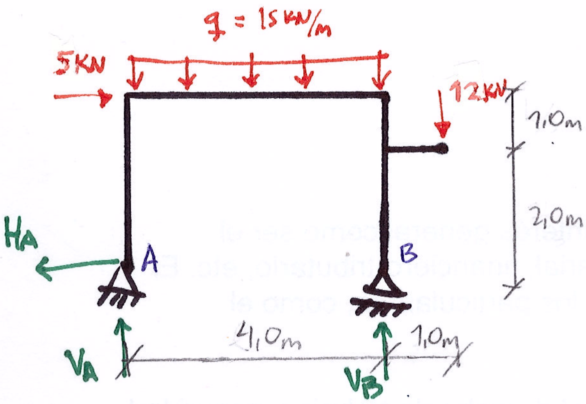
Resistencia de Materiales 1 - Examen de julio de 2016 - MC.



ⓐ $\Sigma M_A = 0 \Leftrightarrow 5\,kN \times 3\,m + 15\,\frac{kN}{m} \times 4{,}0\,m \times 2{,}0\,m + 12\,kN \times 5{,}0\,m - V_B \cdot 4\,m = 0$

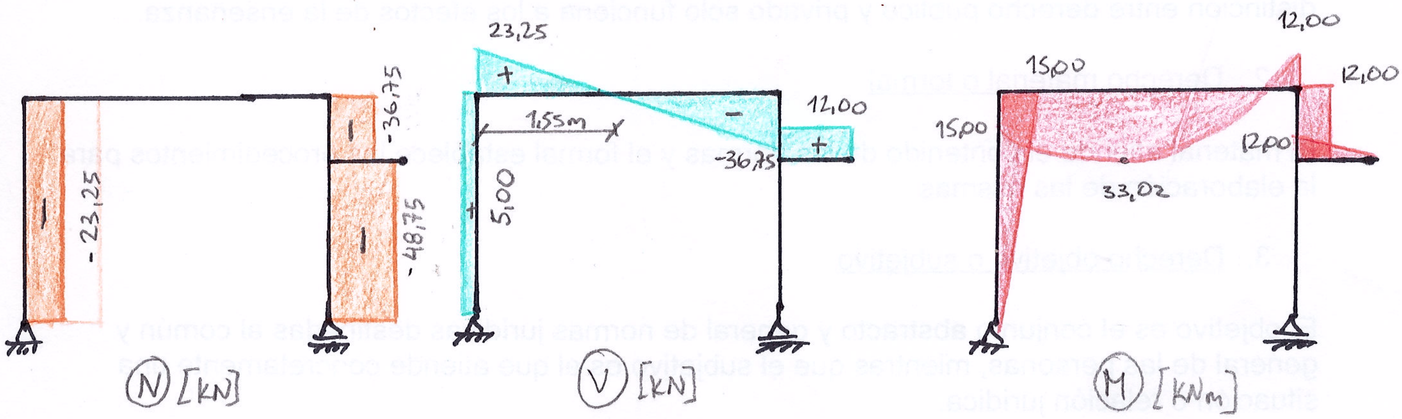
$\Rightarrow \boxed{V_B = \frac{15 + 120 + 60}{4}\,kN = 48{,}75\,kN}$

$\Sigma F_V = 0 \Leftrightarrow V_A + 48{,}75\,kN - 15{,}0\,kN/m \times 4{,}0\,m - 12\,kN = 0$

$\Rightarrow \boxed{V_A = 23{,}25\,kN}$

$\Sigma F_H = 0 \Leftrightarrow 5\,kN - H_A = 0 \Rightarrow \boxed{H_A = 5\,kN}$

ⓑ



ⓒ $\sigma_{adm} = 140\,MPa$. $A = 2 \times 15 \times 1\,cm^2 + A_{PNI}$, $h_{tot} = 2\,cm + h_{PNI}$

