Ejercicio 1

$$\left(\begin{array}{cccc|c} 1 & 1 & -1 & 2 & 1 \\ 0 & 2 & 1 & 0 & 2\lambda \\ 1 & -2 & -1 & 2 & 1-2\lambda \end{array}\right)_{\leftarrow -F_1} \sim \left(\begin{array}{cccc|c} 1 & 1 & -1 & 2 & 1 \\ 0 & 2 & 1 & 0 & 2\lambda \\ 0 & -3 & 0 & 0 & -2\lambda \end{array}\right)\begin{array}{l} \\ .3 \\ .2 \end{array} \sim \left(\begin{array}{cccc|c} 1 & 1 & -1 & 2 & 1 \\ 0 & 6 & 3 & 0 & 6\lambda \\ 0 & -6 & 0 & 0 & -4\lambda \end{array}\right)_{\leftarrow +F_2} \sim \left(\begin{array}{cccc|c} 1 & 1 & -1 & 2 & 1 \\ 0 & 6 & 3 & 0 & 6\lambda \\ 0 & 0 & 3 & 0 & 2\lambda \end{array}\right)$$

Siempre es compatible indeterminado!

$$\begin{cases} x+y-z+2w=1 \\ 6y+3z=6\lambda \\ 3z=2\lambda \end{cases} \rightarrow \begin{cases} x+y-z+2w=1 \\ 6y=6\lambda-2\lambda \\ z=2\lambda/3 \end{cases} \rightarrow \begin{cases} x+(2\lambda/3)-(2\lambda/3)+2w=1 \\ y=2\lambda/3 \\ z=2\lambda/3 \end{cases} \rightarrow \begin{cases} x=1-2w \\ y=2\lambda/3 \\ z=2\lambda/3 \end{cases}$$

Solución: $\begin{pmatrix} x \\ y \\ z \\ w \end{pmatrix} = \begin{pmatrix} 1-2w \\ 2\lambda/3 \\ 2\lambda/3 \\ w \end{pmatrix} = \begin{pmatrix} 1 \\ 2\lambda/3 \\ 2\lambda/3 \\ 0 \end{pmatrix} + w \begin{pmatrix} -2 \\ 0 \\ 0 \\ 1 \end{pmatrix}$

un grado de libertad

[15 puntos]

Ejercicio 2

(1) $\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \overset{P}{\begin{pmatrix} 2 \\ 4 \\ 2 \end{pmatrix}} + \lambda . \overset{\overrightarrow{QR}}{\begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix}}$ ecuaciones paramétricas $\longrightarrow \begin{cases} x-2=z-2 \\ y=4 \end{cases} \sim \begin{cases} x=z \\ y=4 \end{cases}$ ecuaciones reducidas [7 puntos]

(2) $\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \overset{S}{\begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix}} + \lambda . \overset{\overrightarrow{SP}}{\begin{pmatrix} 1 \\ 3 \\ 2 \end{pmatrix}} + \mu \overset{\overrightarrow{SR}}{\begin{pmatrix} 2 \\ 3 \\ 3 \end{pmatrix}}$ ecuaciones paramétricas $\rightarrow \begin{vmatrix} x-1 & y-1 & z \\ 1 & 3 & 2 \\ 2 & 3 & 3 \end{vmatrix} = 9(x-1)+4(y-1)+3z-6z-3(y-1)-6(x-1) =$

$= 9x-9+4y-4+3z-6x-3y+3-6z+6 = \boxed{3x+y-3z-4=0}$ ecuación reducida (7 puntos)

(3) mismo vector normal $\rightarrow 3x+y-3z=d \longrightarrow$ pasa por $(0,0,0) \Rightarrow d=0 \Rightarrow \boxed{3x+y-3z=0}$ (6 puntos)

(4) sustituyo (r) en (s):

$$\begin{cases} 2.(2+\lambda)+4-(2+\lambda)=1 \\ (2+\lambda)+3.(2+\lambda)=2 \end{cases} \sim \begin{cases} \lambda=-5 \\ \lambda=-3/2 \end{cases}$$ la intersección es vacía (5 puntos) [25 puntos]

