

Evaluation of Resistivity Tests for Design of Station Grounds in Nonuniform Soil

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Summary: The actual soil formation at a prospective station site usually can be approximated by an assumed 2-layer stratification. The method of establishing characteristic parameters of such a formation by 4-electrode tests is reviewed. As the main result of this study, curves are presented to determine, from these parameters, the apparent resistivity values to be used in computing the resistance of a grounding grid in a 2-layer soil. During the design of such a grid, the maximum touch and step voltages that might occur during a ground fault have to be evaluated in addition to the station ground resistance.

METHODS FOR EVALUATING soil-resistivity test results and for incorporating the findings into the design of an adequate station grounding system are well established for the case where the soil at a prospective station site is uniform. However, station sites where the soil has a uniform resistivity throughout and to a considerable depth can seldom be found. More often than not, there are several layers, each having a different resistivity. Lateral changes may also occur—for example, from variations in profiles of the layers. In this paper, after a brief review of the design methods for uniform soil conditions, it will be shown how these procedures can be adapted to certain non-uniform soil formations that occur most frequently.

Throughout the paper, the grounding system of a station is considered to consist of a grid of conductors, buried at a depth of a few feet. This assumption is justified because, in most large generating or transformer stations, such a grid is the dominant element of the grounding system. The results of the discussions can, however, be extended to all cases where the horizontal dimension of a ground electrode is much larger than

the vertical—for example, in extensive rodbeds.

Evaluation of Tests for Uniform Soil Conditions

For measuring the electrical resistivity of soils, the 4-electrode method^{1,2} is most often used. Four probes are driven into the earth along a straight line at equal intervals, and the potential difference between the two inner probes is measured while current is passed through the two outer electrodes. Often a 4-terminal earth tester is used, both as a current source and a meter. If the reading on the instrument, representing the ratio of the above potential difference and current, is R_m (ohms), the soil resistivity ρ (ohm-meters) can be computed by the equation

$$\rho = 2\pi s R_m \quad (1)$$

where s (meters) is the probe spacing. A slight correction is needed if the resistance of the voltage probe is not negligible compared with that of the instrument's voltage circuit.

If a number of tests over the station site do not reveal significant variations in the measured ρ values, the soil resistivity may be considered uniform, and the ρ value obtained is the actual value of resistivity.

The ground resistance R of a grid, characterized by radius a of a circle having the same area as the area covered by the grid, is given by

$$R = \frac{\rho}{2\pi a} A \quad (2)$$

where the coefficient A can be determined from a modified form of Laurent's approximation:³

$$A = \frac{\pi}{2} + 0.6 \frac{t}{a} \quad (3)$$

t being an average side-length of the squares that make up the grid. It is hereby assumed that the length-to-width ratio of the grid is not far from unity, and that the depth of burial is

small compared with a . If either of these assumptions does not apply, a different approximation for A might be necessary, such as that given by Schwarz.⁴

If I_0 is the largest current expected to flow into the ground during a power system fault through a station's ground electrode of resistance R , the maximum voltage rise of the station's ground bus over the potential of a remote point equals the product of I_0 and R . This voltage appears across communication or control lines leaving the station and is, therefore, the basis for selecting appropriate voltage ratings for these facilities and associated protective equipment. On the other hand, it was found that the highest touch or step voltage in transformer stations, with conventional grounding grids buried in uniform soil, ranges from 20% to 30% of the total station potential rise, the lower values being valid for larger stations and vice versa.⁵ This is the potential difference that must be considered when evaluating personal hazards.

Based on the foregoing, station grounding design can be started by specifying a value, either for the permissible maximum touch voltage or for the tolerable largest total voltage rise of the station ground bus. In its new Grounding Guide, for example, The Hydro-Electric Power Commission of Ontario specifies an upper limit of 5,000 volts for the total station potential rise, with a recommended limit of 3,000 volts wherever economically possible. Knowing the value of I_0 from system analysis, the permissible maximum of the station ground resistance can be established. Then, using the value of soil resistivity obtained from tests, the main dimensions of a grounding grid can be determined by equations 2 and 3 so that the ground resistance of this electrode will be equal to, or less than, the specified maximum value.

