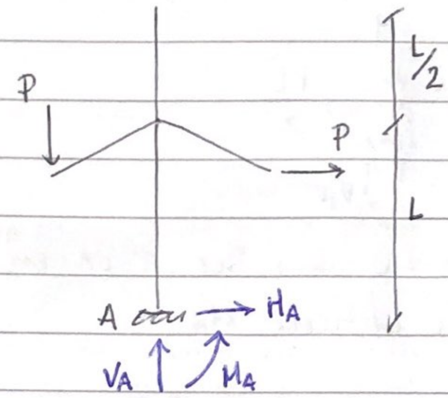
EXAMEN RESISTENCIA de MATERIALES 1

Diciembre 2020

TEÓRICO

a)



→ Equilibrio

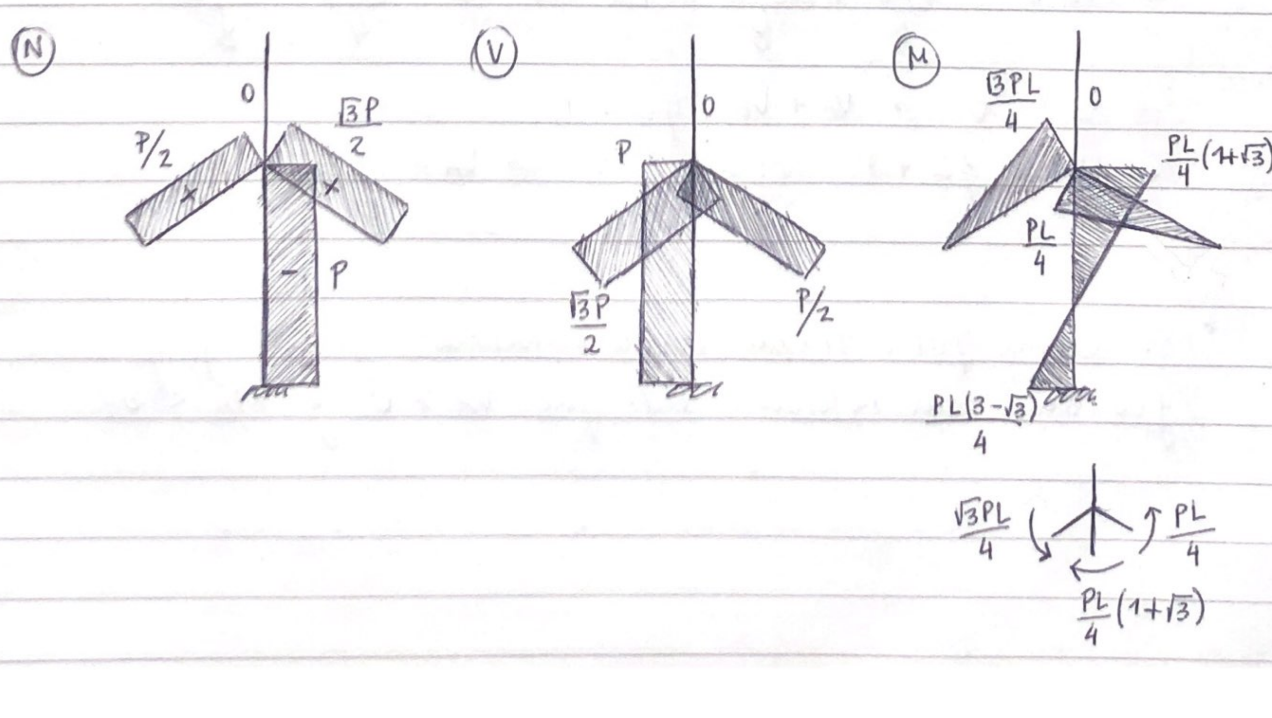
$$\Sigma V = 0 \Rightarrow -P + V_A = 0 \Rightarrow V_A = P$$

