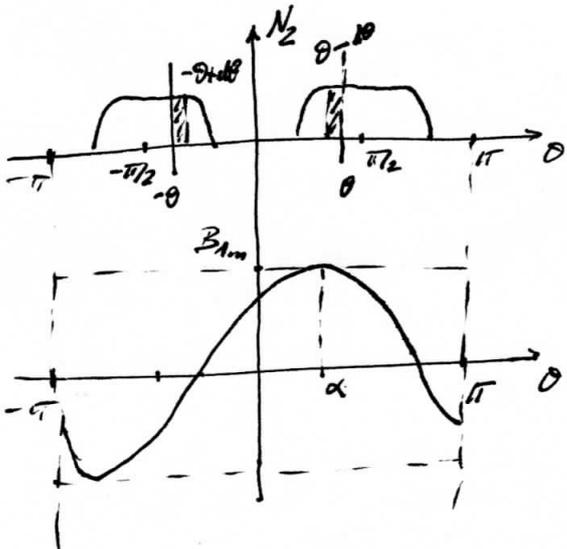
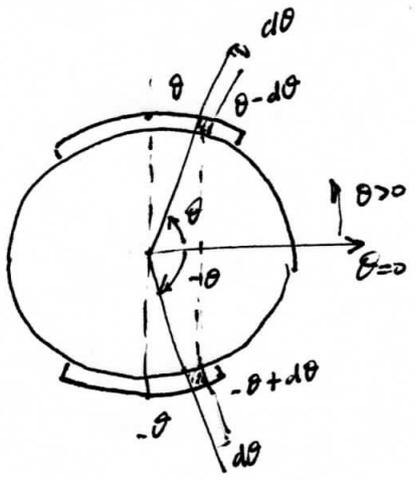


Flujo de enlanchamiento en un bobinado distribuido



Entendamos  $e = d\theta$

Hip. 1) Bobinado distribuido identificado en sub-dato 2,

$$N_2(\theta) = \sum_{h=1}^{\infty} a_{2h-1} \sin((2h-1)\theta)$$

le toma el eje  $\theta=0$  en el plano de simetría del bobinado. (como se hace usualmente para la parte a de un bobinado tri-fásico).

2) Existe en el entrelaño una inducción  $B_1(\theta)$  de distribución sinusoidal en  $\theta$ , en el instante  $t$  en que se analiza el flujo.

$$B_1(\theta) = B_{1m} \cos(\theta - \alpha)$$

$$\text{con } \alpha = \alpha(t) = \omega_m t + \alpha_0$$

Se considera una "espira" ficticia comprendida entre  $-\theta + d\theta$  y  $\theta - d\theta$ . Esta espira es de paso  $\approx 2\theta$  ( $= 2\theta - d\theta$ ). No importa que los conductores se cierra de esa forma u otra, a los efectos de la forma de entrelaño generada, y del flujo enlazado a lo entrelaño.

Espira ficticia tiene  $N_2(\theta) \cdot d\theta$  "conductores".

$$d\psi_{21} = N_2(\theta) d\theta \int_{-\theta}^{+\theta} L R B_1(\theta) d\theta$$

flujo enlazado entre  $-\theta$  y  $\theta$

(integrando en la superficie del entrelaño, donde  $B_1(\theta)$  es  $\perp$  a la superficie).

$$d\psi_{21} = N_2(\theta) d\theta \cdot LR \cdot \int_{-\theta}^{+\theta} B_{1m} \cos(x-\alpha) dx$$

$$= LR B_{1m} N_2(\theta) d\theta \left[ \sin(x-\alpha) \right]_{-\theta}^{+\theta}$$

$$= LR B_{1m} N_2(\theta) d\theta \cdot \left[ \sin(\theta-\alpha) - \sin(-\theta-\alpha) \right]$$

$$\left[ \sin(\theta+\alpha) + \sin(\theta-\alpha) \right]$$

$$\sin\theta \cos\alpha + \sin\alpha \cos\theta + \sin\theta \cos\alpha - \sin\alpha \cos\theta$$

$$\boxed{d\psi_{21} = 2LR B_{1m} N_2(\theta) d\theta \sin\theta \cos\alpha}$$

Flujo de "1 espira" de ancho  $d\theta$ .