Nonuniform Soils—the Concept of Apparent Resistivity

The ground resistance of any ground electrode in uniform soil is proportional to the resistivity of the soil. Hence, if ρ_1 is the soil resistivity, the equation for the ground resistance R_1 of an electrode will be of the form

$$R_1 = \rho_1 f(g) \quad (4)$$

where g represents the rest of the factors, all geometrical, on which ground resistance depends. The same electrode, buried in nonuniform earth, will still have a definite resistance to ground.

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To make possible the calculation of this resistance by equations identical in form to those for uniform soil conditions, the concept of equivalent uniform soil is introduced. The resistivity of this, ρ_a , is defined by

$$R = \rho_a f(g) \quad (5)$$

where R is the measured resistance of the electrode and $f(g)$ is the same function as in equation 4. The resistivity ρ_a will be called "apparent resistivity;" its value will, of course, depend on all the parameters that describe the non-uniformities at the location being studied. Unlike the value for uniform soil, however, ρ_a will depend also on the size and configuration of the electrode (in the case of a grounding grid, these are characterized in equations 2 and 3 by the equivalent radius a and the "density factor" t/a).

In the following analysis it will be assumed that, for a grid, the effects on ρ_a of the electrode configuration or of density are small compared with that of the radius a . Therefore, the equivalent of equation 2 now can be written as follows:

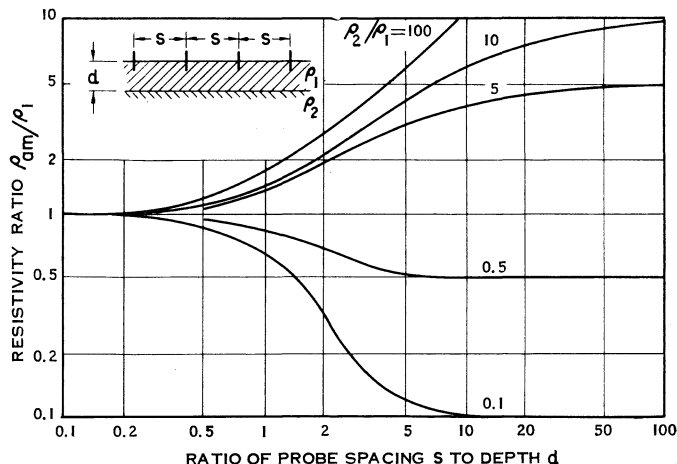
$$R = \frac{\rho_a(a, \nu)}{2\pi a} A \quad (6)$$

where ν stands for all parameters representing the nonuniformities of the soil.

The resistivities evaluated by equation 1 from 4-electrode tests, too, cease to be unique if the soil is not uniform. The measured resistivity ρ_{am} will depend on both s , the probe spacing, and ν . This is because, as the probe spacing is increased, the test current penetrates to more and more distant areas, both in vertical and horizontal directions, no matter how distorted are the current lines because of the varying resistivity. The value of ρ_{am} , therefore, will be more and more influenced by the resistivities of distant soil sections and deeper layers.

The question is: How can the proper ρ_a value for a given electrode be determined from a set of ρ_{am} values obtained by 4-electrode measurements at different probe spacings? Once ρ_a is established for the ground electrode in question under given soil conditions, the electrode's ground resistance can be calculated. The problem of station grounding design is, however, even more involved. Given a set of ρ_{am} values as measured, plus the desired ground resistance value, the size of a grid that will have the given resistance to ground has to be determined. In the following, these problems will be dealt with on the

Fig. 1. Measured resistivity ρ_{am} , as obtained by 4-electrode tests in 2-layer soils



assumption that the soil structure is of the simplest nonuniform type—namely, a 2-layer stratification.