$$\Sigma H = 0 \Rightarrow P + H_A = 0 \Rightarrow H_A = -P$$

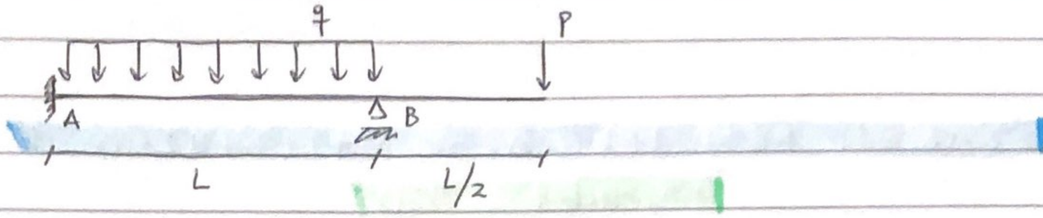
$$\Sigma M^A = 0 \Rightarrow P \cdot \frac{L}{2}\cos(30) - P\left(L - \frac{L}{2}\,\mathrm{sen}(30)\right) + M_A = 0$$

$$\Rightarrow \frac{\sqrt{3}\,PL}{4} - P\left(L - \frac{L}{4}\right) + M_A = 0 \Rightarrow M_A = \frac{PL}{4}\left(3 - \sqrt{3}\right)$$

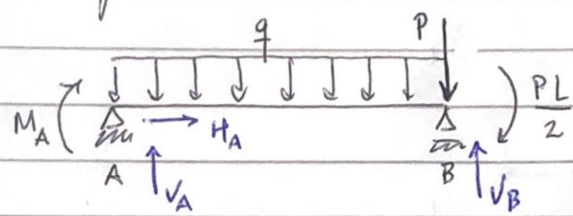
→ Diagramas



b)



→ Trabajo con la estructura auxiliar.



→ Se debe suponer $\theta_A = 0$ por ser A un empotramiento. Con esta condición se obtiene M_A.

$$\Rightarrow \frac{M_A L}{3EI} + \frac{qL^3}{24EI} - \frac{PL^2/2}{6EI} = 0$$

$$\Rightarrow \frac{M_A}{3} + \frac{qL^2}{24} - \frac{PL}{12} = 0 \Rightarrow \boxed{M_A = \frac{PL}{4} - \frac{qL^2}{8}}$$

→ Realizando equilibrio se obtienen las reacciones V_A y V_B.

$$\Rightarrow \Sigma M^A = M_A + \frac{qL^2}{2} + \frac{3PL}{2} - V_B L = 0$$

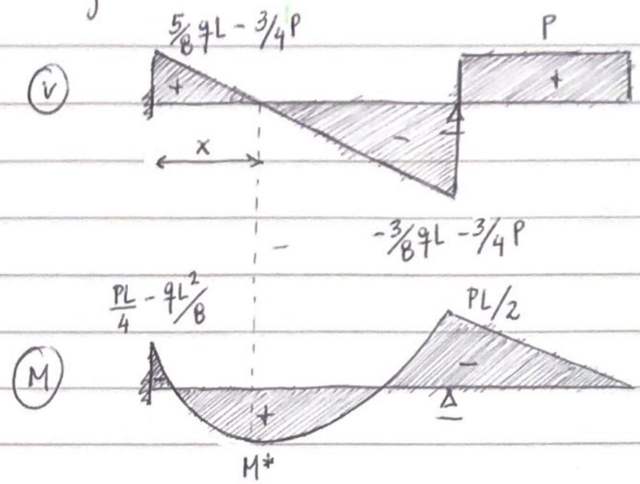
$$\Rightarrow V_B L = \frac{7PL}{4} + \frac{3qL^2}{8} \Rightarrow V_B = \frac{7P}{4} + \frac{3qL}{8}$$

$$\Rightarrow \Sigma V = 0 \Rightarrow V_A + V_B = qL + P$$

$$\Rightarrow V_A = qL + P - \frac{3qL}{8} - \frac{7P}{4} \Rightarrow V_A = \frac{5}{8}qL - \frac{3}{4}P$$

* Se asume para trazar los diagramas que P y q guardan una relación tal que $V_A > 0$ y $M_A < 0$.

