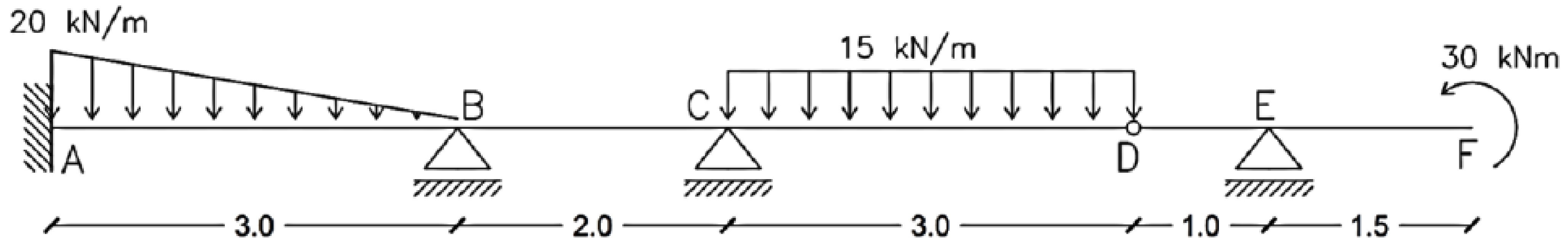
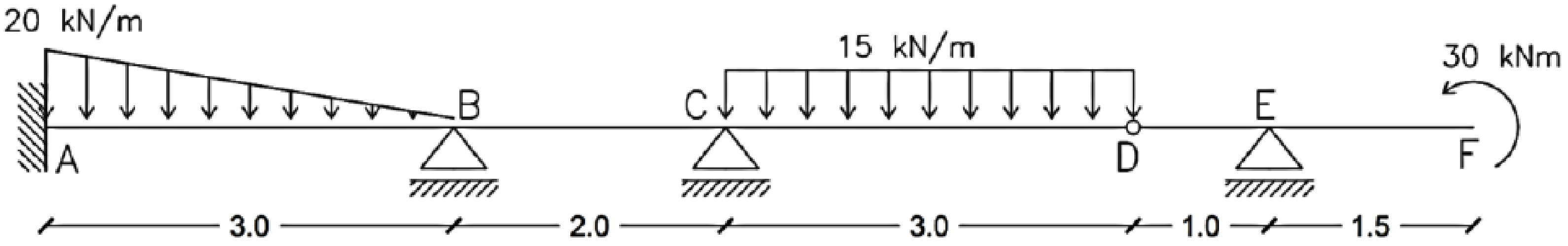
Examen Feb 2023

EXAMEN- 13 de febrero de 2023

Ejercicio 1

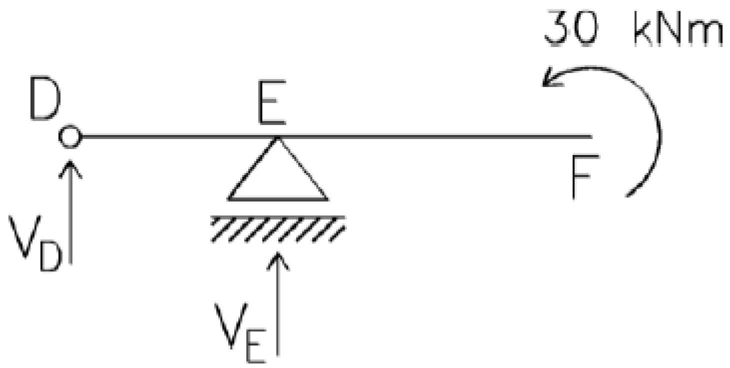
Se tiene la estructura de la Figura 1. En ella hay aplicadas: una carga distribuida constante de 15 kN/m hacia abajo aplicada en la barra CD, una carga distribuida lineal hacia abajo aplicada en AB cuyo valor en el nodo A es de 20 kN/m y un momento puntual de 30 kNm en sentido antihorario aplicado en el nodo G. Para dicha estructura se pide:





i) Hallar las reacciones de la estructura.

Separamos en estructuras tipo. Por un lado, analizamos el tramo DEF:



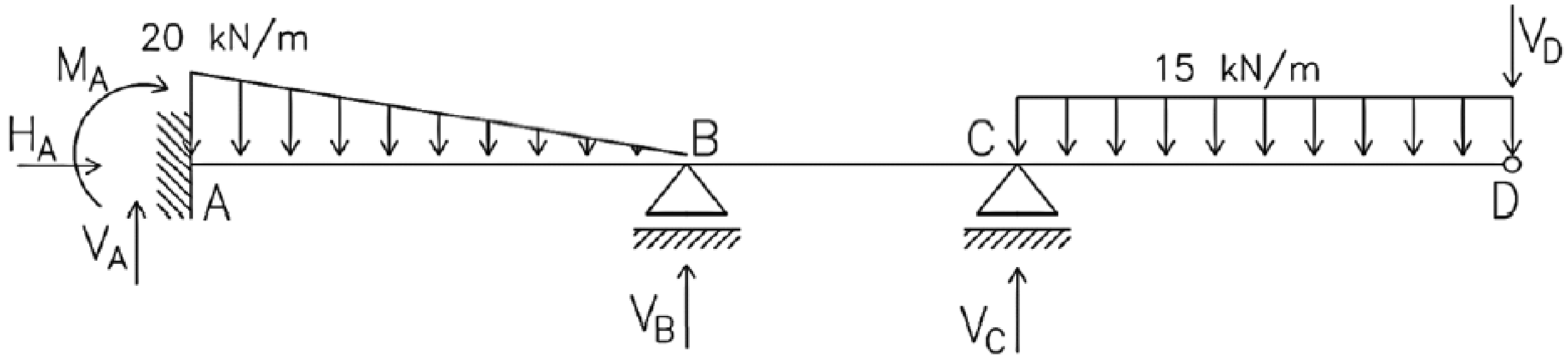
- $\sum M_D = 0 \leftrightarrow V_E \cdot 1\,m + 30\,kNm = 0$

