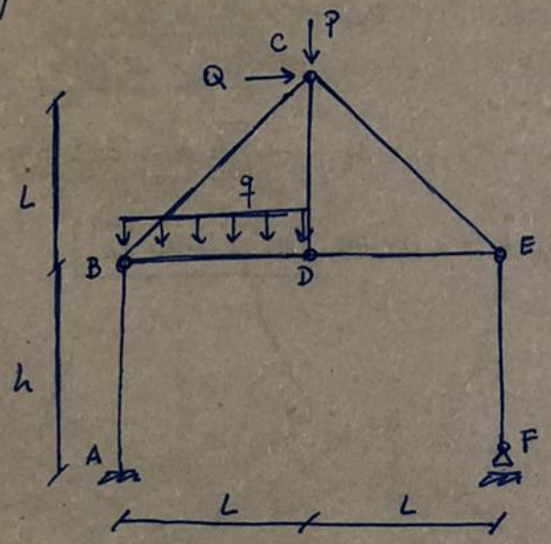
Ejercicio Examen



Datos:

$P = 30\ KN$

$q = 15\ KN/m$

$Q = 10\ KN$

$L = 1{,}5\ m$

$h = 2{,}0\ m$

a) Hallar reacciones y diagramas de solicitaciones (N, V, M)

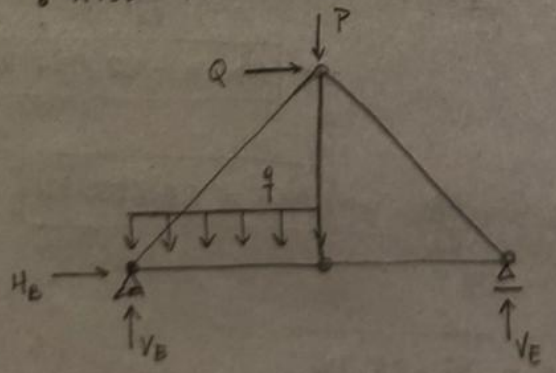
- Barra EF

$\Sigma M_F = 0 \Rightarrow H_E \cdot h = 0 \Rightarrow H_E = 0$

$\Sigma H = 0 \Rightarrow H_E + H_F = 0 \rightarrow \boxed{H_F = 0}$

$\Sigma V = 0 \Rightarrow V_E - V_F = 0 \rightarrow V_E = V_F \quad (I)$

- Aislo reticulado BCDE



$\Sigma H = 0 \Rightarrow H_B + Q = 0 \rightarrow H_B = -Q$

$\Sigma M_B = 0$

$\Sigma H = 0 \Rightarrow H_B + Q = 0 \Rightarrow H_B = -Q$

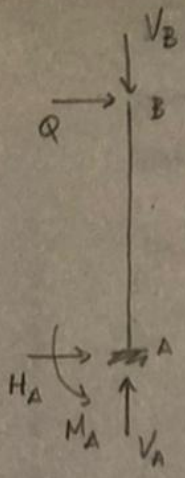
$\Sigma M_B = 0 \Rightarrow V_E \, 2L - Q L - P L - q L^2/2 = 0$

$\Rightarrow V_E = \frac{Q+P}{2} + \frac{qL}{4}$

$\Rightarrow (I)\ V_F = \frac{Q+P}{2} + \frac{qL}{4} \Rightarrow \boxed{V_F = 25{,}625\ KN}$

