SIMPLIFICACIÓN DE LAS ECUACIONES APLICANDO LA TRANSFORMACIÓN DE PARK.

A todas las variables del estator le aplicaremos la transf. de Park modificada $[P_1(\theta)]$:

Transf. modificada de Park : $[P_1(\theta)]$

$$[X_{dqo}] = \begin{bmatrix} X_d \\ X_q \\ X_o \end{bmatrix} \qquad [X_{abc}] = \begin{bmatrix} X_a \\ X_b \\ X_c \end{bmatrix} \qquad [X_{dqo}] = \boxed{P_1(\theta)}\,[X_{abc}]$$

$$[X_{abc}] = \boxed{P_1^{-1}(\theta)}\,[X_{dqo}]$$

$$\boxed{P_1(\theta)} = \sqrt{\frac{2}{3}} \begin{bmatrix} \cos\theta & \cos(\theta - 2\pi/3) & \cos(\theta + 2\pi/3) \\ -\operatorname{sen}\theta & -\operatorname{sen}(\theta - 2\pi/3) & -\operatorname{sen}(\theta + 2\pi/3) \\ 1/\sqrt{2} & 1/\sqrt{2} & 1/\sqrt{2} \end{bmatrix}$$

$$\boxed{P_1^{-1}(\theta)} = \sqrt{\frac{2}{3}} \begin{bmatrix} \cos\theta & -\operatorname{sen}\theta & 1/\sqrt{2} \\ \cos(\theta - 2\pi/3) & -\operatorname{sen}(\theta - 2\pi/3) & 1/\sqrt{2} \\ \cos(\theta + 2\pi/3) & -\operatorname{sen}(\theta + 2\pi/3) & 1/\sqrt{2} \end{bmatrix}$$

Cambio de variables en el estator:

Se definen las nuevas variables:

$$[v_{dqo}] = \begin{bmatrix} v_d \\ v_q \\ v_o \end{bmatrix} \qquad [i_{dqo}] = \begin{bmatrix} i_d \\ i_q \\ i_o \end{bmatrix} \qquad [\Psi_{dqo}] = \begin{bmatrix} \Psi_d \\ \Psi_q \\ \Psi_o \end{bmatrix}$$

$$[i_{dqo}] = \boxed{P_1(\theta)}\,[i_{abc}] \qquad [v_{dqo}] = \boxed{P_1(\theta)}\,[v_{abc}] \qquad [\Psi_{dqo}] = \boxed{P_1(\theta)}\,[\Psi_{abc}]$$

$$[i_{abc}] = \boxed{P_1^{-1}(\theta)}\,[i_{dqo}] \qquad [v_{abc}] = \boxed{P_1^{-1}(\theta)}\,[v_{dqo}] \qquad [\Psi_{abc}] = \boxed{P_1^{-1}(\theta)}\,[\Psi_{dqo}]$$

Transformación de las ecuaciones:

$$[\psi_{abc}] = \boxed{L_{ss}}\,[i_{abc}] + \boxed{M_{sR}}\,[i_{fk}] \quad \longleftarrow \times \boxed{P_1(\theta)}$$

$$[\psi_{fk}] = \boxed{M_{Rs}}\,[i_{abc}] + \boxed{L_{RR}}\,[i_{fk}]$$

$$[v_{abc}] = \boxed{R_s}\,[i_{abc}] + \frac{d}{dt}[\psi_{abc}] \quad \longleftarrow \times \boxed{P_1(\theta)}$$

$$[v_{fk}] = \boxed{R_r}\,[i_{fk}] + \frac{d}{dt}[\psi_{fk}]$$

$$\boxed{P_1(\theta)}\,[\psi_{abc}] = \boxed{P_1(\theta)}\,\boxed{L_{ss}}\,\boxed{P_1^{-1}(\theta)}\,\boxed{P_1(\theta)}\,[i_{abc}] + \boxed{P_1(\theta)}\,\boxed{M_{sR}}\,[i_{fk}]$$

$$[\psi_{fk}] = \boxed{M_{Rs}}\,\boxed{P_1^{-1}(\theta)}\,\boxed{P_1(\theta)}\,[i_{abc}] + \boxed{L_{RR}}\,[i_{fk}]$$

$$\boxed{P_1(\theta)}\,[v_{abc}] = \boxed{P_1(\theta)}\,\boxed{R_s}\,\boxed{P_1^{-1}(\theta)}\,\boxed{P_1(\theta)}\,[i_{abc}] + \boxed{P_1(\theta)}\,\frac{d}{dt}\left[\boxed{P_1^{-1}(\theta)}\,\boxed{P_1(\theta)}\,[\psi_{abc}]\right]$$

$$[v_{fk}] = \boxed{R_r}\,[i_{fk}] + \frac{d}{dt}[\psi_{fk}]$$

$$[\psi_{dq0}] = \boxed{P_1(\theta)}\,\boxed{L_{ss}}\,\boxed{P_1^{-1}(\theta)}\,[i_{dq0}] + \boxed{P_1(\theta)}\,\boxed{M_{sR}}\,[i_{fk}]$$

$$[\psi_{fk}] = \boxed{M_{Rs}}\,\boxed{P_1^{-1}(\theta)}\,[i_{dq0}] + \boxed{L_{RR}}\,[i_{fk}]$$

$$[v_{dq0}] = \boxed{P_1(\theta)}\,\boxed{R_s}\,\boxed{P_1^{-1}(\theta)}\,[i_{dq0}] + \boxed{P_1(\theta)}\,\frac{d}{dt}\left[\boxed{P_1^{-1}(\theta)}\,[\psi_{dq0}]\right]$$