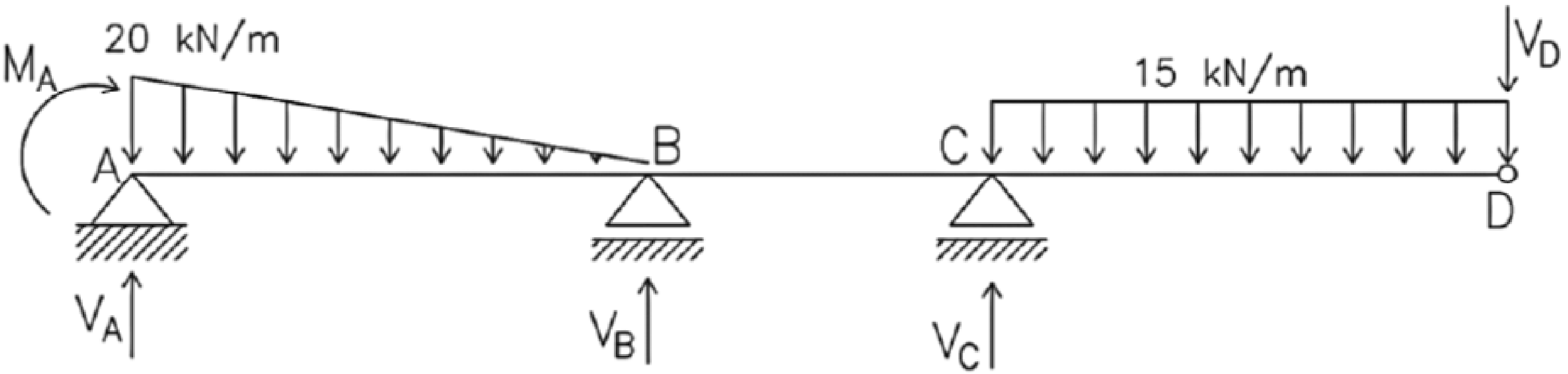
  $\boldsymbol{V_E = -30\,kN}$

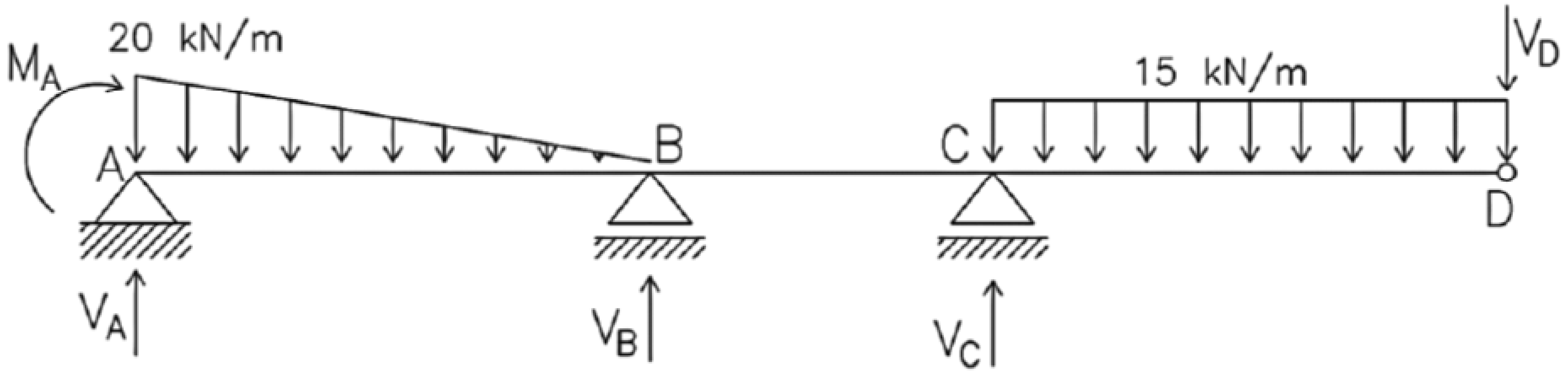
- $\sum V = 0 \leftrightarrow V_E + V_D = 0$

  $V_D = 30\,kN$

Estudiamos el tramo ABCD:





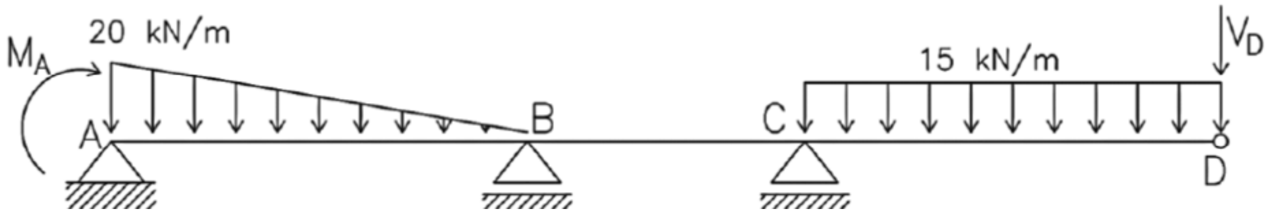


Imponemos giro nulo en A:

$$\theta_A^1 = \alpha_{0,A}^1 + M_A \cdot \alpha_A^1 + M_B \cdot \beta^1 - \Psi^1 = 0$$

$$\theta_A^1 = \frac{20\ kN/m \cdot (3\ m)^3}{45EI} + M_A \cdot \frac{3\ m}{3EI} + M_B \cdot \frac{3\ m}{6EI} = 0$$

$$12 + M_A + \frac{M_B}{2} = 0$$



<u>Aplicamos ecuación de 3 momentos en el nodo B:</u>

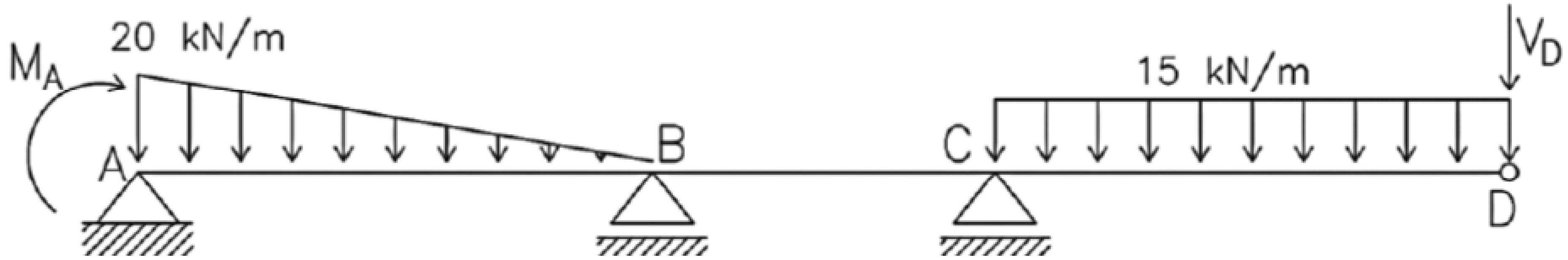
$$M_A \cdot \beta^1 + M_B \cdot (\alpha_B^1 + \alpha_B^2) + M_C \cdot \beta^2 + \alpha_{0,B}^1 + \alpha_{0,B}^2 - \Psi^1 + \Psi^2 = 0$$

