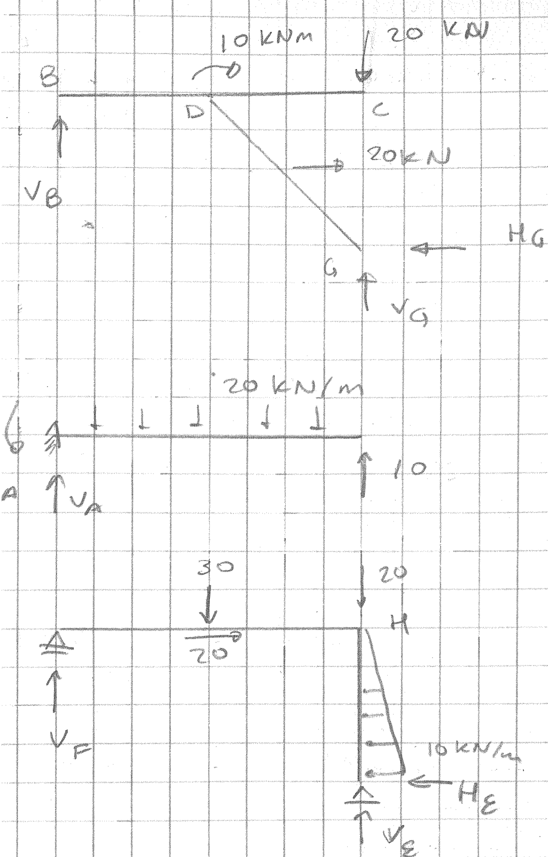
EJERCICIO 1



a) $M_G = V_B\, 2 + 10 + 20 \times 0{,}5 = 0 \Rightarrow V_B = -10$ KN

$V_G = 20 - V_B = 20 + 10 = 30$ KN

$H_G = 20$ KN KN

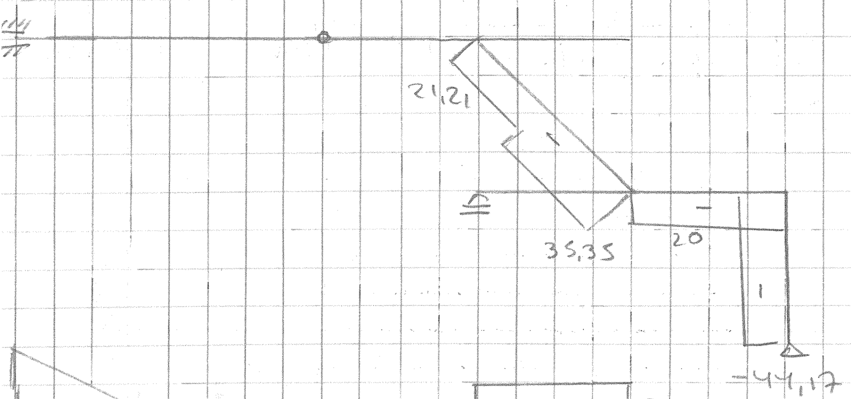
$V_A = 20 \times 2 - 10 = 30$ KN

$M_A = \frac{20 \times 2^2}{2} - 10 \times 2 = 20$ KNm

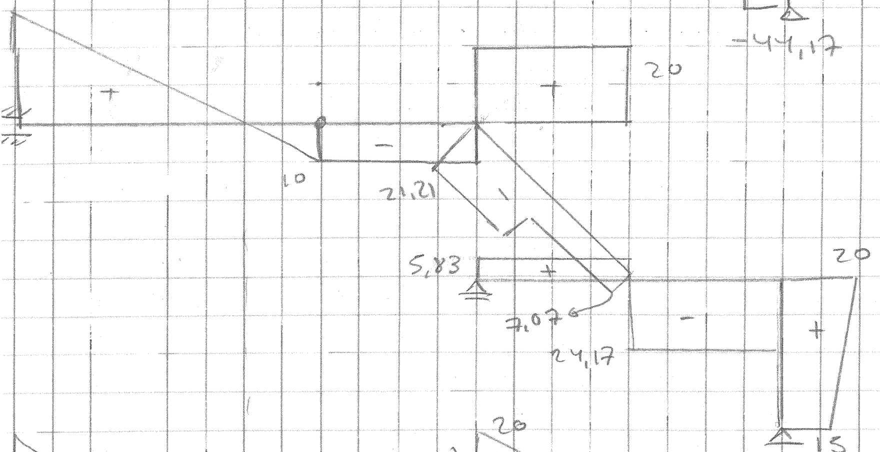
$H_E = 20 - \frac{10 \times 1}{2} = 15$ KN

