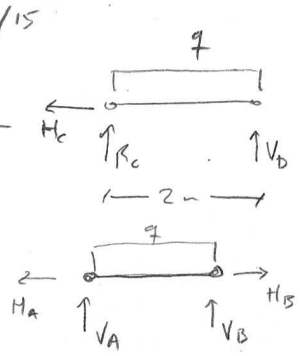
R1 18/2/15

EJ 1

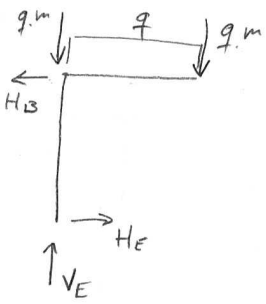


$R_C = V_D = \frac{q \cdot 2m}{2} = q \cdot m$

$H_C = 0$

$V_A = V_B = \frac{q \cdot 2m}{2} = q \cdot m$

$H_A = H_B$



$V_E = 2\,qm + qm = 3qm$

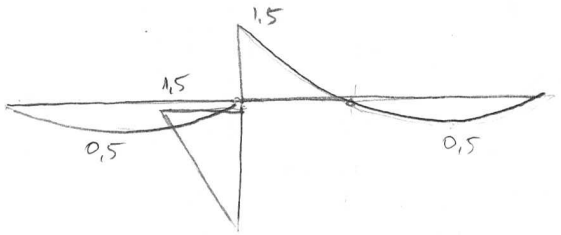
$\Sigma M_E = 0$:

$\frac{q m^2}{2} + q m^2 - H_B \cdot m = 0$

$H_B = 3/2 \cdot q m$

$H_E = 3/2 \cdot q m$

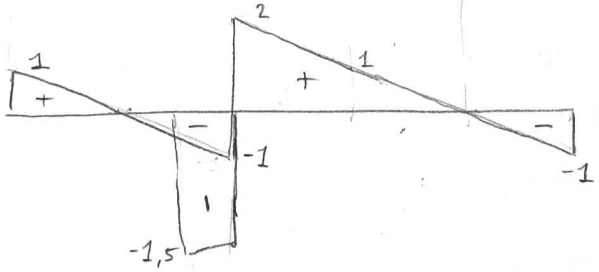
M [qm^2]



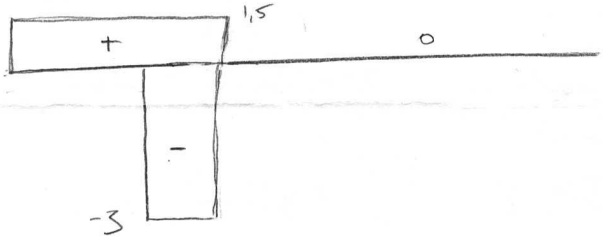
$M_B = 3/2\ q\ m^2$

$M_{CD} = \frac{q(2m)^2}{8} = 1/2\ q\ m^2 = M_{AB}$

V [qm]



N [$q.m$]



b) Dimensiones en el nudo B

$M = 1,5\ q\ m^2 \Rightarrow M = 15\ KN\,m$

$N = -3\ q.m \Rightarrow N = -30\ KN$

Aprox: $\frac{15\ KN\,m}{W} < 140\ MPa$

$\Rightarrow W > \frac{15\ KN\,m}{140\ MN/m^2} = 0,107 \times 10^{-3}\ m^3$

$W > 107\ cm^3 \Rightarrow$ 2 PNC 12

Verif.:

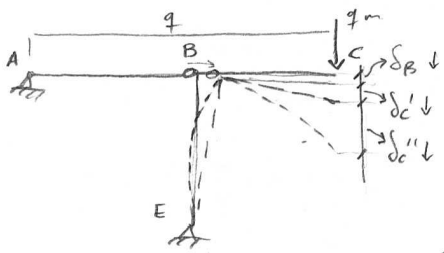
$\frac{15\ KN\,m}{121,4\ cm^3} + \frac{30\ KN}{34\ cm^2} = 123,6\ MPa + 8,8\ MPa$