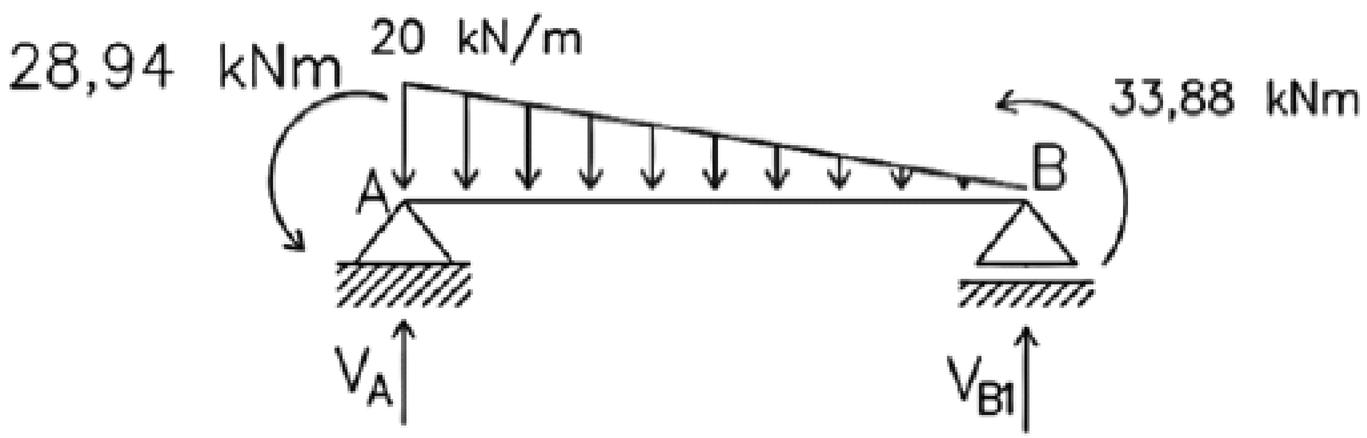
$$M_A \cdot \frac{3\,m}{6EI} + M_B \cdot \left(\frac{3\,m}{3EI} + \frac{2\,m}{3EI}\right) - 157{,}5\,kNm \cdot \frac{3\,m}{6EI} + \frac{7 \cdot 20\,kN/m \cdot (3\,m)^3}{360EI} = 0$$

$$\frac{M_A}{2} + M_B \cdot \frac{5}{3} - 42 = 0$$

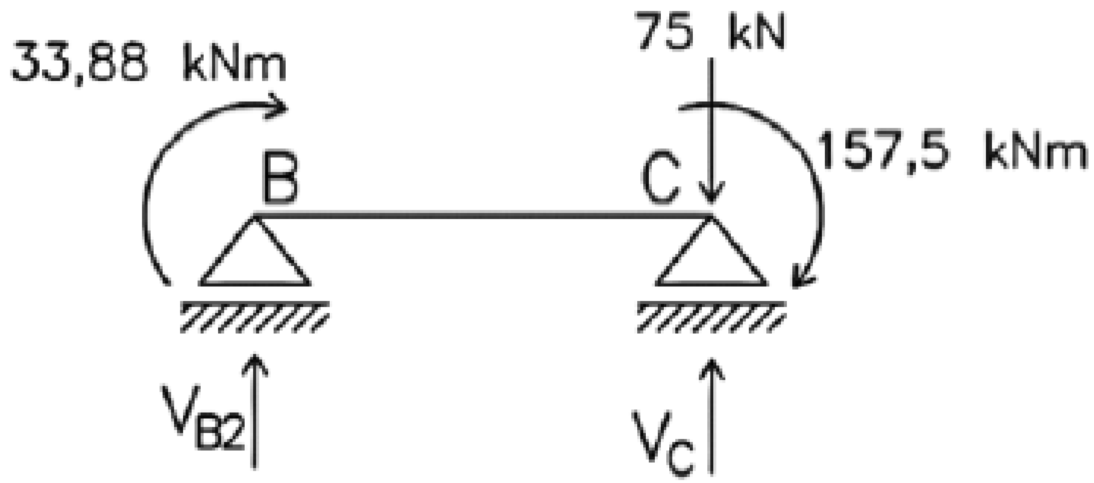
A partir de ambas ecuaciones se obtienen M_A y M_B:

$$\boldsymbol{M_A = -28{,}94\,kNm}$$
$$M_B = 33{,}88\,kNm$$





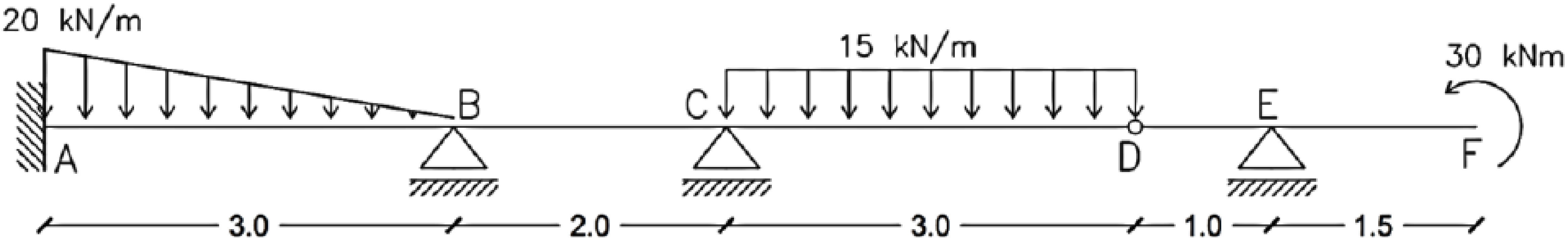
- $\boldsymbol{V_A = 40,94\ kN}$
- $\boldsymbol{V_B} = V_{B1} + V_{B2} = -10,94 - 95,69$

