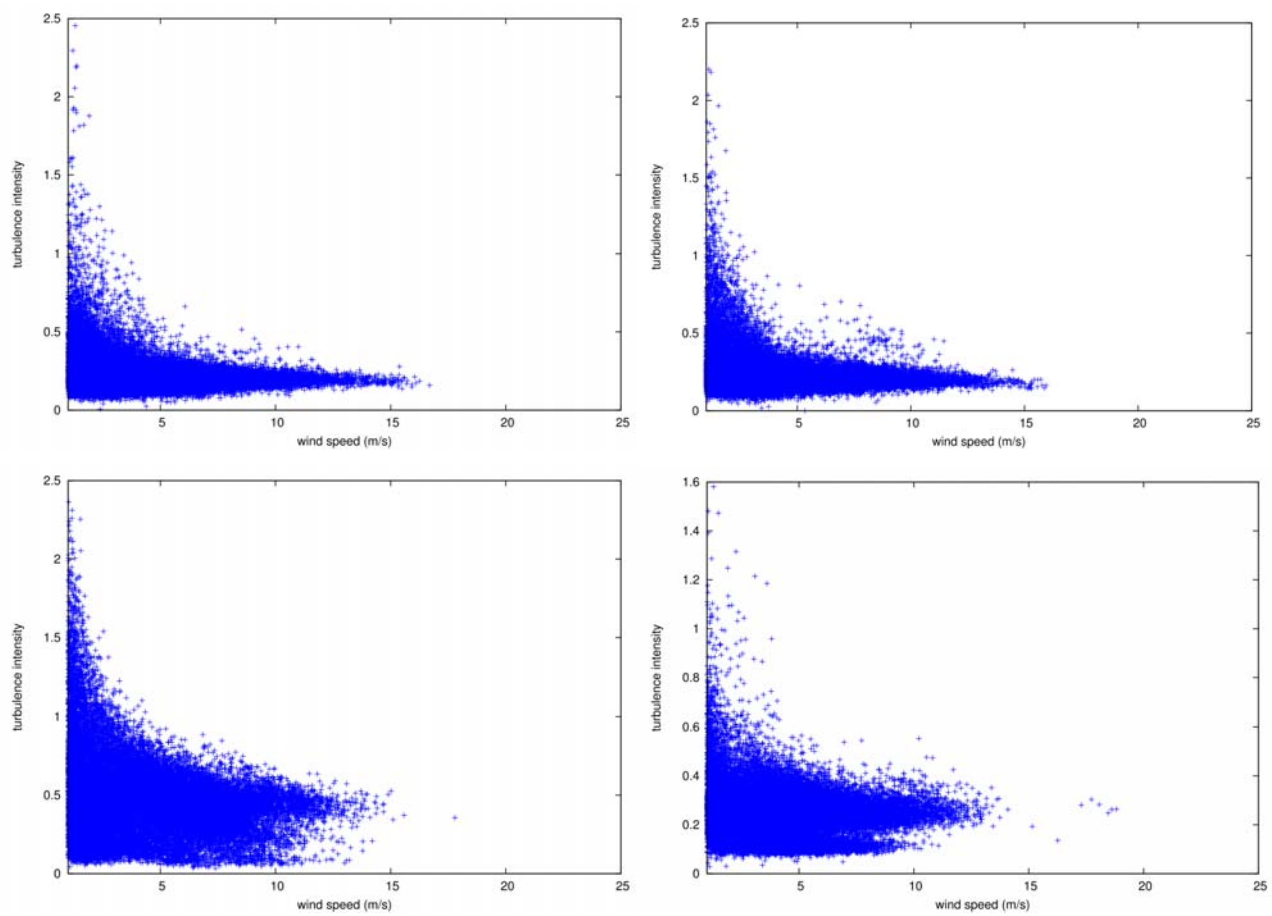
where g is the gravity acceleration, $\overline{T}$ is the mean atmospheric temperature, Γ is the lapse rate, $\Gamma_a = g/c_p$ is the dry adiabatic lapse rate, c_p is the specific heat at constant pressure, $Q_o = H_o/(\rho c_p)$, H_o is the vertical heat flux (positive upwards).