$$[v_{fk}] = \boxed{R_r}\,[i_{fk}] + \frac{d}{dt}[\psi_{fk}]$$

$$\boxed{P_1(\theta)}\,\boxed{R_s}\,\boxed{P_1^{-1}(\theta)} = r_s\,\boxed{P_1(\theta)}\,\boxed{I}\,\boxed{P_1^{-1}(\theta)} = r_s\,\boxed{P_1(\theta)}\,\boxed{P_1^{-1}(\theta)} = r_s\,\boxed{I} = \boxed{R_s}$$

$$\left[\boxed{P_1(\theta)}\,\boxed{M_{sR}}\right]^t = \boxed{M_{sR}}^t\,\boxed{P_1(\theta)}^t = \boxed{M_{Rs}}\,\boxed{P_1^{-1}(\theta)}$$

$$[\psi_{dqo}] = \boxed{P_1(\theta)}\ \boxed{L_{SS}}\ \boxed{P_1^{-1}(\theta)}\ [i_{dqo}] + \boxed{P_1(\theta)}\ \boxed{M_{SR}}\ [i_{fk}]$$

$$[\psi_{fk}] = \left[\boxed{P_1(\theta)}\ \boxed{M_{SR}}\right]^t [i_{dqo}] + \boxed{L_{RR}}\ [i_{fk}]$$

$$[v_{dqo}] = \boxed{R_s}\ [i_{dqo}] + \boxed{P_1(\theta)}\ \frac{d}{dt}\left[\boxed{P_1^{-1}(\theta)}\ [\psi_{dqo}]\right]$$

$$[v_{fk}] = \boxed{R_r}\ [i_{fk}] + \frac{d}{dt}[\psi_{fk}]$$

Cálculo de :

$$\boxed{P_1(\theta)}\ \boxed{L_{SS}}\ \boxed{P_1^{-1}(\theta)}$$

$$\boxed{L_{SS}} = \begin{bmatrix} L_{SO} & M_{SO} & M_{SO} \\ M_{SO} & L_{SO} & M_{SO} \\ M_{SO} & M_{SO} & L_{SO} \end{bmatrix} + L_{SV} \begin{bmatrix} \cos 2\theta & \cos 2(\theta + 2\pi/3) & \cos 2(\theta - 2\pi/3) \\ \cos 2(\theta + 2\pi/3) & \cos 2(\theta - 2\pi/3) & \cos 2\theta \\ \cos 2(\theta - 2\pi/3) & \cos 2\theta & \cos 2(\theta + 2\pi/3) \end{bmatrix} = \boxed{L_{SS}^{0}} + \boxed{L_{SS}^{\theta}}$$

$$\boxed{P_1(\theta)}\ \boxed{L_{SS}}\ \boxed{P_1^{-1}(\theta)} = \boxed{P_1(\theta)}\left[\boxed{L_{SS}^{0}} + \boxed{L_{SS}^{\theta}}\right]\boxed{P_1^{-1}(\theta)} = \boxed{P_1(\theta)}\ \boxed{L_{SS}^{0}}\ \boxed{P_1^{-1}(\theta)} +$$

$$+ \boxed{P_1(\theta)}\ \boxed{L_{SS}^{\theta}}\ \boxed{P_1^{-1}(\theta)}$$

$$\frac{2}{3} \begin{bmatrix} \cos\theta & \cos(\theta - 2\pi/3) & \cos(\theta + 2\pi/3) \\ -\sin\theta & -\sin(\theta - 2\pi/3) & -\sin(\theta + 2\pi/3) \\ 1/\sqrt{2} & 1/\sqrt{2} & 1/\sqrt{2} \end{bmatrix} \begin{bmatrix} X & Y & Y \\ Y & X & Y \\ Y & Y & X \end{bmatrix} \begin{bmatrix} \cos\theta & -\sin\theta & 1/\sqrt{2} \\ \cos(\theta - 2\pi/3) & -\sin(\theta - 2\pi/3) & 1/\sqrt{2} \\ \cos(\theta + 2\pi/3) & -\sin(\theta + 2\pi/3) & 1/\sqrt{2} \end{bmatrix}$$

$$= \begin{bmatrix} X-Y & 0 & 0 \\ 0 & X-Y & 0 \\ 0 & 0 & X+2Y \end{bmatrix} \qquad \boxed{P_1(\theta)}\ \boxed{L_{SS}^{0}}\ \boxed{P_1^{-1}(\theta)} = \begin{bmatrix} L_{SO}-M_{SO} & 0 & 0 \\ 0 & L_{SO}-M_{SO} & 0 \\ 0 & 0 & L_{SO}+2M_{SO} \end{bmatrix}$$

$$\frac{2}{3}L_{sv}\begin{bmatrix} \cos\theta & \cos(\theta-2\pi/3) & \cos(\theta+2\pi/3) \\ -\operatorname{sen}\theta & -\operatorname{sen}(\theta-2\pi/3) & -\operatorname{sen}(\theta+2\pi/3) \\ 1/\sqrt{2} & 1/\sqrt{2} & 1/\sqrt{2} \end{bmatrix} \cdot \begin{bmatrix} \cos 2\theta & \cos 2(\theta+2\pi/3) & \cos 2(\theta-2\pi/3) \\ \cos 2(\theta+2\pi/3) & \cos 2(\theta-2\pi/3) & \cos 2\theta \\ \cos 2(\theta-2\pi/3) & \cos 2\theta & \cos 2(\theta+2\pi/3) \end{bmatrix} \begin{bmatrix} \cos\theta & -\operatorname{sen}\theta & 1/\sqrt{2} \\ \cos(\theta-2\pi/3) & -\operatorname{sen}(\theta-2\pi/3) & 1/\sqrt{2} \\ \cos(\theta+2\pi/3) & -\operatorname{sen}(\theta+2\pi/3) & 1/\sqrt{2} \end{bmatrix}$$