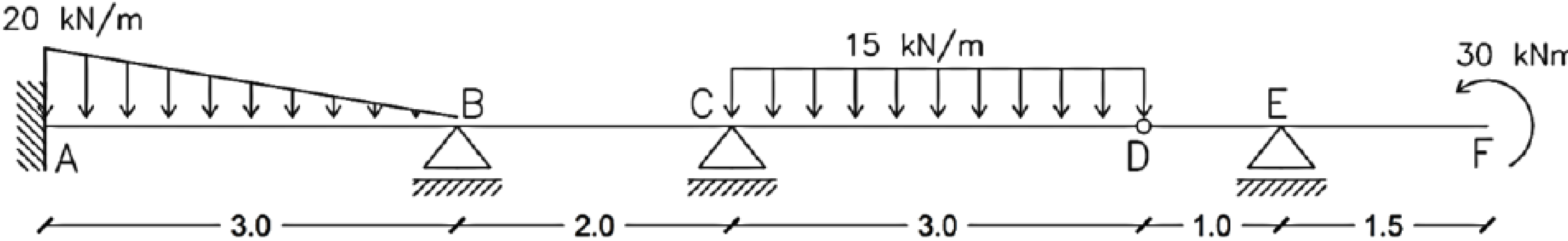


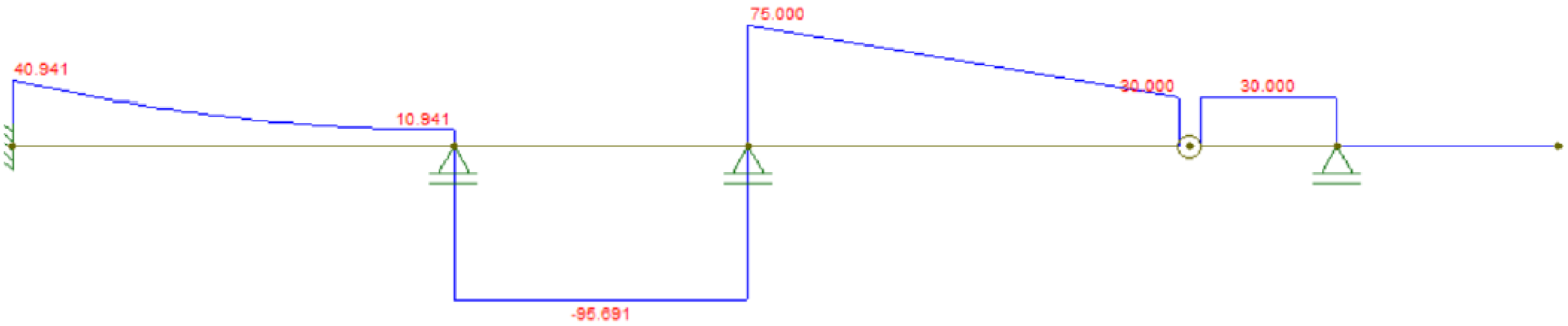
- $\boldsymbol{V_B} = V_{B1} + V_{B2} = -10{,}94 - 95{,}69 = \boldsymbol{-106{,}63\ kN}$
- $\boldsymbol{V_C = 170{,}69\ kN}$

Resumen de reacciones:

- $V_A = 40{,}94\ kN\ \uparrow$
- $H_A = 0\ kN$
- $M_A = 28{,}94\ kN\ \circlearrowleft$
- $V_B = 106{,}63\ kN\ \downarrow$
- $V_C = 170{,}69\ kN\ \uparrow$
- $V_E = 30\ kN\ \downarrow$




20 kN/m
15 kN/m
30 kNm
A
B
C
D
E
F
3.0
2.0
3.0
1.0
1.5


40.941
10.941
75.000
30.000
30.000
-95.691

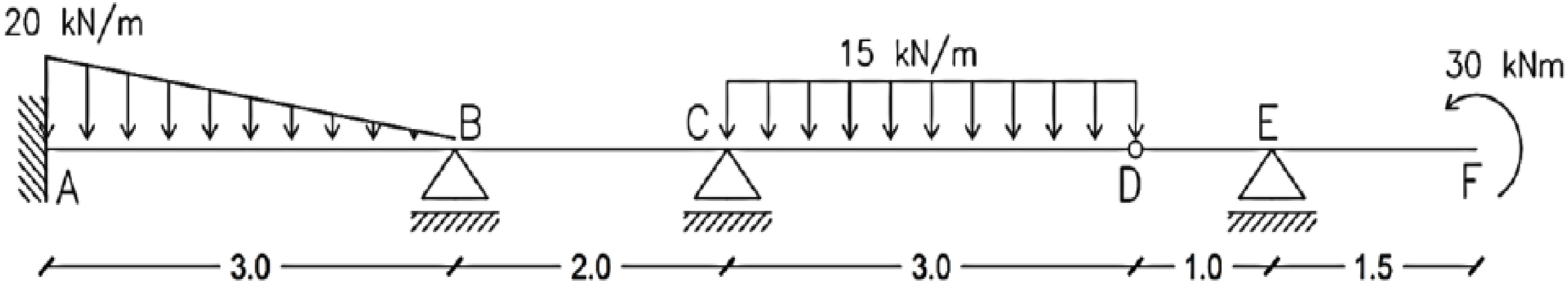
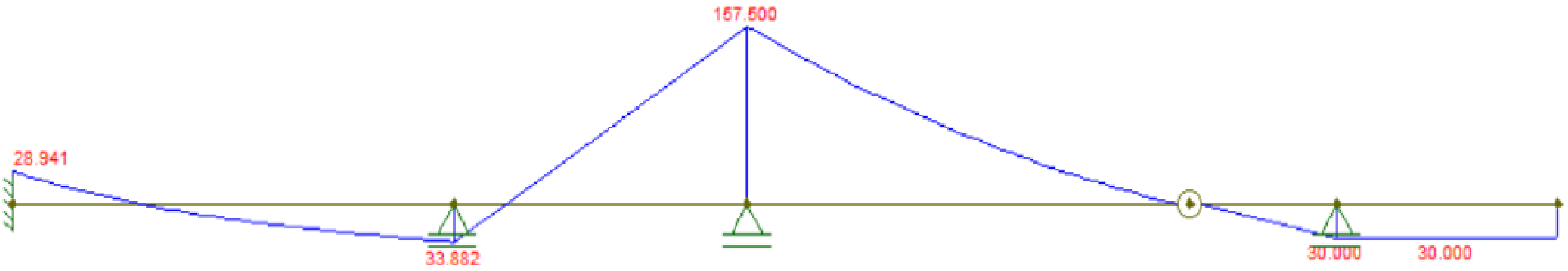


Diagrama de momento flector:



iii) *Dimensionar toda la estructura con un único perfil IPN. Verificar tensiones normales y rasantes* ($\sigma_{adm} = 140\ MPa;\ \tau_{adm} = 70\ MPa$).