Atmospheric thermal stratification	Ri	1/L
Neutral	0	$\rightarrow \infty$
Stable	> 0	> 0
Unstable	< 0	< 0

Several theoretical and empirical models exist to represent the intensity and the psdf of the three turbulence components. The following figure shows the main properties of turbulence with reference to the atmospheric thermal stratification.

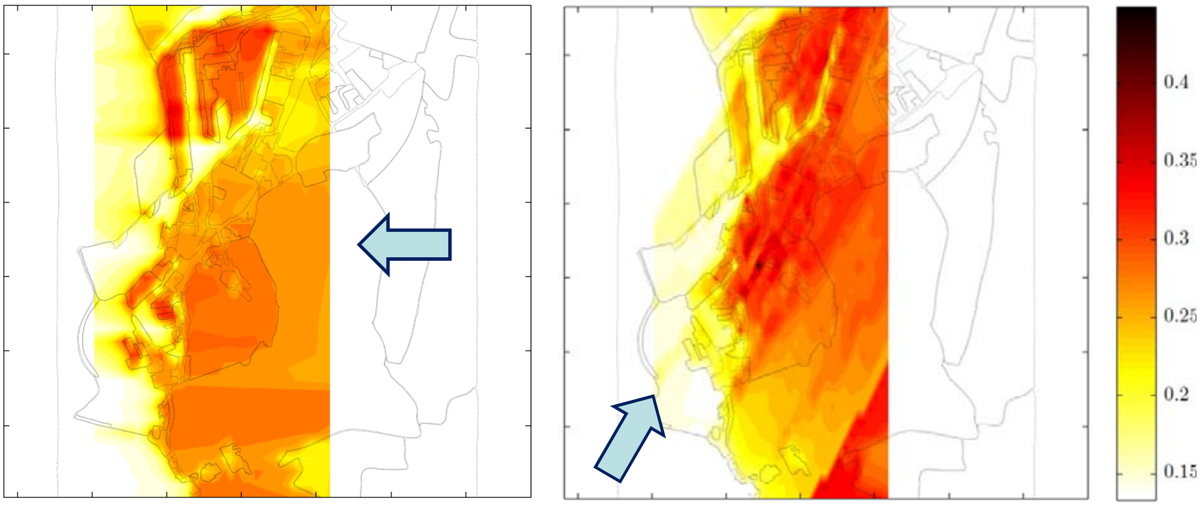


The following figures show some field measurements of the longitudinal turbulence intensity as a function of the mean wind velocity.