2-Layer Stratification— Determination of Parameters

Real soil conditions frequently can be approximated with sufficient accuracy by assuming that (1) the lateral changes in resistivity are gradual compared with the vertical—in other words, the soil resistivity is a function of the depth below the surface only, and (2) as far as the vertical changes are concerned, the soil consists of an upper layer of depth d and resistivity ρ_1 , overlying a lower part of infinite depth and resistivity ρ_2 . Characteristic parameters for a soil formation of this type then are: d , ρ_1 , and ρ_2 . The first question is how to determine these quantities by conventional 4-electrode tests.

It was shown by Sunde⁶ that the ratio ρ_{am}/ρ_1 depends on only two variables, s/d and ρ_2/ρ_1 , in a manner illustrated by a few typical curves in Fig. 1. These were taken from Fig. 2.5 of reference 6 which contains curves for a wide range of ρ_2/ρ_1 values. As would be expected, ρ_{am} is near ρ_1 for small s/d values where only the upper layer is appreciably involved, and tends towards ρ_2 for large s/d values where the effects of the lower layer become dominant.

If values of d , ρ_1 , and ρ_2 at a given location must be established, the curve of ρ_{am} must be plotted against s , as obtained from a sufficient number of tests; then analyzed through comparison with curves shown in Fig. 1. If the plotted curve shows a trend similar to the curves in Fig. 1, a fair assumption is that a 2-layer stratification well represents the actual soil conditions. In such case, the ρ_{am} ordinates at the left side of the plotted chart converge to ρ_1 as

the curve becomes horizontal. Similarly, at the right side of the chart, ρ_{am} converges to ρ_2 . Near the departure from the first horizontal section, the abscissa that corresponds to the abscissa $s/d=1$ of Fig. 1 marks the probe spacing where $s=d$, thus giving an estimation of the third characteristic parameter d .

2-Layer Stratification—Grounding Design

The basic equation for determining main dimensions of a station grounding system is 6, which contains ρ_a , the apparent resistivity of the 2-layer soil formation. With values known for characteristic parameters ρ_1 , ρ_2 , and d , ρ_a may be evaluated from the curves in Fig. 2, representing the relation

$$\frac{\rho_a}{\rho_1} = N \left(\frac{a}{d}, \frac{\rho_2}{\rho_1} \right) \quad (7)$$

among the three dimensionless factors involved. These curves are valid for electrodes where the dominant extension is horizontal; in Fig. 2, this extension is characterized by the equivalent radius a . The curves were developed by evaluating, with the aid of a digital computer, the equations describing the potential field around a horizontal ring electrode placed in the middle of the upper soil layer. The analytical work involved is summarized in the Appendix. Fig. 2 indicates that, as already mentioned, ρ_a depends also on the ground electrode size. If a/d is small, ρ_a is near ρ_1 , and for large a/d values ρ_a approaches ρ_2 .

Modification is necessary if $\rho_2/\rho_1 < 1$ —that is, if the underlying soil has a lower resistivity than the top layer; and if the station ground electrode is buried in the lowest 10% of the upper layer. As discussed in the Appendix in connection with an equivalent scheme (Fig. 5), the first argument of N must, in such

