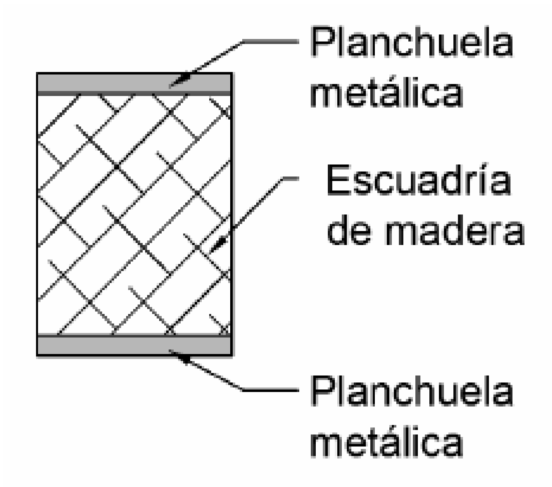
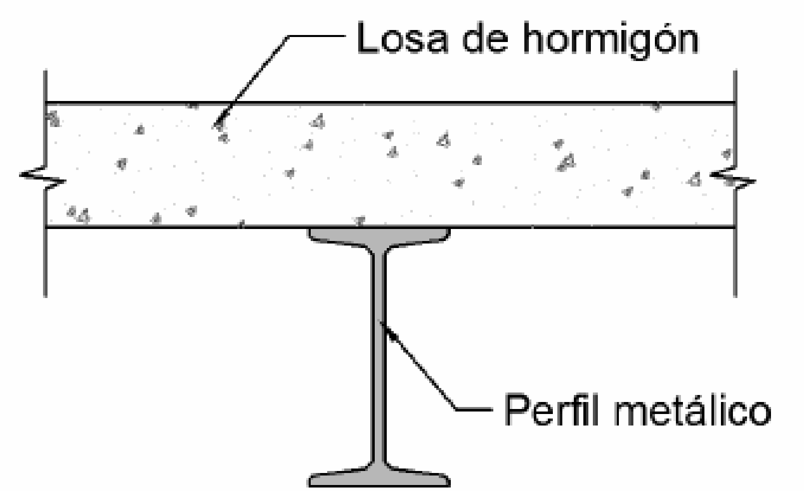
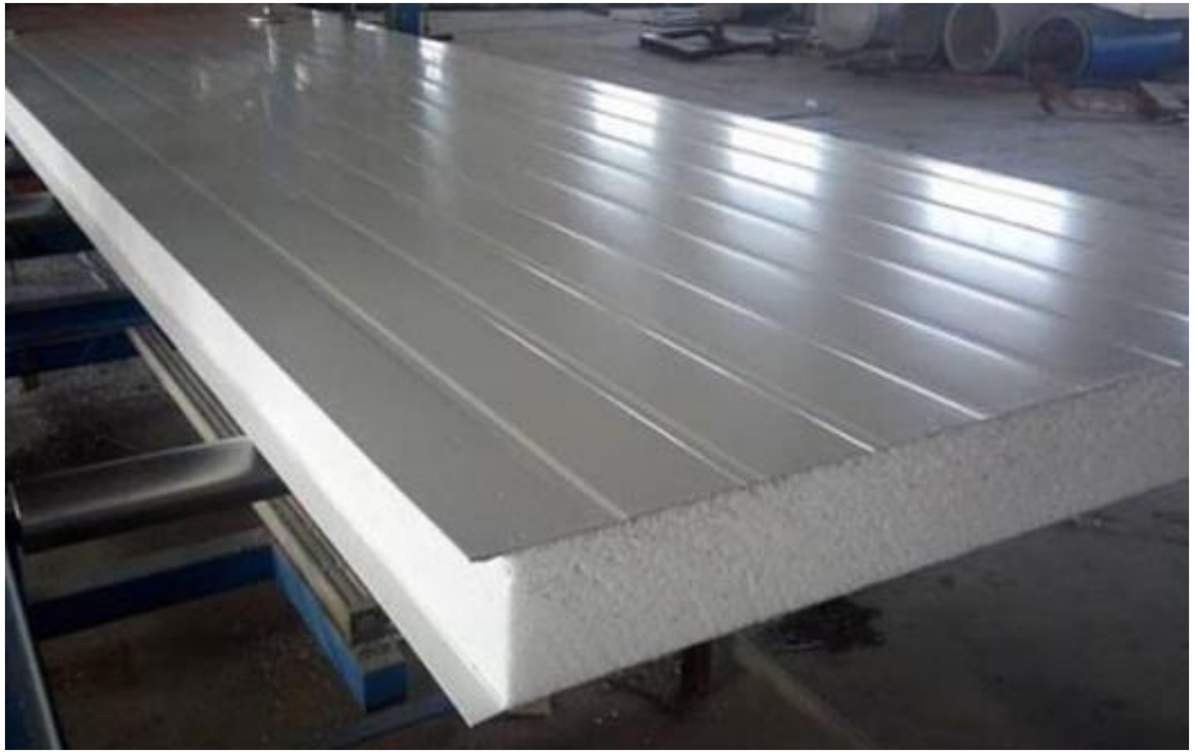
Secciones Compuestas