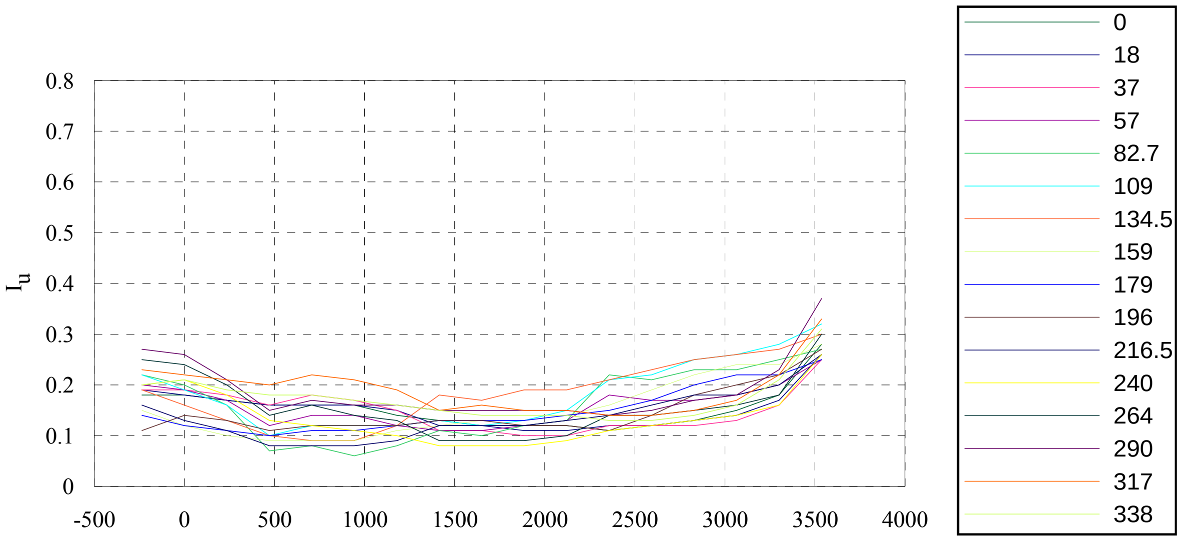


Complex terrains

All the previous discussion has been referred to flat homogeneous terrains. Computer programs are available to schematize turbulence parameters in complex terrains. One of these programs has been developed by ESDU to estimate turbulence intensity in terrains involving roughness changes and simple topographic features. This program has been improved and generalized by WinDyn taking into account, among other aspects, any complex terrain. The following figure shows the longitudinal turbulence intensity at 10 m height in the Port of Livorno.



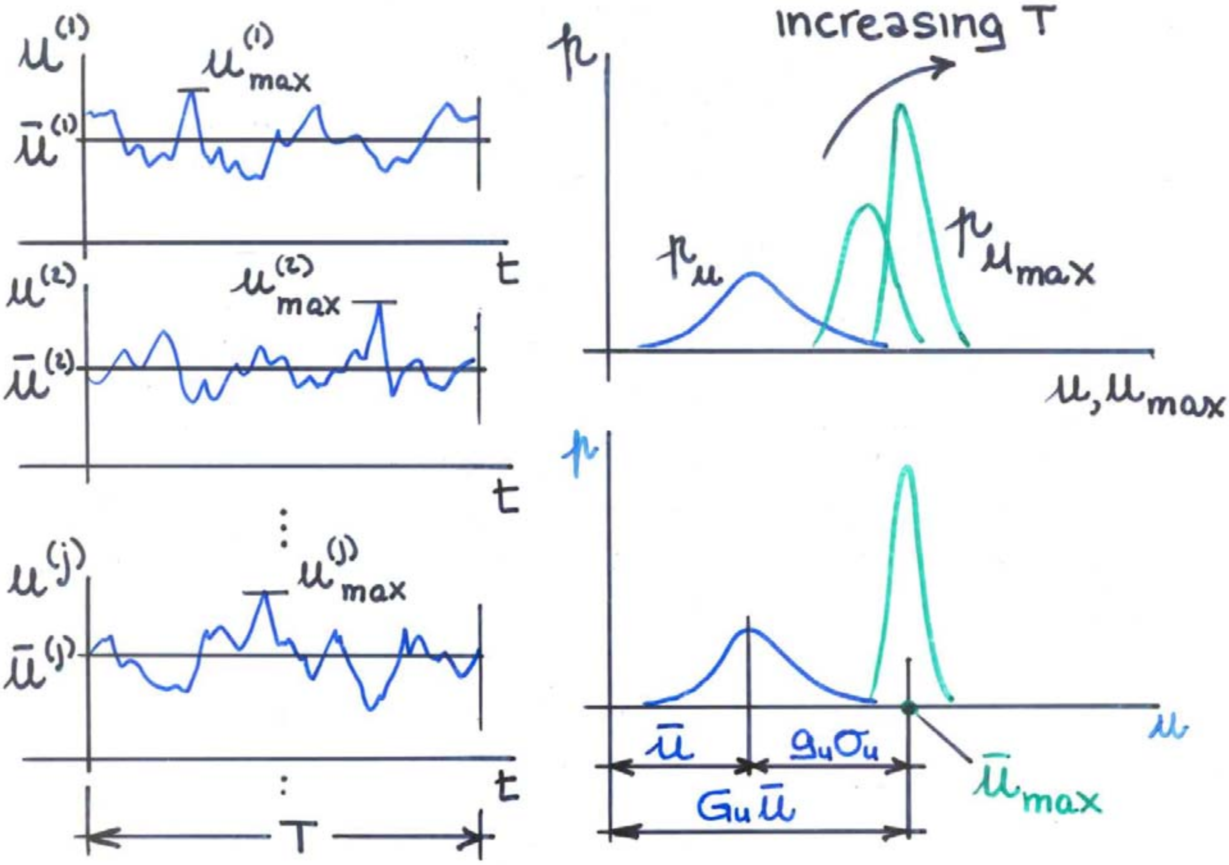
The following figure shows the longitudinal turbulence intensity as evaluated along the axis of the deck of the Messina Strait Bridge, on varying the wind direction. Atmosphere is neutrally stratified.