$\Sigma V = 0 \Rightarrow V_B + V_E - P - qL = 0$

$\Rightarrow V_B = P + qL - V_E$



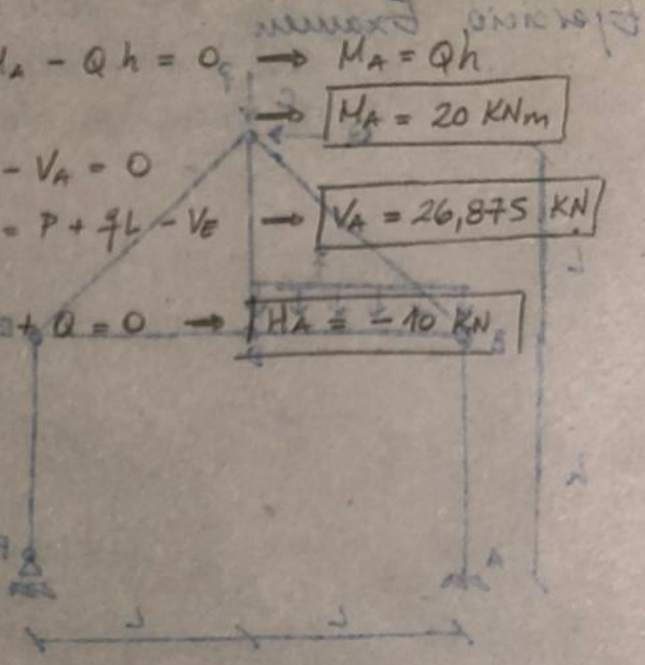
$\rightarrow \sum M_A = 0 \rightarrow M_A - Q\,h = 0 \rightarrow M_A = Q\,h$

$\rightarrow \boxed{M_A = 20\ KNm}$

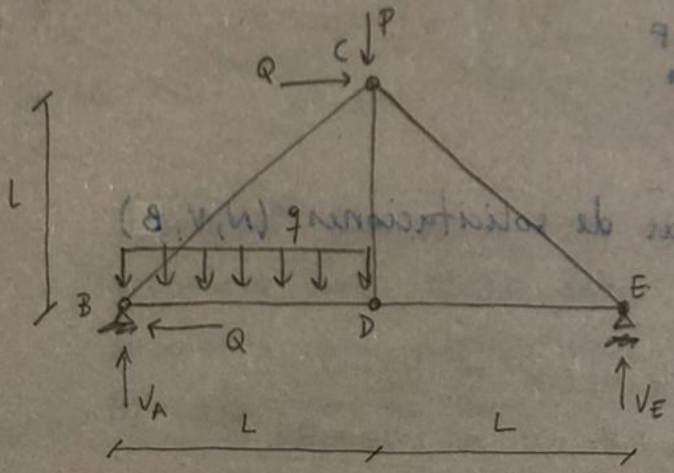
$\rightarrow \sum V = 0 \rightarrow V_B - V_A = 0$

$\rightarrow V_A = P + \frac{3}{4}qL - V_E \rightarrow \boxed{V_A = 26{,}875\ KN}$

$\rightarrow \sum H = 0 \rightarrow H_A + Q = 0 \rightarrow \boxed{H_A = -10\ KN}$



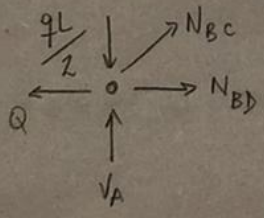
- Resolución estructura BCDE



- Nudo D

$N_{DC} = \frac{3}{4}qL/2 \rightarrow \boxed{N_{DC} = 11{,}25\ KN}$

Nudo B



$\sum V = 0 \rightarrow \frac{3}{4}qL = \frac{N_{BC}}{\sqrt{2}} + V_A \rightarrow N_{BC} = \frac{\frac{3}{4}qL}{\sqrt{2}} - \sqrt{2}\,V_A$

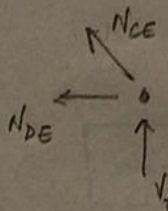
$\rightarrow \boxed{N_{BC} = -22{,}097\ KN}$

$\sum H = 0 \Rightarrow \frac{N_{BC}}{\sqrt{2}} + N_{BD} = Q \rightarrow N_{BD} = Q - \frac{N_{BC}}{\sqrt{2}} \rightarrow \boxed{N_{BD} = 25{,}625\ KN}$

Nudo D

$\sum H = 0 \Rightarrow N_{DB} = N_{DE} \Rightarrow \boxed{N_{DE} = 25{,}625\ KN}$