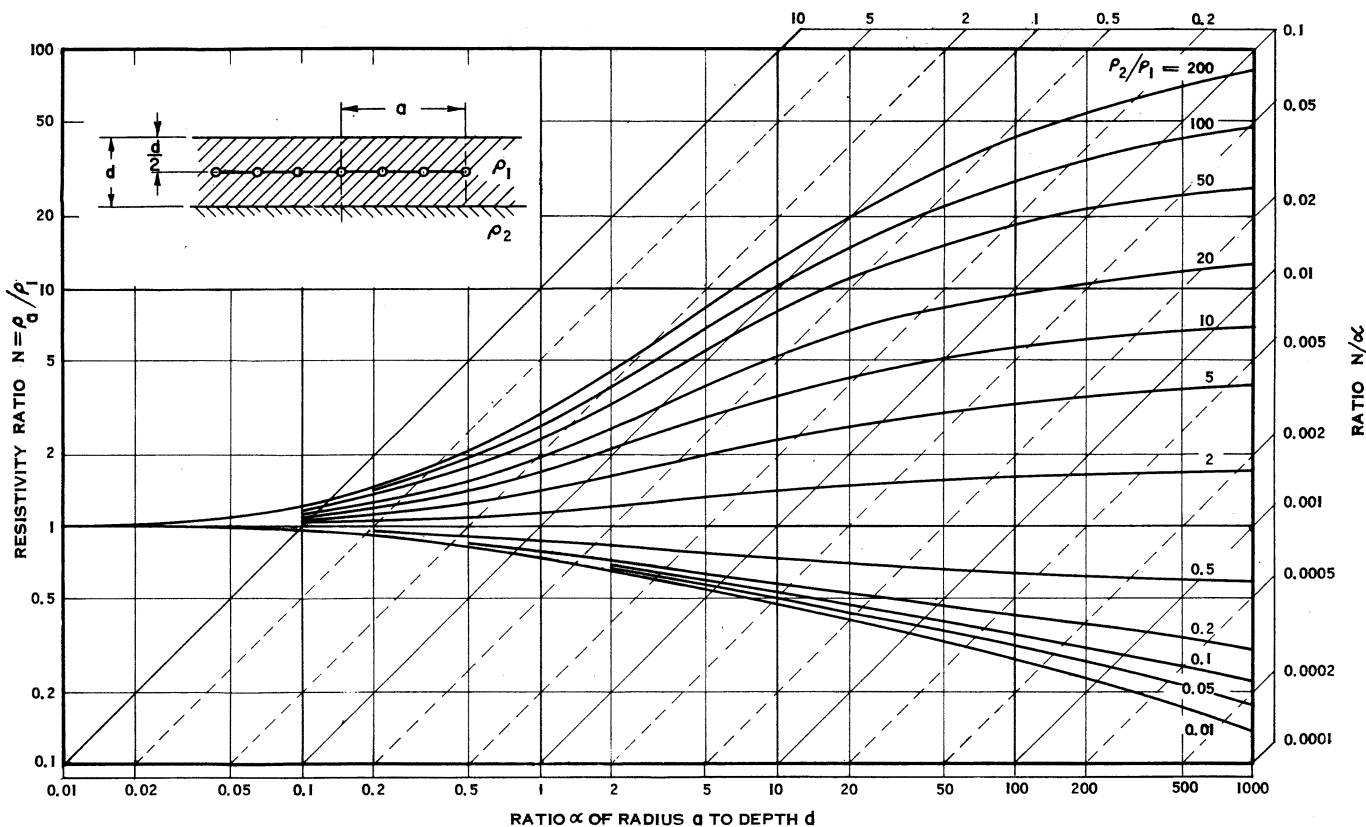


Fig. 2. Apparent resistivity ρ_a in 2-layer soils

cases, be taken at the value of $a/2(d-h)$, where h is the depth of burial of the electrode, rather than at a/d if unnecessarily conservative results are to be avoided.

