SIMPLIFIED EQUATIONS FOR MESH AND STEP VOLTAGES IN AN AC SUBSTATTION
Baldev Thapar $\quad$ Victor Gerez Arun Balakrishnan
Senior Member Senior Member Student Member
Montana State University
Bozeman, Montana Bozeman, Montana

Donald A. Blank
Bonneville Power Administration
Portland, Oregon


#### Abstract

A large number of grounding grids in substations have shapes other than a square or a rectangle, whereas the simplified formulas for mesh and step voltages available in the literature are derived for square or rectangular grids. Improvements in the simplified equations for determining mesh and step voltages are presented in this paper. With the modified simple equations mesh and step voltages at a substation of any practical shape can be estimated with reasonable accuracy. The results obtained with the improved equations have been compared with the accurate results obtained from the computer.


Keywords: Grounding, Touch Voltage, Mesh Voltage, Step Voltage, Grounding Grids, Substation Grounding.

## INTRODUCTION

Ground resistance, mesh and step voltages are important parameters in the design of grounding grids for ac substations. The shape of the grounding grid depends on the shape of the substation area. It may be square, rectangular, triangular, L-shaped, $T$-shaped or of any other shape. For evaluation of the ground resistance, the authors have already developed and proposed an equation, which is applicable to any practical shape of the grounding grid.[1]. However, the formulas available in the literature and recommended in IEEE Standard 80 for determining mesh and step voltage are derived for only square and rectangular grids. $[2,3]$.

Computer algorithric solutions have been proposed to determine mesh and step voltages of a complex grounding system made of linear conductors.[4-9]. The most common algorithm is based on finite element analysis in which the ground conductors are divided into small straight linear segments. Self and mutual resistances of all such segments are used to obtain the current dissipated by each segment. Voltage at any point on the surface of the ground is then evaluated by superposing the voltage produced by each segment. These methods are accurate but require feeding in of a lot of data regarding length, diameter, depth of burial, position and orientation of each subsection of the grid and resistivity of the soil in the computer for extensive computations. This is expensive, inconvenient and time consuming, especially when high degree of accuracy is not required in the analysis and design of the grounding systems.

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Simplified approximate equations for evaluation of the mesh and step voltage are preferred. However the error in simplified formulas, available in literature, when applied to grids of different shapes is not known. It is therefore necessary to have a closer look at the error in these formulas and make improvements so that they should give a better estimate of the mesh and step voltages for various shapes of the grounding grids found in practice.

This paper presents the modifications in formulas given in IEEE Standard 80 [3] so that they can be directly used to determine the mesh and step voltage of a grounding grid which may not be square or rectangular. The results obtained with these modified formulas have been compared with the accurate results obtained from the computer.

## COMPUTER ANALYSIS

A computer program, RESIS, based on finite element analysis, was developed by the authors to determine the ground resistance of a grounding system made of straight linear conductors laid in three mutually perpendicular directions.[1]. The scope of the program has been extended to calculate the mesh and step voltages. Appendix A gives the steps used in determining these voltages. The program is good for horizontal grids with or without vertical rods. RESIS gives accurate value of mesh and step voltages. A comparison of the results obtained with RESIS for square grids, with those reported in the literature is given in Table I. It is observed that the mesh and step voltages obtained from RESIS are very close to the respective values reported in the literature.

Table I - MESH AND STEP VOLTAGES OF SQUARE GRIDS

| Depth of Burial | $=h, m$ |
| :--- | :--- |
| Diameter of conductor | $=d, \mathrm{~mm}$ |
| Number of meshes | $=n$ |

$$
\begin{array}{ll}
\text { SGSYS: } & \text { Substation Grounding System Analysis, } \\
& \text { EPRI Computer Program [9] } \\
\text { J-P: } & \text { Joy, et al [7] }
\end{array}
$$

mesh and step voltages are given as percentage of grr

| GRID |  |  |  | MESH VOLTAGE |  |  | STEP VOLTAGE |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $\begin{gathered} \text { SIZE } \\ \mathrm{m} \end{gathered}$ | n | d | h | $\overline{\text { RESIS }}$ | SGSYS | J-P | RESIS | SGSYS | $J-M$ |
| 8X8 | 1 | 14 | 0.50 | 45.54 | 47.00 | - | 19.80 | 18.54 |  |
| 24X24 | 9 | 14 | 0.50 | 28.12 | 28.70 | - | 14.38 | 14.13 | - |
| $30 \times 30$ | 1 | 10 | 0.50 | 55.56 | 56.93 | 58 | 15.75 | 14.96 | 15.0 |
| 40X40 | 16 | 10 | 0.25 | 29.28 | 30.12 | 32 | 14.94 | 17.34 | 17.5 |
| $50 \times 50$ | 25 | 10 | 0.25 | 27.18 | 27.88 | 28 | 13.40 | 16.27 | 16.0 |
| 60x60 | 36 | 10 | 1.00 | 22.40 | 22.94 | 23 | 7.05 | 6.40 | 7.0 |
| $70 \times 70$ | 49 | 10 | 1.00 | 21.18 | 21.80 | 21 | 6.51 | 6.12 | 6.7 |
| 80X80 | 64 | 10 | 1.00 | 20.22 | 20.87 | - | 6.14 | 5.89 | - |