$\sigma = \frac{N}{A} \pm \frac{M}{W} \leqslant \sigma_{adm}$; $W = \frac{I}{h_{tot}/2} = \left[2 \times \left(\frac{15 \times 1^3}{12}\,cm^4 + 15 \times 1 \times \left(0{,}5 + \frac{h_{PNI}}{2}\right)^2 cm^4\right) + I_{PNI}\right] \cdot \frac{1}{h_{tot}/2}$

Diseña el pto con $\begin{cases} M = 33{,}02\,kNm \\ N = 0\,kN \end{cases} \Rightarrow$

PNI	A cm^2	I cm^4	W cm^3	σ MPa	¿Cumple?
120	44,2	1598	228,3	144,64	No
140	48,3	2263	282,9	116,73	Sí

$\Rightarrow \boxed{PNI\ 140}$

ⓓ La tensión rasante máxima se da en el punto de cortante máximo.

$\tau_{máx} = \frac{V_{máx} \cdot \mathcal{M}}{I \cdot b} = \frac{36{,}75\,kN \times 112{,}5\,cm^3}{2263\,cm^4 \times 6{,}6\,cm} = 0{,}2768\,kN/cm^2 \Rightarrow \boxed{\tau_{máx} = 2{,}77\,MPa}$

$\mathcal{M} = 15 \times 1 \times \left(\frac{14}{2} + 0{,}5\right) cm^3 = 112{,}5\,cm^3$

ⓔ $M_{máx} = 15 + \frac{V_A \cdot x}{2}$, $V_B = \frac{15 + 2 \times 4q + 60}{4} = 2q + 18{,}75\,kN \Rightarrow V_A = -V_B + 4q + 12\,kN = 2q - 6{,}75\,kN$, $V_A - qx = 0 \Rightarrow x = \frac{V_A}{q}$

$= 15 + \frac{V_A \cdot V_A}{2q}$

$= 15 + \frac{(2q - 6{,}75)^2}{2q}$

$= 15 + \frac{4q^2 - 27q + 45{,}5625}{2q}$

$= 15 + 2q - 13{,}5 + \frac{22{,}78125}{q}$

$= 1{,}5 + 2q + \frac{22{,}78125}{q} \leqslant \sigma_{adm} \times W$

$\leqslant \frac{14 \times 282{,}88}{100}\,kNm = 39{,}6032\,kNm$

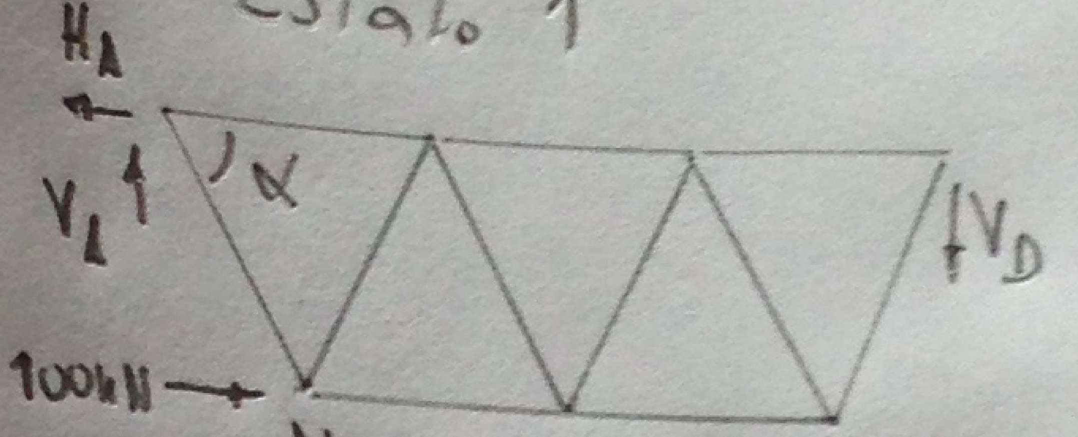
$q + \frac{11{,}3906}{q} \leqslant 19{,}0516$

$q^2 - 19{,}0516q + 11{,}3906 = 0 \Leftrightarrow q = \frac{19{,}0516 \pm \sqrt{19{,}0516^2 - 4 \times 11{,}3906}}{2}$