$132,4\ MPa \leq 140\ MPa$ ✓

$\Rightarrow$ uso 2 PNC 12

↳ $A = 34\ cm^2$ // $I = 728\ cm^4$

c)



desp. B: $\vec{\delta_B} = \frac{N \cdot L}{AE} = \frac{1,5\,q\,m \cdot 2m}{AE} = \frac{3\,q\,m^2}{AE}$

$\delta_B \downarrow = \frac{N_{BE} \cdot L}{AE} = \frac{3\,q\,m \cdot 1m}{AE} = \frac{3\,q\,m^2}{AE} = \frac{3 \cdot 10^3\ N/m \cdot m^2}{34 \cdot 10^{-4}\ m^2 \cdot 210 \times 10^9\ N/m^2} = 4,2 \times 10^{-5}\ m$

giro x desp B: $\vec{\theta_B'} = \frac{\vec{\delta_B}}{1m} = \frac{3\,q\,m^2}{AE \cdot m} = \frac{3\,q\,m}{AE} \Rightarrow \vec{\theta_B'} = \frac{3 \cdot 10^3\ N/m \cdot m}{34 \cdot 10^{-4}\ m^2 \cdot 210 \times 10^9\ N/m^2}$

$\vec{\theta_B} = 4,2 \times 10^{-5}$

giro x acciones en BE: $\vec{\theta_B''} = \frac{1,5\,q\,m^2 \cdot 1m}{3 \cdot EI} = \frac{q\,m^3}{2\,EI} \Rightarrow \vec{\theta_B''} = \frac{10 \cdot 10^3\ N/m \cdot m^3}{2 \cdot 210 \times 10^9\ N/m^2 \cdot 728 \cdot 10^{-8}\ m^4}$

$\vec{\theta_B''} = 0,00327$

$\Rightarrow$ giro en B: $\vec{\theta_B} = \vec{\theta_B'} + \vec{\theta_B''} = 0,00327 + 0,000004 \Rightarrow \vec{\theta_B} = 0,00331$

descenso C x giro B: $\delta_C' \downarrow = \theta_B \cdot 1m \Rightarrow \delta_C' \downarrow = 0,00331\ m$

descenso C x acciones BC: $\delta_C'' \downarrow = \frac{q\,(1m)^4}{8EI} + \frac{q\,m \cdot (1m)^3}{3EI} = \frac{11}{24}\frac{q\,m^4}{EI} = \frac{11 \cdot 10^3\ N/m \cdot m^4}{24 \cdot 210 \cdot 10^9\ N/m^2 \cdot 728 \cdot 10^{-8}\ m^4} = 0,00300\ m$

$\Rightarrow \delta_C \downarrow = \delta_B \downarrow + \delta_C' \downarrow + \delta_C'' = 0,000042 + 0,00331 + 0,00300 = 0,00635\ m \downarrow$

$\delta_C \downarrow = 6,35\ mm$

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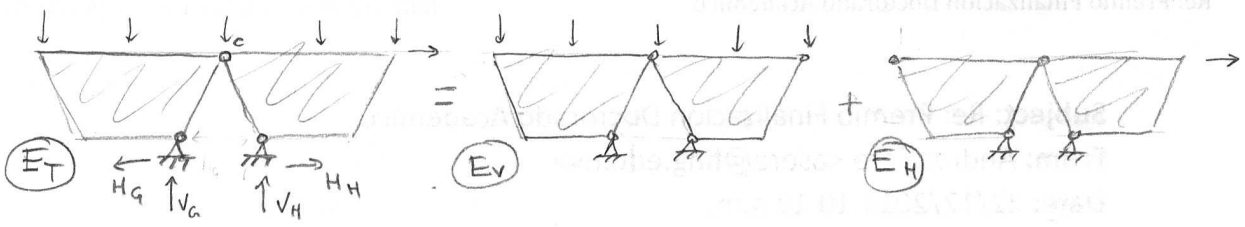
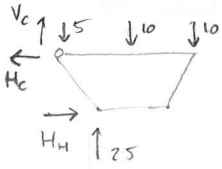
Ej. 2