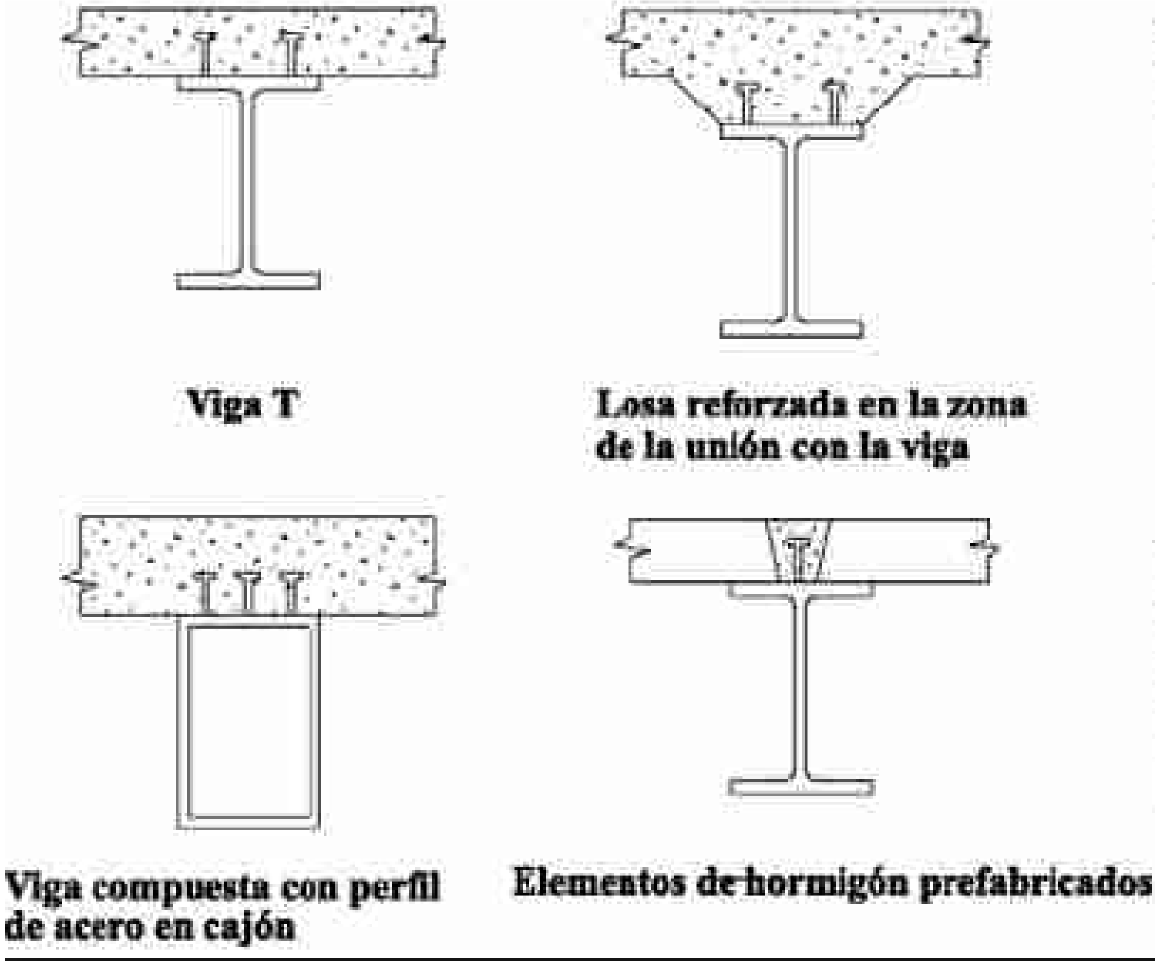




Paneles Sandwich

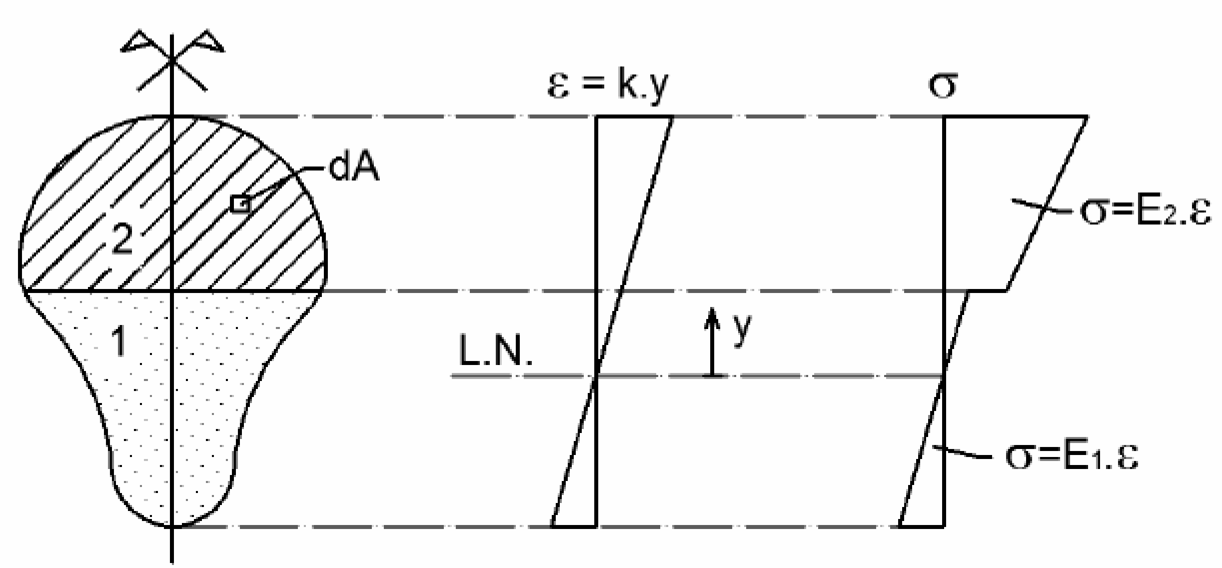


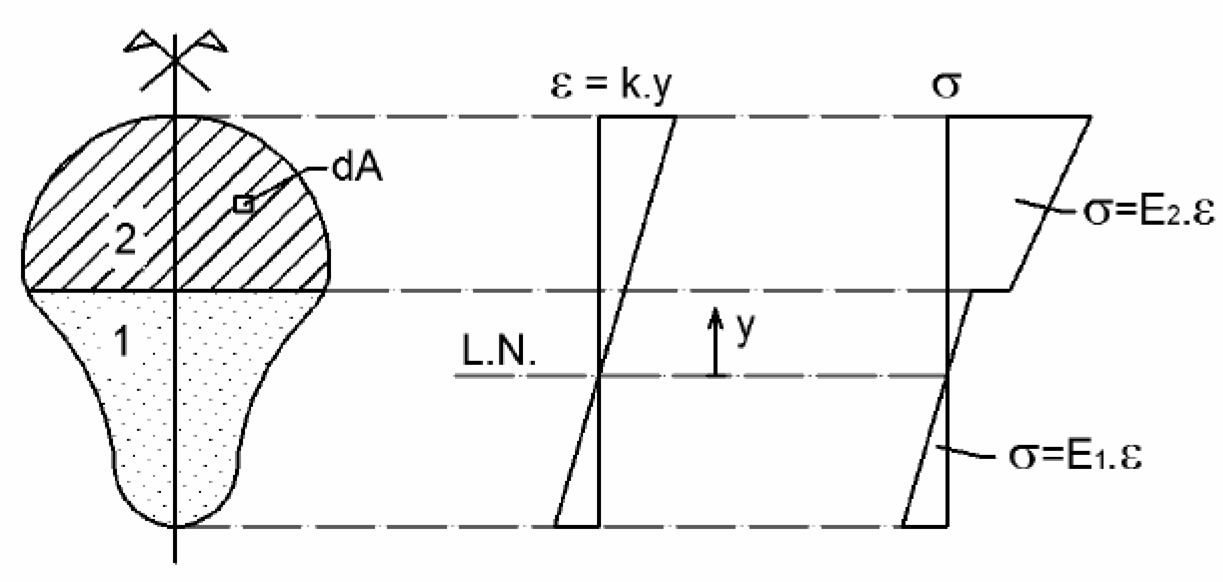
Sección compuesta



Hipótesis

Las secciones planas se mantienen **planas y perpendiculares** al eje de la viga luego de la flexión (Navier). Por lo que las deformaciones unitarias serán proporcionales a la distancia a la línea neutra. Ambos materiales también cumplen con la **Ley de Hooke.**





E2> E1

Material 2
Módulo de elasticidad E2

Material 1
Módulo de elasticidad E1

Supondremos que E1< E2,

Las def. unitarias: ε= k.y

→ Al cumplir la ley de Hooke: $\sigma 1=\varepsilon.E1=k.y.E1$
$\sigma 2=\varepsilon.E2=k.y.E2$

Para determinar la LN y la constante k, igualamos los esfuerzos internos a las fuerzas externas

$$N = \int_{A_1} \sigma_1 dA + \int_{A_2} \sigma_2 dA \qquad M = \int_{A_1} \sigma_1 . ydA + \int_{A_2} \sigma_2 . ydA$$

$$N = \int_{A_1} \sigma_1 dA + \int_{A_2} \sigma_2 dA = \int_{A_1} E_1 kydA + \int_{A_2} E_2 kydA$$

$$N = kE_1 \left[\int_{A_1} ydA + \frac{E_2}{E_1} \int_{A_2} ydA \right]$$

$$M = \int_{A_1} \sigma_1 .ydA + \int_{A_2} \sigma_2 .ydA = \int_{A_1} E_1 ky.ydA + \int_{A_2} E_2 ky.ydA$$

$$M = kE_1 \left[\int_{A_1} y^2 dA + \frac{E_2}{E_1} \int_{A_2} y^2 dA \right]$$

Definiendo $\frac{E_2}{E_1} = n$

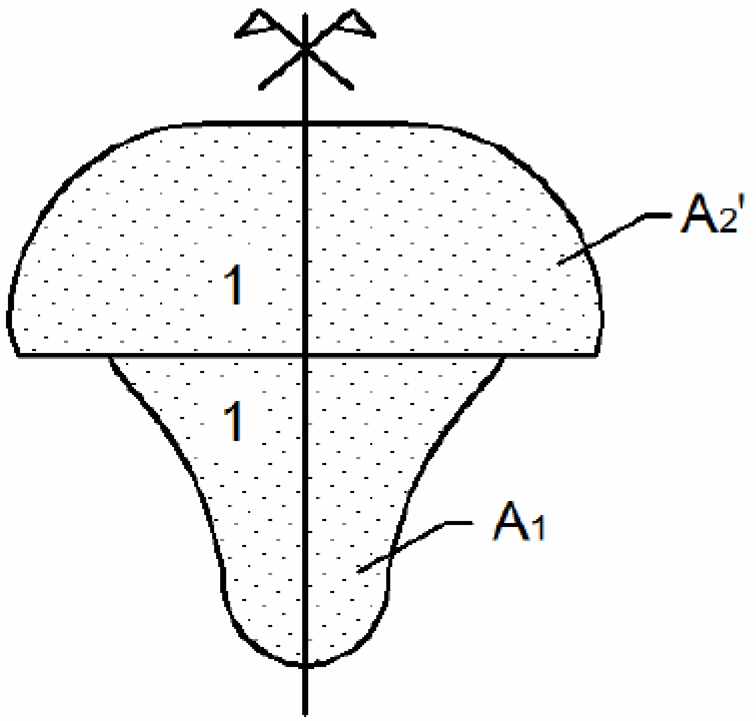
$$N = kE_1\left[\int_{A_1} ydA + n\int_{A_2} ydA\right]$$

$$M = kE_1\left[\int_{A_1} y^2dA + n\int_{A_2} y^2dA\right]$$

$$n\int_{A_2} ydA = \int_{A_2'} ydA \qquad n\int_{A_2} y^2dA = \int_{A_2'} y^2dA$$

Multiplicando el ancho de la **zona 2 por n**, el problema original de 2 materiales puede ser sustituido por un problema equivalente con un material solo de módulo E1 y área Igual a **A1+A2'**, lo que es más sencillo y sabemos resolver.

Sección homogenea equivalente



Cambiamos los ejes al baricentro

→ $$\varepsilon = \varepsilon_G + ky$$

$$\sigma_1 = E_1\varepsilon = E_1 ky + E_1\varepsilon_G$$
$$\sigma_2 = E_2\varepsilon = E_2 ky + E_2\varepsilon_G$$

$$N = \int_{A1} \sigma_1 dA + \int_{A2} \sigma_2 dA = \int_{A1} E_1(ky + \varepsilon_G)dA + \int_{A2} E_2(ky + \varepsilon_G)dA$$

$$= kE_1\left[\int_{A1} ydA + \frac{E_2}{E_1}\int_{A2} ydA\right] + \varepsilon_G E_1\left[\int_{A1} dA + \frac{E_2}{E_1}\int_{A2} dA\right]$$

$$= kE_1\left[\int_{A1} ydA + \frac{E_2}{E_1}\int_{A2} ydA\right] + \varepsilon_G E_1\left[\int_{A1} dA + \frac{E_2}{E_1}\int_{A2} dA\right]$$