$\Rightarrow \boxed{q_{máx} = 18{,}44\,kN/m \rightarrow M_{máx} = 39{,}61\,kNm}$

Solución Examen Julio de 2016

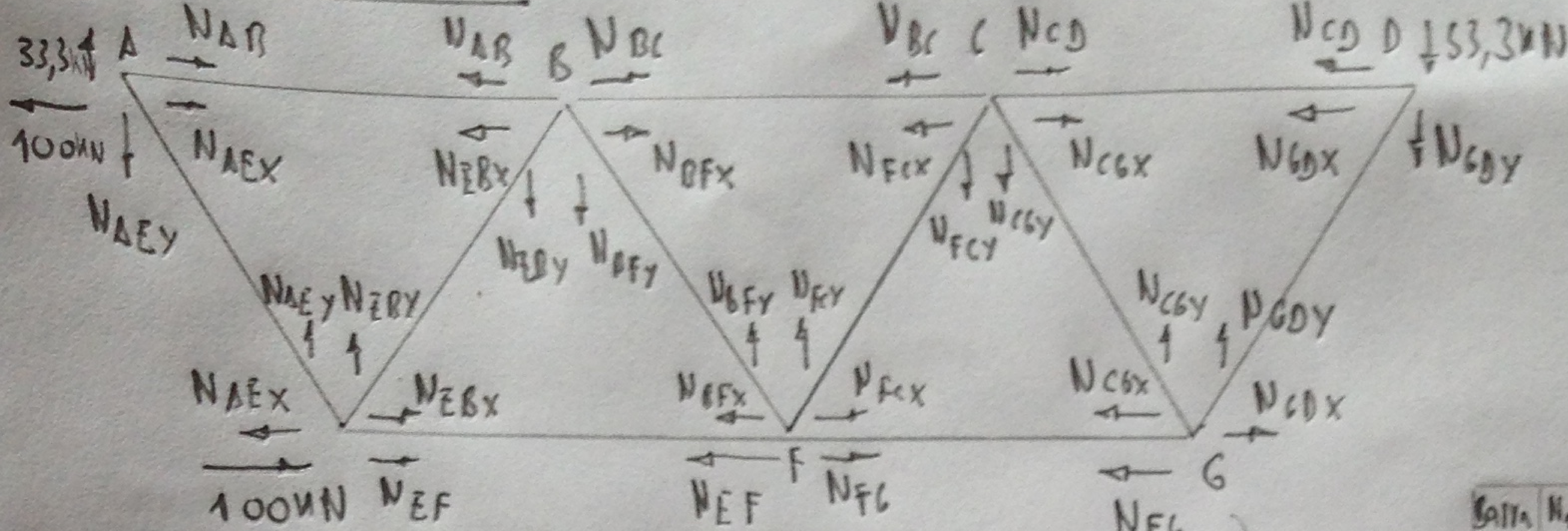
a) Estado 1



$tg\alpha = 2$

$\Sigma M_A = 0 \rightarrow 100 \cdot 1 - V_D \cdot 3 = 0 \rightarrow V_D = 33,3\,kN$

$\Sigma Vert = 0 \rightarrow \boxed{V_A = 33,3\,kN}$; $\Sigma hor = 0 \rightarrow \boxed{H_A = 100\,kN}$



Nudo A: $\boxed{N_{AEY} = 33,3\,kN} \rightarrow \boxed{N_{AEX} = 16,7\,kN}$ $\frac{1}{tg\alpha} = \frac{1}{2}$

