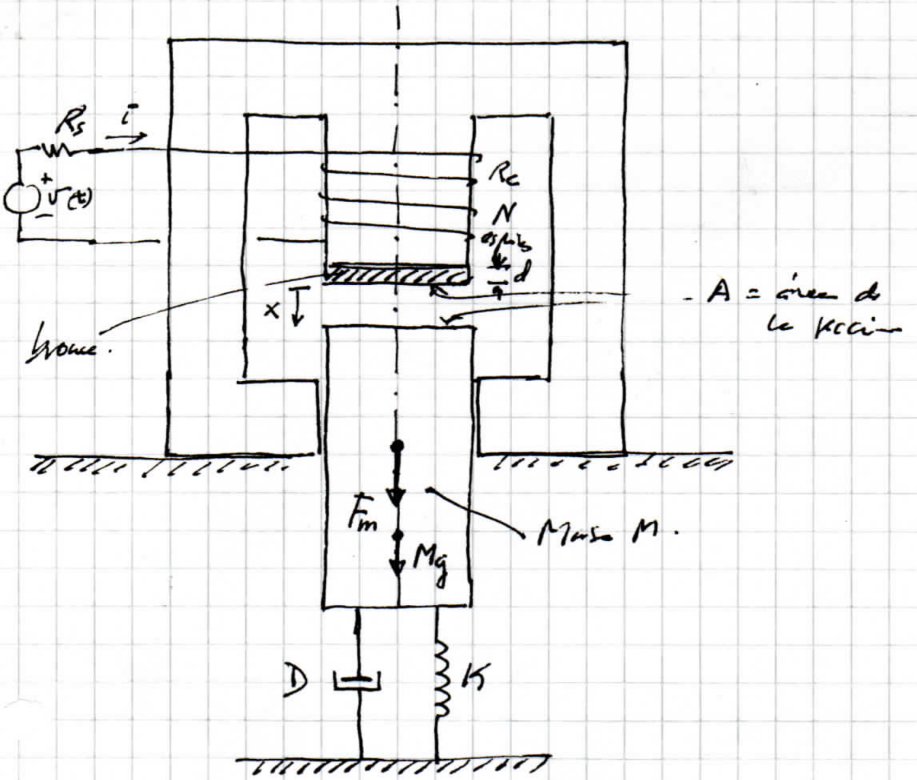


Eq. eléctrica: $\quad v(t) = (R_s + R_c)\, i(t) + \dfrac{d\psi}{dt} \quad (1)$

Eq. mecánica: $\quad M\ddot{x} = F_m + Mg - K(x - b_0) - D\dot{x} \quad (2)$

si $\begin{cases} x > b_0 & \uparrow F_K \\ x < b_0 & \downarrow F_K. \end{cases}$

Eq. eléctrica:

$$\left.\begin{aligned} &\psi = L i \\ &L = \frac{N^2}{\mathcal{R}(x)} \\ &\mathcal{R}(x) = \frac{1}{\mu_0}\frac{(d+x)}{A} \end{aligned}\right\} \quad L(x) = \frac{\mu_0 N^2 A}{(d+x)} \qquad \text{def} \quad \boxed{a = \frac{\mu_0 N^2 A}{2}}$$

$$\boxed{L(x) = \frac{2a}{(d+x)}}$$

$$\frac{d\psi}{dt} = \frac{d}{dt}\left(L(x)\, i\right) = \frac{dL}{dx}\cdot\frac{dx}{dt}\, i + L(x)\cdot\frac{di}{dt} = -\frac{2a}{(d+x)^2}\cdot \dot{x}\, i + \frac{2a}{(d+x)}\cdot\frac{di}{dt}$$

$$\boxed{v(t) = (R_s + R_c)\, i(t) + \frac{2a}{(d+x)}\frac{di}{dt} - \frac{2a}{(d+x)^2}\,\dot{x}\cdot i} \quad (1)$$

$$F_m = \frac{1}{2}\, i \left(\frac{dL}{dx}\right) i = -\frac{1}{2} i^2 \frac{2a}{(d+x)^2} = -\frac{a i^2}{(d+x)^2}$$