Tensión normal máxima:

$$\sigma_{máx} = \frac{M_{máx}}{W} = \frac{157{,}5\ kNm}{W} \leq \sigma_{adm} = 140\ MPa$$

$$\frac{157{,}5\ kNm}{140\ MPa} = 1125\ cm^3 \leq W \rightarrow Usamos\ un\ PNI\ 380$$

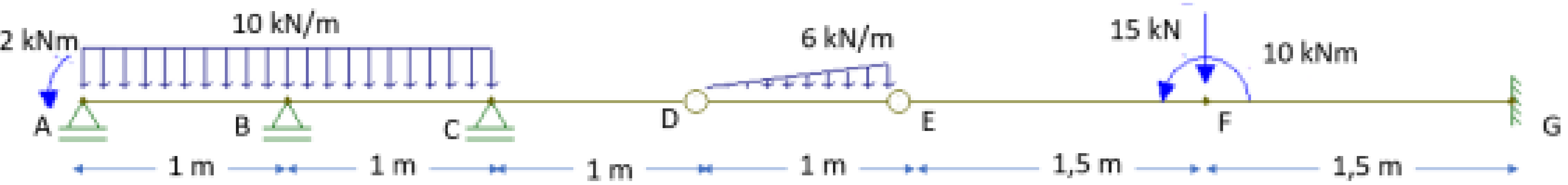
Verificamos que la tensión rasante máxima no supere la admisible para dicho perfil:

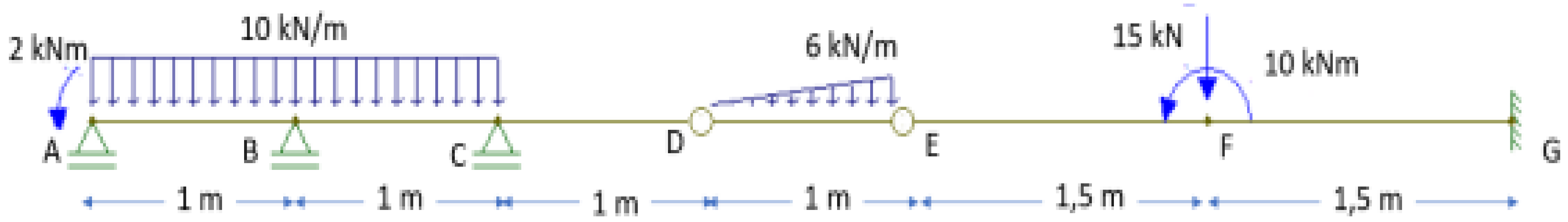
$$\tau_{máx} = \frac{V_{máx} \cdot \mu}{I \cdot b} = \frac{95{,}7\ kN \cdot 741\ cm^3}{19610\ cm^4 \cdot 1{,}37\ cm} = 26{,}40\ MPa \leq \tau_{adm} = 70\ MPa$$

Examen 02/23

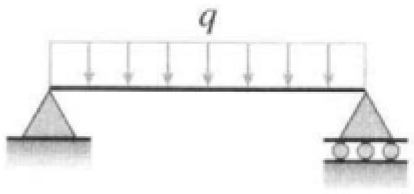
Se pide:

1) Hallar las reacciones.
2) Trazar los diagramas de solicitaciones.
3) Dimensionar con un IPN.
4) Hallar la flecha del punto E.

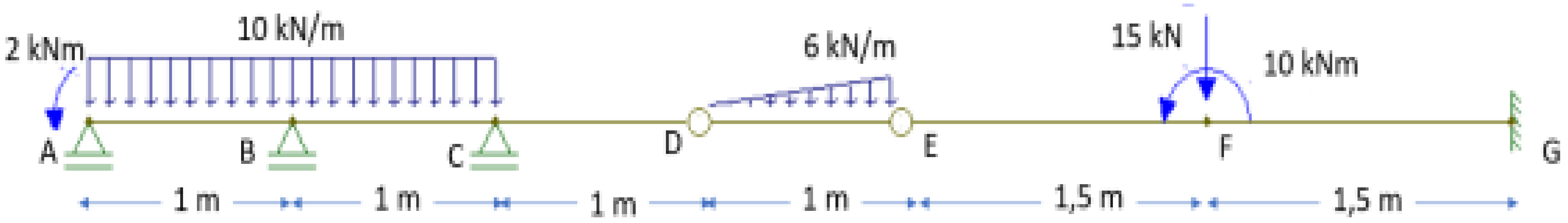




$$M_A\beta^1 + M_B\left(\alpha_B^1 + \alpha_B^2\right) + M_C\beta^2 + \alpha_{0B}^{\ 1} + \alpha_{0B}^{\ 2} - \psi^1 + \psi^2 = 0$$



$$\theta_A = \theta_B = \frac{qL^3}{24EI}$$



$$M_A\beta^1 + M_B\left(\alpha_B^1 + \alpha_B^2\right) + M_C\beta^2 + \alpha_{0B}^1 + \alpha_{0B}^2 - \psi^1 + \psi^2 = 0$$

