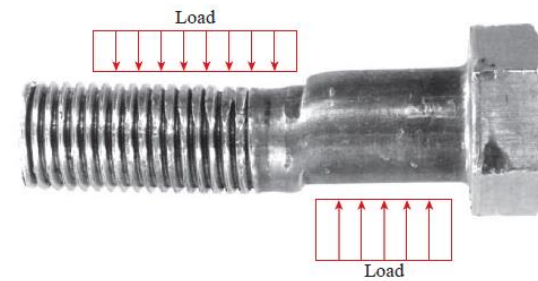
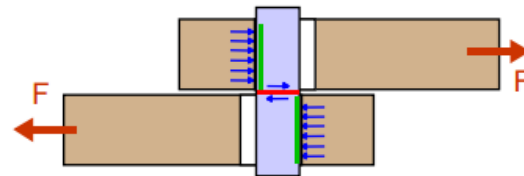
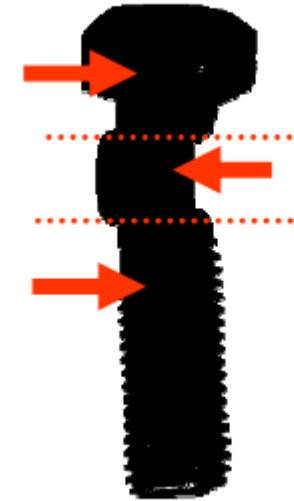
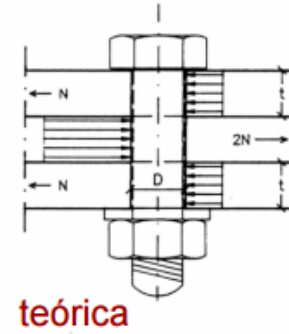
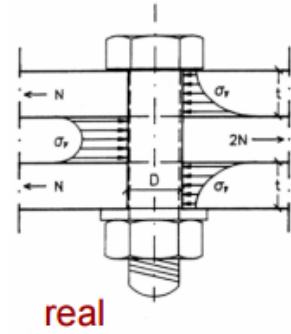
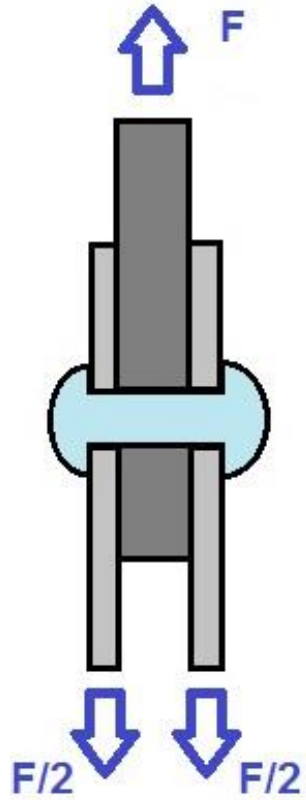
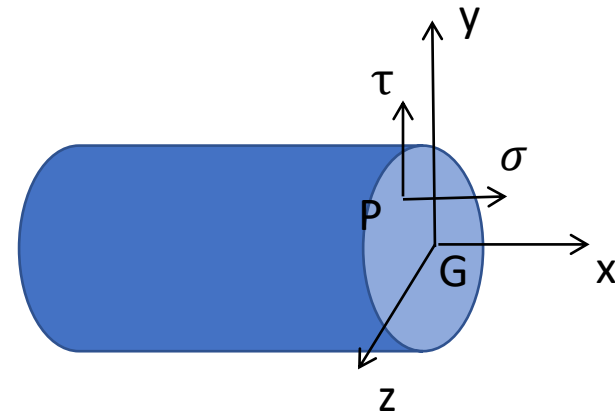
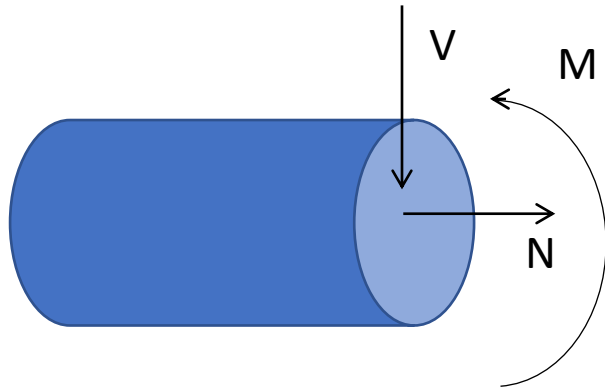
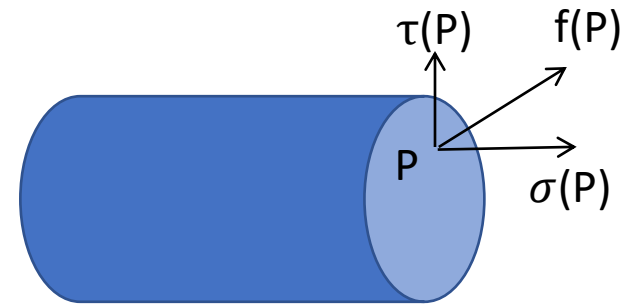
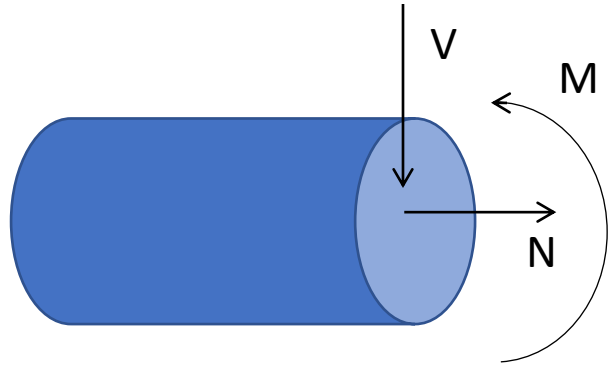