$a=b=20 \mathrm{~m}$ to 150 m

Square

$b: a=1: 4$ to $1: 16$ $b=5 \mathrm{~m}$ to 30 m Rectangular
 $b=20 \mathrm{~m}$ to 100 m Triangular
a
a
$a: b=2: 1$ to $1: 6$
$d: c=4: 1$ to $1: 8$
$a$ and $d=10 \mathrm{~m}$ to 80 m

d
$a: b=3: 1$ to $1: 4$
$a: d=1: 1$ to $1: 2$
$\mathrm{d}: \mathrm{c}=1: 1$ to $1: 5$
a and $d=5 \mathrm{~m}$ to 100 m
T-shaped

Figure 1 - Shape and size of grounding grids.

The shape of the grounding grids found in practice can be approximated to one of the following shapes: Square, Rectangle, Triangle, L-shape, and T-shape These five shapes were considered for analysis. Mesh and step voltages for more than 100 grids of different configurations and sizes were calculated. The variation of parameters considered for different shapes of the grids is given in Figure 1. These sizes cover almost all practical substation grounding grids

Mesh and step voltages are not very sensitive to the change in radius of the conductor within the practical range. The radius of the conductor for all cases was taken as 5 mm . The depth of burial for most cases was taken as 0.5 m and a few cases with 1.0 m depth of burial were also considered. Meshes in all the grids were considered to be square.

In case of the triangular, L-shaped and T-shaped grids, maximum touch voltage in each of the corner meshes was determined. The largest of these voltages was taken as the mesh voltage for a particular grid. Also the step voltage for these three shapes was taken to be the largest of the step voltages determined at all corners of the grid.

Accurate mesh and step voltages of a few representative cases, as obtained with the computer program RESIS are given in Table II.

## AVAILABLE SIMPLIFIED FORMULAS

The best available simple formulas for mesh and step voltages are those developed by Sverak and recommended in IEEE Standard $80 .[2,3]$. Mesh voltage, $\mathrm{E}_{\mathrm{m}}$, is given by

$$
\begin{equation*}
\mathrm{E}_{\mathrm{m}}=\rho \mathrm{I}_{\mathrm{G}} \mathrm{~K}_{\mathrm{m}} \mathrm{~K}_{\mathrm{i}} / \mathrm{L} \tag{1}
\end{equation*}
$$

Where $\quad \rho=$ soil resistivity, ohm-m
$I_{G}=$ maximum grid current that flows between ground grid and surrounding earth, A.
$\mathrm{L}=$ Total length of grounding system conductor, m .
$\mathrm{K}_{\mathrm{i}}=0.656+0.172^{*} \mathrm{n}$
$\mathrm{n}=$ number of parallel conductors in one direction

$$
\begin{equation*}
K_{m}=\frac{1}{2 \pi}\left[\ln \left(\frac{D^{2}}{16 h d}+\frac{(D+2 h)^{2}}{8 D d}-\frac{h}{4 d}\right)+\frac{K_{i i}}{K_{h}} \ln \frac{8}{\pi(2 n-1)}\right] \tag{3}
\end{equation*}
$$

$D=$ Spacing between parallel conductors, m
$\mathrm{h}=$ Depth of ground grid conductor, m
$\mathrm{d}=$ diameter of grid conductor, m
$\mathrm{K}_{\mathrm{ii}}=1 /(2 \mathrm{n})^{2 / \mathrm{n}}$
$\mathrm{K}_{\mathrm{h}}=\sqrt{1+\mathrm{h}}$
step voltage, $\mathrm{E}_{\mathrm{S}}$, is given by

$$
\begin{equation*}
E_{S}=\rho I_{g} K_{S} K_{i} / L \tag{6}
\end{equation*}
$$

where:

$$
\begin{equation*}
\mathrm{K}_{\mathrm{S}}=\frac{1}{\pi}\left[\frac{1}{2 h}+\frac{1}{\mathrm{D}+\mathrm{h}}+\frac{1}{\mathrm{D}}\left(1-0.5^{\mathrm{n}-2}\right)\right] \tag{7}
\end{equation*}
$$

For rectangular grids with square meshes the value of $n$ for computations should be:
$\mathrm{n}=\sqrt{\mathrm{n}_{\mathrm{A}} \mathrm{n}_{\mathrm{B}}}$ for mesh voltage
and $n=$ greater of $n_{A}$ and $n_{B}$ for step voltage
where $n_{A}$ and $n_{B}$ are the number of Parallel conductors in the two directions.