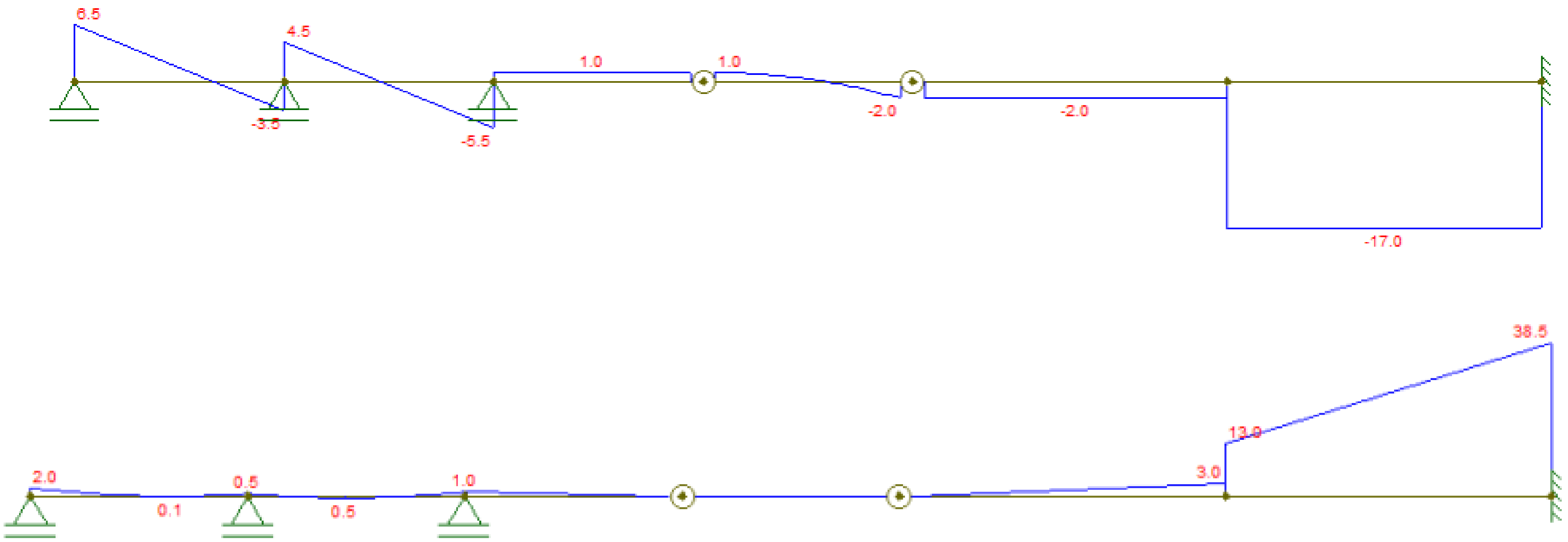
$L_{AB}/(6EI)$

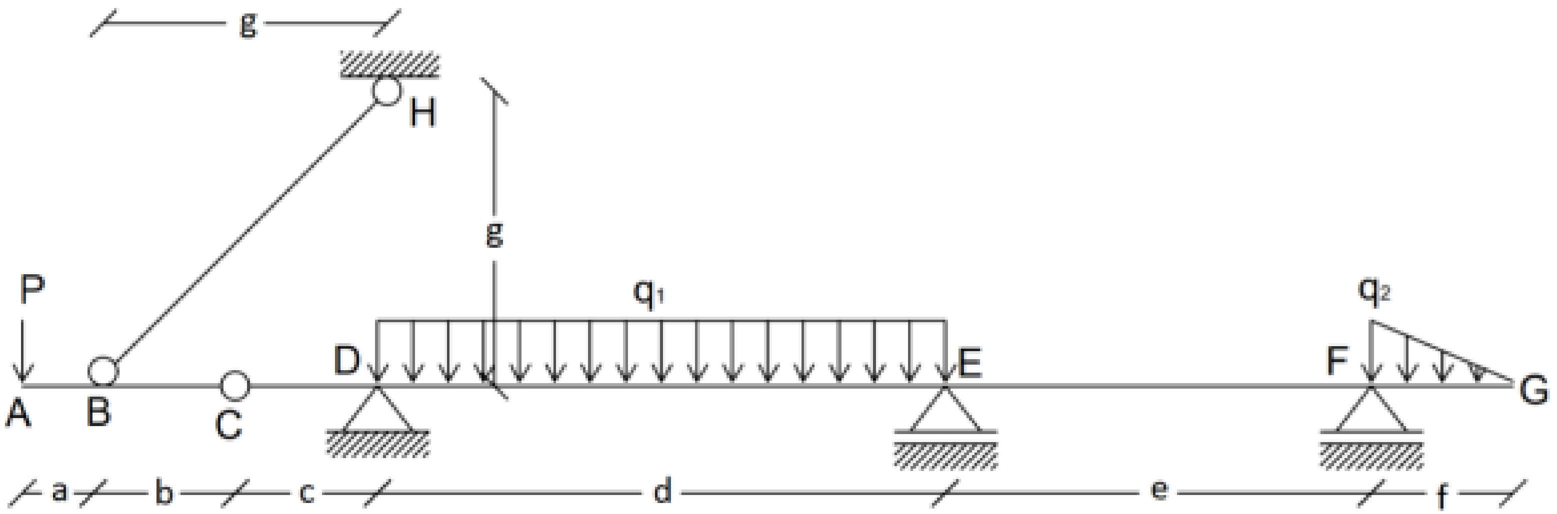
$L_{AB}/(3EI)+L_{BC}/(3EI)$

$L_{BC}/(6EI)$

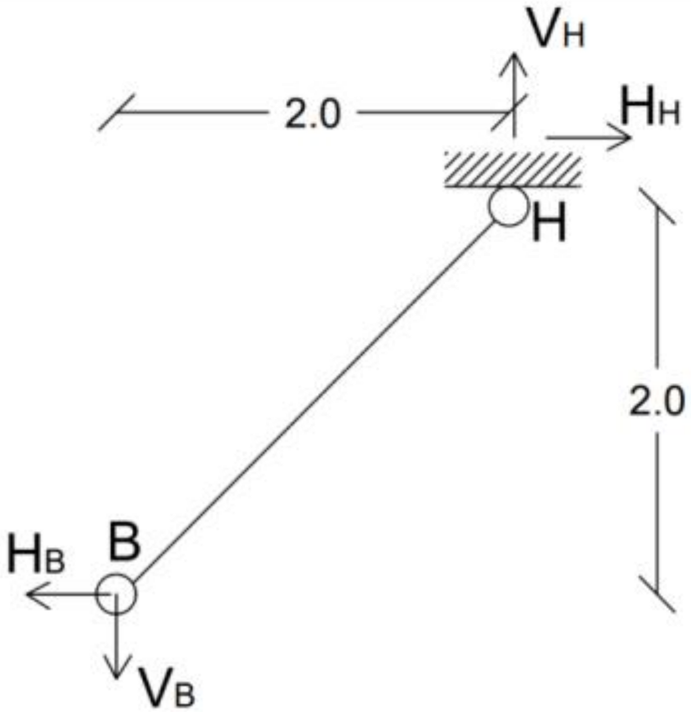
$$\theta_A = \theta_B = \frac{qL^3}{24EI}$$

V(kN)



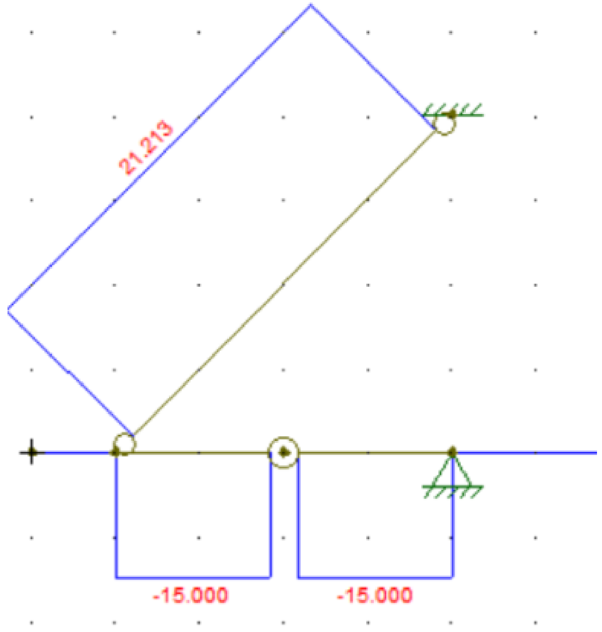

g
H
g
P
q1
q2
D
E
F
A
B
C
G
a
b
c
d
e
f

Estudiamos el tramo BH:

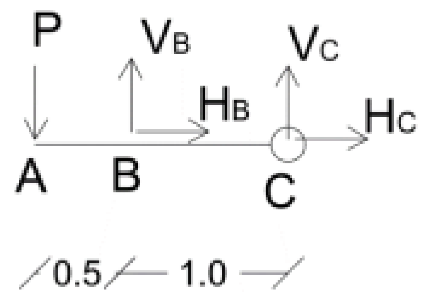


- $\sum M_H = 0 \rightarrow 2\,m \cdot V_B = 2\,m \cdot H_B$

  $H_B = 15\,kN$
- $V_H = 15\,kN$
- $H_H = 15\,kN$



Primero el tramo ABC:

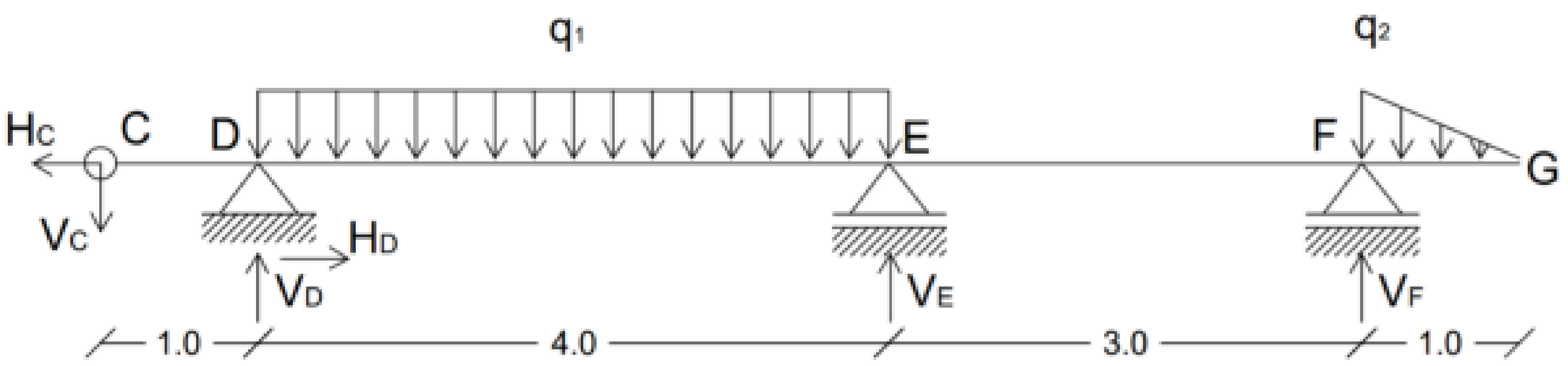


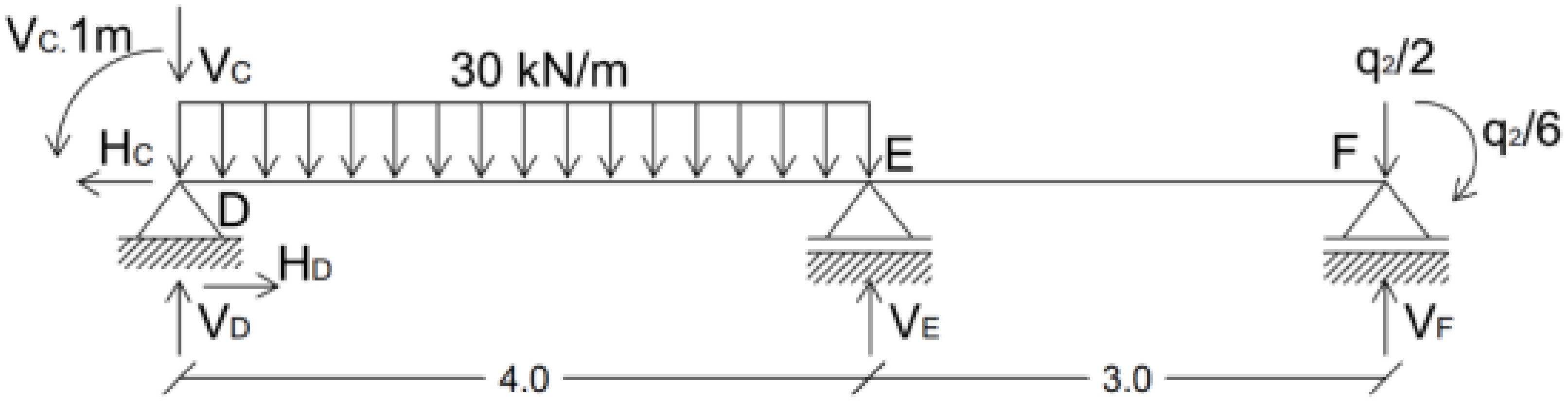
- $\sum M_C = 0 \rightarrow 1\,m \cdot V_B = 1{,}5\,m \cdot 10\,kN$

  $V_B = 15\,kN$
- $\sum V = 0 \rightarrow V_B + V_C = 10\,kN$

  $V_C = -5\,kN$
- $\sum H = 0 \rightarrow H_B = -H_C$

Tramo CDEFG:





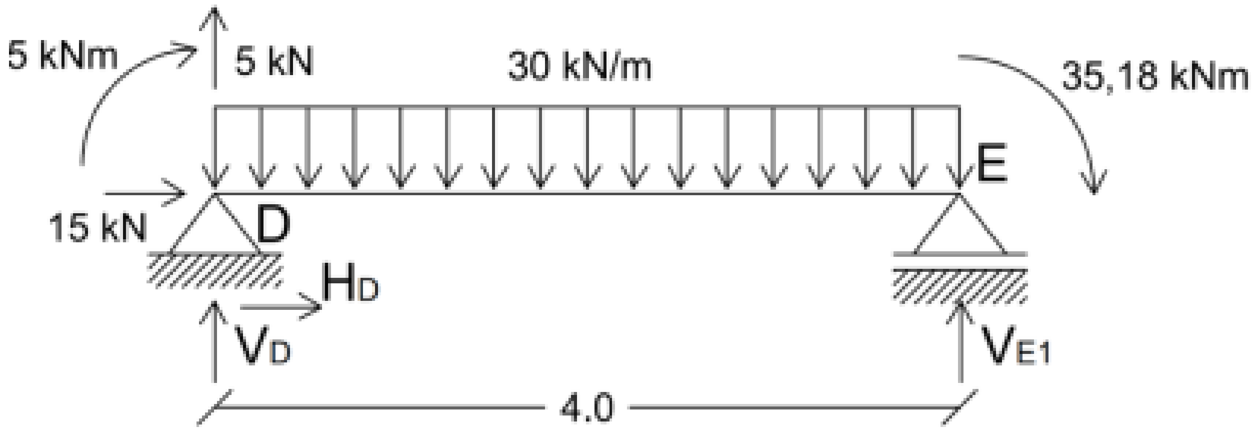
Debido a que se trata de una estructura hiperestática, para resolverlo planteamos ecuación de 3 momentos en el nodo E.