(Flujo creado por la inducción  $B_1$  en la bobina 2).

$\psi_{21}$  = flujo total del bobinado 2, debido a  $B_1$ .

$$\psi_{21} = \int_{-\pi/2}^{+\pi/2} d\psi_{21}, \text{ integrado en } \theta \Rightarrow \text{resultado } \psi_{21} = \psi_{21}(\alpha).$$

↪ se integra en un ángulo total  $\pi$ , si se integra en todo el tiempo,  $2\pi$ , sería  $\psi_{21} = 0$ .

$$\psi_{21} = \int_{-\pi/2}^{+\pi/2} 2LR B_{1m} N_2(\theta) \sin\theta \cos\alpha d\theta$$

$$\boxed{\psi_{21} = 2LR B_{1m} \cos\alpha \int_{-\pi/2}^{+\pi/2} N_2(\theta) \sin\theta d\theta}$$

$$\text{con } N_2(\theta) = \sum_{h=1}^{\infty} a_{2h-1} \sin[(2h-1)\theta]$$

$$\Rightarrow \boxed{\psi_{21} = 2LR B_{1m} \cos\alpha \sum_{h=1}^{\infty} a_{2h-1} \int_{-\pi/2}^{+\pi/2} \sin[(2h-1)\theta] \sin\theta d\theta}$$

$$\sin a \sin b = \frac{1}{2} [\cos(a-b) - \cos(a+b)]$$

$$\sin [(2h-1)\theta] \sin \theta = \frac{1}{2} [\cos (2h-2)\theta - \cos 2h\theta]$$

$$= \frac{1}{2} [\cos 2(h-1)\theta - \cos 2h\theta]$$

$$\int_{-\pi/2}^{\pi/2} [\cos 2(h-1)\theta - \cos 2h\theta] d\theta = 0$$

↑  
 porque  $2h$  y  $2(h-1)$  son múltiplos  
 pares de  $\theta$ , para todo  $h \neq 1$   
 ( $h=2, 3, \dots$ )

Si  $h=1$       $\sin [(2h-1)\theta] \sin \theta = \sin^2 \theta = \frac{1 - \cos 2\theta}{2}$

$$\int_{-\pi/2}^{\pi/2} \sin [(2h-1)\theta] \sin \theta d\theta = \int_{-\pi/2}^{\pi/2} \frac{1 - \cos 2\theta}{2} d\theta = \frac{1}{2} \int_{-\pi/2}^{\pi/2} d\theta = \frac{\pi}{2}$$

$$\psi_{21} = 2LR B_{1m} \cos \alpha \cdot a_1 \cdot \frac{\pi}{2}$$

$$\psi_{21}(\alpha) = \pi LR B_{1m} a_1 \cos \alpha$$

$$B = B_m \cos(\theta - \alpha)$$

$$N_{2a}(\theta) = \sum_{k=1}^{\infty} a_{2k-1} \sin(2k-1)\theta$$

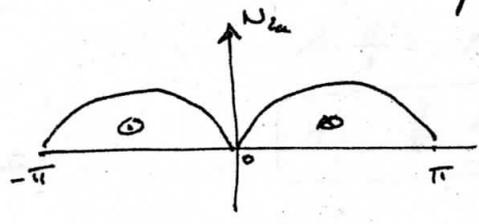
$$\boxed{\psi_a = \psi_a(\alpha) = \pi L R B_m a_1 \cos \alpha}$$

1)

Si el bobinado es "sincrotróticamente repartido", y la inducción es creada por  $2n_2$  bobinados también de repartido sincrotróticamente

$$\boxed{B_m = \mu_0 \frac{n_2 i_a}{e}}$$

$2n_2$  conductores en cada "unidad de la máquina".



$$\int_0^{\pi} N_{2a}(x) dx = 2n_2$$

$$N_{2a}(x) = a_1 \sin x$$

$$2n_2 = \int_0^{\pi} a_1 \sin x dx = a_1 [-\cos x]_0^{\pi} = a_1 [ -(-1) + (1) ] = 2a_1 \Rightarrow \boxed{a_1 = n_2}$$

$$\boxed{\psi_a(\alpha) = \pi L R \mu_0 \frac{n_2 i_a}{e} n_2 \cos \alpha}$$

inductancia propia fase a:  $k=0$

$$L_{aa} = \frac{\psi_{aa}}{i_a} = \frac{1}{2} \pi L R n_2^2$$

$$\boxed{L_{aa} = \mu_0 \left( \frac{\pi L R}{e} \right) n_2^2} = \frac{1}{2} \frac{n_2^2}{P_e}$$

