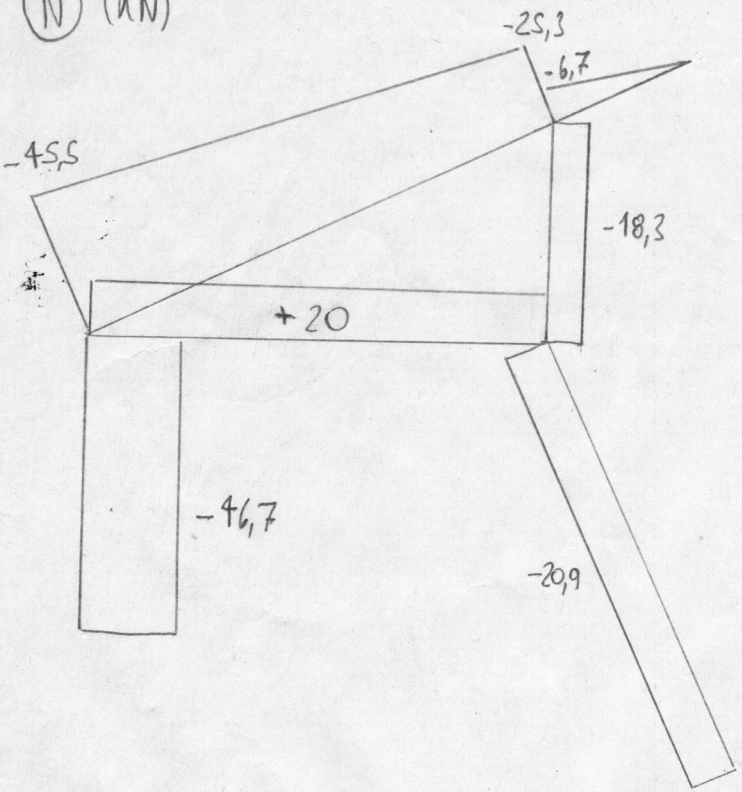
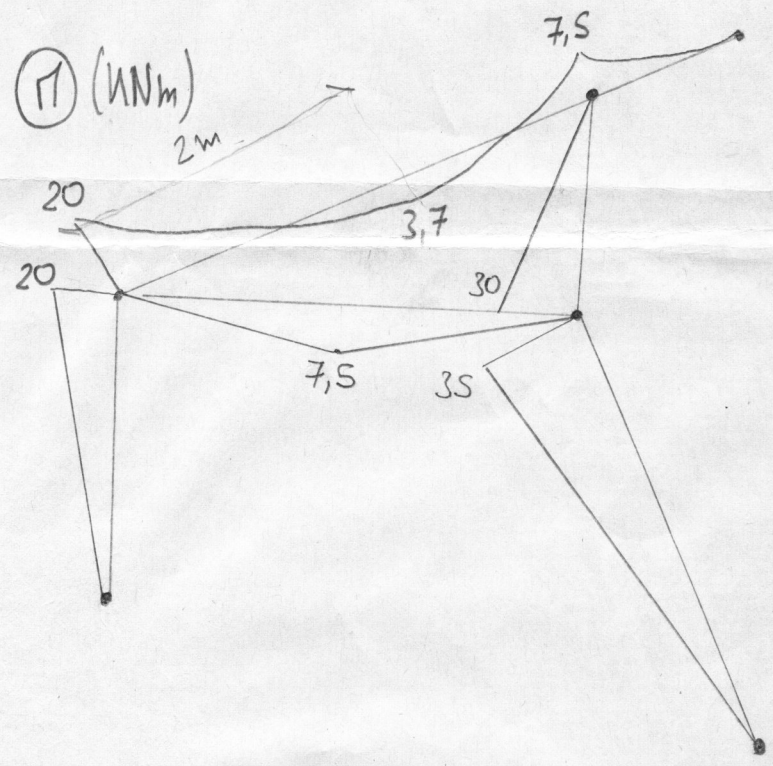