$A_1 + n.A_2$

0= momento de primer orden respecto al baricentro

$$N = \varepsilon_G E_1 A_h$$

N=0 → $\varepsilon_G = 0$

$$M = \int_{A1} \sigma_1 . ydA + \int_{A2} \sigma_2 . ydA$$

$$= \int_{A1} E_1 (ky + \varepsilon_G) ydA + \int_{A2} E_2 (ky + \varepsilon_G) ydA$$

$$M = kE_1\left[\int_{A1} y^2 dA + \frac{E_2}{E_1}\int_{A2} y^2 dA\right] + \varepsilon_G E_1\left[\int_{A1} y dA + \frac{E_2}{E_1}\int_{A2} y dA\right]$$

$$M = kE_1 I_h$$

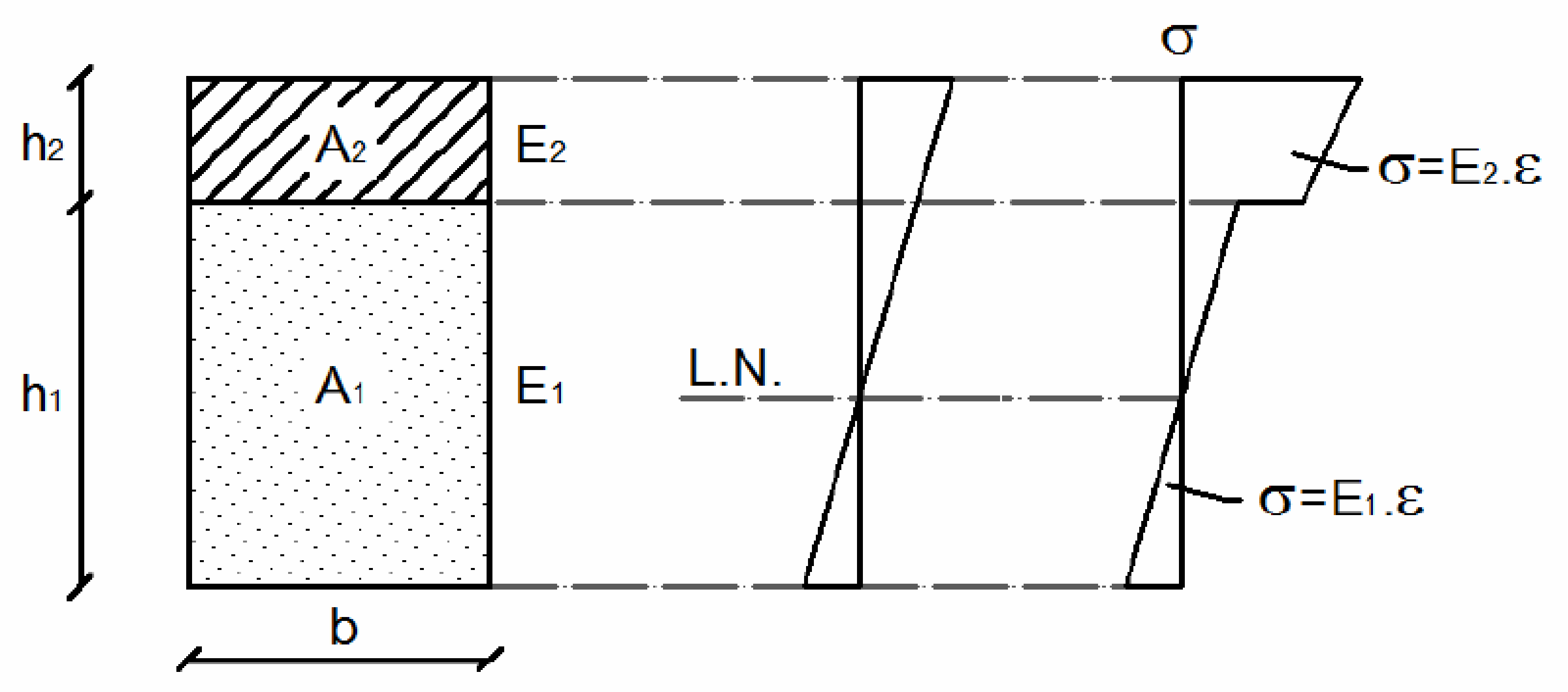
Tensiones Normales

$$M = kE_1 I_h \quad \rightarrow \quad k = \frac{M}{E_1 I_h}$$

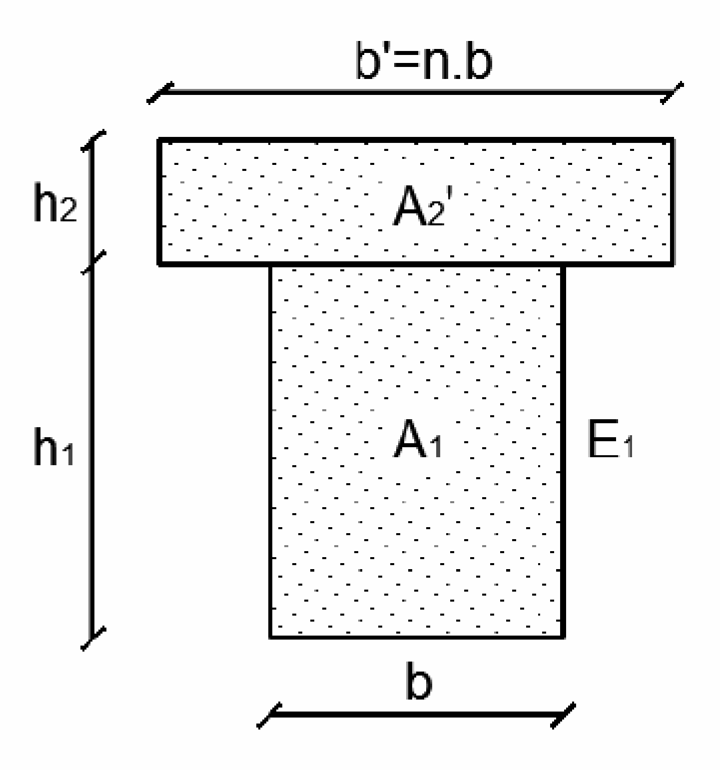
$$\sigma_1(y) = \frac{M.y}{I_h} + E_1 \varepsilon_G = \frac{M.y}{I_h} + \frac{N}{A_h}$$

$$\sigma_2(y) = \frac{E_2}{E_1}\frac{M.y}{I_h} + \frac{E_2}{E_1} E_1 \varepsilon_G = n \cdot \left(\frac{M.y}{I_h} + \frac{N}{A_h} \right)$$

Secciones en bandas horizontales



Sección homogeneizada



Tensiones Rasantes

$$\tau = \frac{V.\mu}{I.b}$$

V es el valor del cortante en la sección en estudio.

μ es el momento de primer orden del tramo de sección que se encuentra por encima de la fibra donde se está hallando la tensión, respecto del baricentro.

I es la inercia de la sección considerada (respecto de su baricentro).

b es el ancho de la sección a la altura de la fibra donde se está hallando la tensión.

$$\tau = \frac{V.\mu_h}{I_h.b}$$

V es el valor del cortante en la sección en estudio.

μ_h es el momento de primer orden del tramo de sección homogénea que se encuentra por encima de la fibra donde se está hallando la tensión, respecto del baricentro de la sección homogénea.

I_h es la inercia de la sección homogénea (respecto de su baricentro).

Dimensionado

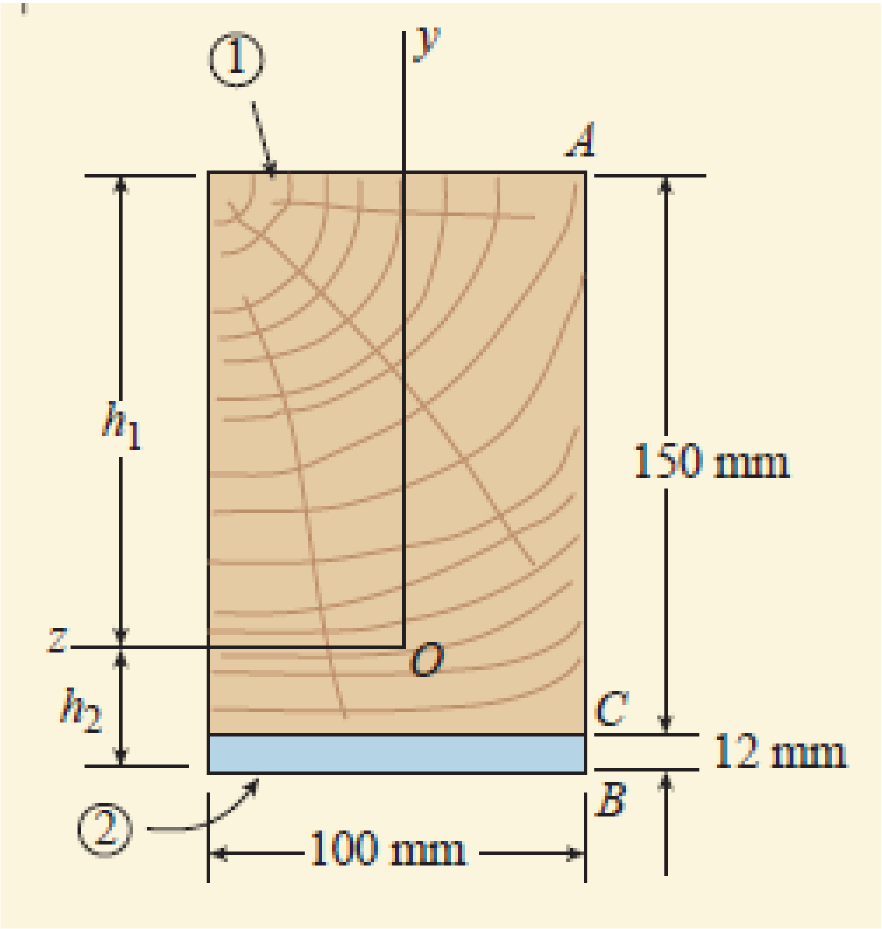
Llamaremos "R" a la fuerza rasante en una longitud s de la viga