If the ground resistance of a given grounding grid of radius a is sought, the procedure is to use the curve chosen from Fig. 2 as having the proper ρ_2/ρ_1 value; then, N 's value can be read from the diagram at the abscissa $\alpha = a/d$. The apparent resistivity for equation 6 will be

$$\rho_a(a, \nu) = \rho_1 N \left(\frac{a}{d}, \frac{\rho_2}{\rho_1} \right)$$

Finally, 6 will yield the ground resistance of the electrode. If the required ground resistance is given and the main dimension of a grid that will comply with the resistance requirement is sought, the calculation can be accomplished by employing the co-ordinates standing at a 45-degree angle in Fig. 2. After A is determined from equation 3 by choice of a realistic value for t/a , the left-hand side of the equation

$$\frac{2\pi R d}{\rho_1 A} = \frac{N \left(\frac{a}{d}, \frac{\rho_2}{\rho_1} \right)}{\frac{a}{d}}$$

can be computed. Since from equations 6 and 7 the foregoing equality stands, the actual ratio of N to a/d is thereby determined. Then, using the proper co-ordinates, the value of a/d can be read directly from Fig. 2.

To test their suitability, the N curves were applied to a few cases where all required data were known, including soil-resistivity test results and the measured station ground resistance. Predictions made from the curves were quite satisfactory; only for $\rho_2/\rho_1 < 1$ were the calculated resistance values somewhat high. This discrepancy may, however, be attributable partly to some uncertainties in the evaluation of the resistance measurements.

Other Stratifications

If the nonuniformity of the soil structure cannot be reasonably approximated by a 2-layer stratification, the mathematical treatment becomes rather complicated and the necessary procedure quite involved. Sometimes the approximation can be improved by assuming a 3-layer stratification. Construction of the corresponding N curve is comparatively simple if the uppermost layer is much thinner than the intermediate

layer or vice versa. This is discussed in some detail in reference 6, which also deals with the most general case where stratification is arbitrary with a continuous variation in resistivity or with any number of discontinuities. It must be emphasized, however, that approximation of actual conditions by a 2-layer stratification is almost always possible and within the limits of reasonable accuracy.

Touch and Step Voltages Under Nonuniform Soil Conditions

As noted previously, when conventional grounding grids are buried in uniform soils, the largest touch or step voltage is 20% to 30% of the total station ground voltage rise. No such unique relationship exists between these quantities, however, if the soil is not uniform, and the above voltage ratio may vary over a much wider range. The total voltage rise is determined by the station ground resistance which is, in turn, proportional to the resistivity of the surrounding soil. Under nonuniform soil conditions, the apparent resistivity ρ_a must be used in calculating the station ground resistance and it was shown that the value of ρ_a is greatly influenced by

the resistivity of lower soil layers. The touch and step voltages are also proportional to soil resistivity, but it can be proved that, for stratified soils, the resistivity of the top layer ρ_1 rather than ρ_a must be used in determining these potentials in the vicinity of a grounding grid. In other words, for stratified soils, the ratio of the touch voltages to the total rise may approach ρ_1/ρ_a times the same ratio under uniform soil conditions.⁵

For two different soil structures, the effect just described acquires particular significance.

First, if the resistivity of the top soil is much larger than that of underlying layers, the touch voltages arising during ground faults may be much higher than normally expected, and only extra precautions can assure safety. For example, special gradient-control rings or meshes may be installed around structures accessible to persons standing on the ground, or all areas regularly approached by station personnel may be covered with extra-thick layers of crushed stone.

Second, if high-resistivity bedrock is covered with a low resistivity but thin overburden, it is often impossible to meet ground-resistance requirement because of the unfavorably high resistivity of the lower layer. Unduly expensive measures can be avoided, however, by recognizing that, if a ground fault occurred, the actual touch voltages would be a smaller proportion of the total station voltage rise than that expected on the assumption of

uniform soil conditions. Nevertheless, caution must be exercised in taking advantage of this circumstance, since the resistivity of the top soil, unlike that of the deeper layers, depends to a considerable extent on weather conditions and shows quite marked seasonal variations.² If the top soil becomes frozen, its resistivity will assume a fairly high value, as will, during a ground fault, the surface gradients—proportional, in turn, to the touch and step voltages. Therefore, although a higher touch voltage would be at the same time tolerable, a rather pessimistic approach should be adopted for calculating and evaluating these potentials.