Tensiones Rasantes

Tensiones Rasantes



Tensiones internas

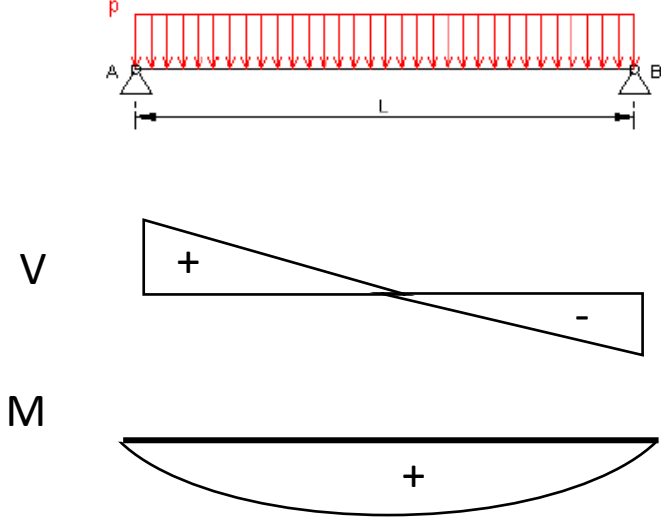
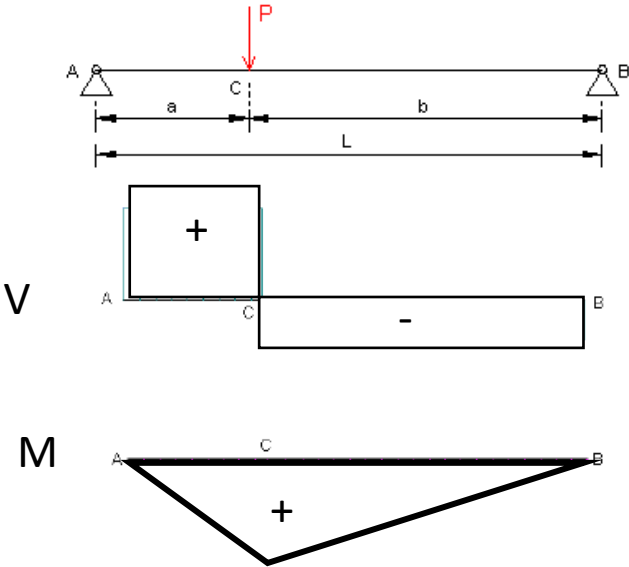