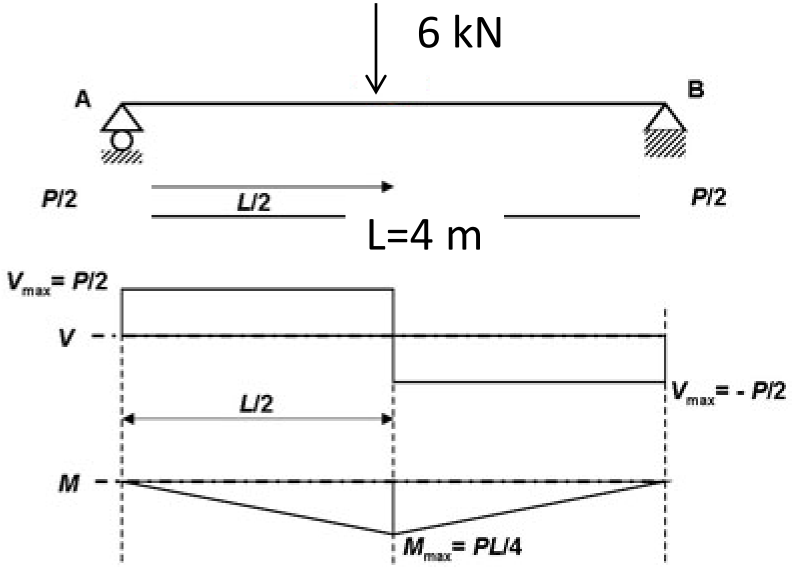
$$R = \tau . b . s = \frac{V . \mu_h}{I_h . b} . b . s \rightarrow R = \frac{V . \mu_h}{I_h} . s$$

En general nos interesará determinar el valor de la fuerza R en la superficie de contacto entre un material y otro, a los efectos de dimensionar la unión que deberá transmitir dicho esfuerzo entre las partes.

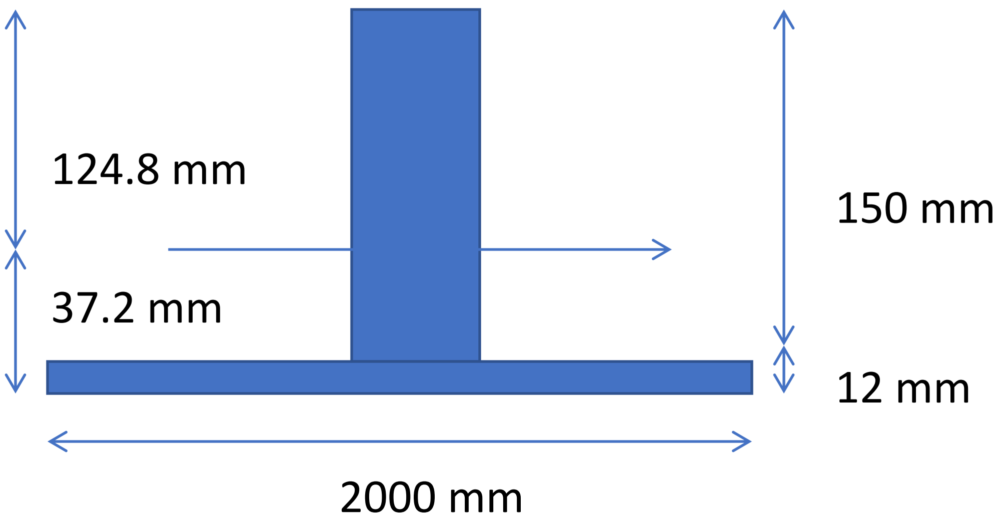
Sección Compuesta

E_{Madera}= 10.5 GPa E_{Acero}= 210 GPa





Sección Compuesta



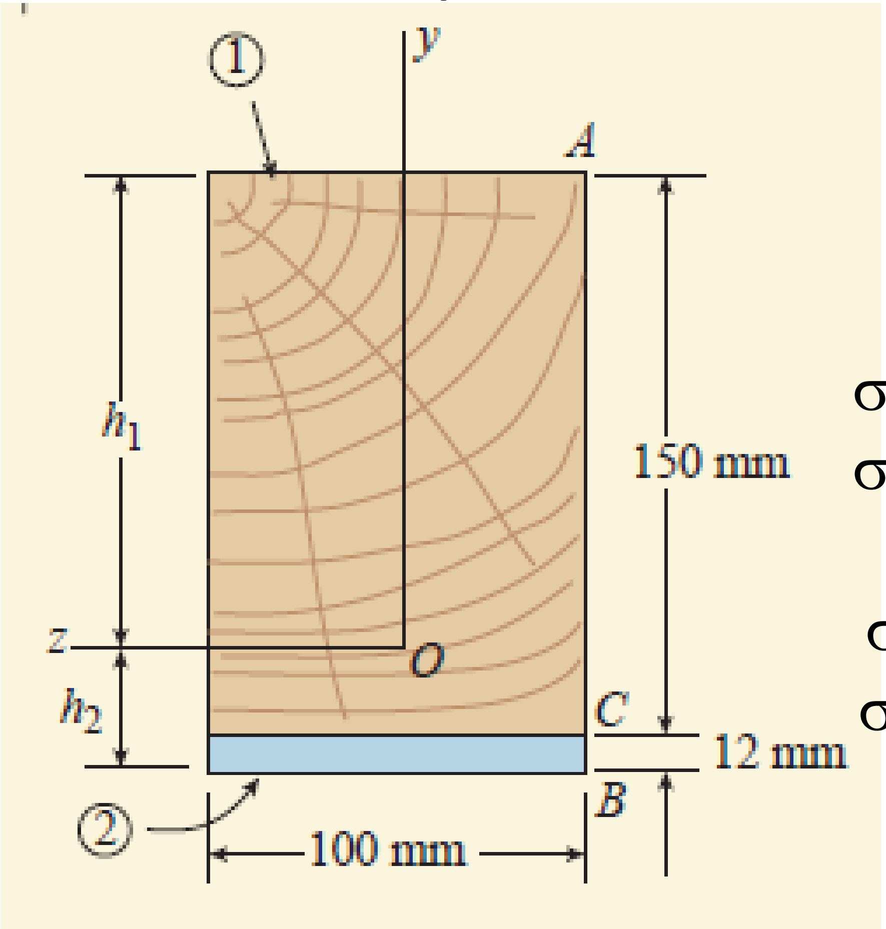
$$y_G = \frac{(150.100).75+(2000.12).156}{(150.100)+(2000.12)}$$

y_G=124.8 mm

I_{homog}= 100.(150)^3/12 +((124.8-75)^2.(100.150)+ 2000. 12^3/12+ 12.2000 .(37.2-6)^2

I_{homog}= 8.898 x 10^7 mm^4

Sección Compuesta



$\sigma_A = E_1.\varepsilon$ $\quad$ $k = M/EI$

$\sigma_A = E_1.\varepsilon = E_1.k.y = E_1.y.M/(E_1 I)$

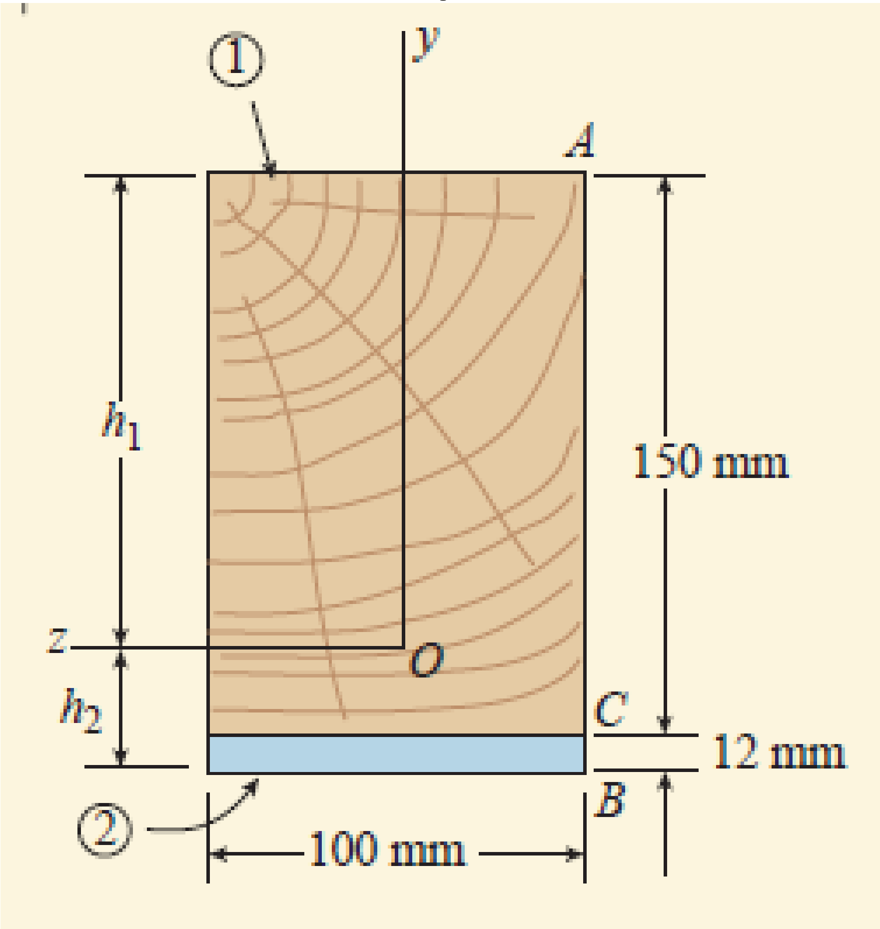
$\sigma_A = -\ 10.5\ GPa.6kNm\ 0.1248/(E_1 I_{homg})$

