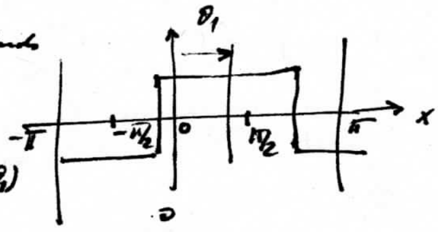
Fmm. de entrehierro y campo magnético creado por la bobina 1

Hip: Bobina 1 no distribuida, sino concentrada diametral
$2N_1$ espiras.

$$\mathcal{E}_1(x,t) = \begin{cases} N_1 i_1(t) \\ -N_1 i_1(t) \end{cases} \leftarrow \text{onda cuadrada}$$



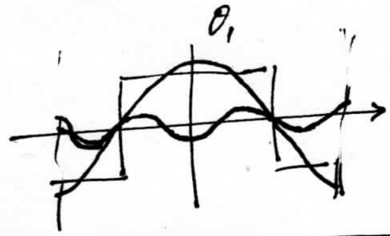
Fundamental $\mathcal{E}_{11}(x,t) = \frac{4}{\pi} N_1 i_1(t) \cos(x-\theta_1)$

En realidad se va a poder desarrollar en una serie de Fourier de cosenos impares

$$\mathcal{E}_1(x,t) = \sum_{k=1}^{\infty} N_1 i_1(t)\, a_{2k-1} \cos(2k-1)(x-\theta_1) \qquad \text{con } a_1 = 4/\pi$$

$$= N_1 i_1 \sum_{k=1}^{\infty} a_{2k-1} \cos(2k-1)(x-\theta_1)$$

$$= N_1 i_1(t) \left[a_1 \cos(x-\theta_1) + a_3 \cos(3x-3\theta_1) + \dots \right.$$



Hip: Si dejamos sólo el fundamental

$$\boxed{\mathcal{E}_{11}(x,t) = N_1 i_1 \cdot \frac{4}{\pi} \cos(x-\theta_1)}$$

$$\Rightarrow \boxed{B_1(x,t) = \frac{4}{\pi} N_1 i_1(t) \left[P_0 \cos(x-\theta_1) + \frac{1}{2} P_2 \cos(x+\theta_1) + \frac{1}{2} P_2 \cos(3x-\theta_1) \right]}$$

y quedaría todo igual, salvo por con un coef $\frac{4}{\pi}$ delante.
Si bobina 2 con distrib. sinusoidal de conductores:

$$\Rightarrow \psi_{21} = \frac{4}{\pi} \cdot \frac{\pi}{2} L R N_1 N_2 i_1 \left[2P_0 \cos(\theta_2-\theta_1) + P_2 \cos(\theta_2+\theta_1) \right]$$

$$\boxed{\psi_{21} = 2 L R N_1 N_2 i_1 \left[2P_0 \cos(\theta_2-\theta_1) + P_2 \cos(\theta_2+\theta_1) \right]}$$

Bobina 1 diametral →
Bobina 2 distribuida sinusoidalmente.

F.m.m. de entrehierro y campo magnético creado por la bobina 1

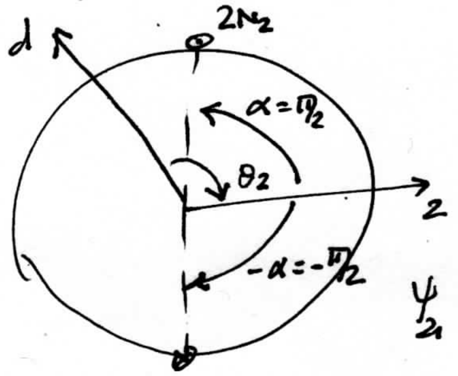
Hip. Bobina 1 distribuida sinusoidalmente

$$\boxed{B_1(x,t) = N_1 i_1 \left[P_0 \cos(x-\theta_1) + \frac{1}{2} P_2 \cos(x+\theta_1) + \frac{1}{2} P_2 \cos(3x-\theta_1) \right]}$$

(Cálculo anterior).

→ Cálculo del flujo que atraviesa el bobinado 2, del campo magnético creado por la bobina 1.