These equations are recommended to be used for square and rectangular grids with the following limits

$$
\begin{aligned}
0.25 \mathrm{~m} & \leq \mathrm{h} \leq 2.5 \mathrm{~m} \\
\mathrm{~d} & <0.25 \mathrm{~h} \\
\mathrm{D} & >2.5 \mathrm{~m}
\end{aligned}
$$

To determine the validity of these equations when applied to grids of different shapes, mesh and step voltages were calculated with the use of these equations for the grids for which the accurate values of the mesh and step voltages had been determined with RESIS. Representative cases are given in Table II. The following observations are made:

1. Mesh voltages obtained from the equations given above have less than $30 \%$ error for square and triangular grids. For other shapes the error may be high and in some cases, as high as $70 \%$.
2. Step voltages obtained from equations given above have less than $30 \%$ error for square grids. For other shapes the error may be high, and in some cases, even higher than $100 \%$.

## MODIFICATIONS

Equations (1), (3), (6) and (7) are based on a model comprising equally spaced infinitely long parallel conductors with no cross connections, dissipating uniform current. Correction factors $\mathrm{K}_{\mathrm{i}}$, $K_{i i}$, and $K_{h}$ are used to account for the difference in the actual grids and the model. $K_{i}$, as given in equation (2) is based on the limited data obtained from the experimental work of koch on square grids.[10]. There is a scope of using a better expression for $K_{i}$ even for square meshes.[11].

The number of parallel conductors in one direction, $n$, is one of the factors used in the simplified equations. It may not be same in the two directions except in case of square grids. For rectangular grids, IEEE Standard 80 recommends the use of equation (8) to determine the value of $n$ to be used in the computations. This does not give good results for all sizes of rectangular grids. Moreover, no guidelines are available for the selection of the value

Table II - Mesh and Step Voltages of Grids of Different Shapes
Grid current, ${ }^{\text {I }}$ Resistivity of
Resistivity of Soil
Diameter of conductor
Depth of burial
1 kA

- $100 \mathrm{ohm}-\mathrm{m}$

Dimensions of the grid $=a, b, c, d$, meters
= $n$

Th : Thapar, et. al. [modified equations given in this paper]
80 : IEEE standard 80, 1986