$\sigma_A = -\ 8.4\ MPa$

$\sigma_{C1} = 10.5GPa.6kNm\ 0.0252/(E_1 I_{homg})$

$\sigma_{C1} = 1.7\ MPa$

Sección Compuesta



$\sigma_A = E_1.\varepsilon$ $k = M/EI$

$\sigma_A = E_1.\varepsilon = E_1.k.y = E_1.y.M/(E_1 I)$

$\sigma_{C1} = 1.7$ MPa

$\sigma_{C1} = 10.5GPa.6kNm\ 0.0252/(E_1 I_{homg})$

$\sigma_{C2} = 210GPa.6kNm\ 0.0252/(E_1 I_{homg})$

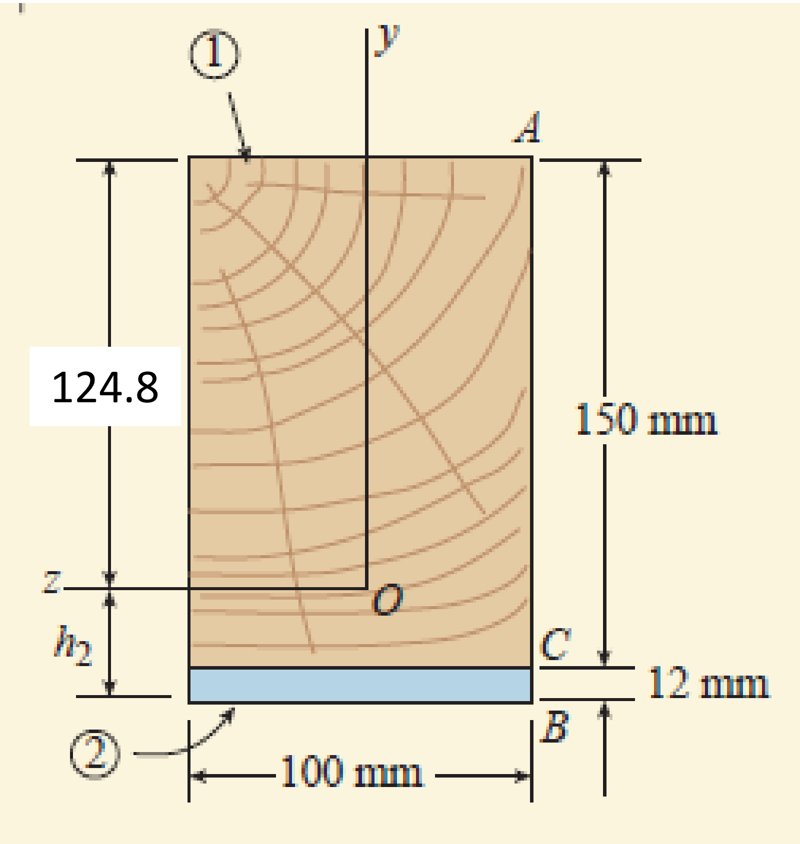
$\sigma_{C2} = 34$ MPa

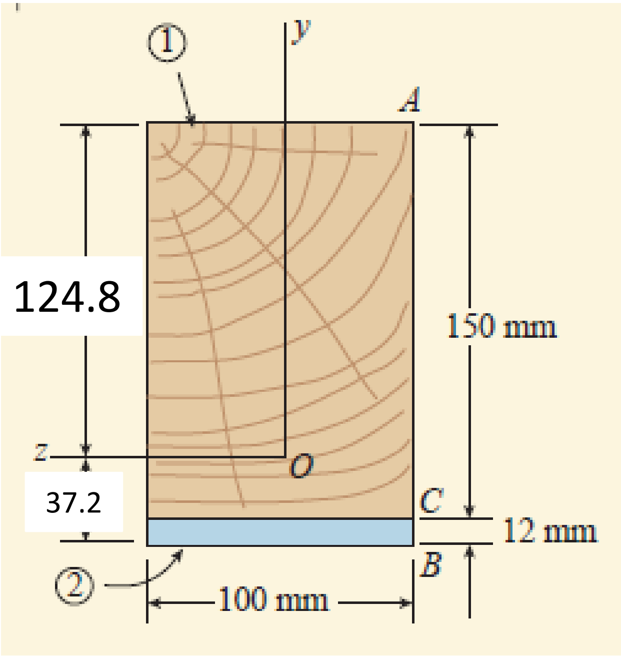
$\sigma_B = 210GPa.\ 6kNm\ 0.0372/(E_1 I_{homg})$

$\sigma_B = 50.2$ MPa

Sección Compuesta

$$\tau_G = 3kN * (124.8*100)*124.8/2/(I_{homog}*b) = 262 \text{ kPa}$$



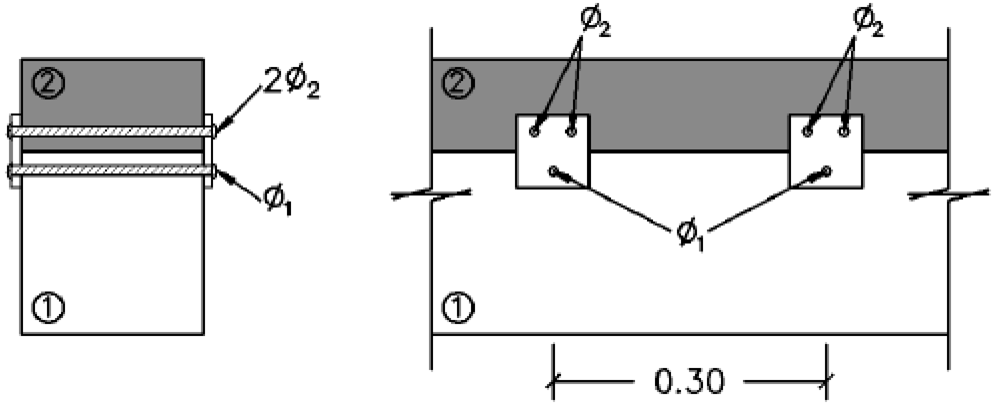


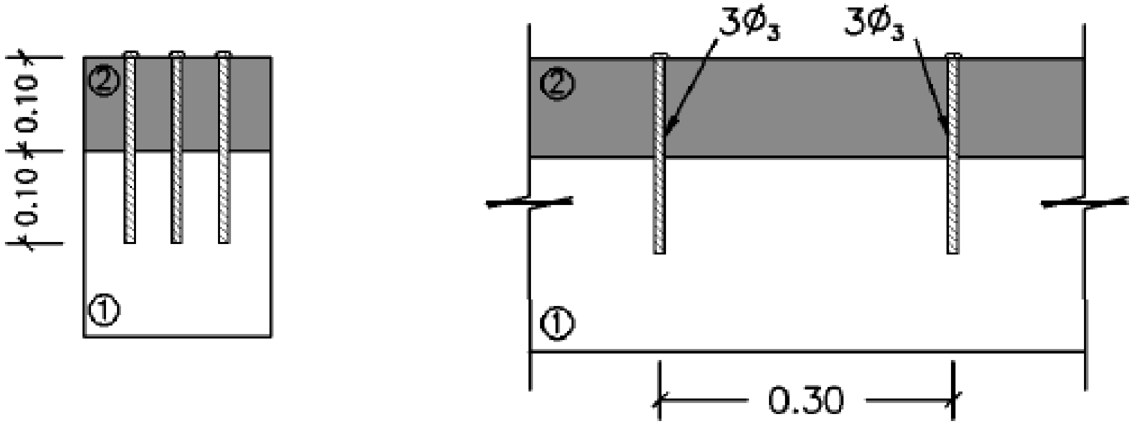
τ_{C1}= 3kN. ((124.8*100)*124.8/2)-

(25.2*100)*25.2/2)/(I_{homog}*b)=252 kPa

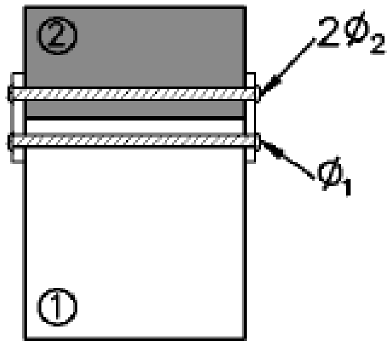
τ_{C2}= 3kN. ((2000*12)*(25.2+6)/(I_{homog}*b)= 252 kPa