$$= L_{sv}\begin{bmatrix} \cos\theta & \cos(\theta-2\pi/3) & \cos(\theta+2\pi/3) \\ -\operatorname{sen}\theta & -\operatorname{sen}(\theta-2\pi/3) & -\operatorname{sen}(\theta+2\pi/3) \\ 1/\sqrt{2} & 1/\sqrt{2} & 1/\sqrt{2} \end{bmatrix} \begin{bmatrix} \cos\theta & \operatorname{sen}\theta & 0 \\ \cos(\theta-2\pi/3) & \operatorname{sen}(\theta-2\pi/3) & 0 \\ \cos(\theta+2\pi/3) & \operatorname{sen}(\theta+2\pi/3) & 0 \end{bmatrix} \doteq \begin{bmatrix} \frac{3}{2}L_{sv} & 0 & 0 \\ 0 & -\frac{3}{2}L_{sv} & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$$\boxed{P_1(\theta)}\ \boxed{L_{ss}^{\theta}}\ \boxed{P_1^{-1}(\theta)} = \begin{bmatrix} \frac{3}{2}L_{sv} & 0 & 0 \\ 0 & -\frac{3}{2}L_{sv} & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

definimos:
$$L_d \triangleq L_{so} - M_{so} + \tfrac{3}{2}L_{sv}$$
$$L_q \triangleq L_{so} - M_{so} - \tfrac{3}{2}L_{sv}$$
$$L_o \triangleq L_{so} + 2M_{so}$$

$$\boxed{P_1(\theta)}\ \boxed{L_{ss}}\ \boxed{P_1^{-1}(\theta)} = \begin{bmatrix} L_{so}-M_{so}+\frac{3}{2}L_{sv} & 0 & 0 \\ 0 & L_{so}-M_{so}-\frac{3}{2}L_{sv} & 0 \\ 0 & 0 & L_{so}+2M_{so} \end{bmatrix} \triangleq \begin{bmatrix} L_d & 0 & 0 \\ 0 & L_q & 0 \\ 0 & 0 & L_o \end{bmatrix}$$

Relaciones trigonométricas utilizadas:

$$\cos\alpha \cos\beta = \tfrac{1}{2}\left[\cos(\alpha-\beta) + \cos(\alpha+\beta)\right]$$
$$\operatorname{sen}\alpha \cos\beta = \tfrac{1}{2}\left[\operatorname{sen}(\alpha-\beta) + \operatorname{sen}(\alpha+\beta)\right]$$
$$\cos x + \cos(x+4\pi/3) + \cos(x-\tfrac{4\pi}{3}) = 0$$
$$\operatorname{sen} x + \operatorname{sen}(x-2\pi/3) + \operatorname{sen}(x+2\pi/3) = 0$$
$$\operatorname{sen} x \cos x + \operatorname{sen}(x+2\pi/3)\cos(x+2\pi/3) + \operatorname{sen}(x-2\pi/3)\cos(x-2\pi/3) = 0$$
$$\operatorname{sen}^2 x + \operatorname{sen}^2(x+2\pi/3) + \operatorname{sen}^2(x-2\pi/3) = 3/2$$
$$\cos^2 x + \cos^2(x+2\pi/3) + \cos^2(x-2\pi/3) = 3/2$$

Cálculo de: $\boxed{P_1(\theta)}\ \boxed{M_{SR}}$

$$\sqrt{\frac{2}{3}}\begin{bmatrix} \cos\theta & \cos(\theta-2\pi/3) & \cos(\theta+2\pi/3) \\ -\sin\theta & -\sin(\theta-2\pi/3) & -\sin(\theta+2\pi/3) \\ 1/\sqrt{2} & 1/\sqrt{2} & 1/\sqrt{2} \end{bmatrix} \begin{bmatrix} M_{FS}\cos\theta & M_{KDS}\cos\theta & -M_{KQS}\sin\theta \\ M_{FS}\cos(\theta-2\pi/3) & M_{KDS}\cos(\theta-2\pi/3) & -M_{KQS}\sin(\theta-2\pi/3) \\ M_{FS}\cos(\theta+2\pi/3) & M_{KDS}\cos(\theta+2\pi/3) & -M_{KQS}\sin(\theta+2\pi/3) \end{bmatrix} =$$

$$= \begin{bmatrix} \sqrt{\frac{3}{2}}M_{FS} & \sqrt{\frac{3}{2}}M_{KDS} & 0 \\ 0 & 0 & \sqrt{\frac{3}{2}}M_{KQS} \\ 0 & 0 & 0 \end{bmatrix}$$

definimos:
$$M_F \triangleq \sqrt{3/2}\, M_{FS}.$$
$$M_{KD} \triangleq \sqrt{3/2}\, M_{KDS}$$
$$M_{KQ} \triangleq \sqrt{3/2}\, M_{KQS}$$

$$\boxed{P_1(\theta)}\ \boxed{M_{SR}} = \begin{bmatrix} \sqrt{\frac{3}{2}}M_{FS} & \sqrt{\frac{3}{2}}M_{KDS} & 0 \\ 0 & 0 & \sqrt{\frac{3}{2}}M_{KQS} \\ 0 & 0 & 0 \end{bmatrix} \triangleq \begin{bmatrix} M_F & M_{KD} & 0 \\ 0 & 0 & M_{KQ} \\ 0 & 0 & 0 \end{bmatrix}$$