| n | a | b | $c$ | d | Mesh voltage, $v$ \& Error |  |  |  |  | Step Voltage, V |  |  | \% Error |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  |  |  |  | Th | 80 | RESIS | Th | 80 | Th | 80 | RESIS | Th | 80 |
| SQUARE |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| 4 | 20 | 20 |  |  | 1020 | 1098 | 943 | 8.2 | 16.4 | 330 | 356 | 420 | -21.4 | -15.2 |
| 16 | 20 | 20 |  |  | 612 | 670 | 576 | 6.3 | 16.3 | 299 | 327 | 342 | -12.6 | -4.4 |
| 9 | 24 | 24 |  |  | 668 | 727 | 591 | 13.1 | 23.1 | 248 | 270 | 292 | -15.1 | -7.5 |
| 4 | 40 | 40 |  |  | 606 | 653 | 564 | 7.5 | 15.8 | 155 | 167 | 206 | -24.7 | -18.9 |
| 16 | 40 | 40 |  |  | 376 | 411 | 349 | 7.7 | 17.8 | 130 | 143 | 166 | -21.6 | -13.8 |
| 64 | 40 | 40 |  |  | 225 | 251 | 224 | 0.5 | 12.1 | 121 | 135 | 137 | -11.7 | -1.5 |
| 16 | 60 | 60 |  |  | 279 | 326 | 255 | 9.4 | 27.8 | 82 | 90 | 107 | -23.4 | -15.9 |
| 100 | 100 | 100 |  |  | 103 | 115 | 98 | 5.1 | 17.3 | 39 | 44 | 48 | -18.8 | -8.3 |
| 144 | 120 | 120 |  |  | 80 | 90 | 76 | 5.2 | 18.4 | 31 | 35 | 35 | -11.4 | 0.0 |
| 225 | 150 | 150 |  |  | 59 | 67 | 57 | 3.5 | 17.5 | 24 | 27 | 26 | -7.7 | 3.8 |
| RECTANGULAR |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| 16 | 80 | 5 |  |  | 465 | 587 | 476 | -2.3 | 23.3 | 220 | 257 | 257 | -16.8 | 0.0 |
| 4 | 80 | 20 |  |  | 553 | 615 | 543 | 1.8 | 13.2 | 141 | 203 | 189 | -25.4 | 7.4 |
| 16 | 80 | 20 |  |  | 348 | 471 | 332 | 4.8 | 41.8 | 120 | 199 | 154 | -22.1 | 29.2 |
| 16 | 160 | 10 |  |  | 285 | 363 | 290 | -1.7 | 25.1 | 96 | 278 | 125 | -23.2 | 122.4 |
| 16 | 120 | 30 |  |  | 258 | 297 | 245 | 5.3 | 21.2 | 76 | 126 | 100 | -24.0 | 26.0 |
| 16 | 240 | 15 |  |  | 211 | 270 | 214 | -1.4 | 26.2 | 61 | 175 | 82 | -25.6 | 113.4 |
| 64 | 320 | 20 |  |  | 109 | 147 | 107 | 1.9 | 37.4 | 39 | 84 | 50 | -22.0 | 68.0 |
| 144 | 480 | 30 |  |  | 62 | 87 | 60 | 3.3 | 45.0 | 23 | 102 | 29 | -20.7 | 251.7 |
| L - SHAPED |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| 32 | 10 | 60 | 10 | 10 | 353 | 432 | 320 | 10.3 | 35.0 | 182 | 347 | 187 | -2.6 | 85.6 |
| 48 | 20 | 20 | 20 | 20 | 285 | 282 | 266 | 7.1 | 6.1 | 152 | 173 | 162 | -6.2 | 6.8 |
| 52 | 10 | 60 | 60 | 10 | 237 | 367 | 215 | 10.2 | 70.7 | 126 | 216 | 126 | 0.0 | 71.4 |
| 20 | 20 | 50 | 80 | 10 | 294 | 396 | 286 | 2.8 | 38.4 | 104 | 173 | 114 | -8.8 | 51.8 |
| 25 | 30 | 50 | 70 | 10 | 263 | 336 | 265 | -0.8 | 26.8 | 94 | 147 | 102 | -7.8 | 44.1 |
| 28 | 20 | 90 | 80 | 10 | 227 | 334 | 225 | 0.9 | 48.4 | 82 | 128 | 90 | -8.9 | 42.2 |
| 32 | 20 | 120 | 20 | 20 | 219 | 269 | 194 | 12.9 | 38.7 | 79 | 150 | 90 | -12.2 | 66.7 |
| 128 | 20 | 120 | 20 | 20 | 134 | 174 | 124 | 8.1 | 40.7 | 76 | 170 | 75 | 1.3 | 126.7 |
| 34 | 40 | 60 | 60 | 10 | 216 | 265 | 229 | -5.7 | 15.7 | 79 | 111 | 82 | -3.7 | 35.4 |
| 36 | 20 | 80 | 80 | 20 | 202 | 276 | 176 | 14.7 | 56.8 | 73 | 105 | 82 | -10.9 | 28.1 |
| 51 | 30 | 70 | 70 | 30 | 167 | 208 | 145 | 15.1 | 43.4 | 62 | 79 | 69 | -10.1 | 14.5 |