Turbulence intensity along the axis of the bridge deck (deg computed from North)

Peak wind velocity

The following treatment refers to the maximum wind velocity in the wind direction. This quantity is referred to as the peak wind velocity.



The mean value of the peak wind velocity is given by the relationship:

$$\bar{u}_{max} = \bar{u} + g_u \sigma_u$$

where $\bar{u}$ is the mean wind velocity, g_u is the velocity peak coefficient, σ_u is the standard deviation (rms value) of the longitudinal turbulence. It follows that:

$$\bar{u}_{max} = \bar{u}\left(1 + g_u \frac{\sigma_u}{\bar{u}}\right) = \bar{u}\left(1 + g_u I_u\right) \Rightarrow$$

$$\bar{u}_{max} = \bar{u} G_u$$

where G_u is referred to as the velocity gust factor:

$$G_u = 1 + g_u I_u$$

$$g_u = \sqrt{2\ell n\left(\nu_u T\right)} + \frac{0.5772}{\sqrt{2\ell n\left(\nu_u T\right)}}$$

$$\nu_u = \frac{1}{2\pi} \frac{\sigma_{\dot{u}}}{\sigma_u}$$

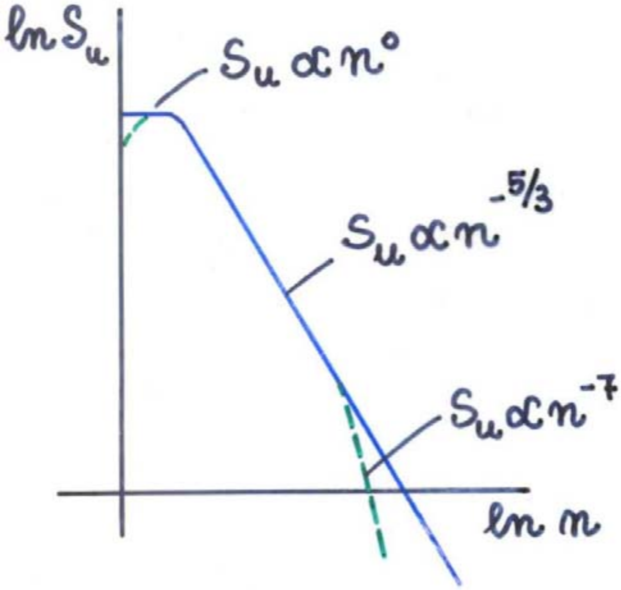
in which T = 10 minutes = 600 s is the length of the temporal interval where the peak wind velocity is determined, ν_u is the expected frequency of the longitudinal turbulence process, $\sigma_{\dot{u}}$ is the rms value of the wind acceleration $\dot{u}(t)$:

$$\sigma_{\dot{u}}^2 = \int_0^\infty S_{\dot{u}}(n)\,dn = \int_0^\infty (2\pi n)^2 S_u(n)\,dn$$

Experience shows that G_u typically varies between 1,3 and 1,7, assuming on average the indicative value G_u = 1,5.