Cálculo de: $\boxed{P_1(\theta)}\dfrac{d}{dt}\left[\boxed{P_1^{-1}(\theta)}\,[\psi_{dq0}]\right]$

$$\boxed{P_1(\theta)}\frac{d}{dt}\left[\boxed{P_1^{-1}(\theta)}\,[\psi_{dq0}]\right] = \boxed{P_1(\theta)}\left[\left[\frac{d}{dt}\boxed{P_1^{-1}(\theta)}\right][\psi_{dq0}] + \boxed{P_1^{-1}(\theta)}\frac{d}{dt}[\psi_{dq0}]\right] =$$

$$\boxed{P_1(\theta)}\left[\frac{d}{dt}\boxed{P_1^{-1}(\theta)}\right][\psi_{dq0}] + \frac{d}{dt}[\psi_{dq0}] = \frac{d\theta}{dt}\boxed{P_1(\theta)}\left[\frac{d}{d\theta}\boxed{P_1^{-1}(\theta)}\right][\psi_{dq0}] + \frac{d}{dt}[\psi_{dq0}]$$

$$\frac{2}{3}\begin{bmatrix} \cos\theta & \cos(\theta-2\pi/3) & \cos(\theta+2\pi/3) \\ -\sin\theta & -\sin(\theta-2\pi/3) & -\sin(\theta+2\pi/3) \\ 1/\sqrt{2} & 1/\sqrt{2} & 1/\sqrt{2} \end{bmatrix} \begin{bmatrix} -\sin\theta & -\cos\theta & 0 \\ -\sin(\theta-2\pi/3) & -\cos(\theta-2\pi/3) & 0 \\ -\sin(\theta+2\pi/3) & -\cos(\theta+2\pi/3) & 0 \end{bmatrix} = \begin{bmatrix} 0 & -1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$$\boxed{P_1(\theta)}\left[\frac{d}{d\theta}\boxed{P_1^{-1}(\theta)}\right] = \begin{bmatrix} 0 & -1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$$\boxed{\frac{d\theta}{dt} = \omega}$$

Siendo ω la velocidad angular del rotor.

$$\boxed{P_1(\theta)}\frac{d}{dt}\left[\boxed{P_1^{-1}(\theta)}\left[\psi_{dqo}\right]\right]=\begin{bmatrix}0&-\omega&0\\ \omega&0&0\\ 0&0&0\end{bmatrix}\begin{bmatrix}\psi_d\\ \psi_q\\ \psi_o\end{bmatrix}+\frac{d}{dt}\begin{bmatrix}\psi_d\\ \psi_q\\ \psi_o\end{bmatrix}$$

En base a los cálculos desarrollados, las ecuaciones transformadas resultan:

$$\begin{bmatrix}\psi_d\\ \psi_q\\ \psi_o\end{bmatrix}=\begin{bmatrix}L_d&0&0\\ 0&L_q&0\\ 0&0&L_o\end{bmatrix}\begin{bmatrix}i_d\\ i_q\\ i_o\end{bmatrix}+\begin{bmatrix}M_F&M_{KD}&0\\ 0&0&M_{KQ}\\ 0&0&0\end{bmatrix}\begin{bmatrix}i_f\\ i_{kd}\\ i_{kq}\end{bmatrix}$$

$$\begin{bmatrix}\psi_f\\ \psi_{kd}\\ \psi_{kq}\end{bmatrix}=\begin{bmatrix}M_F&0&0\\ M_{KD}&0&0\\ 0&M_{KQ}&0\end{bmatrix}\begin{bmatrix}i_d\\ i_q\\ i_o\end{bmatrix}+\begin{bmatrix}L_{fd}&L_{fkd}&0\\ L_{fkd}&L_{kd}&0\\ 0&0&L_{kq}\end{bmatrix}\begin{bmatrix}i_f\\ i_{kd}\\ i_{kq}\end{bmatrix}$$

$$\begin{matrix}(1)\\ (2)\\ (3)\end{matrix}\quad\begin{bmatrix}v_d\\ v_q\\ v_o\end{bmatrix}=\begin{bmatrix}r_s&0&0\\ 0&r_s&0\\ 0&0&r_s\end{bmatrix}\begin{bmatrix}i_d\\ i_q\\ i_o\end{bmatrix}+\begin{bmatrix}0&-\omega&0\\ \omega&0&0\\ 0&0&0\end{bmatrix}\begin{bmatrix}\psi_d\\ \psi_q\\ \psi_o\end{bmatrix}+\frac{d}{dt}\begin{bmatrix}\psi_d\\ \psi_q\\ \psi_o\end{bmatrix}$$

$$\begin{matrix}(4)\\ (5)\\ (6)\end{matrix}\quad\begin{bmatrix}v_f\\ v_{kd}\\ v_{kq}\end{bmatrix}=\begin{bmatrix}r_f&0&0\\ 0&r_{kd}&0\\ 0&0&r_{kq}\end{bmatrix}\begin{bmatrix}i_f\\ i_{kd}\\ i_{kq}\end{bmatrix}+\frac{d}{dt}\begin{bmatrix}\psi_f\\ \psi_{kd}\\ \psi_{kq}\end{bmatrix}$$

(1) $v_d = r_s i_d - \omega\psi_q + \frac{d\psi_d}{dt}$

(2) $v_q = r_s i_q + \omega\psi_d + \frac{d\psi_q}{dt}$

(3) $v_o = r_s i_o + \frac{d\psi_o}{dt}$

(4) $v_f = r_f i_f + \frac{d\psi_f}{dt}$

(5) $v_{kd} = r_{kd} i_{kd} + \frac{d\psi_{kd}}{dt}$

(6) $v_{kq} = r_{kq} i_{kq} + \frac{d\psi_{kq}}{dt}$

Agrupando convenientemente las matrices, las 4 ecuaciones matriciales anteriores se reducen a dos: 6