| 56 | 20 | 120 | 20 | 80 | 144 | 228 | 138 | 4.3 | 65.2 | 53 | 119 | 61 | -13.1 | 95.1 |
| 88 | 70 | 40 | 30 | 60 | 115 | 129 | 107 | 7.5 | 20.6 | 44 | 49 | 52 | -15.4 | -5.8 |
| 1 | 80 | 120 | 120 | 20 | 287 | 306 | 271 | 5.9 | 12.9 | 53 | 56 | 66 | -19.7 | -15.1 |
| 34 | 80 | 120 | 120 | 20 | 132 | 164 | 133 | -0.8 | 23.3 | 36 | 52 | 41 | -12.2 | 26.8 |
| 136 | 80 | 120 | 120 | 20 | 85 | 112 | 83 | 2.4 | 34.9 | 34 | 53 | 36 | -5.6 | 47.2 |
| TRYANGULAR |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| 20 | 40 | 20 |  |  | 517 | 595 | 534 | -3.2 | 11.4 | 261 | 372 | 273 | -4.4 | 36.3 |
| 40 | 80 | 20 |  |  | 299 | 366 | 291 | 2.8 | 25.8 | 156 | 315 | 154 | 1.3 | 104.5 |
| 63 | 90 | 30 |  |  | 226 | 277 | 259 | -12.7 | 6.9 | 123 | 230 | 127 | -3.2 | 81.1 |
| 30 | 100 | 50 |  |  | 228 | 229 | 248 | -8.1 | -7.7 | 86 | 104 | 101 | -14.9 | 2.9 |
| 40 | 160 | 40 |  |  | 186 | 228 | 200 | -7.0 | 14.0 | 67 | 136 | 75 | -10.7 | 81.3 |
| 45 | 150 | 50 |  |  | 176 | 215 | 190 | -7.4 | 13.1 | 64 | 218 | 73 | -12.3 | 61.6 |
| 60 | 200 | 50 |  |  | 140 | 177 | 155 | -9.7 | 14.2 | 52 | 112 | 57 | -8.8 | 96.5 |
| 60 | 100 | 100 |  |  | 145 | 181 | 142 | 2.1 | 27.5 | 54 | 69 | 67 | -19.4 | 2.9 |
| 110 | 200 | 100 |  |  | 98 | 124 | 109 | -10.1 | 13.8 | 46 | 81 | 56 | -17.8 | 44.6 |
| 72 | 160 | 80 |  |  | 123 | 162 | 143 | -13.9 | 13.3 | 38 | 64 | 40 | -5.0 | 60.0 |
| 144 | 320 | 80 |  |  | 77 | 103 | 91 | -15.4 | 13.2 | 30 | 73 | 31 | -3.2 | 135.5 |
| T - Shaped |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| 10 | 5 | 20 | 5 | 5 | 766 | 903 | 723 | 6.0 | 24.9 | 369 | 674 | 391 | -5.6 | 72.4 |
| 32 | 10 | 20 | 30 | 10 | 322 | 467 | 320 | 0.6 | 45.9 | 161 | 273 | 189 | -14.8 | 44.4 |
| 10 | 10 | 20 | 50 | 10 | 446 | 606 | 447 | -0.2 | 35.6 | 151 | 249 | 193 | -21.8 | 29.1 |
| 40 | 10 | 20 | 50 | 10 | 233 | 294 | 274 | -14.9 | 7.3 | 126 | 177 | 159 | -20.1 | 11.3 |
| 30 | 20 | 40 | 40 | 20 | 214 | 299 | 244 | -12.3 | 22.5 | 76 | 126 | 91 | -16.5 | 38.5 |
| 1 | 20 | 40 | 60 | 20 | 384 | 492 | 385 | -0.3 | 27.8 | 97 | 129 | 127 | -23.6 | 1.6 |
| 8 | 20 | 40 | 60 | 20 | 312 | 412 | 311 | 0.3 | 32.5 | 81 | 118 | 111 | -27.1 | 6.3 |
| 32 | 20 | 40 | 60 | 20 | 198 | 291 | 194 | 2.1 | 50.0 | 70 | 118 | 91 | -23.1 | 29.7 |
| 40 | 20 | 40 | 100 | 20 | 171 | 262 | 197 | -13.2 | 33.0 | 61 | 107 | 77 | -20.8 | 39.0 |
| 44 | 20 | 60 | 80 | 20 | 154 | 249 | 150 | 2.7 | 66.0 | 55 | 109 | 71 | -22.5 | 53.5 |
| 66 | 30 | 50 | 60 | 40 | 128 | 163 | 151 | -15.2 | 7.9 | 48 | 79 | 49 | -2.1 | 61.2 |
| 72 | 20 | 80 | 80 | 40 | 116 | 178 | 113 | 2.7 | 57.5 | 43 | 93 | 53 | -18.9 | 75.5 |
| 108 | 60 | 20 | 80 | 60 | 97 | 116 | 91 | 6.6 | 27.5 | 37 | 51 | 44 | -15.9 | 15.9 |
| 48 | 80 | 40 | 80 | 80 | 102 | 126 | 92 | 10.9 | 36.9 | 28 | 36 | 36 | -22.2 | 0.0 |
| 8 | 100 | 200 | 300 | 100 | 89 | 115 | 91 | -2.2 | 26.4 | 16 | 22 | 20 | -20.0 | 10.0 |