Ejercicio 3

(1) $\begin{vmatrix} 0 & 1 & 0 & -1 \\ 0 & 0 & \alpha & 1 \\ 1 & 2 & 3 & 1 \\ 1 & 3 & 2 & \alpha \end{vmatrix} = -\alpha \underbrace{\begin{vmatrix} 0 & 1 & -1 \\ 1 & 2 & 1 \\ 1 & 3 & \alpha \end{vmatrix}}_{1-3+2-\alpha=-\alpha} + \underbrace{\begin{vmatrix} 0 & 1 & 0 \\ 1 & 2 & 3 \\ 1 & 3 & 2 \end{vmatrix}}_{3-2=1} = \alpha^2+1 \neq 0 \ \forall \alpha \Rightarrow$ siempre es invertible (7 puntos)

Calculemos A^{-1}:

$$\left(\begin{array}{cccc|cccc} 0 & 1 & 0 & -1 & 1 & 0 & 0 & 0 \\ 0 & 0 & \alpha & 1 & 0 & 1 & 0 & 0 \\ 1 & 2 & 3 & 1 & 0 & 0 & 1 & 0 \\ 1 & 3 & 2 & \alpha & 0 & 0 & 0 & 1 \end{array}\right)_{\leftarrow -F_3} \sim \left(\begin{array}{cccc|cccc} 0 & 1 & 0 & -1 & 1 & 0 & 0 & 0 \\ 0 & 0 & \alpha & 1 & 0 & 1 & 0 & 0 \\ 1 & 2 & 3 & 1 & 0 & 0 & 1 & 0 \\ 0 & 1 & -1 & \alpha-1 & 0 & 0 & -1 & 1 \end{array}\right)_{\leftarrow -F_1} \sim \left(\begin{array}{cccc|cccc} 0 & 1 & 0 & -1 & 1 & 0 & 0 & 0 \\ 0 & 0 & \alpha & 1 & 0 & 1 & 0 & 0 \\ 1 & 2 & 3 & 1 & 0 & 0 & 1 & 0 \\ 0 & 0 & -1 & \alpha & -1 & 0 & -1 & 1 \end{array}\right)$$ reordeno

$$\left(\begin{array}{cccc|cccc} 1 & 2 & 3 & 1 & 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & -1 & 1 & 0 & 0 & 0 \\ 0 & 0 & -1 & \alpha & -1 & 0 & -1 & 1 \\ 0 & 0 & \alpha & 1 & 0 & 1 & 0 & 0 \end{array}\right)_{\leftarrow +\alpha F_3} \sim \left(\begin{array}{cccc|cccc} 1 & 2 & 3 & 1 & 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & -1 & 1 & 0 & 0 & 0 \\ 0 & 0 & -1 & \alpha & -1 & 0 & -1 & 1 \\ 0 & 0 & 0 & 1+\alpha^2 & -1 & 1 & -1 & 1 \end{array}\right)_{\%(1+\alpha^2)}$$

(hasta acá todo bonito)

(continúa)

Ejercicio 3

llamemos x a $1+\alpha^2$ para facilitar la notación

$$\left(\begin{array}{cccc|cccc} 1 & 2 & 3 & 1 & 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & -1 & 1 & 0 & 0 & 0 \\ 0 & 0 & -1 & \alpha & -1 & 0 & -1 & 1 \\ 0 & 0 & 0 & 1 & \frac{-1}{1+\alpha^2} & \frac{1}{1+\alpha^2} & \frac{-1}{1+\alpha^2} & \frac{1}{1+\alpha^2} \end{array}\right) \underset{\leftarrow -\alpha F_4}{\sim} \left(\begin{array}{cccc|cccc} 1 & 2 & 3 & 1 & 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & -1 & 1 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 & -1+\frac{\alpha}{x} & \frac{\alpha}{x} & -1+\frac{\alpha}{x} & 1+\frac{\alpha}{x} \\ 0 & 0 & 0 & 1 & \frac{-1}{x} & \frac{1}{x} & \frac{-1}{x} & \frac{1}{x} \end{array}\right) \begin{array}{c} \\ \leftarrow +F_4 \\ \\ \\ \end{array}$$