In any event, if doubts exist as to the magnitude of touch and step voltages to be expected, a few measurements of gradients are advisable at the time station ground-resistance tests are undertaken on the completed grounding system.

Appendix. The Derivation of the *N* Curves

According to equation 7, function *N* is defined as ρ_a/ρ_1 , the ratio of the apparent resistivity to the resistivity of the upper soil layer. If *R* is the resistance of a certain ground electrode buried in the upper part of a 2-layer soil and if *R*₁ is the ground resistance of the same electrode in uniform soil of resistivity ρ_1 , then, by using equations 4 and 5, the expression for *N* becomes

$$N = \frac{R}{R_1} \quad (8)$$

Obviously, *N* is a function of soil characteristics ρ_1 , ρ_2 , and *d*, and also, in the case of a grounding grid, of such factors as the equivalent radius *a*, electrode configuration, density and size of conductors, and depth of burial. Within the practical range of all the parameters, however, the influence of all other factors can be assumed small compared to that of *a*, ρ_1 , ρ_2 , and *d*. Hence, in evaluating *N*, a single electrode configuration, with only its main dimension varying, can represent all other electrodes. In the following analysis, chosen for the purpose is a horizontal ring-electrode of diameter *D*, buried at depth *h*, as illustrated in Fig. 3. By the definition of the equivalent radius, $a = D/2$. The ratio of *D* to the conductor diameter *d* is assumed to be 14,000 to 1.

Zaborsky⁷ has shown that the potential *V*₀ at the surface of a horizontal ring, buried at depth *h* in the upper layer of a 2-layer soil and discharging current *I* into the ground, may be calculated by using the method of images. According to this, the ring in the 2-layer soil can be substituted by an infinite series of images placed in a uniform medium of resistivity ρ_1 , and produced by both boundary planes between the air and the upper soil layer and the upper and lower soil layers, as shown in Fig. 3. The original ring and its first image "upwards" will discharge current *I*; the rest can be grouped in sets of fours, each ring

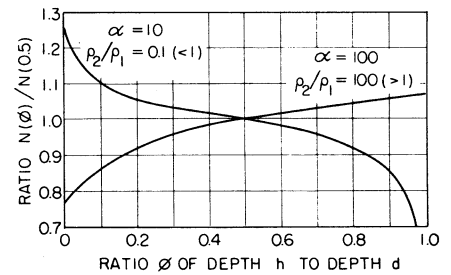


Fig. 4. Effect of relative depth of electrode on value of *N*

in the set discharging μI , $\mu^2 I$, ... amperes, where

$$\mu = \frac{\rho_2 - \rho_1}{\rho_2 + \rho_1} \quad (9)$$

By this method, the expression obtained for *V*₀ is

$$V_0 = \frac{I\rho_1}{2\pi^2} \left[\frac{K_{01} + K_{03}}{b_{01} + b_{03}} + \sum_{m=1}^{\infty} \sum_{n=1}^4 \mu^m \frac{K_{mn}}{b_{mn}} \right] \quad (10)$$

where the *K*_{*mn*}'s are complete elliptic integrals of the first kind, with the corresponding arguments

$$k_{mn} = \frac{2\sqrt{a\left(a + \frac{d_0}{2}\right)}}{b_{mn}} \approx \frac{2a}{b_{mn}} \quad (11)$$

and the *b*_{*mn*}'s are defined in Fig. 3. Since $b_{03} = 2a + d_0/2$ where the second term is much smaller than the first, *k*₀₃ is near 1 and *K*₀₃ can be approximated as

$$K_{03} \approx \ln \frac{4}{\sqrt{1 - k_{03}^2}} \approx \ln \frac{16a}{d_0} \quad (12)$$