↳ Diagramas.



$$\circ\ q x = \frac{5}{8} qL - \frac{3P}{4} \longrightarrow x = \frac{5L}{8} - \frac{3}{4}\frac{P}{q}$$

$$\circ\ M^* = \frac{PL}{4} - \frac{qL^2}{8} + \left(\frac{5}{8}qL - \frac{3P}{4}\right)\cdot\frac{x}{2} = \frac{PL}{4} - \frac{qL^2}{8} + \left(\frac{5}{8}qL - \frac{3P}{4}\right)\left(\frac{5L}{8} - \frac{3}{4}\frac{P}{q}\right)\frac{1}{2}$$

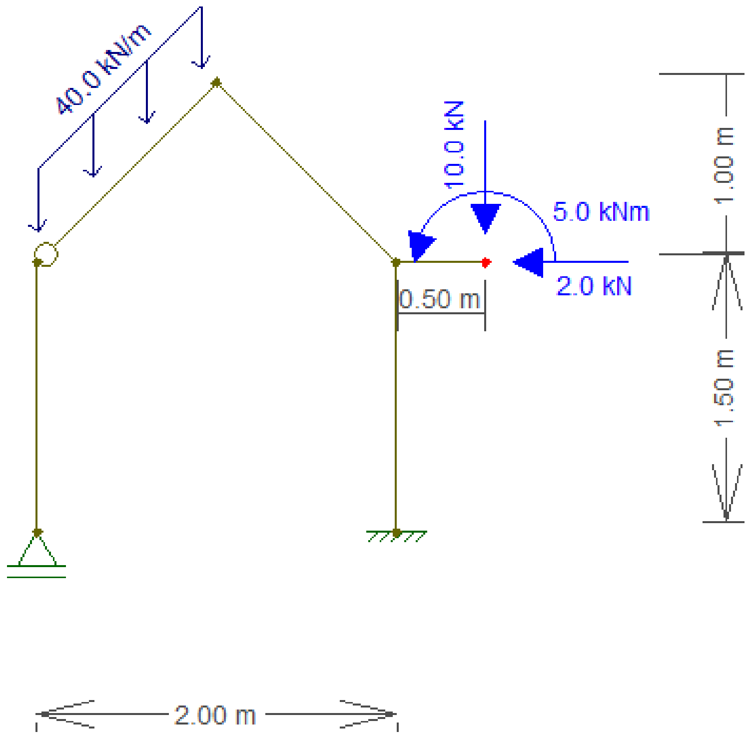
$$M^* = \frac{PL}{4} - \frac{qL^2}{8} + \frac{25}{128}qL^2 - \frac{15}{64}PL - \frac{15}{64}PL + \frac{9P^2}{32q}$$

$$M^* = -\frac{14PL}{64} + \frac{9}{128}qL^2 + \frac{9P^2}{32q} \Longrightarrow \boxed{M^* = -\frac{7PL}{32} + \frac{9qL^2}{128} + \frac{9P^2}{32q}}$$

Ejercicio 2 – Práctico

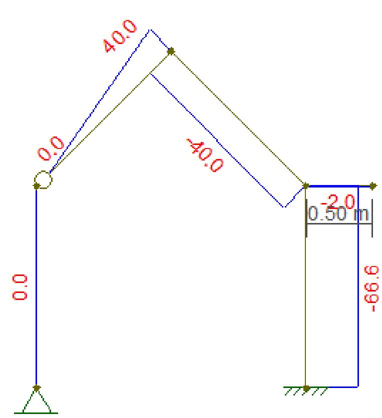
a) Las reacciones de la estructura son:

a. $V_A = 0\ kN$
b. $H_F = 2\ kN$
c. $V_F = 66.6\ kN$
d. $M_F = 87.9\ kNm$ **(horario)**