It should be remembered that, for sake of simplicity, for lack of knowledge and on the safe side, all the psdf currently available assume that, in the high frequency range:

$$S(n) \propto n^{-5/3}$$



It follows that, for n tending to infinite:

$$S_{\dot{u}}(n) = (2\pi n)^2 S_u(n) \propto n^2 n^{-5/3} = n^{1/3}$$

Thus, $\sigma_{\dot{u}}^2$ assumes an infinite value, and the theory of maximum cannot be applied.

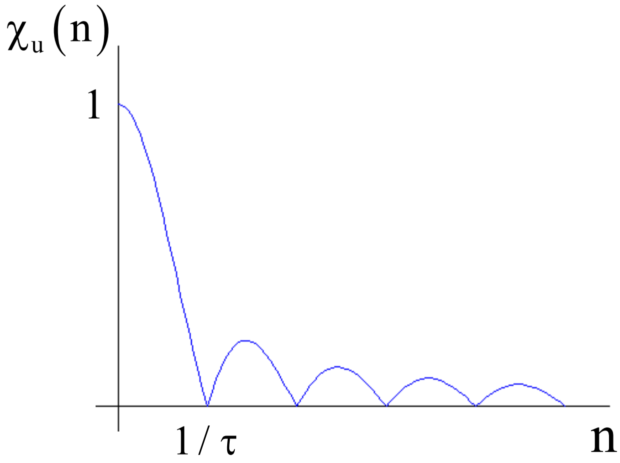
This problem is usually overcome by replacing the instantaneous velocity history u(t) by its mobile average over a short time interval τ. It is possible to demonstrate that such operation is equivalent to replace the power spectral density function $S_u(n)$ by the modified function:

$$S_u^*(n) = S_u(n)\chi_u^2(n)$$

where χ_u is a frequency filter defined as:

$$\chi_u(n) = \frac{\left|\sin(\pi n \tau)\right|}{\pi n \tau} \qquad n \geq 0$$

Since this operation is aimed at defining the gust peak, τ is also called the duration of the gust peak.



Based on this approach (Solari 1993):

$\bar{u}_{max} = \bar{u} G_u^*$

$G_u^* = 1 + g_u^* I_u \sqrt{P_0}$

$g_u^* = \sqrt{2\ell n(\nu_u^* T)} + \dfrac{0.5772}{\sqrt{2\ell n(\nu_u^* T)}}$; $\nu_u^* = \dfrac{\bar{u}}{L_u}\sqrt{\dfrac{P_1}{P_0}}$

$P_0 \cong \dfrac{1}{1+0.56\tilde{\tau}^{0.74}}$; $\dfrac{P_1}{P_0} \cong \dfrac{0.032}{\tilde{\tau}^{1.44}}$; $\tilde{\tau} = \dfrac{\tau\bar{u}}{L_u}$

Example

$z_0 = 0.05\,\text{m}, z = 10\,\text{m}, \bar{u}(z) = 25\,\text{m/s}$

$I_u(z) \cong 1/\ell n(z/z_0) = 1/\ell n(10/0.05) = 0.19$

$L_u(z) = 300(z/200)^{\nu}; \nu = 0.67 + 0.05\ell n(z_0) = 0.52 \Rightarrow L_u(z) = 63\,\text{m}$

$T = 600\,\text{s}, \tau = 1\,\text{s} \Rightarrow \tilde{\tau} = 1\times 25/63 = 0.4$