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$$
\begin{bmatrix} \psi_d \\ \psi_q \\ \psi_0 \\ \psi_f \\ \psi_{kd} \\ \psi_{kq} \end{bmatrix}
=
\begin{bmatrix}
L_d & 0 & 0 & M_F & M_{KD} & 0 \\
0 & L_q & 0 & 0 & 0 & M_{KQ} \\
0 & 0 & L_0 & 0 & 0 & 0 \\
M_F & 0 & 0 & L_{fd} & L_{fkd} & 0 \\
M_{KD} & 0 & 0 & L_{fkd} & L_{kd} & 0 \\
0 & M_{KQ} & 0 & 0 & 0 & L_{kq}
\end{bmatrix}
\begin{bmatrix} i_d \\ i_q \\ i_0 \\ i_f \\ i_{kd} \\ i_{kq} \end{bmatrix}
$$

$$
\begin{bmatrix} v_d \\ v_q \\ v_0 \\ v_f \\ v_{kd} \\ v_{kq} \end{bmatrix}
=
\begin{bmatrix}
r_s & 0 & 0 & 0 & 0 & 0 \\
0 & r_s & 0 & 0 & 0 & 0 \\
0 & 0 & r_s & 0 & 0 & 0 \\
0 & 0 & 0 & r_f & 0 & 0 \\
0 & 0 & 0 & 0 & r_{kd} & 0 \\
0 & 0 & 0 & 0 & 0 & r_{kq}
\end{bmatrix}
\begin{bmatrix} i_d \\ i_q \\ i_0 \\ i_f \\ i_{kd} \\ i_{kq} \end{bmatrix}
+
\begin{bmatrix}
0 & -\omega & 0 & 0 & 0 & 0 \\
\omega & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 \\
-0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0
\end{bmatrix}
\begin{bmatrix} \psi_d \\ \psi_q \\ \psi_0 \\ \psi_f \\ \psi_{kd} \\ \psi_{kq} \end{bmatrix}
+ \frac{d}{dt}
\begin{bmatrix} \psi_d \\ \psi_q \\ \psi_0 \\ \psi_f \\ \psi_{kd} \\ \psi_{kq} \end{bmatrix}
$$

Eliminando los flujos

$$
\begin{bmatrix} v_d \\ v_q \\ v_0 \\ v_f \\ v_{kd} \\ v_{kq} \end{bmatrix}
=
\begin{bmatrix}
r_s & -\omega L_q & 0 & 0 & 0 & -\omega M_{KQ} \\
\omega L_d & r_s & 0 & \omega M_F & \omega M_{KD} & 0 \\
0 & 0 & r_s & 0 & 0 & 0 \\
0 & 0 & 0 & r_f & 0 & 0 \\
0 & 0 & 0 & 0 & r_{kd} & 0 \\
0 & 0 & 0 & 0 & 0 & r_{kq}
\end{bmatrix}
\begin{bmatrix} i_d \\ i_q \\ i_0 \\ i_f \\ i_{kd} \\ i_{kq} \end{bmatrix}
+
\begin{bmatrix}
L_d & 0 & 0 & M_F & M_{KD} & 0 \\
0 & L_q & 0 & 0 & 0 & M_{KQ} \\
0 & 0 & L_0 & 0 & 0 & 0 \\
M_F & 0 & 0 & L_{fd} & L_{fkd} & 0 \\
M_{KD} & 0 & 0 & L_{fkd} & L_{kd} & 0 \\
0 & M_{KQ} & 0 & 0 & 0 & L_{kq}
\end{bmatrix}
\frac{d}{dt}
\begin{bmatrix} i_d \\ i_q \\ i_0 \\ i_f \\ i_{kd} \\ i_{kq} \end{bmatrix}
$$

Utilizando notación operacional, siendo p el operador $\frac{d}{dt}$:

$$
\begin{bmatrix} v_d \\ v_q \\ v_0 \\ v_f \\ v_{kd} \\ v_{kq} \end{bmatrix} =
\begin{bmatrix}
r_s + L_d p & -\omega L_q & 0 & M_F p & M_{KD} p & -\omega M_{KQ} \\
\omega L_d & r_s + L_q p & 0 & \omega M_F & \omega M_{KD} & M_{KQ} p \\
0 & 0 & r_s + L_0 p & 0 & 0 & 0 \\
M_F p & 0 & 0 & r_f + L_{fd} p & L_{fkd} p & 0 \\
M_{KD} p & 0 & 0 & L_{fkd} p & r_{kd} + L_{kd} p & 0 \\
0 & M_{KQ} p & 0 & 0 & 0 & r_{kq} + L_{kq} p
\end{bmatrix}
\begin{bmatrix} i_d \\ i_q \\ i_0 \\ i_f \\ i_{kd} \\ i_{kq} \end{bmatrix}
$$

Comentarios sobre los resultados obtenidos con la transformación.

Simplificación importante de la matriz de inductancias.

	MATRIZ DE INDUCTANCIAS EN COORD. DE FASE	MATRIZ DE INDUCTANCIAS TRANSFORMADA
ELEMENTOS VARIABLES	27	0
ELEMENTOS CTES ≠ DE CERO	5	14
ELEMENTOS NULOS	4	22

Se reduce la complejidad del sistema de ecuaciones diferenciales.
Si $\omega = \frac{d\theta}{dt} = cte$ el sistema de ecuaciones diferenciales tiene coeficientes constantes.
La transformación, cuya matriz es ortogonal, conserva la invariancia de la potencia instantánea, la matriz de inductancias es simétrica y realizable.