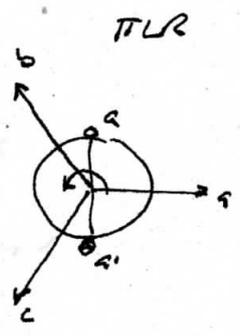
inductancias mutuas

con  $P_e = \frac{1}{\mu_0} \cdot \frac{e}{2\pi R L}$

Si  $B(\theta)$  es creado por fase  $\left. \begin{array}{l} b \Rightarrow \alpha = \pi/3 \\ c \Rightarrow \alpha = -2\pi/3 \text{ (o } 4\pi/3) \end{array} \right\}$

$$\cos \alpha = -\frac{1}{2}$$

$$\boxed{\begin{array}{l} L_{ab} = M_{ab} = -\frac{1}{2} L_{aa} \\ L_{ac} = M_{ac} = -\frac{1}{2} L_{aa} \end{array}}$$



$$\psi_a(\alpha) = \frac{4}{\pi} \left( \frac{\mu_0 L R}{e} \right) \left( \frac{\pi^2}{18} \right) \cdot (2n_2)(2n_2) i_a \cos \alpha$$

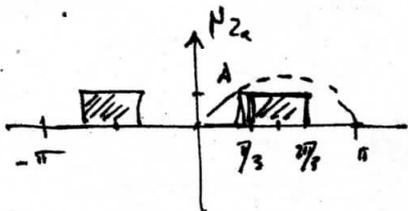
" 0,6168

2) bobinado distribuido en 1 capa

$$B_m = \frac{4}{\pi} \mu_0 \frac{n_1' i_1}{e}$$

$$n_1' = k_b n_2$$

$$k_b = k_d k_r k_i$$



$$\int_0^\pi N_{2a}(x) dx = 2n_2$$

$$N_{2a}(x) = \begin{cases} 0 & 0 \leq x \leq \pi/3 \\ A & \pi/3 \leq x \leq 2\pi/3 \\ 0 & 2\pi/3 \leq x \leq \pi \end{cases}$$

$$A \cdot \frac{\pi}{3} = 2n_2$$

$$A = \frac{3}{\pi} 2n_2$$

Hay que determinar el fundamental

$$a_1 = \frac{2}{2\pi} \int_{-\pi}^{\pi} N_{2a}(x) \sin x dx = \frac{1}{\pi} 2 \int_0^\pi N_{2a}(x) \sin x dx$$

$$= \frac{1}{\pi} 2 \int_{\pi/3}^{2\pi/3} \frac{3}{\pi} 2n_2 \sin x dx$$

$$= \frac{2}{\pi} \cdot \frac{3}{\pi} 2n_2 [-\cos x]_{\pi/3}^{2\pi/3}$$

$$= \frac{2}{\pi} \cdot \frac{3}{\pi} 2n_2 [ -(-\frac{1}{2}) + (\frac{1}{2}) ] = \frac{4}{\pi} \cdot \frac{3}{\pi} 2n_2$$

pero - este caso  $k_b = k_d = \frac{3}{\pi}$

$$a_1 = \frac{12}{\pi^2} n_2$$

$$\psi_a(x) = \frac{1}{\pi} L R \cdot \frac{4}{\pi} \mu_0 \overbrace{k_b n_2' i_a}^{n_1' i_1} \cdot \frac{12}{\pi^2} n_2 \cos x$$

$$\psi_a(x) = \frac{4}{\pi} \left( \frac{\mu_0 L R}{e} \right) \left( 4 n_1' \cdot \left( \frac{3}{\pi} n_2 \right) i_a \cos x \right)$$

$$d=0 \quad \psi_a = \frac{4}{\pi} \left( \frac{\mu_0 L R}{e} \right) (2 n_1') (2 n_1') \cos x$$