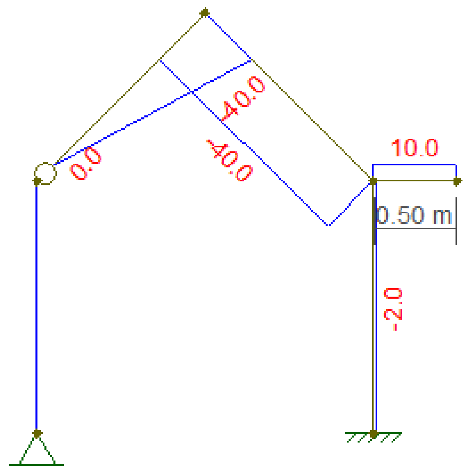


b) Diagramas de solicitaciones

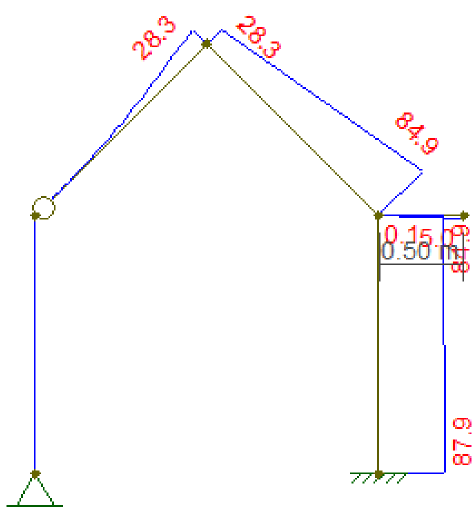
Directa [kN]

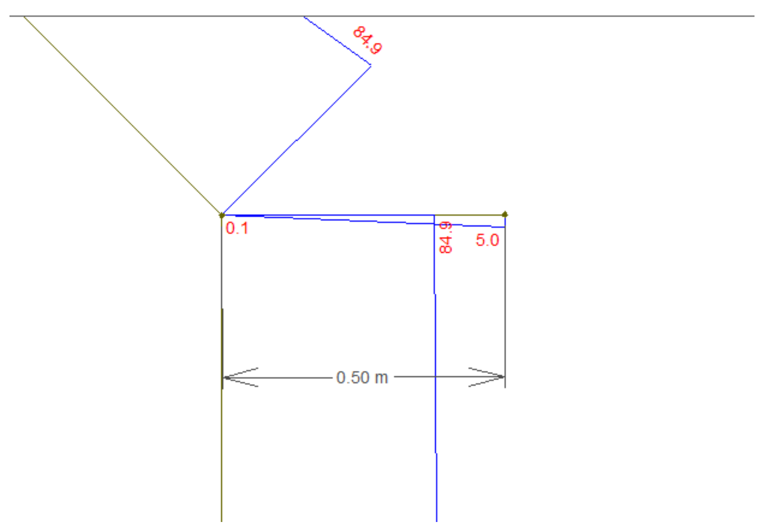


Cortante [kN]



Momento flector [kNm]





c) **Dimensionar con un IPN**

	635.7143	cm3	Requerido solo por M		
W	653	cm3	IPN30		
A	69.1	cm2	134609494.6	9638205	
			M=89kNm	N=66.6 kN	
				144.2477	MPa
			IPN 32		
W	782				
A	77.8	cm3	112404092.1	8560411	
				120.9645	MPa

e) Momento máximo para nueva sección.

yG= (16*77.8+15*32.5)/(15+77.8) = 18.67 cm desde abajo

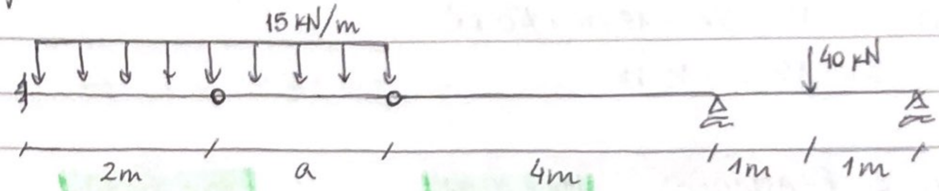
IG= =12512+2.67^2*77.8+15/12+(32.5-18.67)^2*15 =15936.9 cm4