Hip Bobina 2, con $2N_2$ espiras, concentrada diametralmente



En el razonamiento anterior, el único valor posible de α es $\pi/2$

$$\psi_{21} = d\psi_{21} = 2N_2 \int_{\theta_2-\frac{\pi}{2}}^{\theta_2+\frac{\pi}{2}} L R B_1(x,t)\, dx$$

$$\Rightarrow \psi_{21} = 2N_2 L R \int_{\theta_2-\frac{\pi}{2}}^{\theta_2+\frac{\pi}{2}} N_1 i_1 \left[P_0 \cos(x-\theta_1) + \frac{1}{2} P_2 \cos(x+\theta_1) + \frac{1}{2} P_2 \cos(3x-\theta_1) \right] dx$$

$$\psi_{21} = 2LR N_1 N_2 i_1 \left[\int_{\theta_2-\frac{\pi}{2}}^{\theta_2+\frac{\pi}{2}} P_0 \cos(x-\theta_1)\, dx + \int_{\theta_2-\frac{\pi}{2}}^{\theta_2+\frac{\pi}{2}} \frac{1}{2} P_2 \cos(x+\theta_1)\, dx + \int_{\theta_2-\frac{\pi}{2}}^{\theta_2+\frac{\pi}{2}} \frac{1}{2} P_2 \cos(3x-\theta_1)\, dx \right]$$

$$\int_{\theta_2-\frac{\pi}{2}}^{\theta_2+\frac{\pi}{2}} \mathcal{P}_0 \cos(x-\theta_1)\,dx = 2\mathcal{P}_0\cos(\theta_2-\theta_1)\underbrace{\sin\frac{\pi}{2}}_{=1}$$

$$\int_{\theta_2-\frac{\pi}{2}}^{\theta_2+\frac{\pi}{2}} \frac{1}{2}\mathcal{P}_2 \cos(x+\theta_1)\,dx = 2\cdot\frac{1}{2}\mathcal{P}_2\cos(\theta_2+\theta_1)\underbrace{\sin\frac{\pi}{2}}_{=1}$$

$$\int_{\theta_2-\frac{\pi}{2}}^{\theta_2+\frac{\pi}{2}} \frac{1}{2}\mathcal{P}_2 \cos(3x-\theta_1)\,dx = \frac{1}{3}\,\frac{1}{2}\mathcal{P}_2\cdot 2\cos(3\theta_2-\theta_1)\underbrace{\sin\frac{3\pi}{2}}_{=-1}$$

$$\Rightarrow \boxed{\psi_{21} = 2LRN_1N_2\, i_1\left[2\mathcal{P}_0\cos(\theta_2-\theta_1)+\mathcal{P}_2\cos(\theta_2+\theta_1)-\frac{1}{3}\mathcal{P}_2\cos(3\theta_2-\theta_1)\right]}$$

Bobina 1 distribuida sinusoidalmente

Bobina 2 concentrada de paso diametral.

Hip.: Bobina 1 concentrada, paso diametral
Bobina 2 concentrada, paso diametral

De lo anterior:

$$\mathcal{E}_{11}(x,t) = \frac{4}{\pi} N_1 i_1 \cos(x-\theta_1)$$

$$B_1(x,t) = \frac{4}{\pi} N_1 i_1(t) \left[\mathcal{P}_0 \cos(x-\theta_1) + \frac{1}{2} \mathcal{P}_2 \cos(x+\theta_1) + \frac{1}{3} \mathcal{P}_2 \cos(3x-\theta_1) \right]$$

Como la bobina 2 es concentrada:

$$\psi_{21} = 2 N_2 \int_{\theta_2-\frac{\pi}{2}}^{\theta_2+\frac{\pi}{2}} L R B_1(x,t)\, dx$$

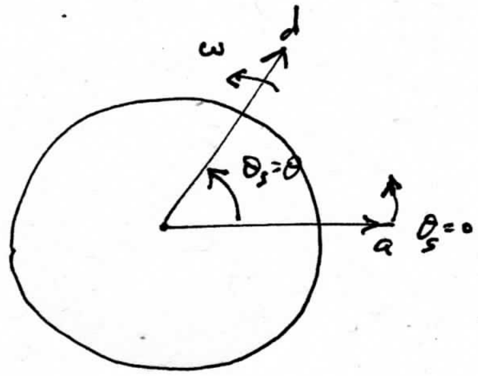
$$\psi_{21} = \frac{4}{\pi} 2 L R N_1 N_2 i_1 \int_{\theta_2-\frac{\pi}{2}}^{\theta_2+\frac{\pi}{2}} \left[\mathcal{P}_0 \cos(x-\theta_1) + \frac{1}{2} \mathcal{P}_2 \cos(x+\theta_1) + \frac{1}{2} \mathcal{P}_2 \cos(3x-\theta_1) \right] dx$$