$$N = \int_{\Omega} \sigma \, d\Omega$$

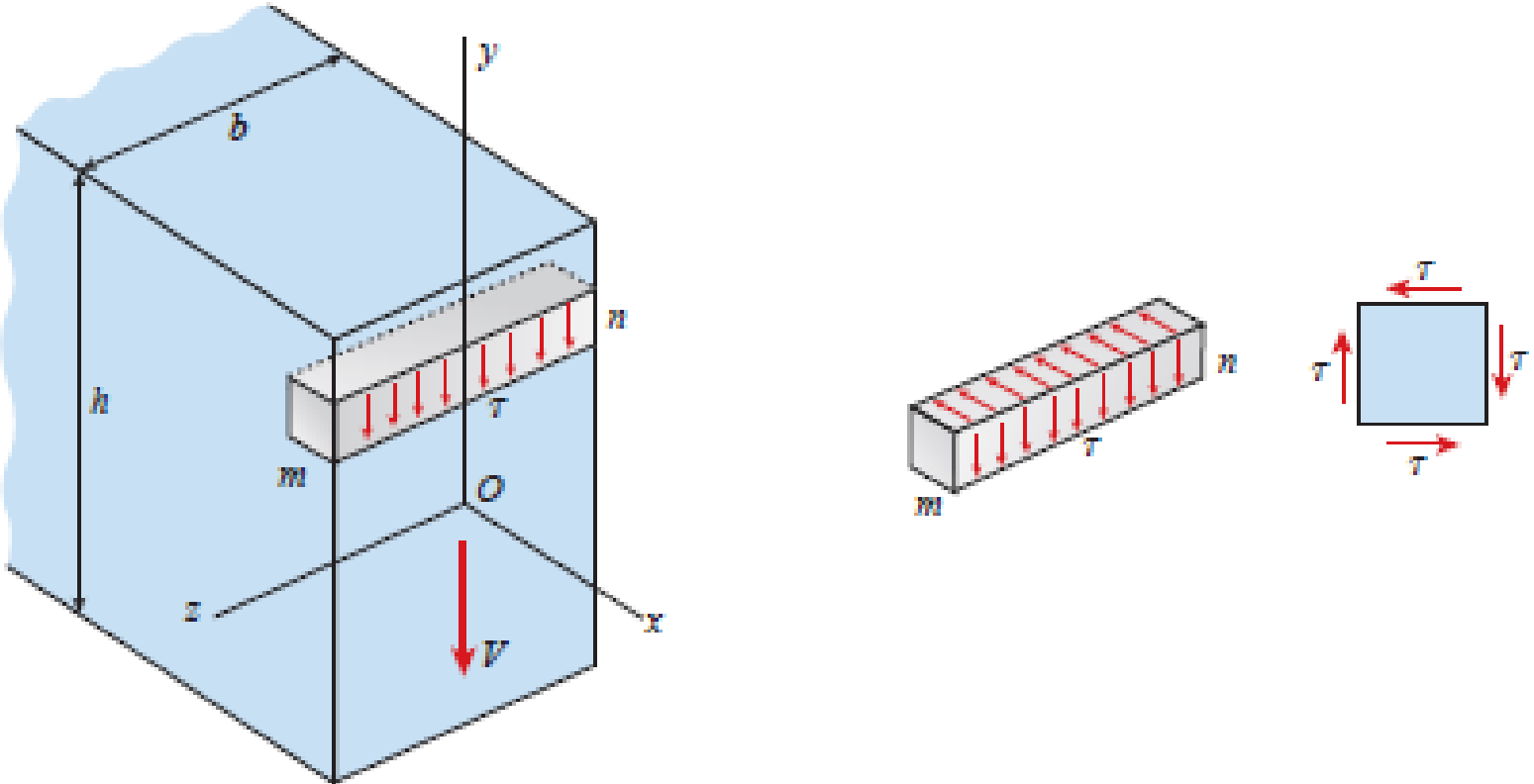
$$V = \int_{\Omega} -\tau \, d\Omega$$

$$M = \int_{\Omega} -\sigma * y \, d\Omega$$

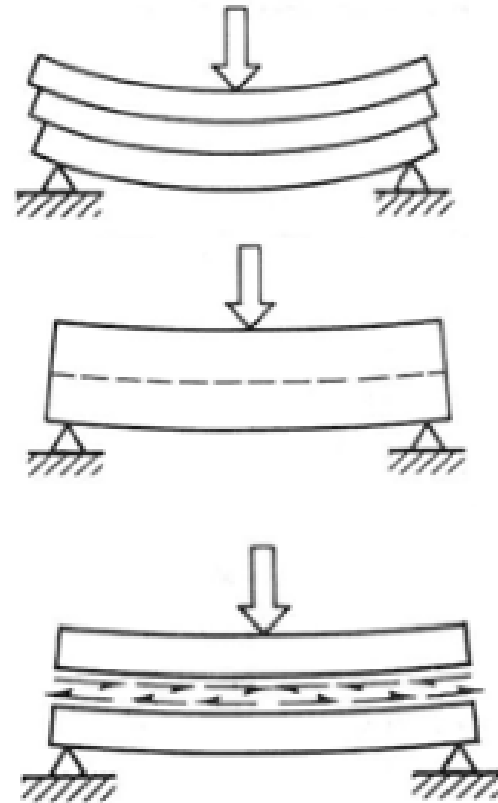
Tensiones internas