Introducing the *c*-coefficients as $1/(2a)$ times the corresponding *b*'s and substituting equation 12, now equation 10 can be rearranged as

$$V_0 = \frac{I\rho_1}{4\pi^2 a} \left[\ln \frac{16a}{d_0} + \frac{K_{0+}}{c_{0+}} + \sum_{m=1}^{\infty} \mu^m \left(\frac{K_{m+}}{c_{m+}} + \frac{2K_m}{c_m} + \frac{K_{m-}}{c_{m-}} \right) \right] \quad (13)$$

where, from the geometry of Fig. 3 and using parameters $\alpha = a/d_0^2$ and $\phi = h/d$

$$c_{m+} = \sqrt{1 + \left(\frac{m+\phi}{\alpha}\right)^2}, \quad c_m = \sqrt{1 + \left(\frac{m}{\alpha}\right)^2},$$

$$c_{m-} = \sqrt{1 + \left(\frac{m-\phi}{\alpha}\right)^2}$$

and from equation 11 the arguments of integrals *K*_{*m*+}, *K*_{*m*}, and *K*_{*m*-} are

$$k_{m+} = \frac{\alpha}{\sqrt{\alpha^2 + (m+\phi)^2}}, \quad k_m = \frac{\alpha}{\sqrt{\alpha^2 + m^2}},$$

$$k_{m-} = \frac{\alpha}{\sqrt{\alpha^2 + (m-\phi)^2}}$$

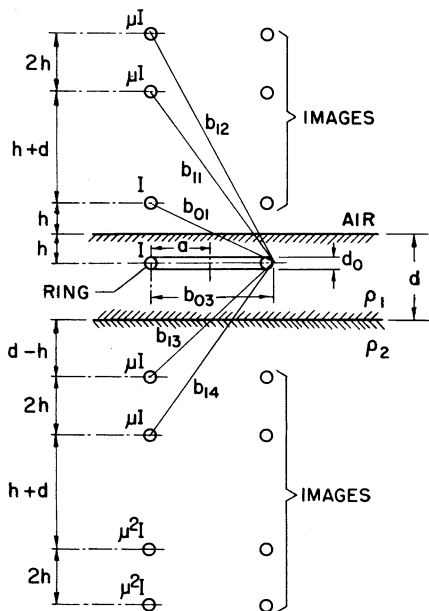


Fig. 3. Horizontal ring electrode buried in upper layer of 2-layer soil, and its images for computing the ring's ground resistance

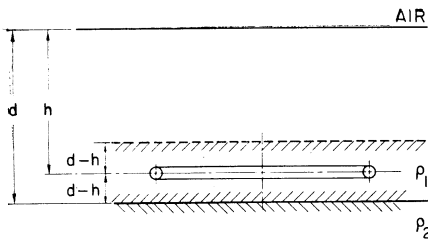


Fig. 5. "Equivalent top soil" of height $2(d-h)$ for instances in which $0.9 < h/d < 1$

The resistance of ring, R , to be substituted in equation 8 to yield N , will be

$$R = \frac{V_0}{I}$$

To find R_1 for the same equation, uniform soil conditions must be assumed; that is, $\rho_2 = \rho_1$. From equation 9, then, $\mu = 0$. Substituting this into equation 13, the potential V_{01} at the ring surface becomes

$$V_{01} = \frac{I\rho_1}{4\pi^2 a} \left(\ln \frac{16a}{d_0} + \frac{K_{0+}}{c_{0+}} \right)$$

and, as before,

$$R_1 = \frac{V_{01}}{I}$$

Finally, computing the ratio R/R_1

$$N = 1 + \frac{\sum_{m=1}^{\infty} \mu^m \left(\frac{K_{m+}}{c_{m+}} + \frac{2K_m}{c_m} + \frac{K_{m-}}{c_{m-}} \right)}{\ln \frac{16a}{d_0} + \frac{K_{0+}}{c_{0+}}} \quad (14)$$