$\boxed{N_{AB} = 83,3\,kN}$

E: $N_{EBy} = -33,3\,kN \rightarrow N_{EBx} = 16,7\,kN$, $N_{EF} = -66,6\,kN$

B: $N_{BFy} = 33,3\,kN \rightarrow N_{BFx} = 16,7\,kN$; $N_{BC} = 50\,kN$

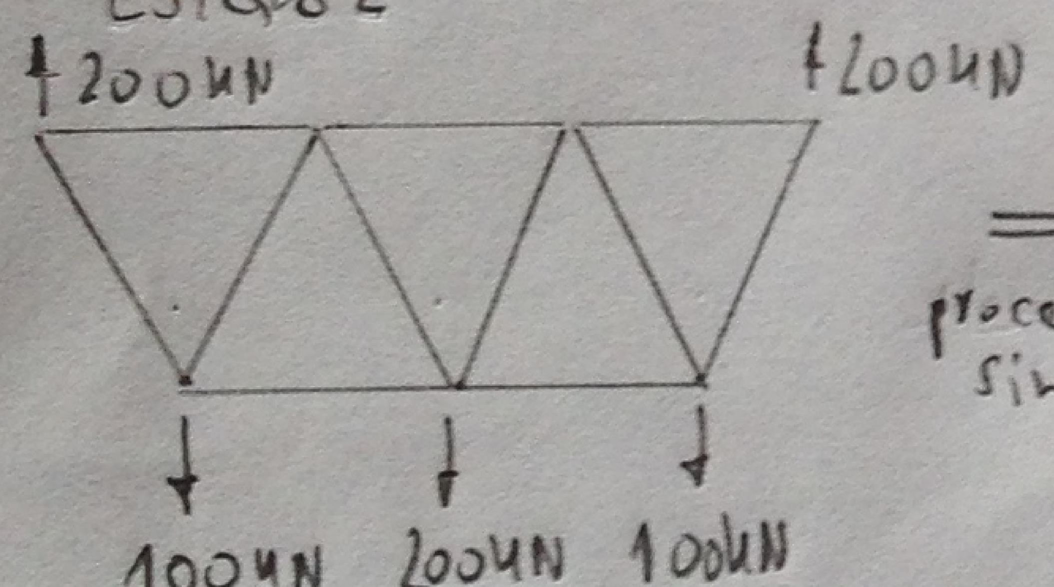
F: $N_{FCy} = -33,3\,kN \rightarrow N_{FCx} = -16,7\,kN$; $N_{FG} = -33,3\,kN$

C: $N_{CGy} = 33,3\,kN \rightarrow N_{CGx} = 16,7\,kN$; $N_{CD} = 16,7\,kN$

G: $N_{GDy} = -33,3\,kN \rightarrow N_{GDx} = 16,7\,kN$

Barra	N_{1i} (/100kN)
AB	+0,833
BC	+0,5
CD	+0,167
AE	0,373
BE	-0,373
BF	+0,373
CF	-0,373
CG	+0,373
DG	-0,373
EF	-0,666
FG	-0,333

Estado 2



$\Rightarrow$ procedimiento similar E1

Barra	N_{2i} (kN)
AB	-100
BC	-200
CD	-100
AE	+223
BE	-112
BF	+112
CF	+112
CG	-112
DG	+223
EF	+150
FG	+150

Como Estado original = E1 + E2 $\Rightarrow$

Barra	N_i (kN)
AB	-16,7
BC	-150,0
CD	-83,3
AE	260,3
BE	-149,3
BF	+149,3
CF	74,7
CG	-74,7
DG	185,7
EF	83,4
FG	116,7

BC: máx. compresión
AE: máx. tracción

b) Secc. circular $\Rightarrow d \geq \sqrt{\frac{260,3 \cdot 4}{\pi \cdot 140 \times 10^3}}\,m \Rightarrow \boxed{d = 4,9\,cm}$

Secc. cuadrada $\Rightarrow l \geq \sqrt{\frac{150}{140 \times 10^3}}\,m \rightarrow \boxed{l = 3,3\,cm}$

c) Utilizando Tma Castigliano y los Estados antes hallados:

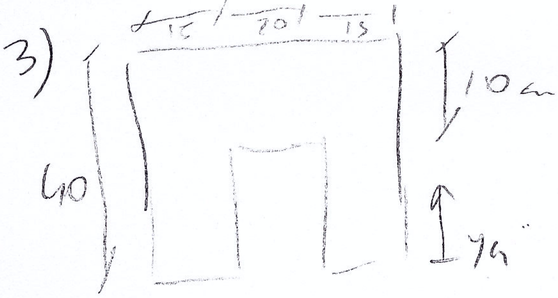
$\frac{\partial U}{\partial P} = \delta_{hor\,E}$; donde $P = 100\,kN$ y $N_i = N_{2i} + N_{1i} \cdot P$

entonces $\left.\frac{\partial U}{\partial P}\right|_{P=100kN} = \sum_{i=1}^{11} \frac{L_i N_{1i}}{E A_i} \underbrace{(N_{2i} + N_{1i} \cdot P)}_{N_i}$

Barra	A_i ($\times 10^{-3}\,m^2$)	L_i (m)	$L_i N_{1i}/EA_i$ ($\times 10^{-5}$ m/kN)	Π ($\times 10^{-5}$ m)
AB	1,09	2	7,28	-121,58
BC	1,09	2	4,37	-655,5
CD	1,09	2	1,46	-121,62
AE	1,89	2,24	2,11	549,23
BE	1,09	2,24	-3,65	544,95
BF	1,89	2,24	2,11	315,02
CF	1,89	2,24	-2,11	-157,62
CG	1,09	2,24	3,65	-272,66
DG	1,89	2,24	-2,11	-391,83
EF	1,89	2	-3,36	-280,22
FG	1,89	2	-1,66	-193,72

$\Rightarrow \delta_{hor\,E} = 7,86 \times 10^{-1}\,mm$ hacia la izquierda

3)



$$y_G = \frac{2\times15\times40\times20 + 20\times10\times35}{2\times15\times40 + 20\times10} = 22{,}1\,cm$$

$$I_x = \left(\frac{15\times40^3}{12} + 15\times40(22{,}1-20)^2\right).2 +$$

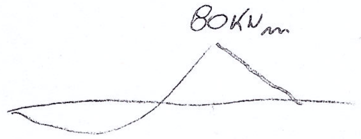
$$\frac{20\times10^3}{12} + 20\times10(22{,}1-35)^2 = 200.241\,cm^4$$

$$W = \frac{I}{y}$$

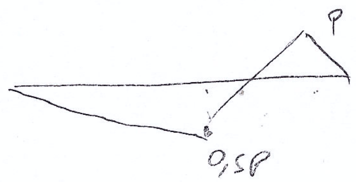
$$W_{inf} = \frac{200.241}{22{,}1} = 9060{,}7\,cm^3$$

$$W_{sup} = \frac{200.241}{(40-22{,}1)} =$$

$$M_{max} = 10MPa \,.\, 9060{,}7\,cm^3 = 90{,}6\,KN\,m$$



$9R_A$



$$R_C = \frac{5}{4}P$$

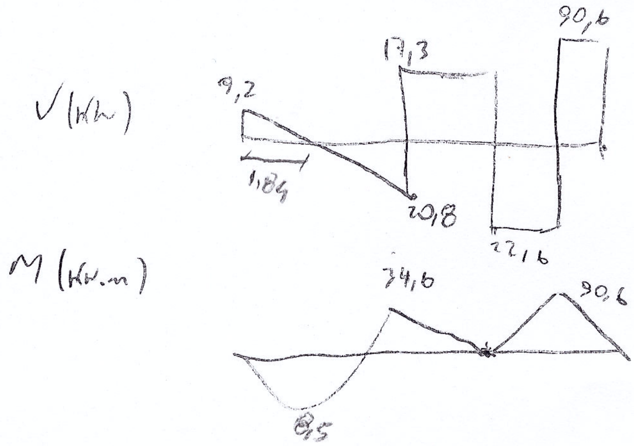
$$R_A = \frac{P}{12}$$

En B

$$80 - 0{,}5P \leq 90{,}6\,KN\,m$$

$$P \geq -21{,}2\,KN$$

En D $\quad P \leq 90{,}6\,KN$



$$\tau = \frac{90{,}6\times10\times50\times12{,}9}{200\,241\times30} = 97\,N/cm^2 = 0{,}97\,MPa$$