of $n$ to be used in the computations for grids of shapes other than square and rectangular. It is therefore desirable to find the value of $n$ to be used in the computation of mesh and step voltages, applicable to grids of all the shapes and sizes being considered.

Modification of $K_{i}$ and selection of a proper
value of $n$ were therefore considered for the
improvement of the results obtained from the simplified improvemen
equations.

For a large number of square and rectangular grids, mesh voltage was obtained by using RESIS. From this computer generated data, the following expression for $K_{i}$ was found to give a better fit in equation (I).

$$
\begin{equation*}
\mathrm{K}_{\mathrm{j}}=0.644+0.148 \star \mathrm{n} \tag{9}
\end{equation*}
$$

Selection of $n$ for use in the computation for grids other than square is complicated because of the following factors:

1. The number of parallel conductors in one direction may be different from that in the other direction.
2. The length of the parallel conductors in the same direction may not be the same.

For different shapes of the grid the selection of the value of $n$ to be used in the computations is possible if $n$ is expressed as a function of the dimensions of the grid and the total length of the conductor used in the grid. From the data generated by the computer for mesh voltages the following expression for $n$ was found to be suitable for all the shapes and sizes that were considered.
$\mathrm{n}=\mathrm{a} * \mathrm{~b} * \mathrm{c} * \mathrm{~d}$
where $a=2 L_{t} / L_{p}$

$$
\begin{equation*}
b=\left[L_{p} / 4 \sqrt{A}\right] 1 / 2 \tag{11}
\end{equation*}
$$

$$
\begin{equation*}
c=\left[\mathrm{L}_{\mathrm{X}} \mathrm{~L}_{\mathrm{Y}} / \mathrm{A}\right]^{0.7 \mathrm{~A} /\left(\mathrm{L}_{\mathrm{X}} \mathrm{~L}_{\mathrm{Y}}\right)} \tag{12}
\end{equation*}
$$

$$
\begin{equation*}
d=D_{m} /\left(L_{z}^{2}+L_{Y}^{2}\right)^{1 / 2} \tag{13}
\end{equation*}
$$

$L_{t}=$ Total length of the conductor in the horizontal grid, m
$L_{p}=$ Peripheral length of the gird, $m$
$A^{p}=$ Area of the grid, sq.m
$\mathrm{L}_{\mathrm{x}}=$ maximum length of the grid in x direction, $m$
$\mathrm{L}_{\mathrm{y}}=$ maximum length of the grid in y direction, $m$
$D_{\mathrm{m}}=$ Maximum distance between any two points on the grid, m.

The dimensions $L_{\mathbf{Y}}, L_{Y}$ and $D_{m}$ are illustrated for $T$ shaped grids in Figure 2 .

The expression for $n$ given in equation (10) is dimensionless. For square grids having square meshes the value of $n$ calculated from this equation is always equal to the number of parallel conductors in any one direction. The factors $b, c$ and $d$ in equation (10) are always equal to 1 for square grids; $c$ and $d$ are always equal to 1 for square and rectangular grids; $d$ is always equal to 1 for square, rectangular and L-shaped grids.

The following changes in the simplified equations recommended in IEEE Standard 80, and given in the previous section of this paper, are suggested.

1. Equation (9) which gives a better value of $K_{i}$ be used in place of equation (2).
2. Equation (10) gives a suitable value of $n$. The value of $n$ obtained from equation (10) be used in equations (3), (4), (7) and (9).

With these modified equations, mesh and step voltages for the cases for which the accurate values of mesh and step voltages had been determined with RESIS, were computed. The results of the representative cases are given in Table II.

Table II also gives the error, for the various cases, in the equations recomended in the IEEE Standard 80 and the modified equations, with respect to the accurate values obtained from the computer program RESIS. From the data presented in Table II the following observations are made.

1. Mesh Voltages.

The error in the modified equations is within $16 \%$
for any size and shape of the grid considered.
The error in the IEEE Standard 80 equations in some cases is more than $50 \%$.
2. Step Voltages.

Error in the modified equations is less than 30\%. The error in the IEEE Standard 80 equations in some cases is more than $100 \%$.

For some cases error in the modified equations is negative. This indicates that the modified equations may give values of mesh or step voltages, for some configurations of the grid, lower than the corresponding values obtained from the computer program RESIS. For mesh voltages the negative error is small and may be ignored. For step voltages the negative error for some cases may approach $-30 \%$. This higher error is because of the fact that the correction factor $K_{i}$ and the expression for ${ }^{n} n$ " were derived for mesh voltage. This error may be acceptable in the design of most grounding grids as, usually, step voltage is not critical. However, if higher degree of accuracy in the calculation of step voltage is desired, different expressions for $K_{i}$ and $n$ are to be developed for use in the computation of step voltage.