Tensiones rasantes en Vigas

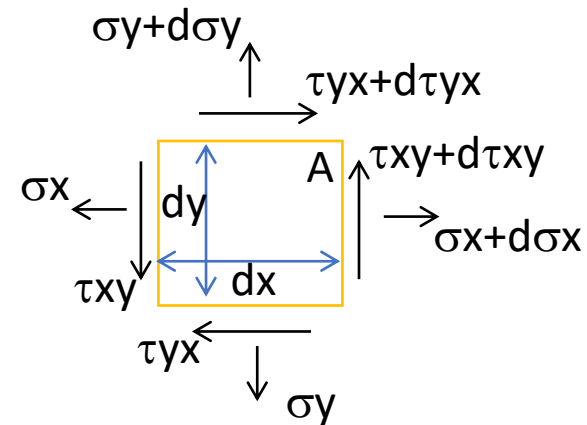
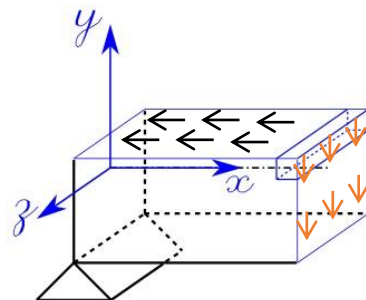
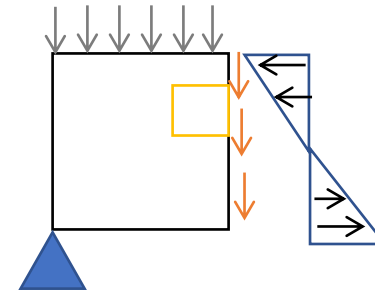
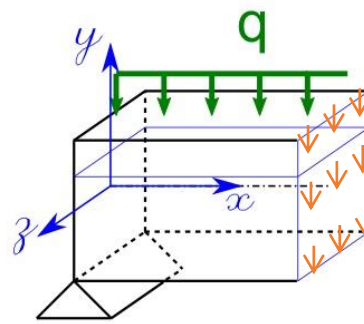
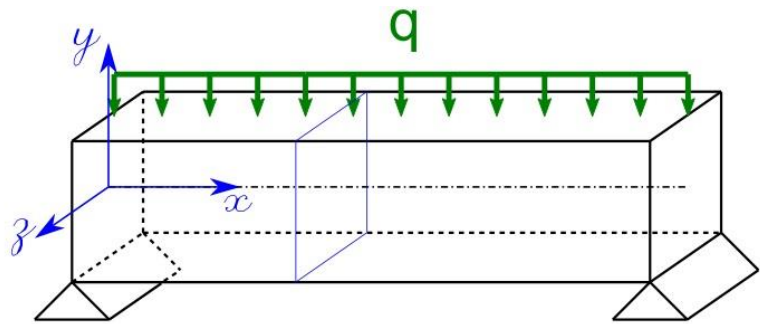