Nudo E

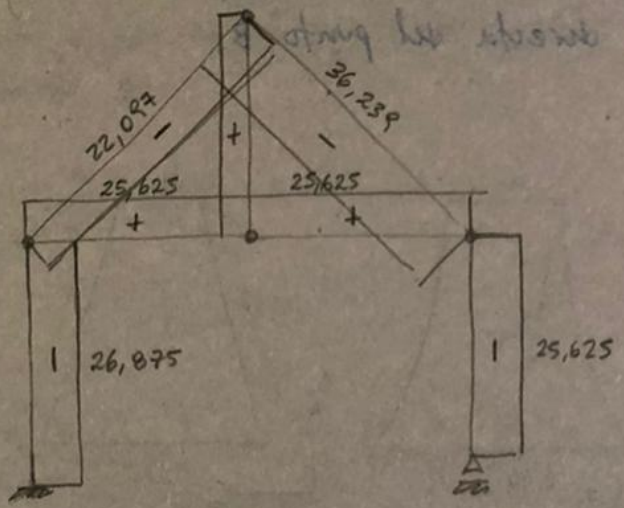


$\Rightarrow \sum V = 0 \rightarrow \frac{N_{CE}}{\sqrt{2}} + V_E = 0 \Rightarrow N_{CE} = -\sqrt{2}\,V_E$

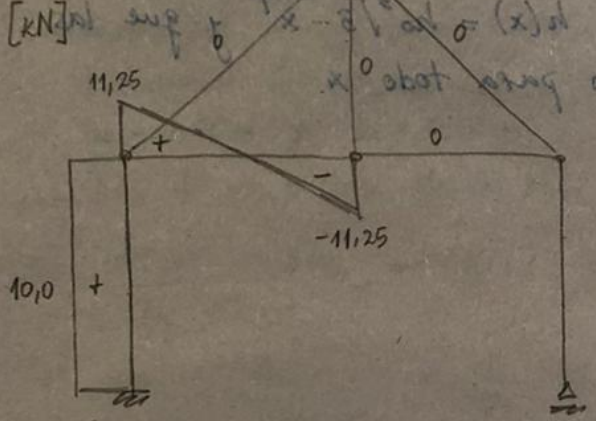
$\rightarrow \boxed{N_{CE} \doteq -36{,}239\ KN}$

Diagramas:

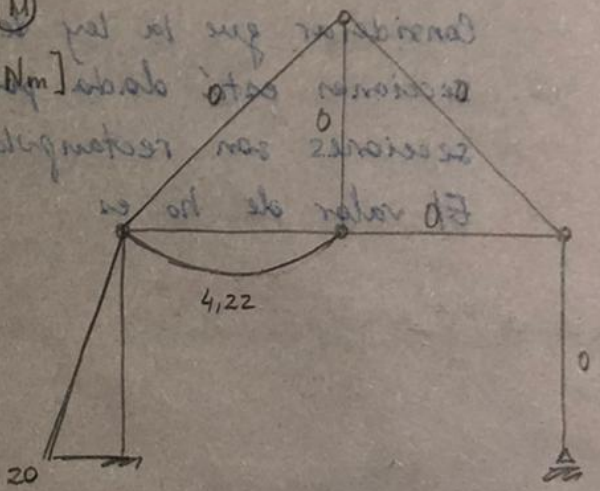
(N) [kN]



(V) [kN]



(M) [kNm]



b) Dimensionar los elementos BD, BC, CE, DE y EF con un único perfil PNI, considerando tensiones normales

Tomo viga BD:

Predimensiono $\left.\begin{array}{l} \frac{M}{W} < 140\ MPa \\ M = 4,22\ KNm \end{array}\right\} \Rightarrow W > 30,1\ cm^3 \Rightarrow PNI\ 10$

PNI 10 ($W = 34,2\ cm^3$; $A = 10,6\ cm^2$)

$\Rightarrow \sigma = \frac{M}{W} + \frac{N_{BD}}{A} < 140\ MPa \Rightarrow \sigma = 147\ MPa$ ✗