y para  $\alpha = \pi$

$$\psi_{a\pi} = -\frac{1}{2} \psi_a$$

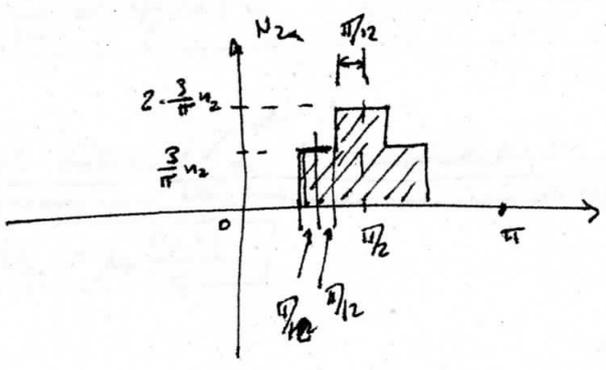
$$\psi_{a\pi} = -\frac{1}{2} \psi_a$$

$$\psi_a = L_a - M = \frac{3}{2} L_a$$

3) Bobinado distribuido en 2 capas y de paso reducido

$$\boxed{B_m = \frac{4}{\pi} \mu_0 \frac{n_2' i_a}{e}}$$

$$n_2' = \underbrace{k_d k_r k_i}_{k_b} n_2$$



$$\Sigma = \pi/12$$

$$\pi/2 - 3 \frac{\pi}{12} = \frac{\pi}{2} - \frac{\pi}{4} = \frac{\pi}{4}$$

$$\frac{\pi}{4} + \frac{\pi}{6} = \frac{6+4}{24} = \frac{10}{24} = \frac{5}{12} \pi$$

$$a_1 = \frac{2}{2\pi} \int_{-\pi}^{\pi} N_{2a}(\alpha) \cos \alpha \, d\alpha = \frac{1}{\pi} \int_0^{\pi} N_{2a}(\alpha) \cos \alpha \, d\alpha$$

$$= \frac{4}{\pi} \left[ \int_{\pi/12}^{5\pi/12} \frac{3}{\pi} n_2 \sin \alpha \, d\alpha + \int_{5\pi/12}^{6\pi/12} \frac{6}{\pi} n_2 \sin \alpha \, d\alpha \right]$$

$$= \frac{4n_2}{\pi} \left[ \frac{3}{\pi} [-\cos \alpha]_{\pi/12}^{5\pi/12} + \frac{6}{\pi} [-\cos \alpha]_{5\pi/12}^{6\pi/12} \right]$$

$$= \frac{4}{\pi} \cdot \frac{3}{\pi} n_2 \left[ -\cos \frac{5\pi}{12} + \cos \frac{\pi}{4} + 2 \cos \frac{\pi}{2} + 2 \cos \frac{5\pi}{12} \right]$$

$$= \frac{4}{\pi} \cdot \frac{3}{\pi} n_2 \left[ \frac{\sqrt{2}}{2} + 0,2588 \right] = \frac{4}{\pi} \cdot \frac{3}{\pi} n_2 \cdot \underbrace{0,9659}_{k_r}$$

$$\cos 5\pi/12 = 0,2588$$

$$0,9659$$

$$\cos 15^\circ = 0,9659 = k_r$$

$$k_d = \frac{3}{4}$$

$$\boxed{a_1 = \frac{12}{\pi^2} \cdot 0,9659 n_2}$$

$$\psi_a = \underbrace{K_L R}_{k_r} \cdot \left( \frac{4}{\pi} \mu_0 \frac{n_2' i_a}{e} \right) \cdot \left( \frac{12}{\pi^2} \cdot 0,9659 n_2 \right) \cos \alpha$$

$$\boxed{\psi_a = \frac{4}{\pi} \left( \frac{\mu_0 L R}{e} \right) (2n_2') (2 \underbrace{k_d \cdot k_r}_{n_2'}) i_a \cos \alpha}$$

$$\alpha = \begin{cases} 2\pi/3 \\ -2\pi/3 \end{cases}$$

$$\Rightarrow L_{aa} = \frac{4}{\pi} \left( \frac{\mu_0 L R}{e} \right) (2n_2') (2n_2') \cos \alpha$$

$$\begin{cases} L_{ab} = M_{ab} = -\frac{1}{2} L_{aa} \\ L_{ac} = M_{ac} = -\frac{1}{2} L_{aa} \end{cases}$$

1, pyme d=20