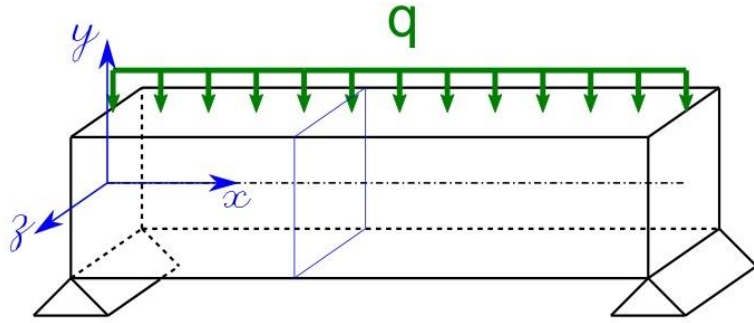
Rasantes en vigas



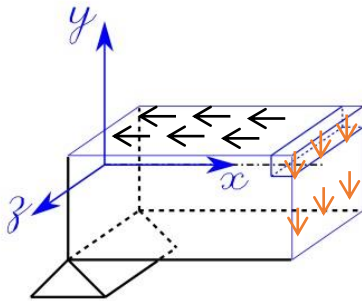
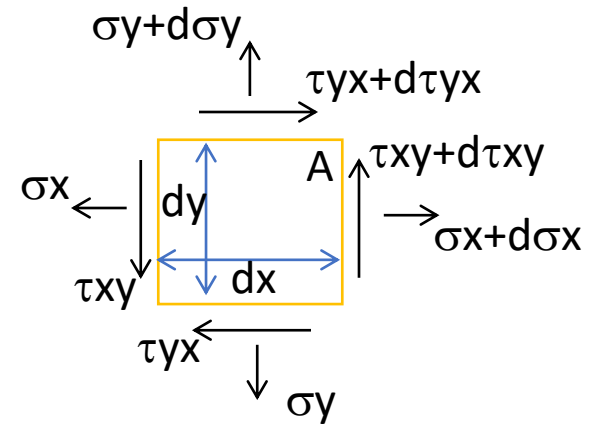
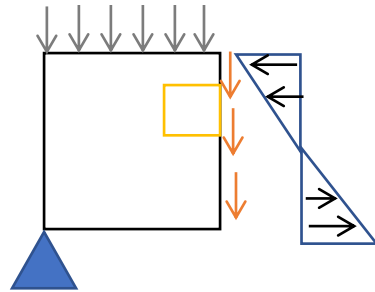
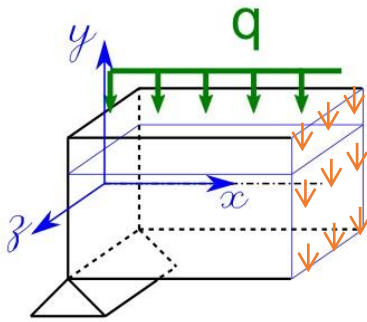
Reciprocidad de las Tensiones Rasantes

Hipótesis :
Las tensiones son uniformes en todo el ancho de la viga.



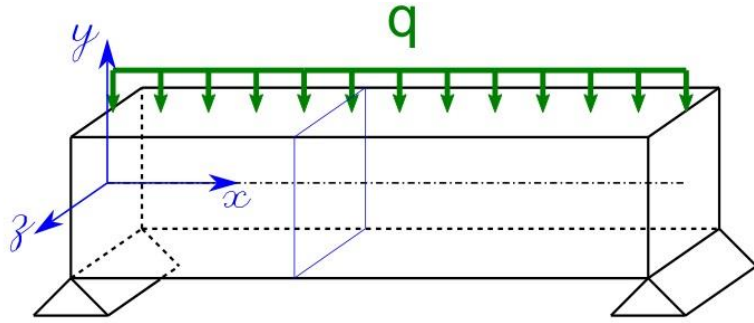


hipótesis :
Las tensiones son uniformes en todo el ancho de la viga.

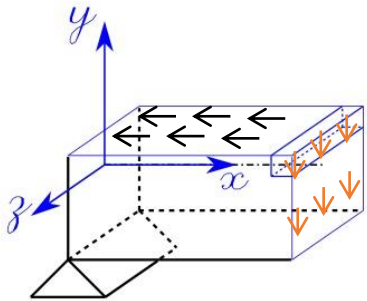
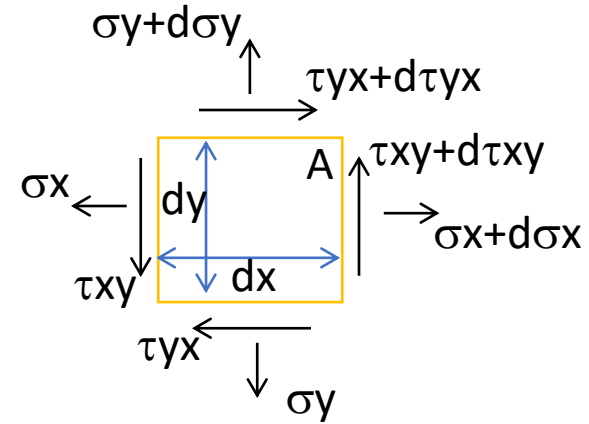
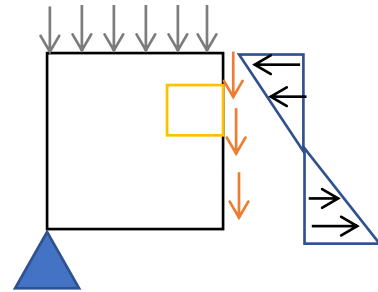
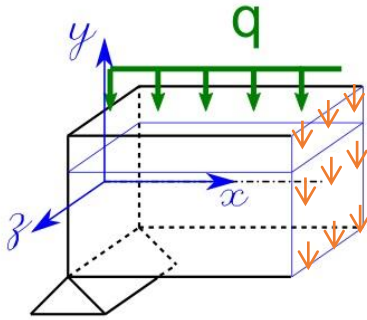


$$\text{Sum}(MA)=0$$

$$\sigma_x \cdot dy \cdot dy / 2 - (\sigma_x + d\sigma_x) \cdot dy \cdot dy / 2 + \dots$$



hipótesis :
Las tensiones son uniformes en todo el ancho de la viga.