POTENCIA ELÉCTRICA :

La potencia eléctrica instantánea consumida por una máquina sincrónica trifásica de la red exterior a la cual está conectada, está dada por la suma de las potencias instantáneas por fase:

$$P_{3\phi}(t) = v_a i_a + v_b i_b + v_c i_c = \left[v_{abc}\right]^t \left[i_{abc}\right]$$

Hallaremos la expresión en función de las variables $dq0$:

$$\left[v_{abc}\right] = \boxed{P_1^{-1}(0)} \left[v_{dq0}\right] \quad y \quad \left[i_{abc}\right] = \boxed{P_1^{-1}(0)} \left[i_{dq0}\right]$$

$$P_{3\phi}(t) = \left[\boxed{P_1^{-1}(0)} \left[v_{dq0}\right]\right]^t \boxed{P_1^{-1}(0)} \left[i_{dq0}\right] = \left[v_{dq0}\right]^t \boxed{P_1^{-1}(0)}^t \boxed{P_1^{-1}(0)} \left[i_{dq0}\right]$$

por ser una transf. ortogonal : $\boxed{P_1^{-1}(0)}^t = \boxed{P_1(0)}$

$$P_{3\phi}(t) = \left[v_{dq0}\right]^t \boxed{P_1(0)} \boxed{P_1^{-1}(0)} \left[i_{dq0}\right] = \left[v_{dq0}\right]^t \boxed{I} \left[i_{dq0}\right] = \left[v_{dq0}\right]^t \left[i_{dq0}\right]$$

$$\boxed{P_{3\phi}(t) = v_d i_d + v_q i_q + v_0 i_0}$$

PAR ELECTROMAGNÉTICO :

$$T = \frac{1}{2} \left[i\right]^t \left[\frac{\partial}{\partial\theta} \boxed{L}\right] \left[i\right]$$

$$T = \frac{1}{2} \left[\left[i_{abc}\right]^t, \left[i_{fk}\right]^t \right] \left(\frac{\partial}{\partial\theta} \begin{bmatrix} L_{SS} & M_{SR} \\ M_{RS} & L_{RR} \end{bmatrix} \right) \begin{bmatrix} \left[i_{abc}\right] \\ \left[i_{fk}\right] \end{bmatrix}$$

siendo: $\frac{\partial}{\partial\theta} \boxed{L_{RR}} = 0$ y $\boxed{M_{RS}} = \boxed{M_{SR}}^t$

$$T = \frac{1}{2} \left[\left[i_{abc}\right]^t, \left[i_{fk}\right]^t \right] \begin{bmatrix} \left[\frac{\partial}{\partial\theta} \boxed{L_{SS}}\right] \left[i_{abc}\right] + \left[\frac{\partial}{\partial\theta} \boxed{M_{SR}}\right] \left[i_{fk}\right] \\ \left[\frac{\partial}{\partial\theta} \boxed{M_{RS}}\right] \left[i_{abc}\right] \end{bmatrix}$$

$$T = \frac{1}{2} \left\{ \left[i_{abc}\right]^t \left[\frac{\partial}{\partial\theta} \boxed{L_{SS}}\right] \left[i_{abc}\right] + \left[i_{abc}\right]^t \left[\frac{\partial}{\partial\theta} \boxed{M_{SR}}\right] \left[i_{fk}\right] + \left[i_{fk}\right]^t \left[\frac{\partial}{\partial\theta} \boxed{M_{RS}}\right] \left[i_{abc}\right] \right\}$$

Al tratarse de matrices de (1×1)

$$\left[i_{abc}\right]^t \left[\frac{\partial}{\partial\theta} \boxed{M_{SR}}\right] \left[i_{fk}\right] = \left\{ \left[i_{abc}\right]^t \left[\frac{\partial}{\partial\theta} \boxed{M_{SR}}\right] \left[i_{fk}\right] \right\}^t = \left[i_{fk}\right]^t \left[\frac{\partial}{\partial\theta} \boxed{M_{SR}}\right]^t \left[i_{abc}\right] =$$

$$= \left[i_{fk}\right]^t \left[\frac{\partial}{\partial\theta} \boxed{M_{RS}}\right] \left[i_{abc}\right]$$

Resulta :

$$\boxed{T = \frac{1}{2} \left[i_{abc}\right]^t \left[\frac{\partial}{\partial\theta} \boxed{L_{SS}}\right] \left[i_{abc}\right] + \left[i_{abc}\right]^t \left[\frac{\partial}{\partial\theta} \boxed{M_{SR}}\right] \left[i_{fk}\right]}$$

Aplicando la transformación de Park modificada a las variables del estator:

$$[i_{abc}] = [P_1^{-1}(\theta)]\,[i_{dqo}] \qquad [i_{abc}]^t = \left([P_1^{-1}(\theta)]\,[i_{dqo}]\right)^t = [i_{dqo}]^t\,[P_1^{-1}(\theta)]^t = [i_{dqo}]^t\,[P_1(\theta)]$$

$$\boxed{T = \frac{1}{2}[i_{dqo}]^t\,[P_1(\theta)]\left[\frac{\partial}{\partial\theta}[L_{SS}]\right][P_1^{-1}(\theta)]\,[i_{dqo}] + [i_{dqo}]^t\,[P_1(\theta)]\left[\frac{\partial}{\partial\theta}[M_{SR}]\right][i_{fk}]}$$