haré $-2F_2+3F_3-F_4$

$$\sim \left(\begin{array}{cccc|cccc} 1 & 2 & 3 & 1 & 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 & 1-\frac{1}{x} & \frac{1}{x} & \frac{-1}{x} & \frac{1}{x} \\ 0 & 0 & -1 & 0 & -1+\frac{\alpha}{x} & \frac{\alpha}{x} & -1+\frac{\alpha}{x} & 1+\frac{\alpha}{x} \\ 0 & 0 & 0 & 1 & \frac{-1}{x} & \frac{1}{x} & \frac{-1}{x} & \frac{1}{x} \end{array}\right)$$

$$\left(\begin{array}{cccc|cccc} 1 & 0 & 0 & 0 & -2\left(1-\frac{1}{\alpha}\right)+3\left(-1+\frac{\alpha}{x}\right)+\frac{1}{\alpha} & \frac{-2}{x}+\frac{3\alpha}{x}-\frac{1}{x} & \frac{-2}{\alpha}+3\cdot\left(-1+\frac{\alpha}{x}\right)+\frac{1}{\alpha} & \frac{-2}{x}+3\left(1+\frac{\alpha}{x}\right)-\frac{1}{x} \\ 0 & 1 & 0 & 0 & 1-1/x & 1/x & -1/x & 1/x \\ 0 & 0 & 1 & 0 & -1+\alpha/x & \alpha/x & -1+\alpha/x & 1+\alpha/x \\ 0 & 0 & 0 & 1 & -1/x & 1/x & -1/x & 1/x \end{array}\right) \begin{array}{c} \\ \\ \cdot(-1) \\ \\ \end{array}$$; por lo que

$$A^{-1} = \left(\begin{array}{c|c|c|c} -5+\frac{3(1+\alpha)}{(1+\alpha^2)} & \frac{3\cdot(\alpha-1)}{1+\alpha^2} & \frac{(3\alpha-1)}{1+\alpha^2}-3 & 3\cdot\left(1+\frac{\alpha-1}{(1+\alpha^2)}\right) \\ 1/(1+\alpha^2) & 1/(1+\alpha^2) & -1/(1+\alpha^2) & 1/1+\alpha^2 \\ -1+\alpha/(1+\alpha^2) & \alpha/(1+\alpha^2) & -1+\alpha/(1+\alpha^2) & 1+\alpha/1+\alpha^2 \\ -1/(1+\alpha^2) & 1/(1+\alpha^2) & -1/(1+\alpha^2) & 1/1+\alpha^2 \end{array}\right)$$

(3 puntos)

[2] $\left|2\begin{pmatrix} a & c & 2b \\ d & f & 2e \\ g & i & 2h \end{pmatrix}\right| = 2^3 \cdot \begin{vmatrix} a & c & 2b \\ d & f & 2e \\ g & i & 2h \end{vmatrix} = 2^3 \cdot 2 \begin{vmatrix} a & c & b \\ d & f & e \\ g & i & h \end{vmatrix} = -2^4 \begin{vmatrix} a & b & c \\ d & e & f \\ g & h & i \end{vmatrix} = -2^5$ [7 puntos]

[3] Llamemos x a $\det(A)$. $x^3 = x \Rightarrow x^3 - x = 0 \Rightarrow x \cdot (x^2-1) = 0 < \begin{matrix} 0 \\ 1 \\ -1 \end{matrix}$ (3 puntos)

$\det(A)$ puede valer $0, 1$ o -1

[20 puntos]

Criterios de Corrección:

EJ 1: Discusión: 10 puntos
Solución: 5 puntos

EJ 2: (1) 7 puntos (3, 4)
(2) 7 puntos (4 y 3)
(3) 6 puntos
(4) 5 puntos

EJ 3: (1) Ver invertible: 7 puntos
Hallar A^{-1}: 3 puntos
(2) 7 puntos
(3) 3 puntos