E_V $\Sigma M_G = 0 \Rightarrow 10\,kN \cdot 5 \cdot 1m - V_H \cdot 2m = 0$

$\Rightarrow V_H = 25\,kN$

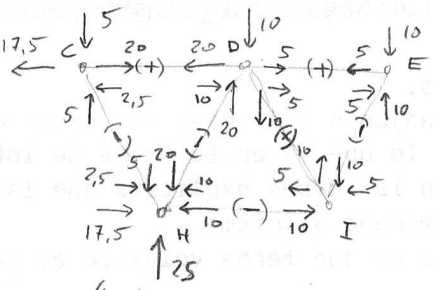
$\Sigma M_c^{der} = 0 \Rightarrow 20kN \cdot 3m - 25kN \cdot 1m - H_H \cdot 2m = 0$

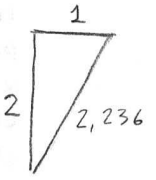


$H_H = 17,5\,kN$

$H_C = 17,5\,kN$

$V_C = 0\,kN$

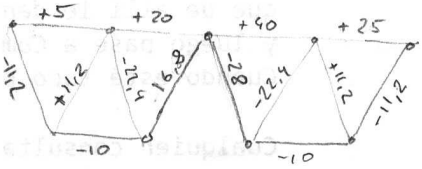




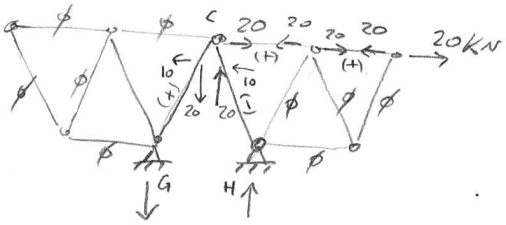
Fuerzas (kN)

E_V: CD = CB = +20
DE = BA = +5
CH = CG = -5,6
HD = GB = -22,4
DI = BF = +11,2
HI = FG = -10
EI = AF = -11,2

$E_T = E_V + E_H$



E_H



E_H: DE = +20
CD = +20
CH = -22,4
GC = 22,4
resto: 0

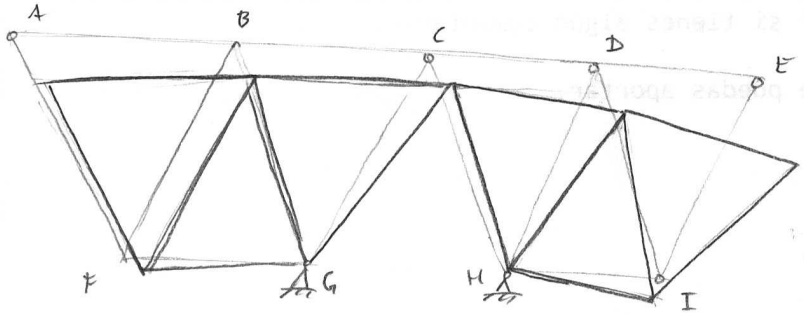
b) tracción max: CD = 40 kN $\Rightarrow A \geq N/\sigma = 40 \cdot 10^3 N / 140 \cdot 10^6 N/m^2 = 2,86\,cm^2$

comp. max: CH = -28 kN

$A = r^2 \pi \Rightarrow r = \sqrt{\frac{2,86\,cm^2}{\pi}} = 0,95\,cm$

$\Rightarrow A = 2,0\,cm^2 \Rightarrow l = 1,4\,cm$ □ le

c)