$$[P_1(\theta)]\left[\frac{\partial}{\partial\theta}[L_{SS}]\right][P_1^{-1}(\theta)] = \begin{bmatrix} 0 & 3L_{SV} & 0 \\ 3L_{SV} & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \qquad [P_1(\theta)]\left[\frac{\partial}{\partial\theta}[M_{SR}]\right] = \begin{bmatrix} 0 & 0 & -\sqrt{\frac{3}{2}}M_{KQS} \\ \sqrt{\frac{3}{2}}M_{FS} & \sqrt{\frac{3}{2}}M_{KDS} & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$$\frac{1}{2}[i_{dqo}]^t\,[P_1(\theta)]\left[\frac{\partial}{\partial\theta}[L_{SS}]\right][P_1^{-1}(\theta)]\,[i_{dqo}] = 3L_{SV}\,i_d\,i_q$$

$$[i_{dqo}]^t\,[P_1(\theta)]\left[\frac{\partial}{\partial\theta}[M_{SR}]\right][i_{fk}] = \sqrt{\frac{3}{2}}\left(M_{FS}\,i_f\,i_q + M_{KDS}\,i_{kd}\,i_q - M_{KQS}\,i_{kq}\,i_d\right)$$

$$\boxed{T = 3L_{SV}\,i_d\,i_q + \sqrt{\frac{3}{2}}\left(M_{FS}\,i_f\,i_q + M_{KDS}\,i_{kd}\,i_q - M_{KQS}\,i_{kq}\,i_d\right)}$$

$$3L_{SV}\,i_d\,i_q = (L_{SO} - M_{SO} + 3/2\,L_{SV})\,i_d\,i_q - (L_{SO} - M_{SO} - 3/2\,L_{SV})\,i_d\,i_q = L_d\,i_d\,i_q - L_q\,i_d\,i_q$$

$$T = L_d\,i_d\,i_q - L_q\,i_d\,i_q + \sqrt{\frac{3}{2}}M_{FS}\,i_f\,i_q + \sqrt{\frac{3}{2}}M_{KDS}\,i_{kd}\,i_q - \sqrt{\frac{3}{2}}M_{KQS}\,i_{kq}\,i_d$$

$$T = \left(L_d\,i_d + \sqrt{\frac{3}{2}}M_{FS}\,i_f + \sqrt{\frac{3}{2}}M_{KDS}\,i_{kd}\right)i_q - \left(L_q\,i_q + \sqrt{\frac{3}{2}}M_{KQS}\,i_{kq}\right)i_d$$

siendo: $\Psi_d = L_d\,i_d + \sqrt{\frac{3}{2}}M_{FS}\,i_f + \sqrt{\frac{3}{2}}M_{KDS}\,i_{kd}$

$\Psi_q = L_q\,i_q + \sqrt{\frac{3}{2}}M_{KQS}\,i_{kq}$

resulta $\boxed{T = \Psi_d\,i_q - \Psi_q\,i_d}$

Desarrollo del cálculo de: $\boxed{P_1(\theta)}\left[\frac{\partial}{\partial\theta}\boxed{L_{SS}}\right]\boxed{P_1^{-1}(\theta)}$

$$\boxed{L_{SS}} = \begin{bmatrix} L_{SO} & M_{SO} & M_{SO} \\ M_{SO} & L_{SO} & M_{SO} \\ M_{SO} & M_{SO} & L_{SO} \end{bmatrix} + L_{SV}\begin{bmatrix} \cos 2\theta & \cos 2(\theta+2\pi/3) & \cos 2(\theta-2\pi/3) \\ \cos 2(\theta+2\pi/3) & \cos 2(\theta-2\pi/3) & \cos 2\theta \\ \cos 2(\theta-2\pi/3) & \cos 2\theta & \cos 2(\theta+2\pi/3) \end{bmatrix}$$

$$\frac{\partial}{\partial\theta}\boxed{L_{SS}} = -2L_{SV}\begin{bmatrix} \operatorname{sen} 2\theta & \operatorname{sen} 2(\theta+\frac{2\pi}{3}) & \operatorname{sen} 2(\theta-\frac{2\pi}{3}) \\ \operatorname{sen} 2(\theta+\frac{2\pi}{3}) & \operatorname{sen} 2(\theta-\frac{2\pi}{3}) & \operatorname{sen} 2\theta \\ \operatorname{sen} 2(\theta-\frac{2\pi}{3}) & \operatorname{sen} 2\theta & \operatorname{sen} 2(\theta+\frac{2\pi}{3}) \end{bmatrix}$$

$$\frac{4}{3}L_{SV}\begin{bmatrix} \cos\theta & \cos(\theta-2\pi/3) & \cos(\theta+2\pi/3) \\ -\operatorname{sen}\theta & -\operatorname{sen}(\theta-2\pi/3) & -\operatorname{sen}(\theta+2\pi/3) \\ 1/\sqrt{2} & 1/\sqrt{2} & 1/\sqrt{2} \end{bmatrix}\begin{bmatrix} \operatorname{sen} 2\theta & \operatorname{sen} 2(\theta+2\pi/3) & \operatorname{sen} 2(\theta-2\pi/3) \\ \operatorname{sen} 2(\theta+2\pi/3) & \operatorname{sen} 2(\theta-2\pi/3) & \operatorname{sen} 2\theta \\ \operatorname{sen} 2(\theta-2\pi/3) & \operatorname{sen} 2\theta & \operatorname{sen} 2(\theta+2\pi/3) \end{bmatrix}\begin{bmatrix} \cos\theta & -\operatorname{sen}\theta & 1/\sqrt{2} \\ \cos(\theta-2\pi/3) & -\operatorname{sen}(\theta-2\pi/3) & 1/\sqrt{2} \\ \cos(\theta+2\pi/3) & -\operatorname{sen}(\theta+2\pi/3) & 1/\sqrt{2} \end{bmatrix}$$