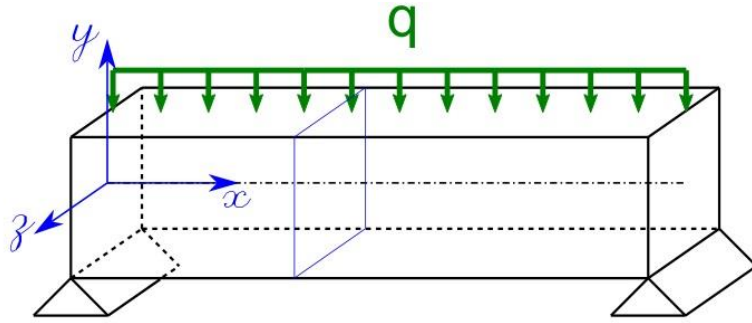


$$\text{Sum}(MA)=0$$

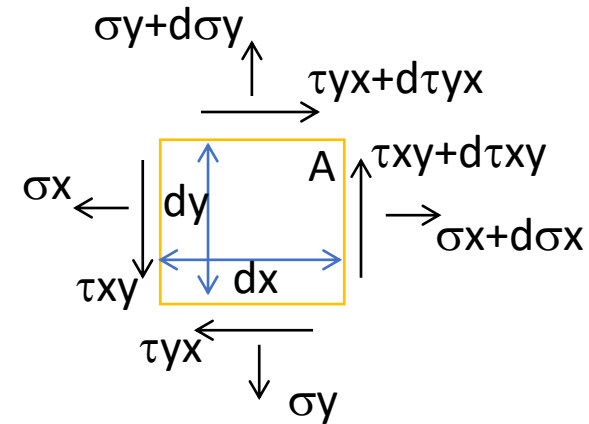
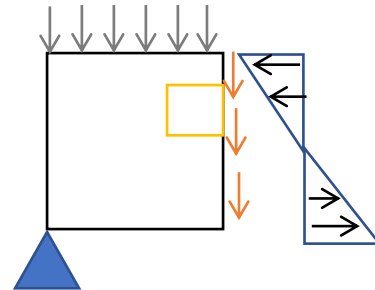
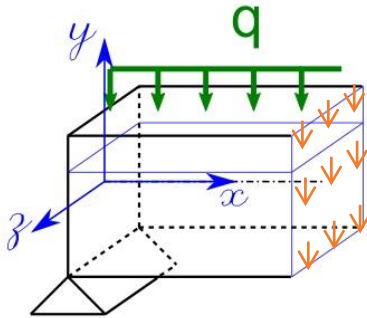
$$\sigma_x \cdot dy \cdot dy / 2 - (\sigma_x + d\sigma_x) \cdot dy \cdot dy / 2 + (\sigma_y + d\sigma_y) \cdot dx \cdot dx / 2 - \sigma_y \cdot dx \cdot dx / 2 + \tau_{yx} \cdot dx \cdot dy - \tau_{xy} \cdot dx \cdot dy = 0 \text{ despreciando terminos de 3er orden}$$

$$\tau_{yx} \cdot dx \cdot dy - \tau_{xy} \cdot dx \cdot dy = 0$$

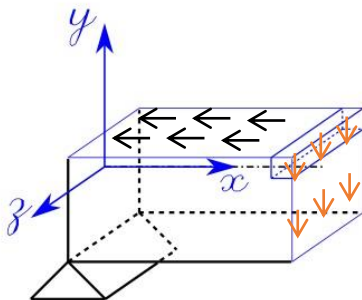
$$\rightarrow \tau_{yx} = \tau_{xy}$$



hipótesis :
Las tensiones son uniformes en todo el ancho de la viga.