140 MPa $\geq$ M/W --> 140 MPa* 15936.9 cm4/18.67cm

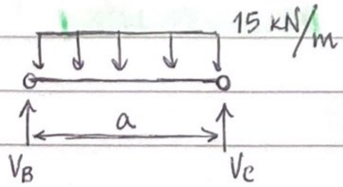
M$\leq$ 119.5 kN.m

PRÁCTICO

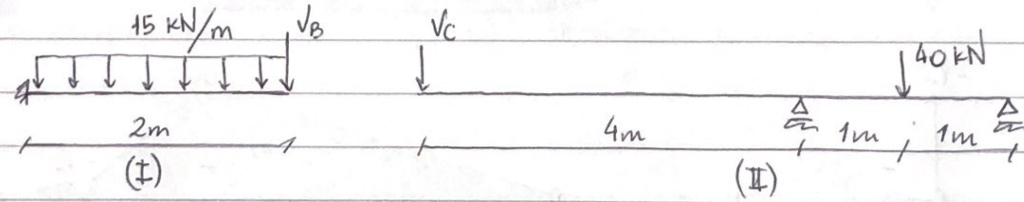
Ejercicio 3



a) Se quiere $M_A = M_D$



$\rightarrow V_B = V_C = \dfrac{15\ kN/m \cdot a}{2}$

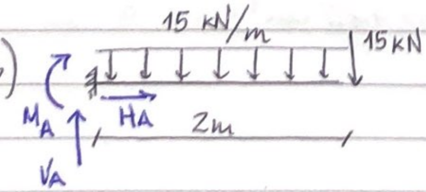


(I) $M_A(-) = \dfrac{15\ kN/m \cdot a \cdot 2m}{2} + \dfrac{15\ kN/m\,(2m)^2}{2} = 15\ kN\,(a + 2m)$

(II) $M_D(-) = \dfrac{15\ kN/m \cdot a \cdot 4m}{2} = 15\ kN \cdot 2a$

$\rightarrow M_A(-) = M_D(-) \longleftrightarrow 15\ kN\,(a + 2m) = 15\ kN \cdot 2a \longleftrightarrow (a + 2m) = 2a$

$\rightarrow M_A(-) = M_D(-) \longleftrightarrow a = 2m$

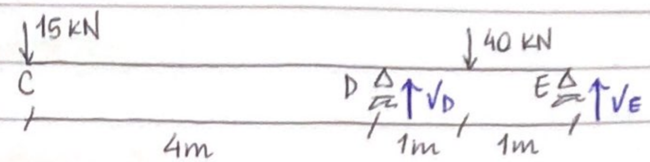
b) 

$\rightarrow \Sigma V = 0 \Rightarrow V_A - 15\ kN/m \cdot 2m - 15\ kN = 0$

$\Rightarrow V_A = 45\ kN$

$\rightarrow$ De (I) se tiene $M_A = -60\ kNm$.

$\rightarrow$ De realizar equilibrio de fuerzas horizontales a la estructura global se obtiene $H_A = 0\ kN$.



$\Rightarrow \sum M_E = 0 \Rightarrow 15\,kN \cdot 6\,m + 40\,kN \cdot 1\,m = V_D \cdot 2\,m.$