$$= -2L_{SV}\begin{bmatrix} \cos\theta & \cos(\theta-2\pi/3) & \cos(\theta+2\pi/3) \\ -\operatorname{sen}\theta & -\operatorname{sen}(\theta-2\pi/3) & -\operatorname{sen}(\theta+2\pi/3) \\ 1/\sqrt{2} & 1/\sqrt{2} & 1/\sqrt{2} \end{bmatrix}\begin{bmatrix} \operatorname{sen}\theta & -\cos\theta & 0 \\ \operatorname{sen}(\theta-2\pi/3) & -\cos(\theta-2\pi/3) & 0 \\ \operatorname{sen}(\theta+2\pi/3) & -\cos(\theta+2\pi/3) & 0 \end{bmatrix} = \begin{bmatrix} 0 & 3L_{SV} & 0 \\ 3L_{SV} & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

Desarrollo del cálculo de:

$$\boxed{P_1(\theta)}\left[\frac{\partial}{\partial\theta}\boxed{M_{SR}}\right]$$

$$\boxed{M_{SR}} = \begin{bmatrix} M_{FS}\cos\theta & M_{KDS}\cos\theta & -M_{KQS}\operatorname{sen}\theta \\ M_{FS}\cos(\theta-2\pi/3) & M_{KDS}\cos(\theta-2\pi/3) & -M_{KQS}\operatorname{sen}(\theta-\frac{2\pi}{3}) \\ M_{FS}\cos(\theta+2\pi/3) & M_{KDS}\cos(\theta+2\pi/3) & -M_{KQS}\operatorname{sen}(\theta+\frac{2\pi}{3}) \end{bmatrix}$$

$$\boxed{\frac{\partial}{\partial\theta}M_{SR}} = \begin{bmatrix} -M_{FS}\operatorname{sen}\theta & -M_{KDS}\operatorname{sen}\theta & -M_{KQS}\cos\theta \\ -M_{FS}\operatorname{sen}(\theta-2\pi/3) & -M_{KDS}\operatorname{sen}(\theta-2\pi/3) & -M_{KQS}\cos(\theta-2\pi/3) \\ -M_{FS}\operatorname{sen}(\theta+2\pi/3) & -M_{KDS}\operatorname{sen}(\theta+2\pi/3) & -M_{KQS}\cos(\theta+2\pi/3) \end{bmatrix}$$

$$\sqrt{\frac{2}{3}}\begin{bmatrix} \cos\theta & \cos(\theta-\frac{2\pi}{3}) & \cos(\theta+\frac{2\pi}{3}) \\ -\operatorname{sen}\theta & -\operatorname{sen}(\theta-\frac{2\pi}{3}) & -\operatorname{sen}(\theta+\frac{2\pi}{3}) \\ 1/\sqrt{2} & 1/\sqrt{2} & 1/\sqrt{2} \end{bmatrix}\begin{bmatrix} -M_{FS}\operatorname{sen}\theta & -M_{KDS}\operatorname{sen}\theta & -M_{KQS}\cos\theta \\ -M_{FS}\operatorname{sen}(\theta-2\pi/3) & -M_{KDS}\operatorname{sen}(\theta-2\pi/3) & -M_{KQS}\cos(\theta-2\pi/3) \\ -M_{FS}\operatorname{sen}(\theta+2\pi/3) & -M_{KDS}\operatorname{sen}(\theta+2\pi/3) & -M_{KQS}\cos(\theta+2\pi/3) \end{bmatrix} = \begin{bmatrix} 0 & 0 & -\sqrt{\frac{2}{3}}M_{KQS} \\ \sqrt{\frac{2}{3}}M_{FS} & \sqrt{\frac{2}{3}}M_{KDS} & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

Otra forma de deducir la expresión del par electromagnético:

De las ecuaciones de Park:

$$v_d = r_s i_d + \frac{d\psi_d}{dt} - \omega \psi_q \qquad \times i_d$$
$$v_q = r_s i_q + \frac{d\psi_q}{dt} + \omega \psi_d \qquad \times i_q$$
$$v_0 = r_s i_0 + \frac{d\psi_0}{dt} \qquad \times i_0$$

y las sumo:

$$v_d i_d + v_q i_q + v_0 i_0 = (r_s i_d^2 + r_s i_q^2 + r_s i_0^2) + \left(i_d \frac{d\psi_d}{dt} + i_q \frac{d\psi_q}{dt} + i_0 \frac{d\psi_0}{dt}\right) + \omega(\psi_d i_q - \psi_q i_d)$$

siendo:

$P_{3\phi}(t) = v_d i_d + v_q i_q + v_0 i_0$ pot. instantánea entregada en bornes del estator a la máquina.-

$P_{Cu} = r_s i_d^2 + r_s i_q^2 + r_s i_0^2$ " " disipada por efecto Joule en los arrollamientos del estator.-

$P_c = i_d \frac{d\psi_d}{dt} + i_q \frac{d\psi_q}{dt} + i_0 \frac{d\psi_0}{dt}$ " " correspondiente a la energía magnética almacenada en la armadura.-

$P_g = \omega(\psi_d i_q - \psi_q i_d)$ pot. instantánea transferida en el entrehierro.-

El par instantáneo puede obtenerse a partir de la potencia transferida en el entrehierro, a través de la siguiente relación: $T = \frac{P_g}{\omega} = \psi_d i_q - \psi_q i_d \Rightarrow T = \psi_d i_q - \psi_q i_d$