Examen Viernes 13 de Diciembre de 2013 Ejercicio 1

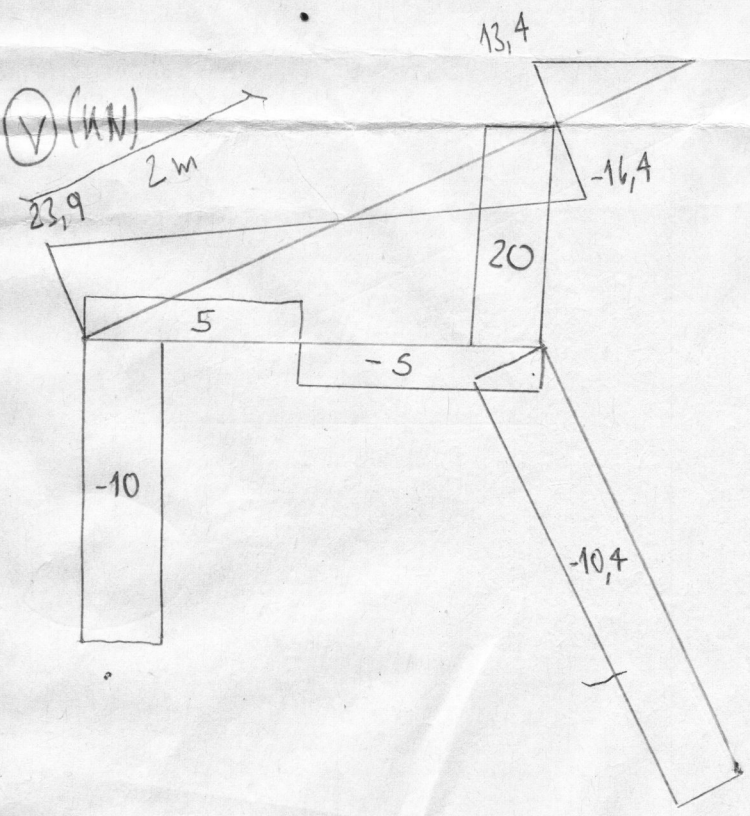
(N) (kN)

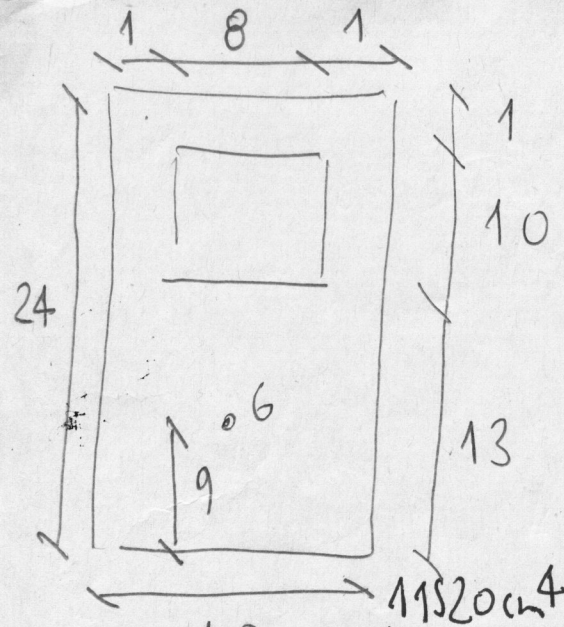


(M) (kNm)



(V) (kN)





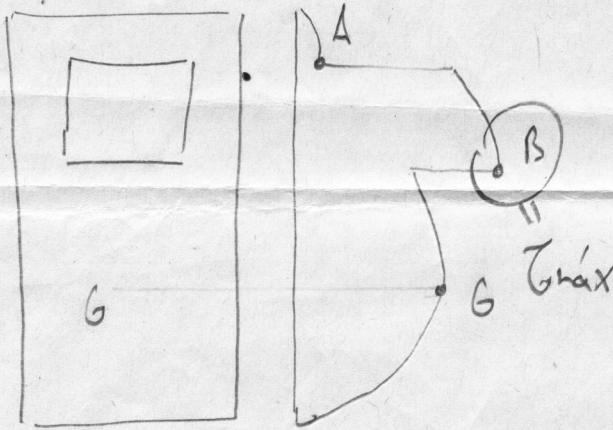
$$A' = 24 \cdot 10 - 8 \cdot 10 = 160 \text{ cm}^2$$

$$y_G = \frac{1}{A'} (24 \cdot 10 \cdot 12 - 8 \cdot 10 \cdot 18)$$

$$y_G = 9 \text{ cm}$$

$$I_x = \frac{10 \cdot 24^3}{12} + \frac{2460 \text{ cm}^4}{12} - \frac{666,67 \text{ cm}^4}{12} - \frac{6480 \text{ cm}^4}{12}$$