Conectores

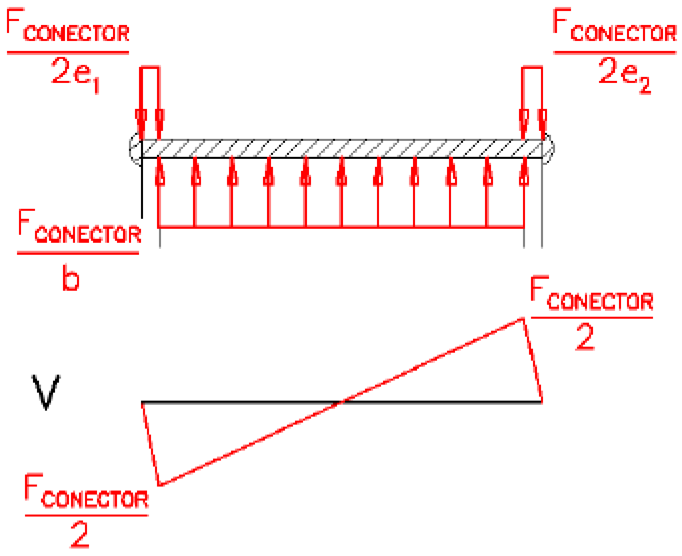




Conectores

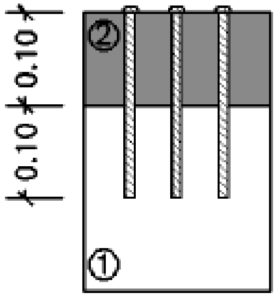


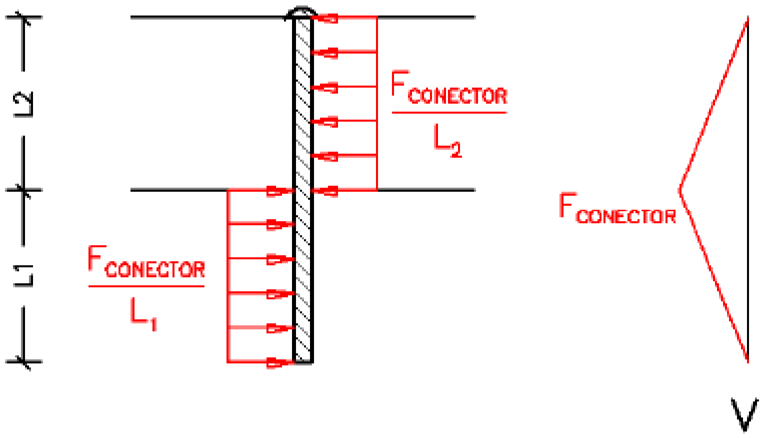
2
1
2Ø2
Ø1



F CONECTOR / 2e1
F CONECTOR / 2e2
F CONECTOR / b
V
F CONECTOR / 2
F CONECTOR / 2

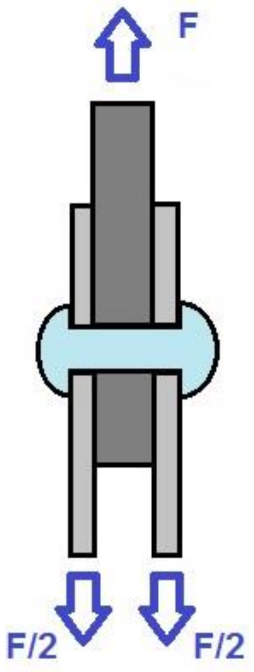
Conectores

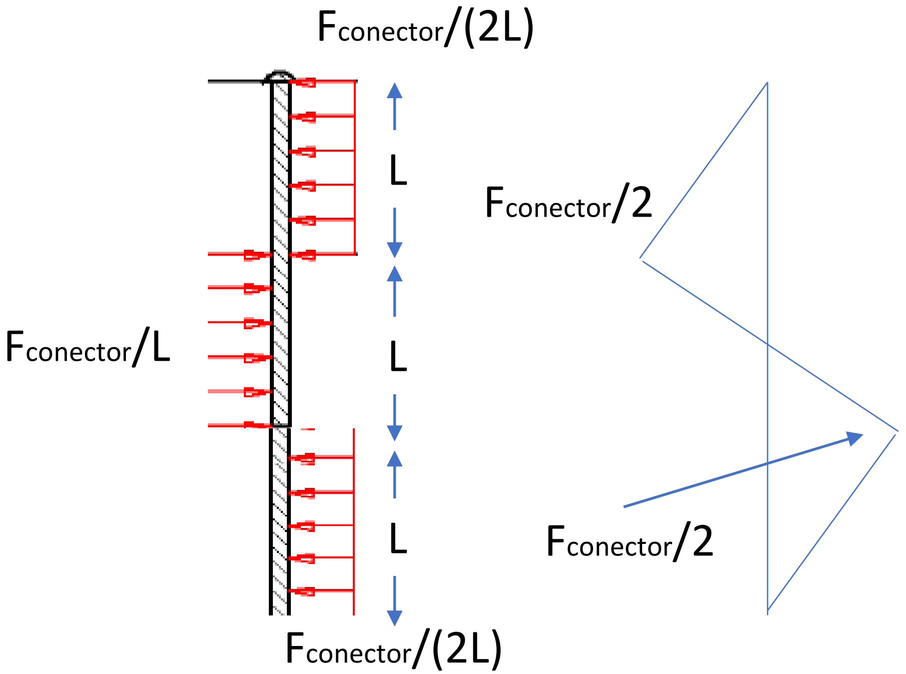




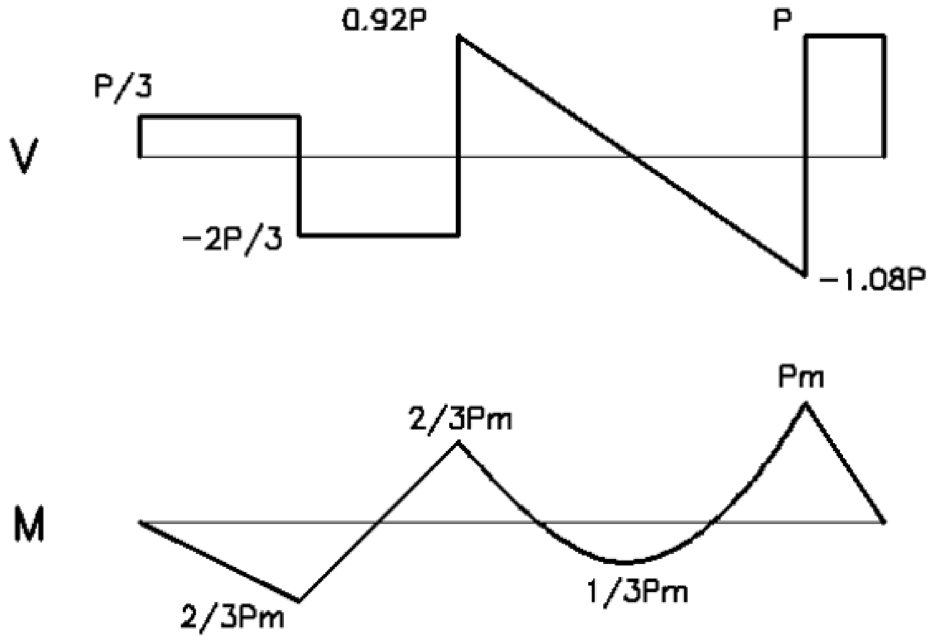
Conectores

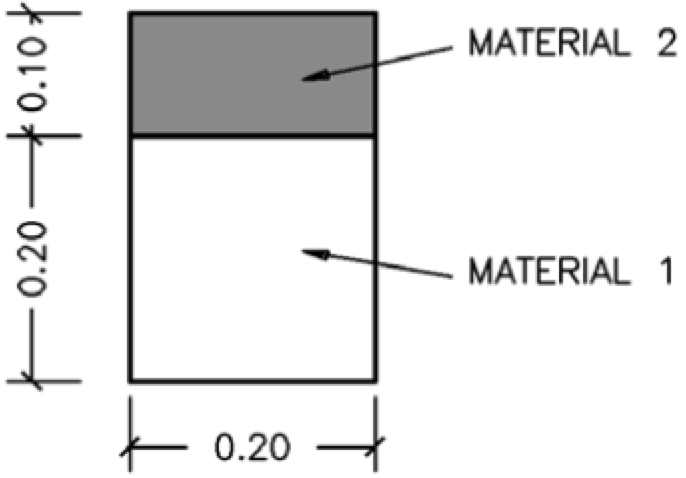
- Corte Doble





Ejemplo

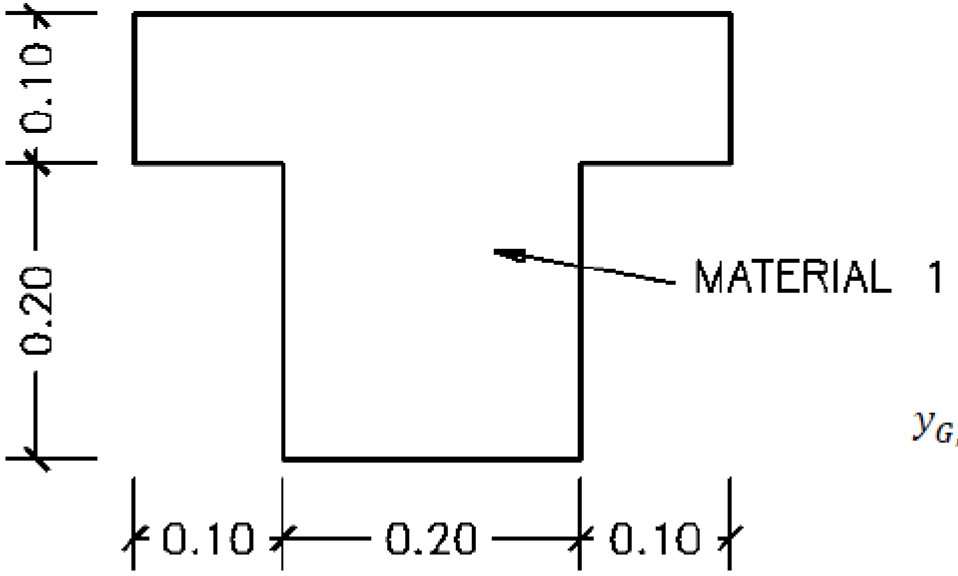




- $E_2/E_1=2$
- P=20 kN
- Tensiones admisibles: $\sigma_{adm,1}$ = 6 MPa y $\sigma_{adm,2}$ = 9 MPa
- Aplastamiento: σ_{adm}^{aplast} (mat.1) = 10 MPa y σ_{adm}^{aplast} (mat. 2) = 15 MPa.