Por los cálculos anteriores:

$$\boxed{\psi_{21} = \frac{4}{\pi} 2 L R N_1 N_2 i_1 \left[2 \mathcal{P}_0 \cos(\theta_2-\theta_1) + \mathcal{P}_2 \cos(\theta_2+\theta_1) - \frac{4}{3} \mathcal{P}_2 \cos(3\theta_2-\theta_1) \right]}$$

Ambas bobinas concentradas y de paso diametral.

Inductancias propias del estator



Fase a

$$N_1 = N_2 = N_s$$
$$i_1 = i_2 = i_a$$
$$\theta_1 = \theta_2 = -\theta_s \;(=-\theta)$$
por facilidad

Hip bobinado fase a distribuido sinusoidalmente

$$\psi_{aa} = \frac{\pi}{2} L R N_s^2 i_a \left[2P_0 \cos\underbrace{(\theta_2-\theta_1)}_{=0} + P_2 \cos\underbrace{(\theta_2+\theta_1)}_{=-2\theta_s} \right]$$

$$\psi_{aa} = \frac{\pi}{2} L R N_s^2 i_a \left[2P_0 + P_2 \cos 2\theta \right]$$

$$L_{aa} = \frac{\psi_{aa}}{i_a} = \underbrace{\pi L R N_s^2 P_0}_{L_{s_0}} + \underbrace{\frac{\pi L R N_s^2 P_2}{2}}_{L_{sv}} \cos 2\theta$$

$$\boxed{L_{aa} = L_{s_0} + L_{sv} \cos 2\theta}, \quad \text{con} \quad \boxed{L_{sv} = \frac{1}{2} L_{s_0}}$$

Fase b : cambiar θ por $\theta - \frac{2\pi}{3}$

Fase c : " " " $\theta - \frac{4\pi}{3}$

$$\boxed{L_{bb} = L_{s_0} + L_{sv} \cos 2\left(\theta - \frac{2\pi}{3}\right)}$$

$$\boxed{L_{cc} = L_{s_0} + L_{sv} \cos 2\left(\theta - \frac{4\pi}{3}\right)}$$

Inductancias mutuas del estator

$N_1 = N_2 = N_3$

fase a : bobina 2 $\Rightarrow$ $i_2 = 0$, $\theta_2 = -\theta$
fase b : " 1 $\Rightarrow$ $i_1 = i_b$, $\theta_1 = -(\theta - \frac{2\pi}{3}) = -\theta + \frac{2\pi}{3}$

$$\psi_{21} = \frac{\pi}{2} L R N_s^2 i_b \left[2 P_0 \underbrace{\cos\left(\cancel{-\theta} \cancel{+\theta} - \frac{2\pi}{3}\right)}_{\cos\frac{2\pi}{3} = -\frac{1}{2}} + P_2 \underbrace{\cos\left(-\theta - \theta + \frac{2\pi}{3}\right)}_{\cos(2\theta - \frac{2\pi}{3})} \right]$$

$$L_{ab} = \frac{\psi_{21}}{i_b} = \frac{\pi}{2} L R N_s^2 \left[-\frac{2P_0}{2} + P_2 \cos\left(2\theta - \frac{2\pi}{3}\right) \right]$$

$$L_{ab} = -\left(\frac{\pi L R N_s^2 P_0}{2}\right) + \frac{\pi L R N_s^2 P_2}{2} \cos\left(2\theta - \frac{2\pi}{3}\right)$$

$$\boxed{L_{ab} = M_{s0} + L_{sv} \cos\left(2\theta - \frac{2\pi}{3}\right)} \quad \text{con} \quad \boxed{M_{s0} = -\frac{1}{2} L_{s0}}$$

Fases b y c, modificando los ángulos θ y las corrientes:

$$\Rightarrow \boxed{\begin{aligned} L_{ba} &= L_{ab} = M_{s0} + L_{sv} \cos\left(2\theta - \frac{2\pi}{3}\right) \\ L_{bc} &= L_{cb} = M_{s0} + L_{sv} \cos 2\theta \\ L_{ca} &= L_{ac} = M_{s0} + L_{sv} \cos\left(2\theta - \frac{4\pi}{3}\right) \end{aligned}}$$