$$I_x = 6533,33 \text{ cm}^4$$



$$M_A = 10 \cdot 1 \cdot 14,5 = 145 \text{ cm}^3$$

$$M_B = 10 \cdot 13 \times 2,5 = 325 \text{ cm}^3$$

$$M_G = 9 \cdot 10 \cdot 4,5 = 405 \text{ cm}^3$$

$$M_A / 10 = 14,5 \text{ cm}^2$$

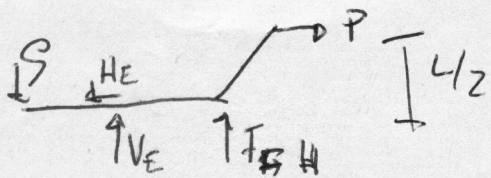
$$M_A / 2 = 72,5 \text{ cm}^2$$

$$M_B / 2 = 162,5 \text{ cm}^2$$

$$M_B / 10 = 32,5 \text{ cm}^2$$

$$M_G / 10 = 40,5 \text{ cm}^2$$

$$\tau_{\max} = \frac{23,9 \text{ kN}}{6533,33 \text{ cm}^4} \cdot 162,5 \text{ cm}^2 = \underline{\underline{5,94 \text{ kPa}}}$$

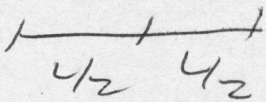


$$H_E = P \quad \sum M_E = 0 \quad F_{HF} \cdot \frac{L}{2} + \frac{PL}{2} - \frac{PL}{2} = 0$$

$$F_{HF} = P - Q$$

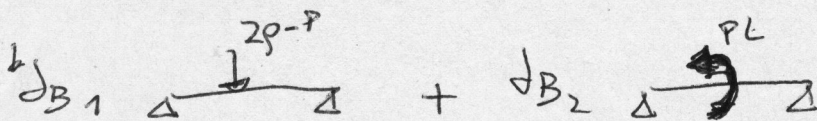
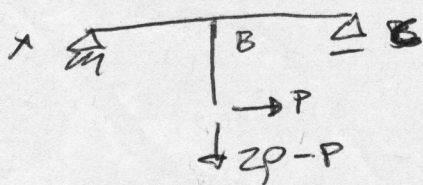
$$V_E + F_{HF} = P \rightarrow V_E = P - F_{HF} = P - P + P = P - P$$

$$V_E = 2P - P$$



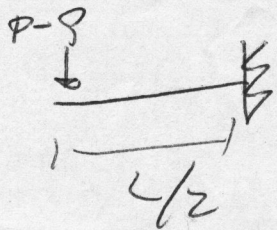
$$\delta_E^L = \delta_B^L$$

$$\delta_F^d = \delta_H^d$$



$$\delta_B = \frac{(2P-P)L^3}{48EI}$$

$$\delta_H^d = \frac{(P-Q)(L/2)^3}{3EI}$$

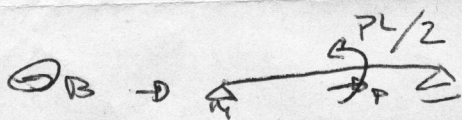


$$\frac{(2P-P)L^3}{48EI} = \frac{(P-Q)L^3}{8 \times 3 \times EI} \rightarrow 2P-P = 2P-2Q$$