Separacion de los conectores s=30 cm

Sección Homogeneizada

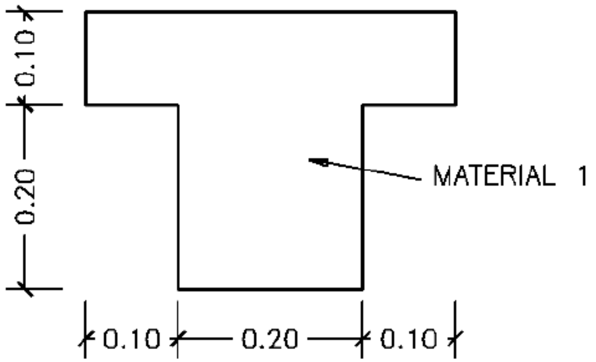


$$y_{G,hom} = \frac{20^2 \cdot 10 + 40 \cdot 10 \cdot 25}{20^2 + 40 \cdot 10} = 17.5\ cm$$

$$I_{x,hom} = \frac{20^4}{12} + 20^2 \cdot (10 - 17.5)^2 + \frac{40 \cdot 10^3}{12} + 40 \cdot 10 \cdot (25 - 17.5)^2 = 61666.7\ cm^4$$

Conectores

$$\tau \cdot b = \frac{V \cdot \mu_{hom}(\text{unión})}{I_{x,hom}}$$



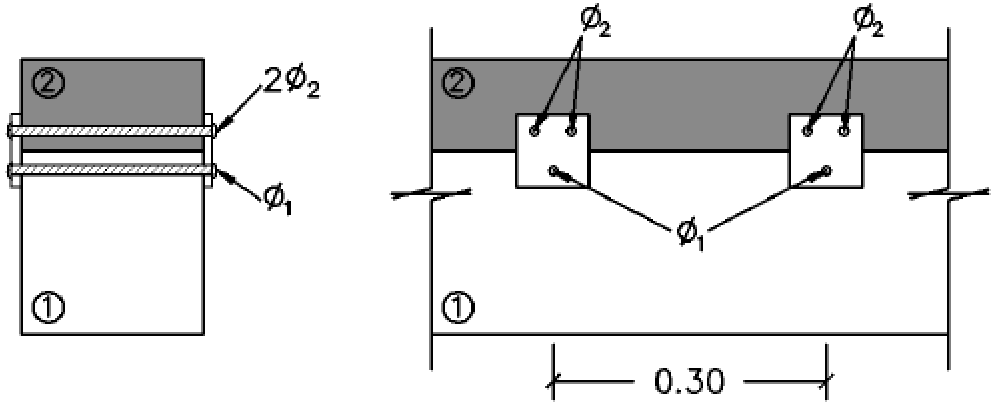
$$\mu_{hom}(\text{unión}) = 40 \cdot 10 \cdot (25 - 17.5) = 3000\ cm^3$$

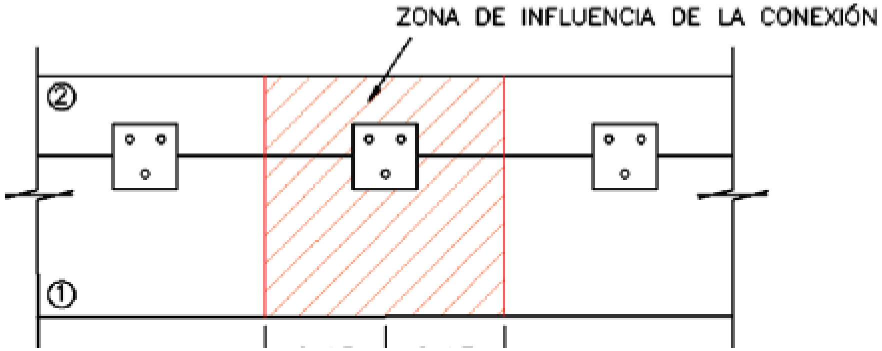
$$I_{x,hom} = 61667\ cm^4$$

$$V = 1.08 \cdot P = 21.6\ kN$$

$$\tau \cdot b = \frac{21.6 \cdot 3000}{61667} = 1.05 \frac{kN}{cm}$$

Conectores



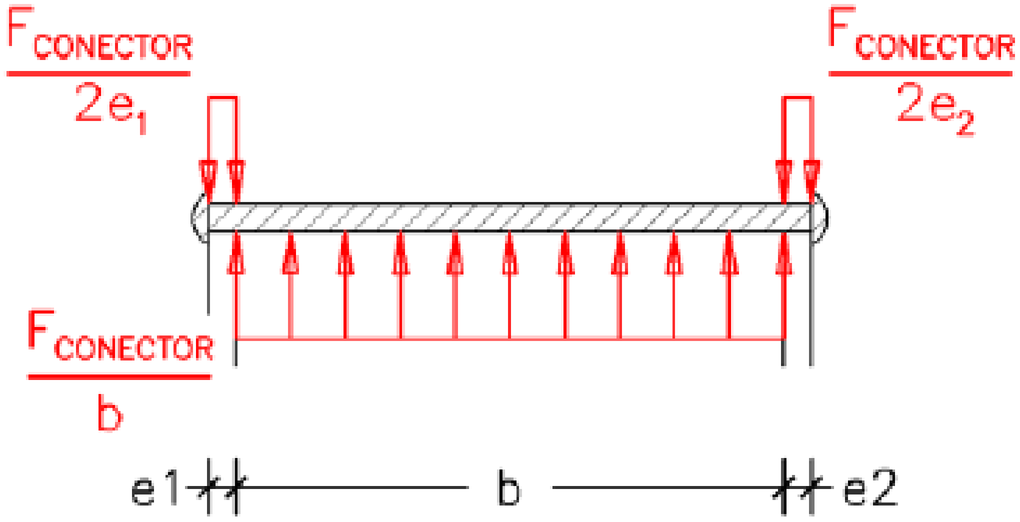


$$F_{conexión} = \tau \cdot b \cdot s = 1.05\frac{kN}{cm} \cdot 30\ cm = 31.5\ kN$$

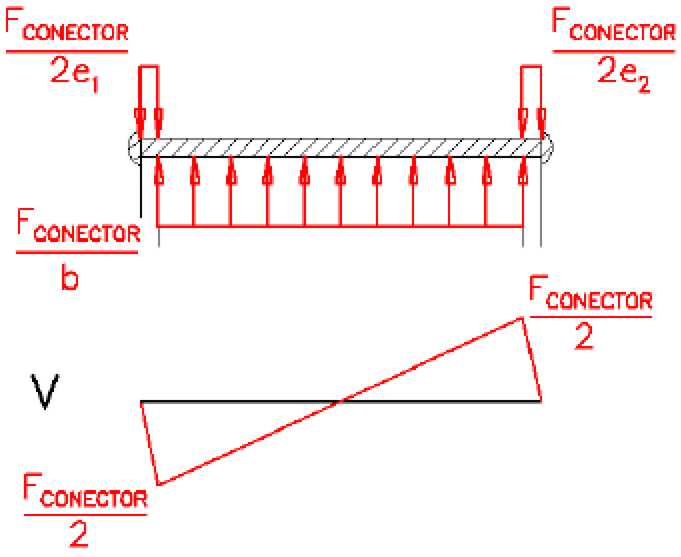
Conectores

$$F_{conectores\ sup} = \frac{F_{conexión}}{2} = \frac{31.5\ kN}{2} = 15.75\ kN$$

$$F_{conectores\ inf} = \frac{F_{conexión}}{1} = \frac{31.5\ kN}{1} = 31.5\ kN$$



Conectores



resistencia admisible al corte ($\tau_{adm} = 70\,MPa$)

$$V_{conector}^{máx}\ (\text{corte doble}) = \frac{F_{conector}}{2}$$

Tenemos que:

$$\tau_{conector\ sup}^{máx} = \frac{V_{conector\ sup}^{máx}}{A_{conector\ sup}} = \frac{\frac{15.75\ kN}{2}}{\pi \cdot \emptyset_2^{\ 2}/_4} \leq \tau_{adm} = 7\frac{kN}{cm^2} \rightarrow \boldsymbol{\emptyset_2 \geq 1.20\ cm}$$

$$\tau_{conector\ inf}^{máx} = \frac{V_{conector\ inf}^{máx}}{A_{conector\ inf}} = \frac{\frac{31.5\ kN}{2}}{\pi \cdot \emptyset_1^{\ 2}/_4} \leq \tau_{adm} = 7\frac{kN}{cm^2} \rightarrow \boldsymbol{\emptyset_1 \geq 1.69\ cm}$$

Aplastamiento del Material

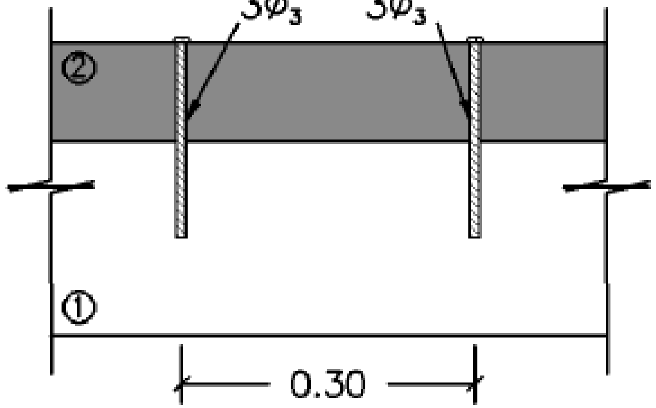
$$\sigma_{aplast} = \frac{F_{conector}}{\emptyset \cdot l}$$