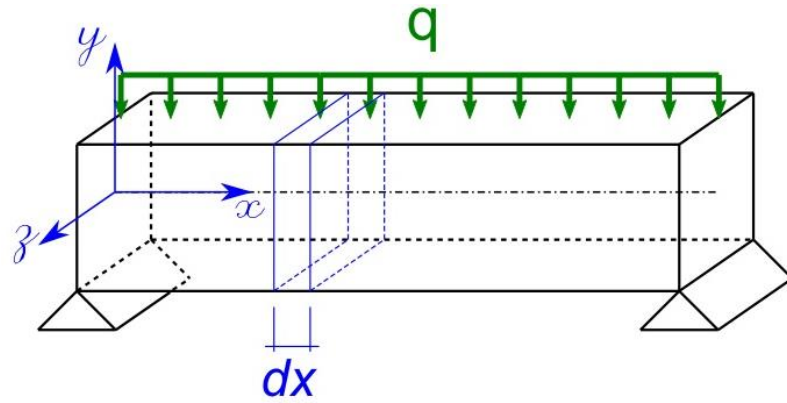


$$\rightarrow \tau_{yx} = \tau_{xy}$$



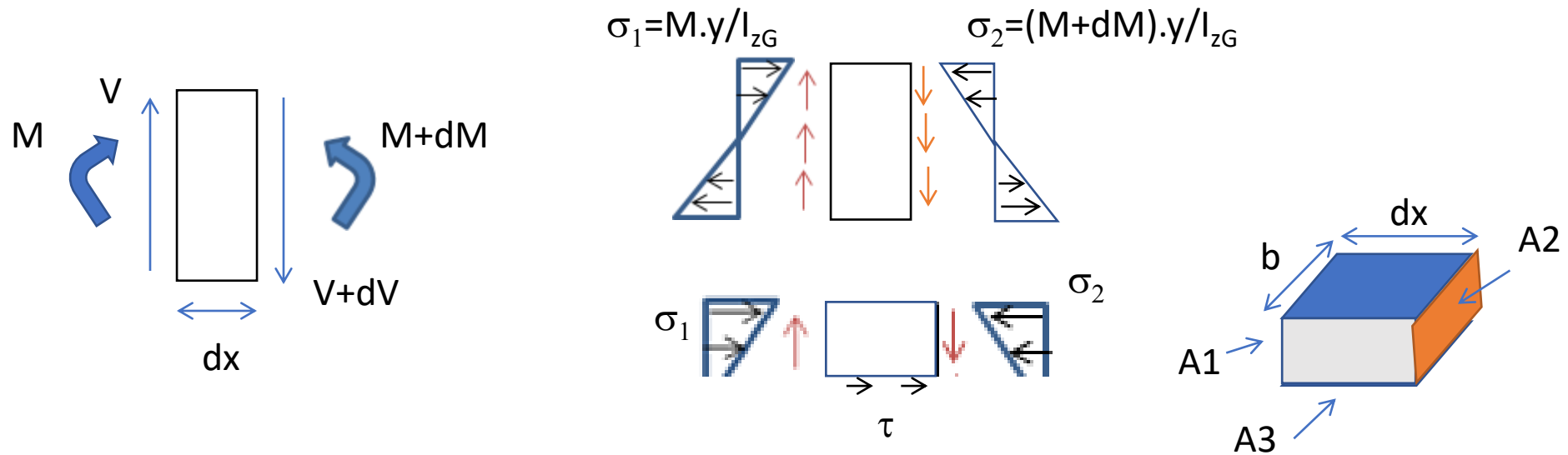
Se concluye que las **tensiones rasantes** sobre **caras adyacentes** de un elemento infinitesimal **son iguales en magnitud**, y tienen sentidos tales que **ambos** esfuerzos se dirigen **hacia el vértice común**, o **ambos esfuerzos se alejan de él**.

Jourawski

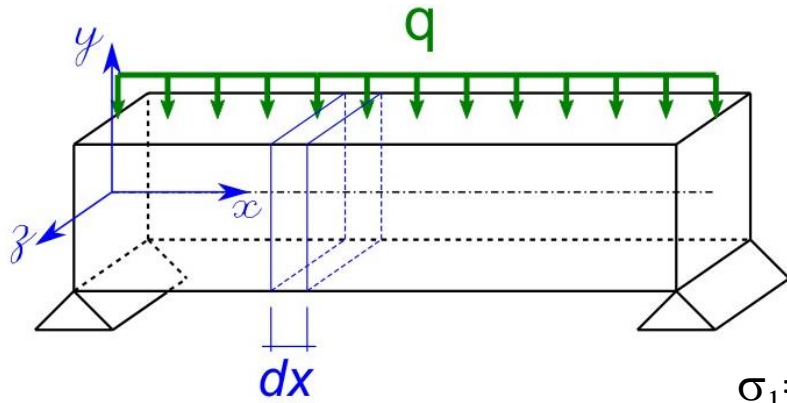


Fórmula de Jourawski:

$$\tau = \frac{V \cdot \mu}{I_{zG} \cdot b}$$

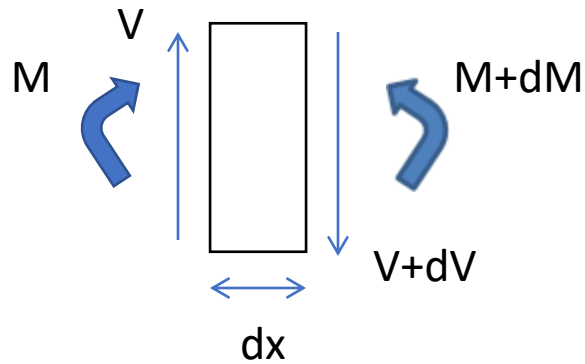


Jourawski

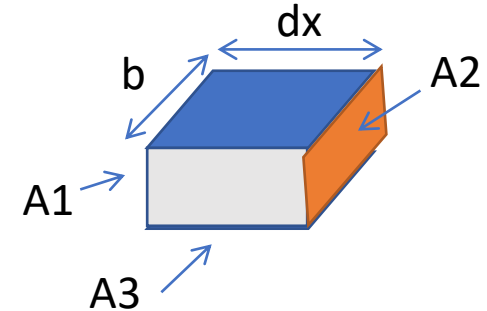
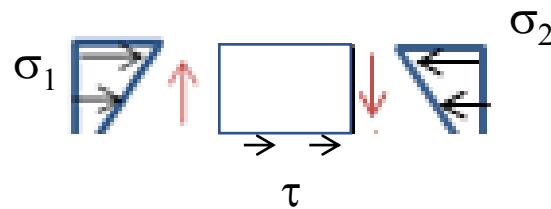
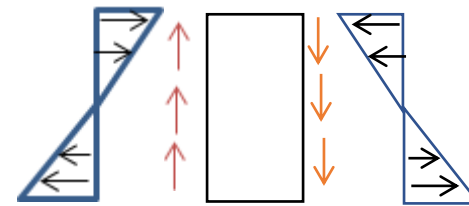


Fórmula de Jourawski:

$$\tau = \frac{V \cdot \mu}{I_{zG} \cdot b}$$



$$\sigma_1 = M \cdot y / I_{zG} \quad \sigma_2 = (M + dM) \cdot y / I_{zG}$$



$$\text{Suma(FH)} = \int_{A1} \sigma_1 dA + \int_{A3} \tau dA - \int_{A2} \sigma_2 dA = 0$$

$$\int_{A1} \frac{M y}{I_{zG}} dA + \tau \cdot b \cdot dx - \int_{A2} \frac{(M + dM) y}{I_{zG}} dA = 0$$

$$\int_{A1} \frac{My}{I} dA + \tau \cdot b \cdot dx - \int_{A2} \frac{(M + dM)y}{I} dA = 0$$

$$\tau \cdot b \cdot dx - \int_{A2} \frac{(dM)y}{I} dA = 0$$

$$\tau \cdot b \cdot dx - \frac{(dM)}{I} \int_{A2} y dA = 0 \qquad \tau = \frac{dM}{dx \cdot b \cdot I} \int_{A2} y dA$$

$$\frac{dM}{dx} = V \qquad \mu = \int_{A2} y dA$$

$$\tau = \frac{V \cdot \mu}{b \cdot I_{ZG}}$$

Ejemplo

Definición:
Flujo cortante

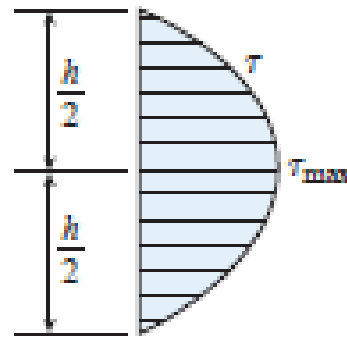
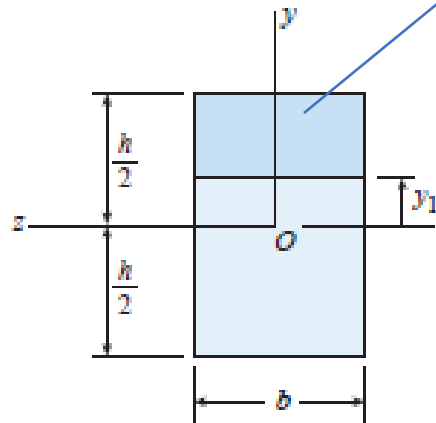
$$q = \frac{V \cdot \mu}{I}$$

El flujo cortante es la fuerza cortante horizontal por unidad de longitud en la dirección del eje de la barra.

Esta definición es más general. Para determinar q no es necesario imponer que las tensiones rasantes sean uniformes en el ancho de la sección

$$\int y dA = \int_{y_1}^{h/2} yb dy = \frac{b}{2} \left(\frac{h^2}{4} - y_1^2 \right)$$

$$\mu: b \left(\frac{h}{2} - y_1 \right) \left(y_1 + \frac{h/2 - y_1}{2} \right) = \frac{b}{2} \left(\frac{h^2}{4} - y_1^2 \right)$$



$$\tau = \frac{V \cdot \mu}{b \cdot I_{ZG}}$$

$$\tau_{\max} = \frac{Vh^2}{8I} = \frac{3V}{2A}$$

Ancho variable / Ej.: PNI