$$4P = 3P$$

$$\delta_E = \delta_E^1 + \delta_B \cdot L/2$$

$$\delta_E^1 = \frac{P(L/2)^3}{3EI} = \frac{PL^3}{24EI}$$



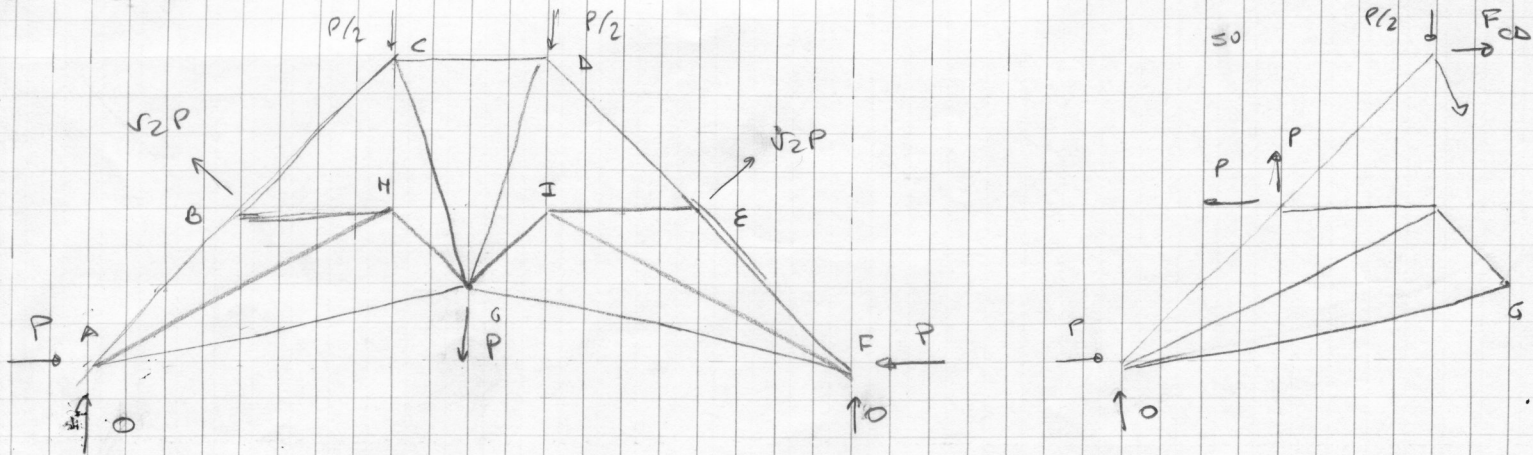
$$V' = -\frac{M_0}{24EI} (L^2 - 12x^2)$$

$$x = L/2$$

$$V'(L/2) = \frac{PL/2}{24EI} (L^2 - \frac{12L^2}{4})$$

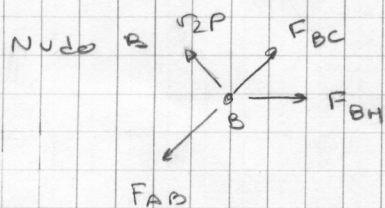
$$\delta_B = -\frac{P}{24EI} \frac{1}{2} (-2L^2) = \frac{PL^2}{24EI}$$

$$\delta_E = \frac{PL^3}{24EI} + \frac{PL^2}{24EI} \cdot \frac{L}{2} = \frac{PL^3}{18EI}$$

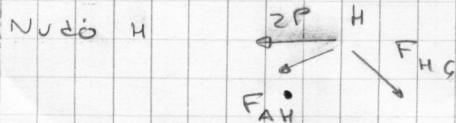


Secciones por el medio $\Rightarrow M_G = 0 \Rightarrow P/2 \cdot 0,5 - P \cdot 1,5 + P \cdot 0,5 + P \cdot 0,5 = F_{CD} \cdot 1,5$

$$F_{CD} = -0,167$$



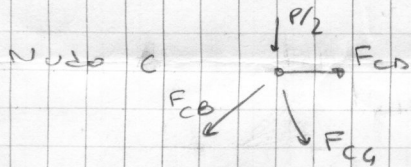
$$\sqrt{2}P = \frac{F_{BH}}{\sqrt{2}} \Rightarrow F_{BH} = 2P = F_{IE}$$



$$\begin{cases} 2P + F_{AH} \frac{2}{\sqrt{3}} = \frac{F_{HG}}{\sqrt{2}} \\ F_{AH} \frac{1}{\sqrt{3}} + \frac{F_{HG}}{\sqrt{2}} = 0 \end{cases} \Rightarrow \begin{cases} 2P + \frac{2}{\sqrt{3}} F_{AH} = -\frac{1}{\sqrt{3}} F_{AH} \\ \frac{3}{\sqrt{3}} F_{AH} = -2P \end{cases}$$

$$F_{AH} = -1,49P = F_{IF}$$

$$\Rightarrow F_{HG} = -\frac{\sqrt{2}}{\sqrt{3}} F_{AH} \Rightarrow F_{HG} = 0,943P = F_{FI}$$

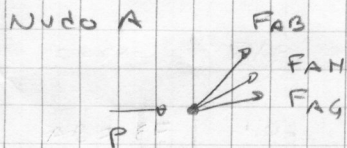


$$\begin{cases} \frac{F_{CB}}{\sqrt{2}} = F_{CD} + F_{CG} \frac{1/2}{\sqrt{10/2}} \\ \frac{F_{CB}}{\sqrt{2}} + F_{CG} \frac{3/2}{\sqrt{10/2}} + \frac{P}{2} = 0 \end{cases} \Rightarrow \begin{cases} -\frac{3}{\sqrt{10}} F_{CG} - \frac{P}{2} = -0,167P + \frac{F_{CG}}{\sqrt{10}} \\ (0,167 - \frac{1}{2})P = \frac{4}{\sqrt{10}} F_{CG} \end{cases}$$