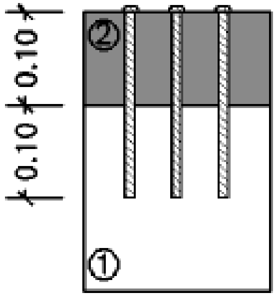
El material 2 está en contacto con los conectores superiores:

$$\sigma_{aplast} = \frac{F_{conectores\ sup}}{\emptyset_2 \cdot l} = \frac{15.75\ kN}{\emptyset_2 \cdot 20\ cm} \leq \sigma_{aplast,adm} = 1.50 \frac{kN}{cm^2} \rightarrow \boldsymbol{\emptyset_2 \geq 0.53\ cm}$$

$$\sigma_{aplast} = \frac{F_{conectores\ inf}}{\emptyset_1 \cdot l} = \frac{31.5\ kN}{\emptyset_1 \cdot 20\ cm} \leq \sigma_{aplast,adm} = 1.00 \frac{kN}{cm^2} \rightarrow \boldsymbol{\emptyset_1 \geq 1.58\ cm}$$

Conectores



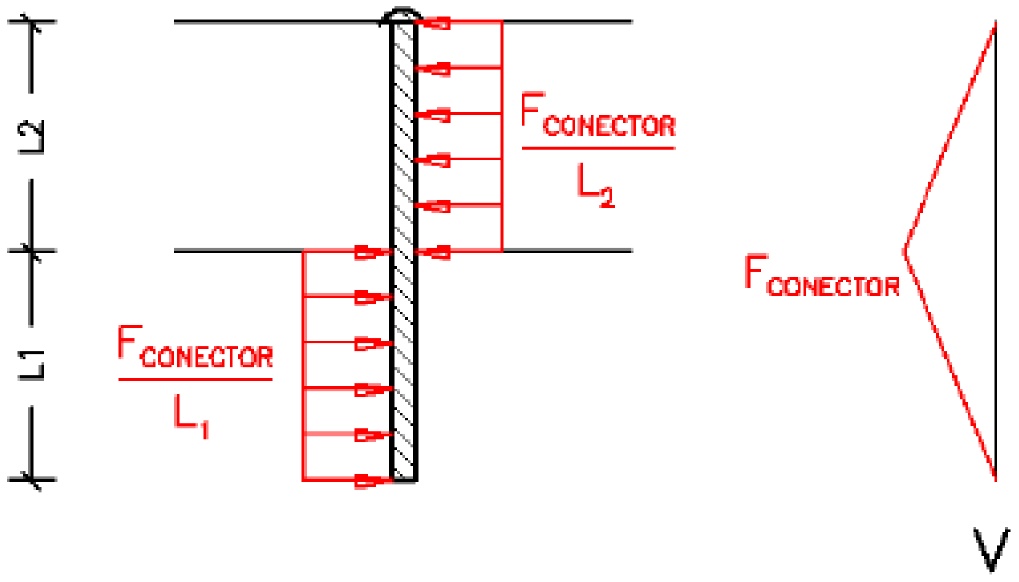


$$\tau \cdot b = \frac{21.6 \cdot 3000}{61667} = 1.05 \frac{kN}{cm}$$

$$F_{conexión} = \tau \cdot b \cdot s = 1.05 \frac{kN}{cm} \cdot 30\ cm = 31.5\ kN$$

Conectores

$$F_{conector} = \frac{F_{conexión}}{3} = \frac{31.5\ kN}{3} = 10.5\ kN$$



$$V_{conector}^{máx}\ (\text{corte simple}) = F_{conector}$$

Conectores

$$\tau_{conector} = \frac{V_{conector}}{A_{conector}}$$

$$\tau_{conector}^{máx} = \frac{V_{conector}^{máx}}{A_{conector}} = \frac{10.5\ kN}{\pi \cdot {\emptyset_3}^2/_4} \leq \tau_{adm} = 7\frac{kN}{cm^2} \ \rightarrow \ \emptyset_3 \geq \mathbf{1.38\ cm}$$

En el material 2:

$$\sigma_{aplast} = \frac{F_{conectores}}{\emptyset_3 \cdot l} = \frac{10.5\ kN}{\emptyset_3 \cdot 10\ cm} \leq \sigma_{aplast,adm} = 1.50\frac{kN}{cm^2} \ \rightarrow \ \emptyset_3 \geq \mathbf{0.70\ cm}$$

En el material 1:

$$\sigma_{aplast} = \frac{F_{conectores}}{\emptyset_3 \cdot l} = \frac{10.5\ kN}{\emptyset_3 \cdot 10\ cm} \leq \sigma_{aplast,adm} = 1.00\frac{kN}{cm^2} \ \rightarrow \ \emptyset_3 \geq \mathbf{1.05\ cm}$$

Tensiones Normales

$$\sigma_1 = E_1\varepsilon = E_1ky = E_1\frac{M}{E_1 \cdot I_{hom}}y = \frac{2000}{61666.7}y = 0.0324y$$

$$\sigma_2 = E_2\varepsilon = E_2ky = E_2\frac{M}{E_1 \cdot I_{hom}}y = 2\frac{2000}{61666.7}y = 0.0649y$$

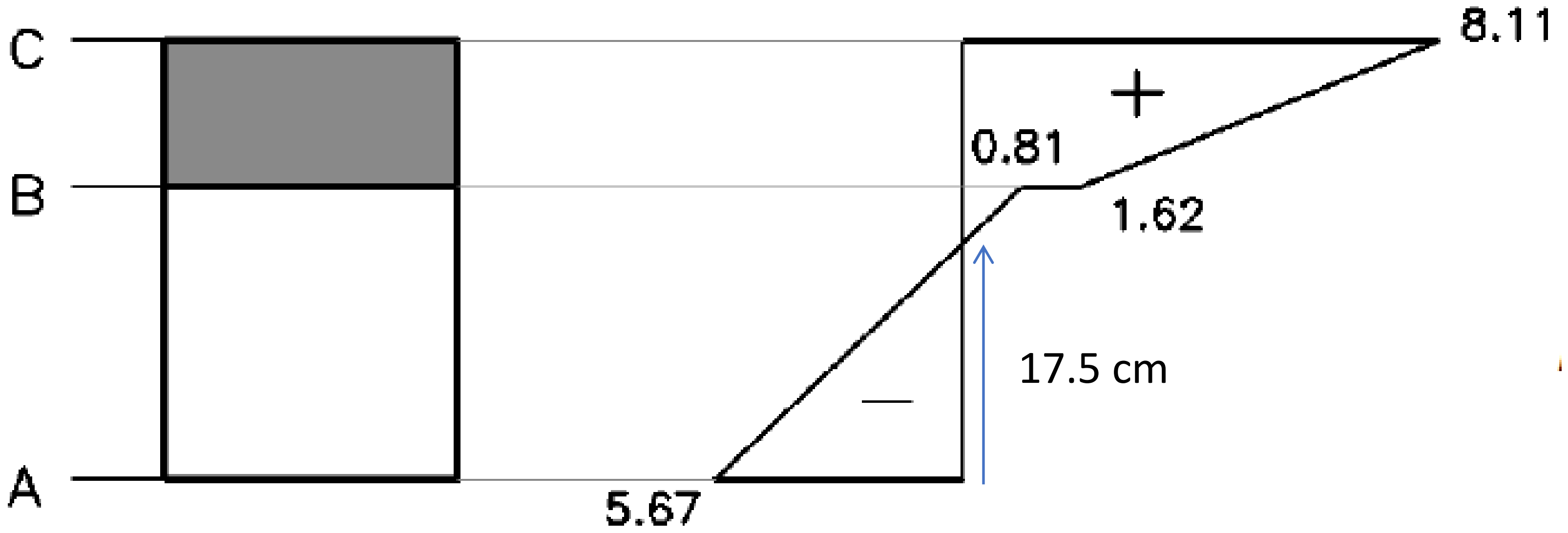
$$\sigma_1\,(A) = \; 0.0324 \cdot 17.5 = 0.567\frac{kN}{cm^2} = 5.67\;MPa < 6\;MPa$$

$$\sigma_1\,(B) = \; 0.0324 \cdot 2.5 = 0.081\frac{kN}{cm^2} = 0.81\;MPa < 6\;MPa$$

$$\sigma_2\,(B) = \; 0.0649 \cdot 2.5 = 0.162\frac{kN}{cm^2} = 1.62\;MPa < 9\;MPa$$

$$\sigma_2\,(C) = \; 0.0649 \cdot 12.5 = 0.811\frac{kN}{cm^2} = 8.11\;MPa < 9\;MPa$$

Tensiones Normales



$$\sigma_1\,(A) = \; 0.0324 \cdot 17.5 = 0.567\,\frac{kN}{cm^2} = 5.67\;MPa < 6\;MPa$$

$$\sigma_1\,(B) = \; 0.0324 \cdot 2.5 = 0.081\,\frac{kN}{cm^2} = 0.81\;MPa < 6\;MPa$$

$$\sigma_2\,(B) = \; 0.0649 \cdot 2.5 = 0.162\,\frac{kN}{cm^2} = 1.62\;MPa < 9\;MPa$$

$$\sigma_2\,(C) = \; 0.0649 \cdot 12.5 = 0.811\,\frac{kN}{cm^2} = 8.11\;MPa < 9\;MPa$$