Otra forma de calcular F_m

$$F_m = -\left.\frac{\partial W_s}{\partial x}\right|_{\psi = cte}.$$

$$W_s = \frac{1}{2}\frac{B^2}{\mu_0}\cdot A(x+d) \quad // \quad F_m = -\left.\frac{\partial W_s}{\partial x}\right|_{\psi = cte} = -\frac{1}{2}\frac{B^2}{\mu_0}A.$$

$$\varphi = B.A \Rightarrow B = \frac{\varphi}{A} \quad // \quad \psi = N\varphi \Rightarrow B = \frac{\psi}{NA}$$

$$F_m = -\frac{1}{2}\frac{1}{\mu_0}\cdot A \cdot \frac{\psi^2}{N^2A^2} = -\frac{1}{2}\frac{1}{\mu_0 N^2 A}\psi^2$$

$$\psi = Li = \frac{2a}{(d+x)}i$$

$$\boxed{F_m = -\frac{1}{2}\underbrace{\frac{1}{\mu_0 N^2 A}}_{= 2a}\cdot\frac{4a^2 i^2}{(d+x)^2} = -\frac{a i^2}{(d+x)^2}}$$

$$F_m = \left.\frac{\partial W_s'}{\partial x}\right|_{i = cte} \qquad W_s' = \frac{1}{2}Li^2 \Rightarrow F_m = \frac{1}{2}i^2\left(\frac{dL}{dx}\right)$$

Eq. mecánica:

$$M\ddot{x} = -\frac{a i^2}{(d+x)^2} + Mg - K(x-b) - D\dot{x} \qquad (2)$$

$$\boxed{M\ddot{x} + D\dot{x} + K(x-b) + \frac{a i^2}{(d+x)^2} = Mg \quad (2)} \quad (2) \qquad M\ddot{x} + D\dot{x} = f_1(x) - f_2(x)$$

Sean

$$\left.\begin{aligned} v(t) &= v_0 \\ x(t) &= x_0 \\ i(t) &= i_0 \end{aligned}\right] \text{ estado estacionario.}$$

$\Rightarrow$ resolvemos las ecs. (1) y (2) con todas las derivadas $= 0$.

(1) $v_0 = (R_s + R_c)i_0$ (3)

(2) $K(x_0 - b) + \dfrac{a i_0^2}{(d+x_0)^2} = Mg$ (4)

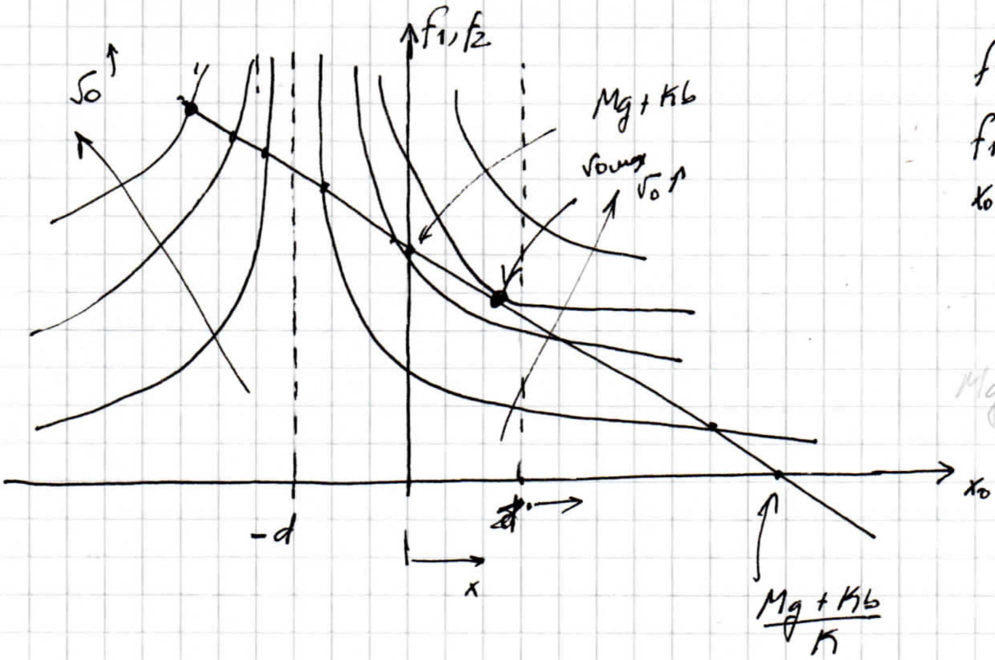
$$(3) \Rightarrow (4) \Rightarrow (5) \quad \boxed{K(x_0 - b) + \frac{a v_0^2}{(d+x_0)^2(R_s+R_c)^2} = Mg}$$