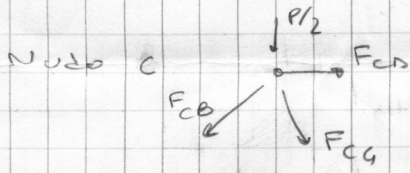
$$F_{CG} = -0,264P = F_{DG}$$

$$\frac{F_{CB}}{\sqrt{2}} = -\frac{P}{2} - \frac{3}{\sqrt{10}} F_{CG} = \left(-\frac{1}{2} + \frac{3}{\sqrt{10}} \cdot 0,264\right)P \Rightarrow F_{CB} = -0,354P = F_{ED}$$

$$\text{Nudo B} \Rightarrow F_{AB} = F_{BC} + \frac{F_{BH}}{\sqrt{2}} = -0,354P + \frac{2P}{\sqrt{2}} \Rightarrow F_{AB} = 1,06P = F_{EF}$$



$$\frac{F_{AB}}{\sqrt{2}} + F_{AH} \frac{1}{\sqrt{3}} + F_{AG} \frac{1/2}{\sqrt{26/2}} = 0 \Rightarrow F_{AG} = \sqrt{26} \left(-\frac{1,06}{\sqrt{2}} + \frac{1,49}{\sqrt{3}}\right)P$$

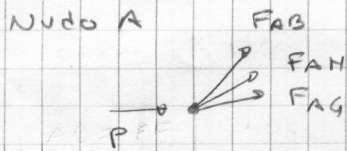


$$\left. \begin{aligned} \frac{F_{CB}}{\sqrt{2}} &= F_{CD} + F_{CG} \frac{1/2}{\sqrt{10/2}} \\ \frac{F_{CB}}{\sqrt{2}} + F_{CG} \frac{3/2}{\sqrt{10/2}} + \frac{P}{2} &= 0 \end{aligned} \right\} \begin{aligned} -\frac{3}{\sqrt{10}} F_{CG} - \frac{P}{2} &= -0,167 P + \frac{F_{CG}}{\sqrt{10}} \\ (0,167 - \frac{1}{2}) P &= \frac{4}{\sqrt{10}} F_{CG} \end{aligned}$$

$$F_{CG} = -0,264 P = F_{DG}$$

$$\frac{F_{CB}}{\sqrt{2}} = -\frac{P}{2} - \frac{3}{\sqrt{10}} F_{CG} = \left(-\frac{1}{2} + \frac{3}{\sqrt{10}} 0,264\right) P \Rightarrow F_{CB} = -0,354 P = F_{ED}$$

Nudo B $\Rightarrow F_{AB} = F_{BC} + \frac{F_{BH}}{\sqrt{2}} = -0,354 P + \frac{2P}{\sqrt{2}} \Rightarrow F_{AB} = 1,06 P = F_{EF}$



$$\frac{F_{AB}}{\sqrt{2}} + F_{AH} \frac{1}{\sqrt{5}} + F_{AG} \frac{1/2}{\sqrt{26/2}} = 0 \Rightarrow F_{AG} = \sqrt{26} \left(-\frac{1,06}{\sqrt{2}} + \frac{1,49}{\sqrt{5}}\right) P$$

$$F_{AG} = -0,425 P = F_{FG}$$

Barra	F/P	L	EA ΔL
AB = EF	1,06	$\sqrt{2}$	1,5
BC = DE	-0,354	$\sqrt{2}$	-0,5
CD	-0,167	1	-0,17
BH = EH	2	1	2
AH = FI	-1,49	$\sqrt{5}$	-3,33
AG = FG	-0,425	$\sqrt{26/2}$	-1,08
HG = IG	0,943	$0,5\sqrt{2}$	0,67
CG = DG	-0,264	$\sqrt{10/2}$	-0,42

} ΔL_{AC} = 1

$$\Delta L = \frac{NL}{EA}$$

Orden puntos Williot = C - D - G - A \Rightarrow

$$\begin{aligned} \uparrow \delta_C &= 3,15 P/EA & \uparrow \delta_D &= 3,15 P/EA \\ \delta_C &= 1,73 P/EA & \delta_D &= 1,9 P/EA \end{aligned}$$