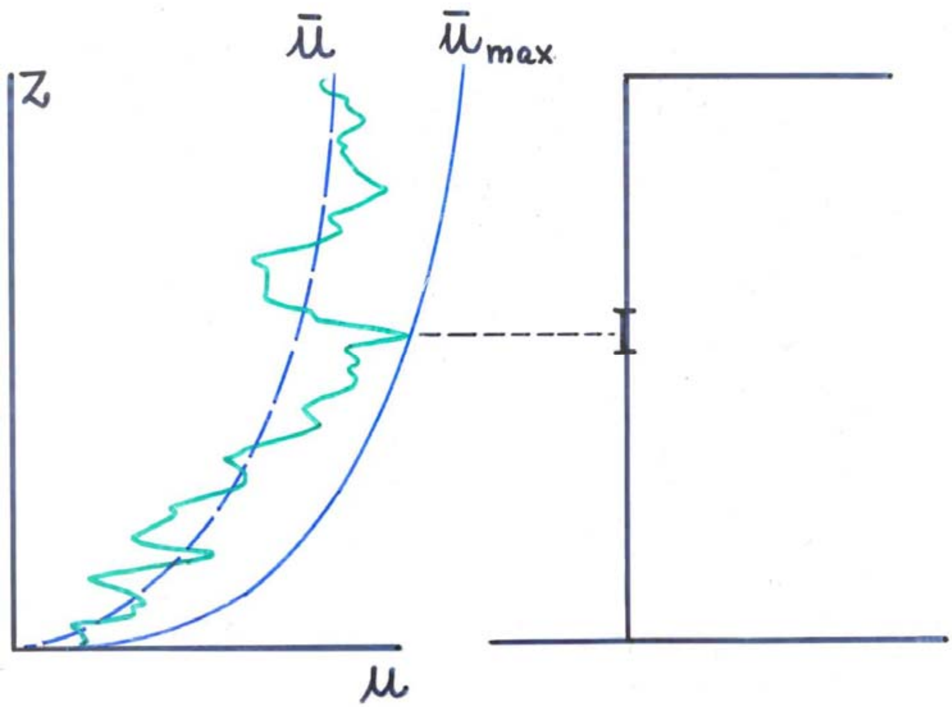
$P_0 = 0.78; P_1 = 0.093 \Rightarrow \nu_u^* = 0.137\ \text{Hz} \Rightarrow g_u^* = 3.164 \Rightarrow$

$G_u^* = 1 + 3.164\times 0.19\times\sqrt{0.78} = 1.53$

$\bar{u}_{max} = 25\times 1.53 = 38.25\ \text{m/s}$

Design wind velocity

The following figure shows the mean wind velocity profile and the peak wind velocity profile. It is worth noting that, due to the partial correlation of turbulence, the peak wind velocities do not occur simultaneously at each height. This means that designing a structure under the peak wind velocity profile is widely prudential and not realistic. Instead, this is definitely correct and even more necessary, when designing a small structural or non structural element. In other words, the design wind velocity depends on the size of the element or the structure to design.



This can be dealt with by correlating the duration τ of the gust peak with the size L of the element or structure considered. On increasing L, τ increases and G_u^* diminishes. This concept can be quantified through the relationship (Greenway 1979, Solari 1993):

$$\tau = \frac{\alpha\sqrt{C_{yu}C_{zu}}L}{\bar{u}}$$

where C_{yu} and C_{zu} are exponential decay coefficients; α = 0,35 – 0,45.

<u>Example</u>

Assume $\bar{u} = 20\,\mathrm{m/s}$, $I_u = 0.2$, $L_u = 75\,\mathrm{m}$, T = 600 s, $C_{yu} = C_{zu} = 10$, $\alpha = 0.4$. The table below reports the design peak velocity for 4 elements or structures with characteristic size L = 3, 5, 10, 20 m, respectively.

L (m)	τ (s)	$\tilde{\tau}$	P_0	P_1/P_0	ν_u^* (Hz)	g_u^*	G_u^*	$\bar{u}_{max}$ (m/s)
3	0,6	0,160	0,874	0,448	0,178	3,245	1,61	32,2
5	1	0,267	0,826	0,214	0,123	3,130	1,57	31,4
10	2	0,533	0,740	0,079	0,075	2,968	1,51	30,2
20	4	1,067	0,630	0,029	0,045	2,792	1,44	28,8

Remembering that the velocity pressure is proportional to the squared wind velocity, passing from L = 3 m to L = 20 m implies that the load is reduced by a factor 0,8.