$M_E = V_F \times 2 + 20 \times 1 - 30 \times 1 - \frac{10 \times 1}{2} \times \frac{1}{3} \Rightarrow V_F = 5{,}83$

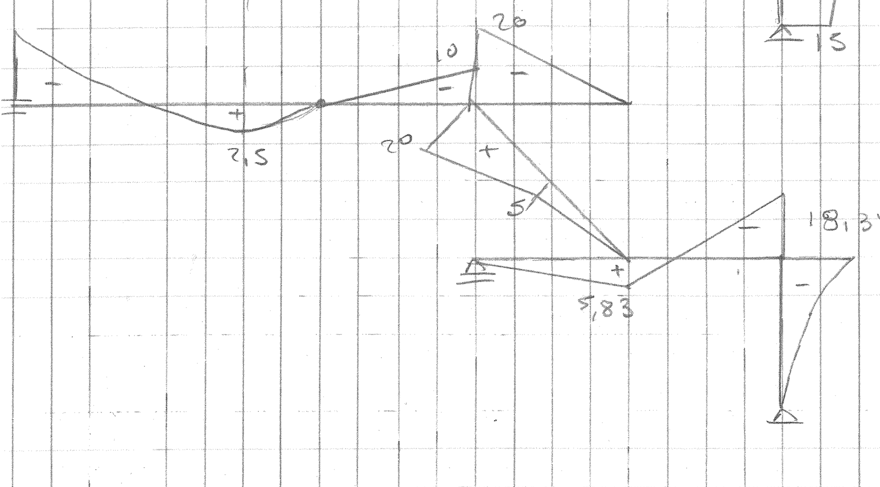
$V_E = 20 + 30 - V_F = 44{,}17$



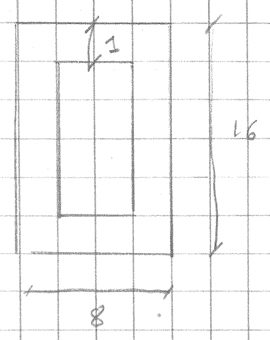
N



V



M



$A = 16 \times 8 - 14 \times 6 = 44\,cm^2$

$W = \left[\frac{8 \times 16^3}{12} - \frac{6 \times 14^3}{12} \right) \frac{1}{8} = 169{,}83\,cm^3$

Sección A y D derecha =

$\left.\begin{array}{l} M = 20\,KNm = 2000\,KNcm \\ N = 0 \end{array}\right\} \Rightarrow \sigma^T = \sigma^C = \frac{2000}{169{,}83} = 11{,}77\,KN/cm^2$

$\sigma^T = \sigma^C = 117{,}7\,MPa$

Sección D inclinada

$\left.\begin{array}{l} M = 20\,KNm \\ N = -21{,}21\,KN \end{array}\right\} \Rightarrow \sigma^T = \frac{2000}{169{,}83} - \frac{21{,}21}{44} = 11{,}29\,KN/cm^2 = 112{,}9\,MPa$

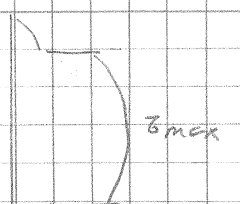
$\sigma^C = \frac{2000}{169{,}83} + \frac{21{,}21}{44} = 12{,}26\,KN/cm^2 = 122{,}6\,MPa$

Sección H derecha

$\left.\begin{array}{l} M = 18{,}34\,KNm \\ N = -44{,}17\,KN \end{array}\right\} \Rightarrow \sigma^T = \frac{1834}{169{,}83} - \frac{44{,}17}{44} = 9{,}79\,KN/cm^2 = 97{,}9\,MPa$

$\sigma^C = \frac{1834}{169{,}83} + \frac{44{,}17}{44} = 11{,}80\,KN/cm^2 = 118{,}0\,MPa$

$\Rightarrow \begin{cases} \sigma^T_{max} = 117{,}7 \text{ e A y D derecha arriba} \\ \sigma^C_{max} = 122{,}6 \text{ e D inclinada arriba} \end{cases}$



$I = \left[\frac{8 \times 16^3}{12} - \frac{6 \times 14^3}{12} \right) = 1358{,}67\,cm^4$

$\mu_{max} = 6 \times 1 \times 7{,}5 + 8 \times 2 \times 4 = 109\,cm^3$

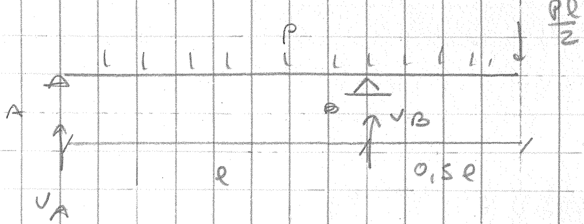
$\mu_{11} = 8 \times 1 \times 7{,}5 = 60\,cm^3$