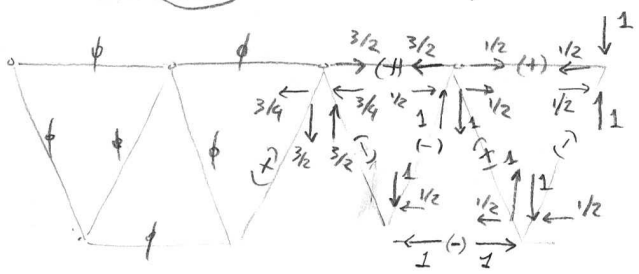


punto de mayor descenso: E

Barra	L (m)	A (cm²)	N_{pi} (kN)	N_{1i}	π
GC	2,24	2,86	16,8	1,68	22,1
CD	2	2,86	40	1,5	42,0
DE	2	2,86	25	0,5	8,7
CH	2,24	2	-28	-1,68	52,7
HD	2,24	2	-22,4	-1,12	56,2
DI	2,24	2,86	11,2	1,12	19,6
HI	2	2	-10	-1	10
IE	2,24	2	-11,2	-1,12	28,1

× castigliano

estado E_{1i}



$\Rightarrow u_{E(\downarrow)} = \Sigma \pi = \frac{187,5}{E} \frac{KN \cdot m}{cm^2}$

$u_{E(\downarrow)} = \frac{187,5 \cdot 10^3 \cancel{N}\,m}{210 \cdot 10^9 \cancel{N/m^2} \cdot 10^{-4}\,m^2} = 0,00893$

$u_{E(\downarrow)} = 8,93\,mm$

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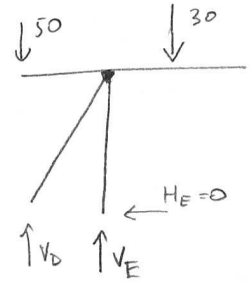
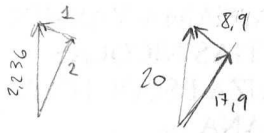
Ej. 3

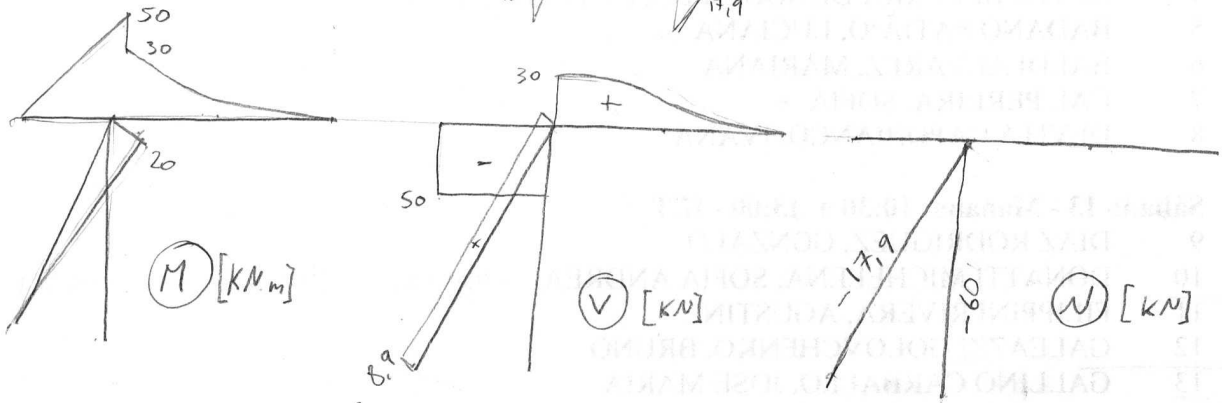
a)