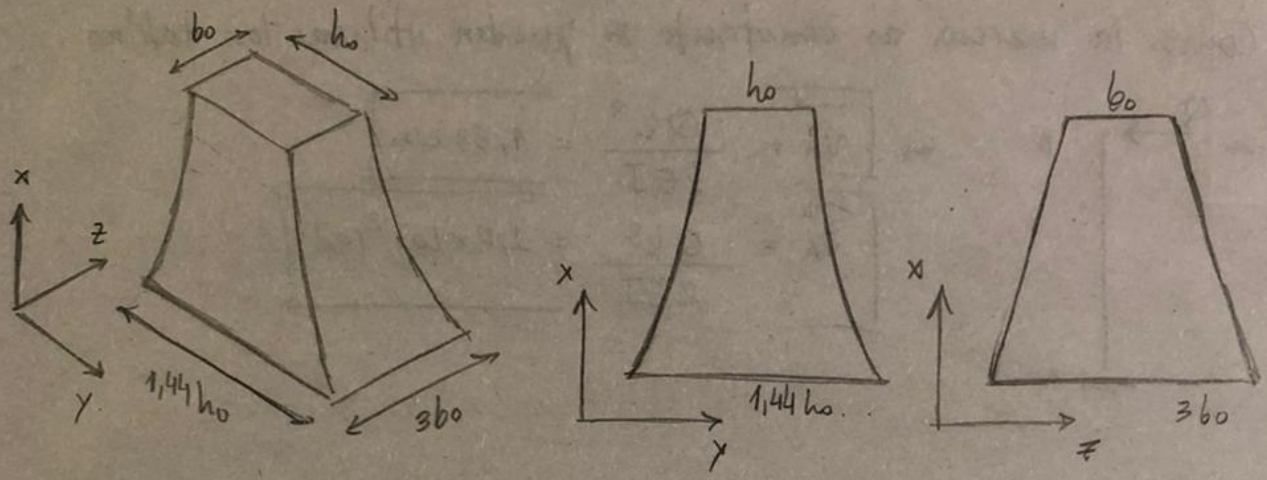
Tomo PNI 12 ($W = 54,7\ cm^3$; $A = 14,2\ cm^2$)

$\Rightarrow \sigma = \frac{M}{W} + \frac{N_{BD}}{A} = 95,2\ MPa < 140\ MPa$ ✓

Verifico: $\sigma = \frac{N_{max}}{A} = \frac{N_{CE}}{A} = 25,5\ MPa < 140\ MPa$ ✓

PNI 12

c) Considerando la sección del pilar AB como se indica a continuación y asumiendo que se verifican las tensiones admisibles en el material, hallar el descenso por directa del punto B.



Considerar que la ley de variación de la altura de las secciones está dada por: $h(x) = h_0 / \sqrt[3]{3-x}$ y que todas las secciones del pilar son rectangulares. Tomar $E = 30\ GPa$; $h_0 = 0{,}20\ m$ y $b_0 = 0{,}10\ m$.

⇒ Ley de variación de la base: $b(x) = b_0 (3-x)$

$$\Rightarrow \frac{du(x)}{dx} = \varepsilon(x) \rightarrow u(L) - u(0) = \int_0^L \varepsilon(x)\,dx \Rightarrow u(L) = \int_0^L \varepsilon(x)\,dx$$

$$\Rightarrow \varepsilon(x) = \frac{N(x).L}{A(x).E} = \frac{NL}{E}\,\frac{1}{A(x)}$$

$$\Rightarrow A(x) = b(x)\,h(x) = b_0 h_0 (3-x)(3-x)^{-1/3} = b_0 h_0 (3-x)^{2/3}$$

$$\Rightarrow u(L) = \int_0^L \frac{NL}{E b_0 h_0 (3-x)^{2/3}}\,dx = \frac{NL}{E b_0 h_0}\int_0^L (3-x)^{-2/3}\,dx$$

$$= \frac{NL}{E b_0 h_0}\left[\frac{-(3-x)^{1/3}}{1/3}\right]_0^L = \frac{3NL}{E b_0 h_0}\left(-(3-L)^{1/3} + 3^{1/3}\right)$$

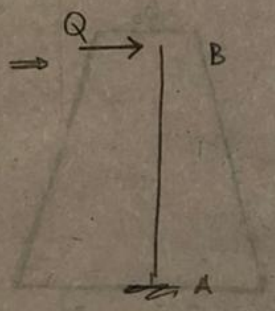
$$\Rightarrow u(L) = \frac{3NL}{E b_0 h_0}\left(3^{1/3} - (3-L)^{1/3}\right) \rightarrow u(L) = 1{,}19 \times 10^{-4}\ m.$$

d) Hallar el giro y la flecha del punto B.

[redacted]

$$I(x) = \frac{b(x)\cdot h(x)^3}{12} = \frac{b_o(3-x)\cdot h_o^3/(3-x)}{12} = \frac{b_o h_o^3}{12}$$

Como la inercia es constante se pueden utilizar las tablas.



$$\overrightarrow{v_A} = \frac{QL^3}{3EI} = 1{,}33\ cm$$

$$\overrightarrow{\theta_A} = \frac{QL^2}{2EI} = 1{,}0 \times 10^{-2}\ rad$$