$V = 30\,KN \Rightarrow \tau_{max} = \frac{30 \times 109}{1358{,}67 \times 2} = 1{,}2\,KN/cm^2 = 12\,MPa$

$\tau_1 = \frac{30 \times 60}{1358{,}67 \times 8} = 0{,}16\,KN/cm^2 = 1{,}6\,MPa$

$\tau_2 = \frac{30 \times 60}{1358{,}67 \times 2} = 0{,}66\,KN/cm^2 = 6{,}6\,MPa$

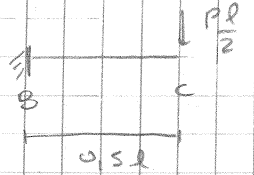
EJERCICIO 2



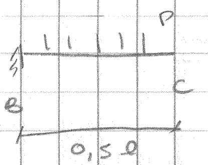
Sup. B no baja

$$V_B = 1{,}5\,pl\;0{,}75 + \frac{pl}{2}\,1{,}5 = \frac{15}{8}\,pl = 1{,}875\,pl$$

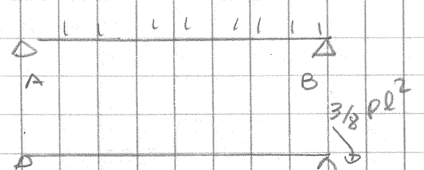
$$V_A = 1{,}5\,pl + 0{,}5\,pl - \frac{15}{8}\,pl = \frac{1}{8}\,pl = 0{,}125\,pl$$



$$\downarrow \delta_C^1 = \frac{pl/2\;(0{,}5l)^3}{3EI} = \frac{pl^4}{48EI}$$



$$\downarrow \delta_C^2 = \frac{p\,(0{,}5l)^4}{8EI} = \frac{pl^4}{128EI}$$



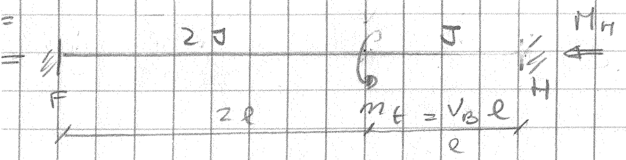
$$\theta_B = \frac{pl^3}{24EI}$$

$$\theta_B = \frac{3/8\,pl^2\,l}{3EI} = \frac{pl^3}{8EI}$$

$$\left\{\begin{array}{l} \theta_B = \left(\frac{1}{8} - \frac{1}{24}\right)\frac{pl^3}{EI} \\ \theta_b = \frac{pl^3}{12EI} \end{array}\right.$$

$$\downarrow \delta_C^3 = \theta_B\,0{,}5l = \frac{pl^4}{24EI}$$

cálculo de descenso de B



$$\left\{\begin{array}{l} M_F + M_H = \frac{15}{8}\,pl^2 \\ \theta_{FG} = \theta_{GH} \end{array}\right.$$

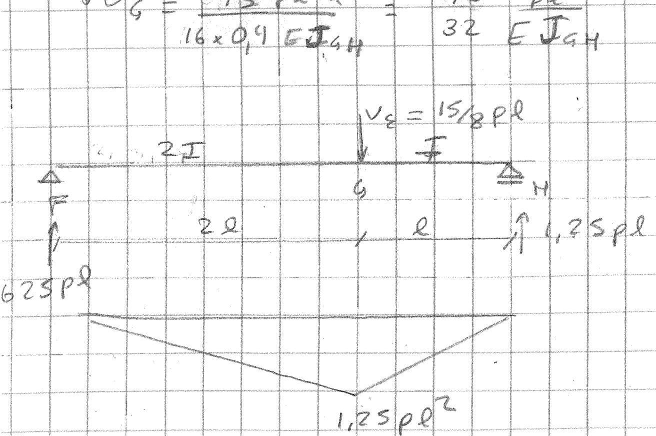
$$J_{FG} = 2\,J_{GH}$$

$$\left.\begin{aligned} \theta_{FG} &= \frac{M_F\, 2\ell}{0{,}4E\; 2\, J_{GH}} \\ \theta_{GH} &= \frac{\left(15/8\, p\ell^2 - M_F\right)\ell}{0{,}4E\; J_{GH}} \end{aligned}\right\} \Rightarrow \frac{2\, M_F}{\cancel{0{,}4}\times 2} = \frac{\left(15/8\, p\ell^2 - M_F\right)}{\cancel{0{,}4}}$$

$$4\, M_F = 2 \times \frac{15}{8}\, p\ell^2 \Rightarrow M_F = \frac{15}{16}\, p\ell^2 = 0{,}9375$$