dado $v_0 \Rightarrow$ $\rightarrow x_0$ que resuelve (5) es el "punto de funcionamiento" o punto de equilibrio.

$\rightarrow$ también se conoce $i_0 = \dfrac{v_0}{R_s + R_c}$.

~~Otra~~ Una forma de encontrar las soluciones x_0, dado v_0, es presentar la ec. (5) como la igualdad de 2 fuerzas que dependen de x_0

$$\underbrace{Mg - K(x_0 - b)}_{f_1(x_0)} = \underbrace{\frac{a v_0^2}{(R_S+R_C)^2}\ \frac{1}{(d+x_0)^2}}_{f_2(x_0)}$$



$f_1(x_0) = -Kx_0 + (Mg+Kb)$

$f_1(0) = Mg + Kb.$

x_0: $f_1 = 0$

$x_0 = \frac{Mg+Kb}{K}$

$Mg - Kx_0 + Kb = 0$

$v_{0\,min}$: si aumentamos $v_0 > v_{0\,min} \Rightarrow \nexists$ soluciones x_0 (puntos de funcionamiento estable)!

$v_0 = v_0(x_0)$

$\frac{dv_0}{dx_0} = 0 \overset{?}{\Rightarrow} v_0\ min$

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v_0 y poniendo las condiciones.

(a) $Mg - K(x_0 - b) = \frac{a v_0^2}{(R_S+R_C)^2}\ \frac{1}{(d+x_0)^2}$

(b) igualdad de pendientes: $-K = -\frac{2a v_0^2}{(R_S+R_C)^2}\ \frac{1}{(d+x_0)^3}$

$$K = \frac{2a v_0^2}{(R_S+R_C)^2}\cdot\frac{1}{(d+x_0)^2}\ \frac{1}{(d+x_0)} \Rightarrow \frac{a v_0^2}{(R_S+R_C)^2}\cdot\frac{1}{(d+x_0)^2} = \frac{K}{2}(d+x_0)$$

$$Mg - K(x_0 - b) = \frac{K}{2}(d+x_0)$$

$$\left.\left(Mg + Kb - \frac{K}{2}d\right) = Kx_0 + \frac{K}{2}x_0 \right\} \Rightarrow \frac{3}{2}Kx_0 = \left(Mg + Kb - \frac{K}{2}d\right)$$

$$\boxed{x_0^* = \frac{2}{3}\left(\frac{M}{K}g + b\right) - \frac{d}{3}}$$

x_0^* corresponde a $V_{0\,máx}$

$$Mg - K(x_0^* - b) = \frac{a\,V_{0\,max}^2}{(R_s + R_c)^2}\,\frac{1}{(d + x_0^*)^2}$$

$$Mg - Kb - \frac{2}{3}K\left(\frac{M}{K}g + b\right) - \frac{dK}{3} = \frac{a\,V_{0\,max}^2}{(R_s+R_c)^2}\,\frac{1}{\left[\frac{2}{3}d + \frac{2}{3}\left(\frac{M}{K}g + b\right)\right]^2}$$

$$\frac{1}{3}Mg - \frac{5}{3}Kb - \frac{Kd}{3} =$$

$$\boxed{V_{0\,max} = \frac{2(R_s+R_c)}{3\sqrt{3\,(a/K)}}\left(\frac{M}{K}g + b + d\right)^{3/2}}$$

$f_2 < 0$, para ...