A computer program was written and run to evaluate equation 14 for a set of α , ϕ , and ρ_2/ρ_1 values. The results, for $\phi = 0.5$, are shown in Fig. 2. The relative depth of the electrode in the upper layer, ϕ , has only a minor effect on N , as shown for two sets of α and ρ_2/ρ_1 values in Fig. 4. Only if $\rho_2/\rho_1 < 1$ and if ϕ approaches 1 is the effect of ϕ significant. This can be corrected by substituting an "equivalent top soil" of $2(d-h)$ depth for the real top layer, as illustrated in Fig. 5, and by assuming that the ring is placed in the middle of this equivalent layer. This arrangement keeps the distance of the ring from the bottom layer the same as it was in the original, so that the influence of the lower layer remains unchanged; at the same time the curves for $\phi = 0.5$ can still be used with only a slight modification; obviously, α now becomes equal to $a/2(d-h)$. Omission of the upper part of the top layer is permissible under the circumstances, since

this high-resistivity portion, if present, would influence the ground resistance of the ring to only a negligible extent.

The effect of conductor diameter d_0 on the value of N is small, within a wide range of possible variations of d_0 . For example, a 5-fold increase or decrease in d_0 would change the value of N , even in the worst case, by less than 7%. The effect of the grid density (t/a) on N might be somewhat more pronounced if ρ_2 is much larger or smaller than ρ_1 .

On the basis of these considerations, it was concluded that a horizontal ring electrode with the given a/d_0 ratio and buried in the middle of the upper soil layer ($\phi = 0.5$) is a satisfactory model for most practical cases, with only a simple modification to be added if $\rho_2/\rho_1 < 1$ and h approaches d . The results of Fig. 2 are, therefore, considered applicable for electrodes of all types in which the main dimension is horizontal and much larger than the maximum vertical extension of the electrode.

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Discussion

Stephen J. Schwarz (Sverdrup and Parcel and Associates, Inc., Consulting Engineers, San Francisco, Calif.): The author presupposes a 2-layer stratification with two discrete resistivity values as an approximation to describe nonuniform soil. This method has certainly some merits. However, the horizontal variations of resistivity

over the ground surface are disregarded. It would be interesting to have the author's opinion on whether this is permissible with respect to the influence of any surface variation in resistivity on touch and step voltages. As the author rightly remarks, increased probe spacings will have an averaging effect on test results, but smaller spacings scattered over an area of variable resistivity will not tend towards a single value of ρ_1 . What averaging method would the author propose here?

J. Endrenyi: The author wishes to thank Mr. Schwarz for his interest. As to the variations of soil resistivity, in most practical cases it was found reasonable to assume that the horizontal changes were gradual compared with the vertical changes, at least over an area the size of an average station site. That is why computations in the paper were based on the assumption that the soil was horizontally stratified.

If a situation to the contrary is experienced, mapping of the prospective station site, by measurements repeated at various points, becomes necessary.

The most likely deviation from the assumed soil formation is where the dividing surface between layers is not quite horizontal. In such a case, it may be possible to select the most favorable area within the site for installing the grounding grid, or, as an alternative, an average depth of the upper layer may be considered. If, on the other hand, there is marked change in resistivity ρ_1 of the upper layer, the apparent resistivity should be established by the method described in the paper using, in turn, the measured minimum and maximum values of ρ_1 and averaging the ρ_a values thus obtained. The two will not differ greatly anyway since it is the resistivity of the deeper layer that predominantly influences the values of ρ_a in most cases.

This leaves the rare cases where both upper and lower layers show a rapid lateral change in resistivity, or where no horizontal layers exist at all and the soil formation could be best represented by vertical layers. For the analytical evaluation of ρ_a under such circumstances, little guidance can be found in the literature; hence, a rather cautious approach is indicated. It may be possible to single out an area where the conditions are favorable and close to uniform; otherwise, a conservatively chosen average-resistivity value must be used. In every case, the local touch and step voltages will be proportional to the ρ_1 values at the same point. Since these voltages cannot be averaged, it is advisable to use over-all the value describing the worst conditions.