$$M_D \cdot \beta^{DE} + M_E \cdot (\alpha_E^{DE} + \alpha_E^{EF}) + M_F \cdot \beta^{EF} + \alpha_{0E}^{DE} + \alpha_{0E}^{EF} + \psi^{EF} - \psi^{DE} = 0$$

$$\frac{-V_C \cdot 4\,m}{6EI} + M_E \cdot (\frac{4\,m}{3EI} + \frac{3\,m}{3EI}) - \frac{2{,}5\,kNm \cdot 3\,m}{6EI} + \frac{30\,kN/m \cdot (4\,m)^3}{24\,EI} = 0$$

$$\boldsymbol{M_E = -35{,}18\,kNm}$$

Se trabaja con las barras DE y EF por separado.



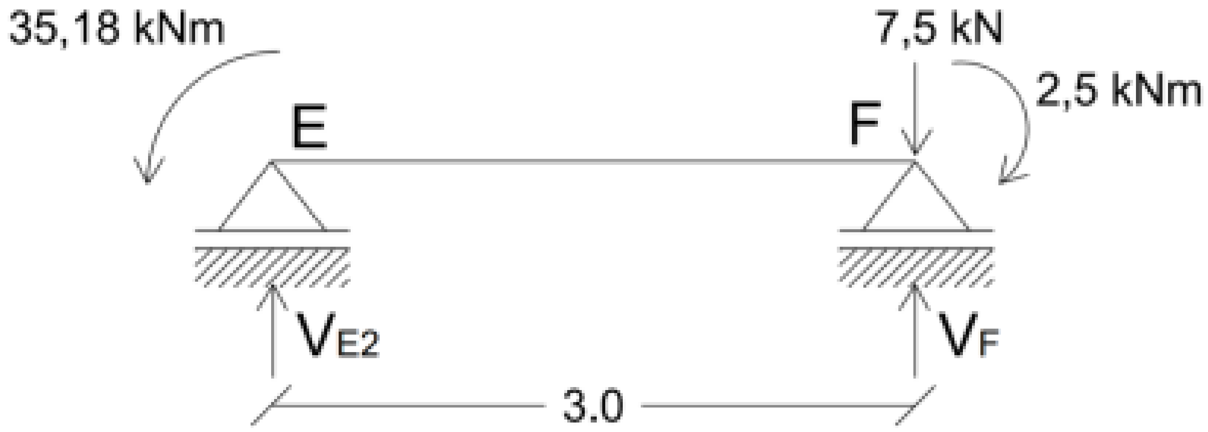
- $\sum M_D = 0 \rightarrow\ 5\ kNm\ +\ 30\ kN/m \cdot 4m \cdot 2m + 35{,}18\ kNm - V_{E1} \cdot 4\ m = 0$

$$V_{E1} = 70{,}045 \text{ kN}$$

- $\sum V = 0 \rightarrow\ V_D + V_{E1}\ + 5\ kN -\ 30\ kN/m \cdot 4\ m = 0$

$$V_D = 44{,}955\ kN$$

- $\sum H = 0 \rightarrow H_D = -15\ kN$



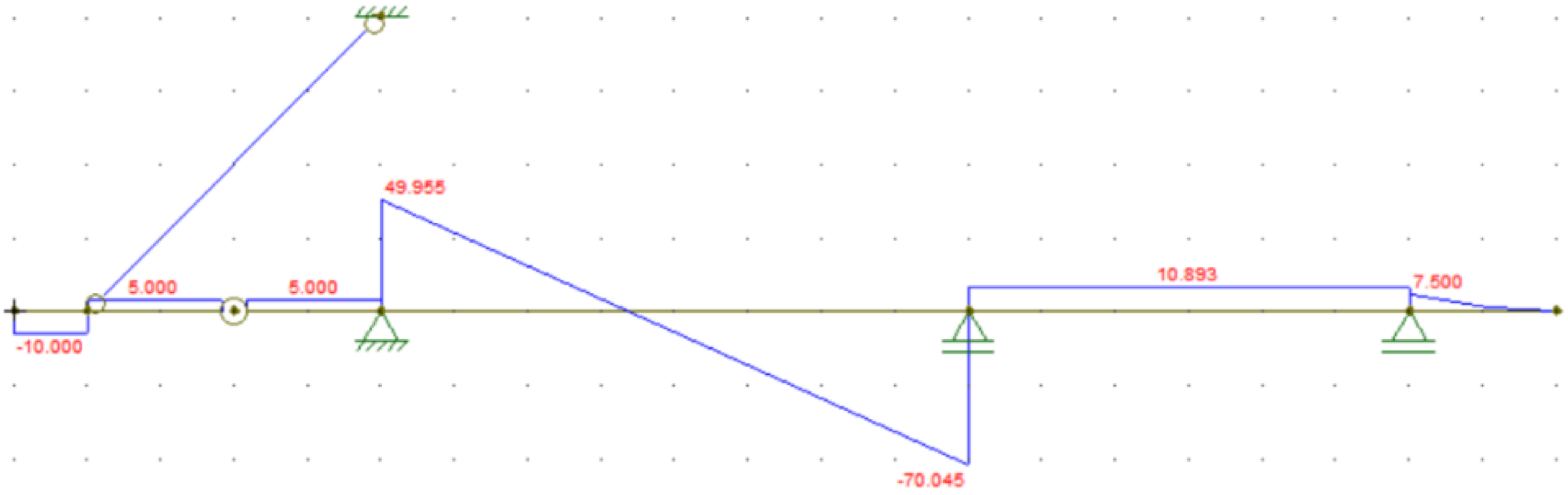
- $\sum M_F = 0 \rightarrow -2{,}5\ kNm + 35{,}18\ kNm - V_{E2} \cdot 3\ m = 0$

  $V_{E2} = 10{,}893$ kN$\rightarrow V_E = V_{E1} + V_{E2} = 80{,}938\ kN$

- $\sum V = 0 \rightarrow V_F + V_{E2} - 7{,}5\ kN = 0$

  $V_F = -3{,}393\ kN$

V(kN)



M(kNm)