CONCLUSIONS

1. The simplified formulas available so far, for the calculation of mesh and step voltages at the substations are derived for grids which are square. When these formulas are used for the grids which are not nearly square, the error in the calculated mesh and step voltage may be high. In some cases the error may be as high as $70 \%$ for mesh voltage and more than $100 \%$ for step voltage.
2. The modifications, proposed in this paper, make the simplified formulas applicable to different shapes and sizes of the grounding grids. The error in the values obtained with these equations is within $16 \%$ for mesh voltage and less than $30 \%$ for step voltage. The shapes and sizes considered in this paper cover almost all the grounding grids found in practice.
3. The formulas proposed in this paper can be used to estimate the mesh and step voltages at the substations. They should prove useful to the engineers engaged in the design and analysis of the grounding systems.

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## APPENDIX A

Procedure for Dete $\frac{{ }^{\text {rmining Mes }} \text { \& }}{}$
The following procedure is adopted for calculating the mesh and step voltages of a substation grid.

1. The grid is divided in $J$ linear segments, each about 5 m in length.
2. Self and mutual ground resistance of each segment is determined by average potential method and a matriz of the ground resistance of the segments, [R], is formulated. This matrix is of dimensions JxJ.
3. When the grid discharges a total current, $I_{G}$, to the ground, the current discharged by each of the $J$ segments of the grid is given by $I_{1}, I_{2}, \ldots, I_{J}$, so that

$$
\begin{equation*}
{\underset{i=1}{J} I_{i}=I_{G}, ~}_{\text {in }} \tag{A1}
\end{equation*}
$$

The voltage drop along the grid conductors is neglected and each segment is assumed to be at voltage $V$.

$$
\begin{equation*}
[\mathrm{R}] \quad[\mathrm{I}]=[\mathrm{V}] \tag{A2}
\end{equation*}
$$

4. For a specified $I_{G}$, $J+1$ equations given by equations (A1) and (A2) are solved to determine $I_{1}$, $I_{2}, \ldots, I_{J}$ and $V . \quad V$ is the ground potential rise (GPR) of the grid.
5. The voltage at any point on the ground surface is determined by obtaining the sum of the voltages due to the currents dissipated by all the segments.
6. On the surface of the ground above the grid, areas where the minimum surface voltage is expected to occur are identified. In the specified areas (may be all the area above the grid), on the surface of the ground, voltage is determined at 0.1 m intervals and the minimum voltage on the surface of the ground above the grid is obtained. The difference between the GPR and this minimum voltage gives the mesh voltage.
7. The step voltage is calculated as the difference in voltage between two points on the surface of the ground, one directly above a corner of the grid and the other one meter away in the direction of the diagonal of the corner mesh extended outwards. In case of irregular meshes, step voltage at all the corners is determined and the highest value is recorded as the step voltage of the grid.


Baldev Thapar ( $\mathrm{M}^{\prime} 60, \mathrm{sm}^{\prime} 62$ ) was born in India on Sept. 1, 1930. He received the B.Sc. (Honours) degree from Banaras Hindu University, M.S. and Ph.D. degrees from Illinois Institute of Technology, in 1953, 1960 and 1963 respectively, all in electrical engineering. From 1953 to 1955 he was with Punjab Public Works Department, India, working in Power System Operation. In 1955 he joined the faculty of Punjab Engineering College, Chandigarh, India, where he was Professor, Electrical Engineering from 1966 to 1985 . In 1985-86 he was a Visiting Professor at Louisiana State University. At present he is a Professor in the faculty of Electrical Engineering Department, Montana State University, Bozeman.

Dr. Thapar is a Fellow of Institutions of Engineers (India). He is a member of Eta Kappa Nu, Tau Beta Pi and Sigma Xi. His research interests are in electric power system analysis, protection and grounding.


Victor Gerez (SM) was born in
Santander, Spain, on April 11, 1934. He received his Engineering degree from National University of Mexico and his M.S. and Ph.D. degrees from the University of California at Berkeley in 1958, 1969 and 1972 respectively, all in electrical engineering.

From 1958 to 1965 he was an electrical design engineer in several Mexican companies. From 1966 to 1973 he was a member of the technical staff of Mexico's National Utility. In 1973 he became chairman of the Mechanical-Electrical Engineering Department at the National University of Mexico. In 1977 he was named director of the power system division in Mexico's Electric Research Institute. He joined the Electrical Engineering Department at Montana State University in 1983 and became chairman in 1984.

Dr. Gerez is the author of several articles on system and power engineering and co-author of six electrical and system engineering textbooks widely used in Spanish speaking countries.

Arun Balakrishnan was born in New Delhi, India, on May 1, 1966. He received the B. Engg. (Honours) degree in electrical engineering from Panjab University, India, in 1987. For about one year he worked as Software Engineer at Electronic Systems Punjab Ltd., India. Currently he is working as a Graduate Research Scholar in electrical engineering at Montana State University.