$\Rightarrow V_D = 65\,kN$

$\Rightarrow \sum V = 0 \Rightarrow V_D + V_E = 15\,kN + 40\,kN$

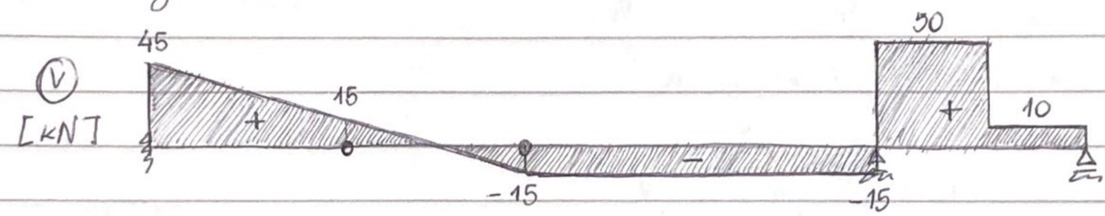
$\Rightarrow V_E = -10\,kN$

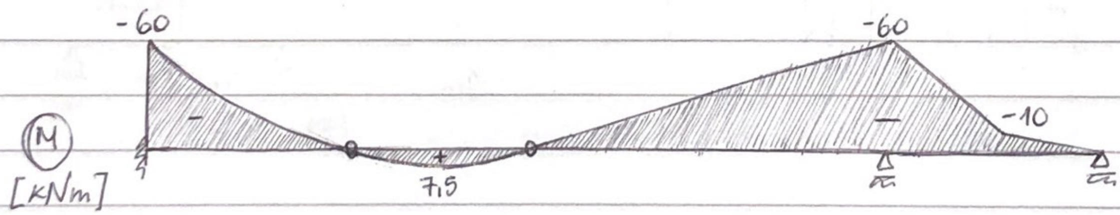
RESUMEN de REACCIONES: $H_A = 0\,kN$ $\quad V_D = 65\,kN$

$V_A = 45\,kN$ $\quad V_E = -10\,kN$

$M_A = -60\,kNm$

c) Diagramas:





d) Tensiones Normales

- $\sigma = \frac{M}{W} < \sigma_{adm} \leftrightarrow W > \frac{M}{\sigma_{adm}}$

$\left.\begin{array}{l} M_{max} = 60\,kNm \\ \sigma_{adm} = 140\,MPa \end{array}\right\} \rightarrow W > 428\,cm^3$

$\rightarrow$ Se toma un IPN 260 $(W = 442\,cm^3)$

Tensiones Rasantes

- $\tau = \frac{V\mu}{Ib} < \tau_{adm}$

$V_{max} = 50\,kN$

$\tau_{adm} = 70\,MPa$

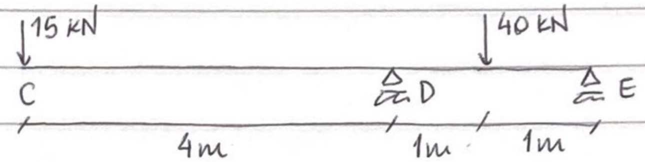
Además, para el IPN 260: $I = 5740\ cm^4$

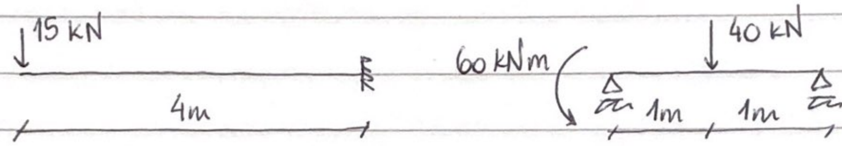
$\mu = 257\ cm^3$

$b = t_w = 0{,}94\ cm$

$\Rightarrow \tau = 23{,}82\ MPa < \tau_{adm}$ ✓

e) y f)





$$\delta_C^1 = \frac{15\,kN \cdot (4m)^3}{3EI} \downarrow \qquad \theta_D^1 = \frac{60\,kNm \cdot 2m}{3EI}$$

$$\theta_C^1 = \frac{15\,kN\,(4m)^2}{2EI} \qquad \theta_D^2 = \frac{40\,kN \cdot (2m)^2}{16EI}$$

$$\theta_D = \frac{30\,kNm^2}{EI}$$

$$\theta_C^2 = \theta_D = \frac{30\,kNm^2}{EI}$$

$$\delta_C^2 = \theta_D \cdot 4m = \frac{120\,kNm^3}{EI} \downarrow$$

$$\Rightarrow \delta_C = \delta_C^1 + \delta_C^2 = \frac{320\,kNm^3}{EI} + \frac{120\,kNm^3}{EI} \downarrow \Rightarrow \delta_C = 3{,}65\,cm \downarrow$$

$$\Rightarrow \theta_C = \theta_C^1 + \theta_C^2 = \frac{120\,kNm^2}{EI} + \frac{30\,kNm^2}{EI} \Rightarrow \theta_C = 0{,}012\,rad$$