$\underline{V_{0\,min}}$: haciendo $x_0 = 0$, equilibrio de fuerzas

$$\Rightarrow \quad Mg + Kb = \frac{a\,V_{0\,min}^2}{(R_s+R_c)^2}\,\frac{1}{d^2}$$

$$\boxed{V_{0\,min} = \frac{d(R_s+R_c)}{\sqrt{a}}\sqrt{(Mg + Kb)}}$$

Si $V_0 = 0 \Rightarrow f_1 = 0$

$$\Rightarrow Mg - K(x_0 - b) = 0$$

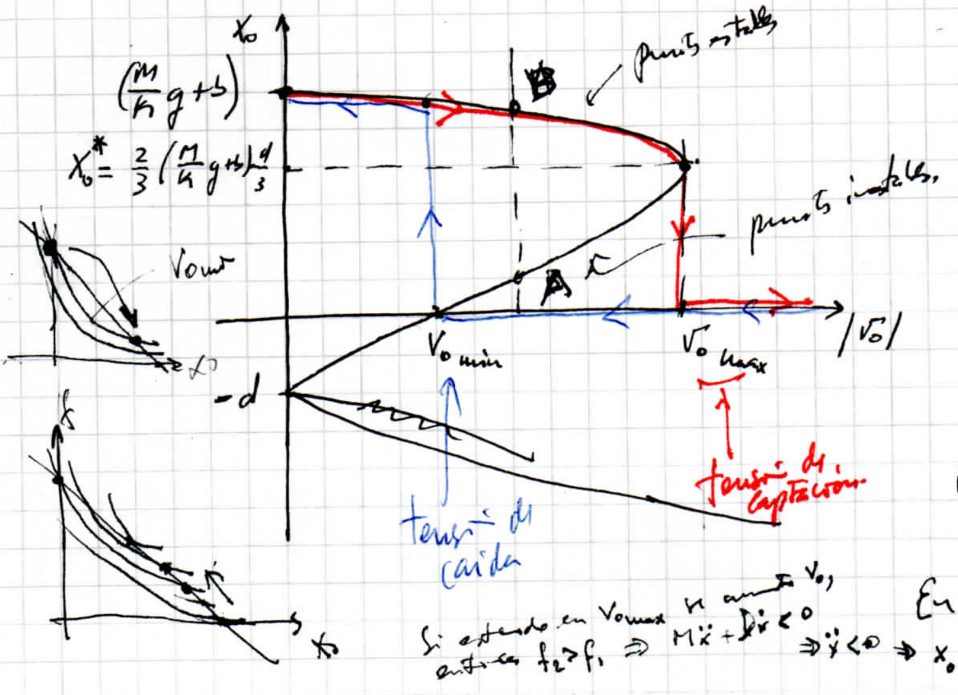
$$K x_0 = Mg + Kb$$

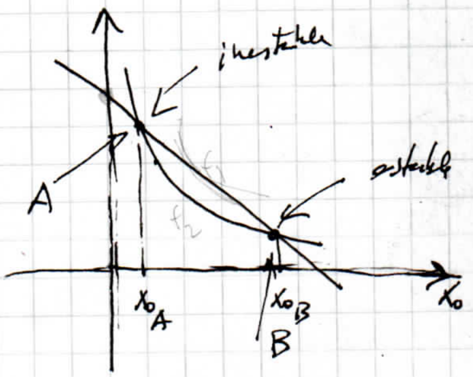
$$\boxed{x_0 = \frac{M}{K}g + b}$$

Obs: en (a) cuando $x_0 \to -d \Rightarrow f_1 \to Mg - K(-d-b) = Mg + K(d+b) > 0$, $\neq 0$, $\neq \infty$

$\Rightarrow$ en f_2 $V_0 \to 0$. para que exista un punto de funcionamiento

$$M\ddot{x} = f_1 - f_2$$





En A, si $x_0 > x_{0A}$, $f_1 > f_2 \Rightarrow \ddot{x} > 0$, $x_0 \nearrow$

En B, si $x_0 > x_B$ $f_1 < f_2 \Rightarrow \ddot{x} < 0$, $x_0 \searrow$

Si estando en $V_{0\,max}$ se aumenta V_0, entonces $f_2 > f_1 \Rightarrow M\ddot{x} + D\dot{x} < 0$ $\Rightarrow \ddot{x} < 0 \Rightarrow x_0 \to 0$