Mr. Balakrishnan is a student member of the IEEE, and its Power Engineering Society. His academic interests are in the field of power system analysis and protection, and power electronics.

Donald A. Blank received the BSEE degree from Seattle University in 1955. Subsequently, he became employed at The Boeing co. in Seattle, Wash., where he designed flight electronics for eight years. He then joined the Substation Engineering Division at the Bonneville Power Administration in Portland, Oregon. There, he has been involved in most aspects and levels of design, most recently in substation grounding.

## Discussion

N. Barbeito (Florida Power Corporation, St. Petersburg, FL): The authors have developed an interesting extension to the information provided in the IEEE Standard 80. The analytical method presented is applicable to homogeneous soil conditions. Noting the fact that soil characteristics influence significantly the conductor current density and therefore the maximum value of mesh potential, how is the accuracy of the authors' method affected by normal (non-homogeneous) soil conditions?

The term " $n$ "" is defined differently in two areas of the paper. It was defined as the number of parallel conductors (IEEE Standard 80 definition) and also as the number of meshes. Was the correct value used in the IEEE Standard 80 formulas to obtain the values listed in Table II?

The discusser calculated the mesh and step voltages for one case of each of the three most commonly occurring substation shapes (square, rectangular and L-shaped) listed in Table II. The values obtained with our method [1], are within 16 percent of the authors' results.

## Reference

1. N. Barbeito and J. Lazzara. Simplified Two Layer Model Substation Ground Grid Design Methodology. IEEE Paper 90 131-3 PWRD, December 27, 1989.

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J. G. Sverak (JGSconsulting, Yorktown, NY): The Authors are to be congratulated for their effort toward expanding the applicability of simplified formulas provided in the IEEE Std 80-1986.
In these days, when giving a Weitek math-coprocessor and the 386 or 486 chips in an engineering station a run for the money often appears as more meritorious than taking the frugal challenge of weeding out the unnecessary work, considering a satisfactory engineering solution for the usual extent of environmental uncertainties, this down-to-earth effort should be applauded by the industry.

Both in this and the preceding paper $/ 1 /$, it has been demonstrated that an evaluation of T-shaped and L-shaped grids, as well as of odd triangular and extremely narrow rectangular grids is possible, if a composite corrective factor is added to these equations. It is both interesting and gratifying to see that the use of an equivalent parameter for the number of grid wires, in the form of

$$
\mathrm{n}_{\mathrm{cq}}=\mathrm{f}(\mathrm{lc}, \mathrm{~A}, \mathrm{~g} 1, \mathrm{~g} 2, \mathrm{~g} 3, \mathrm{~g} 4)
$$

for the mesh and step voltage formulas, and a corrective multiplier in the form of

$$
\mathrm{M}_{\text {corr }}=\mathrm{f}(\mathrm{lc}, \mathrm{~A}, \mathrm{~g} 1)
$$

for the resistance formula, produce satisfactory results in a broad variety of cases involving non-rectangular or extremely narrow grounding areas, due to an introduction of no more than four shape-characterizing parameters g1, g2, g3 and g4, in combination with the total length, lc, and the grounded area, A .
In view of the given results, it appears that a useful enhancement has been achieved in terms of the practical applicability of grounding calculations based on the Std 80. As it has been correctly detected by the Authors in their tests, the simplified equations in the basic form are (intentionally) biased to yield conservative result, in order to minimize the possibility of underdesigning.

Has the effect of peripheral ground rods been analyzed? It would be more logical to expect the use of Kii $=1$, instead of Kii per Eq. 3, for the T-shaped and L-shaped grids, since their layout "naturally", provides for the predominance of peripheral ground rods. In such a case, the Guide recommends the use of Kii $=1$

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B. Thapar, V. Gerez, A. Balakrishnan, D. A. Blank: The authors appreciate the interest expressed by the discussers and sincerely thank them for their comments and questions.

Presence of ground rods reduces both mesh and step voltages. The effect of the ground rods is not considered in this paper. However it should be interesting and useful to analyze the effect of the ground rods in different shapes of the grounding grids.

Soil characteristics definitely influence the current flow in the ground and hence the mesh and step voltages. Multilayer models of ground have been used for analysis of the grounding systems. In real life even the simple multilayer models do not closely represent the actual soil conditions. The resistivity of soil may also change from time to time because of the variation in the temperature and the moisture content. Because of these uncertainties a simple model of the ground having homogeneous soil is considered adequate for arriving at the simplified equations.

Correct value of $n$, number of parallel conductors in one direction, has been used in the equations to obtain the data presented in Tables I and II

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