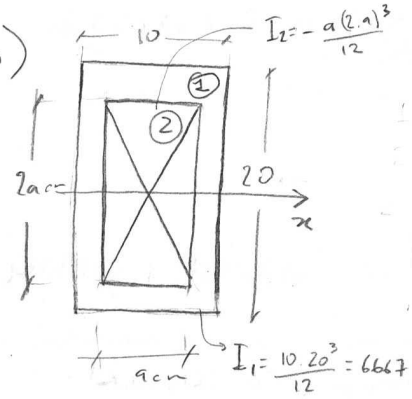
$\Sigma M_D = 0 \Rightarrow 30 \cdot 2 = V_E \cdot 1 \Rightarrow \underline{V_E = 60\,kN}$

$\Sigma V = 0 \Rightarrow V_D + 60 - 50 - 30 = 0 \Rightarrow \underline{V_D = 20\,kN}$





b)



$A = 200\,cm^2 - 2a^2$

$I = 6667\,cm^4 - \frac{2}{3}a^4$

Verifico σ_{adm} en:

I) Max M: B de barra AB
↳ M = 50 kN.m

II) Max N: Barra EB
↳ N = -60 kN

III) comb. M,N: B de barra DB
↳ M = 20 kNm, N = -17,9 kN

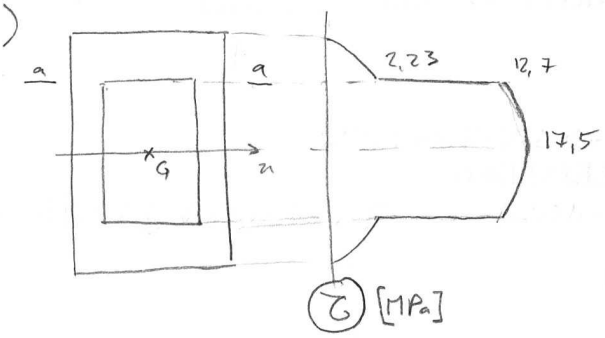
I) $\frac{5000\,kN/cm \cdot 10\,cm}{6667\,cm^4 - 2/3\,a^4\,cm^4} \leq 14\,kN/cm^2 \Rightarrow 2/3\,a^4 \leq 3096 \Rightarrow \underline{a \leq 8{,}25}$

II) $\frac{60\,kN}{200\,cm^2 - 2a^2\,cm^2} = 0{,}94\,kN/cm^2 \leq 14\,kN/cm^2$ ✓
(con $a = 8{,}27$)

III) $\frac{2000\,kN/cm \cdot 10\,cm}{6667\,cm^4 - 2/3\,(8{,}25)^4\,cm^4} + \frac{17{,}9\,kN}{63{,}9} = 5{,}58 + 0{,}28 = 5{,}87\,kN/cm^2 \leq 14{,}0\,kN/cm^2$ ✓

$\Rightarrow$ toma $\underline{a \leq 8{,}25\,cm}$

c)



$I = 3578\,cm^4$

$\mu_a = (10^2 - 8{,}25^2) \cdot 10/2 = 159{,}7\,cm^3$

$q_a = \frac{50\,kN \cdot 159{,}7\,cm^3}{3578\,cm^4} = 2{,}23\,kN/cm$

$\sigma_a^{sup} = \frac{2{,}23\,kN/cm}{10\,cm} = 0{,}223\,kN/cm^2 = \underline{2{,}23\,MPa}$

$\sigma_a^{inf} = \frac{2{,}23}{1{,}75} = 1{,}27\,kN/cm^2 = \underline{12{,}7\,MPa}$

$\mu_G = \mu_a + \frac{8{,}25\,cm^2 \cdot (10 - 8{,}25)\,cm}{2} = 159{,}7 + 59{,}6 = 219{,}3\,cm^3$

$\sigma_G = \frac{50\,kN \cdot 219{,}3\,cm^3}{3578\,cm^4 \cdot 1{,}75\,cm} = 1{,}75\,kN/cm^2 = 17{,}5\,MPa$