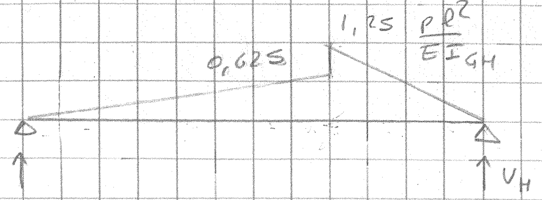
$$M_H = \left(\frac{15}{8} - \frac{15}{16}\right) p\ell^2 = \frac{15}{16}\, p\ell^2$$

$$\theta_G = \frac{15\, p\ell^2 \ell}{16 \times 0{,}4\, E J_{GH}} = \frac{75}{32}\,\frac{p\ell^3}{E J_{GH}} \Rightarrow \downarrow \delta_E' = \theta_G\, \ell = \frac{2{,}3\, p\ell^4}{E J_{GH}}$$



$$I_{FG} = 2\, I_{GH}$$

Viga análoga =

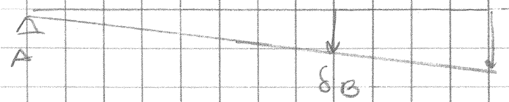


$$V_H = \frac{1}{3l}\left(\frac{0,625 \times 2l}{2} \frac{2}{3} 2l + \frac{1,25 \times l}{2}\left(2 + \frac{1}{3}\right) l \right) \frac{pl^2}{EI_{GH}} = 0,76 \frac{pl^3}{EI_{GH}}$$

$$\downarrow \delta_G = M_G = V_H\, l - 1,25 \frac{pl^2}{EI_{GH}} \frac{l}{2} \frac{1}{3} l = 0,76 \frac{pl^4}{EI_{GH}} - 0,21 \frac{pl^4}{EI_{GH}} = 0,55 \frac{pl^4}{EI_{GH}}$$

$$\Delta l_{EB} = \frac{V_B\, l}{E\Omega} = \frac{15/8\, pl^2}{E\Omega}$$

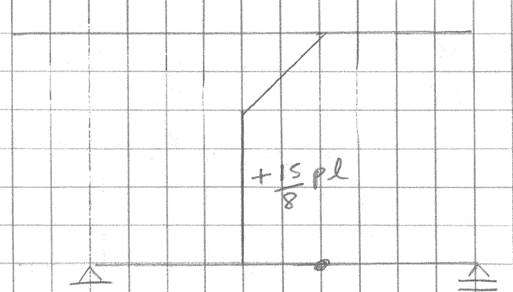
$$\downarrow \delta_B = \downarrow \delta_G + \downarrow \delta_E^1 + \Delta l_{EB} = 0,55 \frac{pl^4}{E\underbrace{I_{GH}}_{= J_{GH}/2}} + 2,3 \frac{pl^4}{EJ_{GH}} + \frac{15 pl^2}{8 E\Omega} = \boxed{3,4 \frac{pl^4}{EJ_{GH}} + \frac{15 pl^2}{8E\Omega} = \downarrow \delta_B}$$



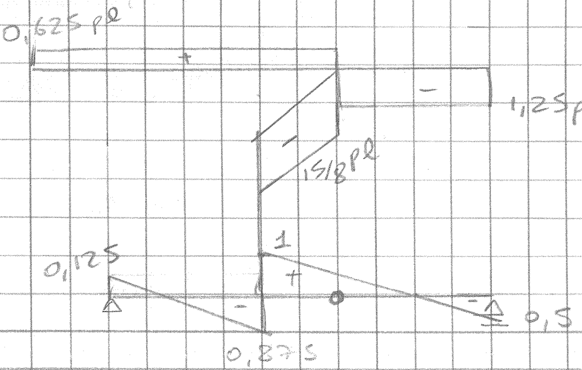
$$\downarrow \delta_c^4 = 1,5 \downarrow \delta_B = 5,1 \frac{pl^4}{EI} + 2,81 \frac{pl^2}{E\Omega}$$

$$\downarrow \delta_c = \delta_c^1 + \delta_c^2 + \delta_c^3 + \delta_c^4 = \left(\frac{1}{48} + \frac{1}{128} + \frac{1}{24} + \frac{3,4}{4}\right)\frac{pl^4}{EI} + 2,81 \frac{pl^2}{E\Omega} = \boxed{0,92 \frac{pl^4}{EI} + 2,81 \frac{pl^2}{E\Omega}}$$

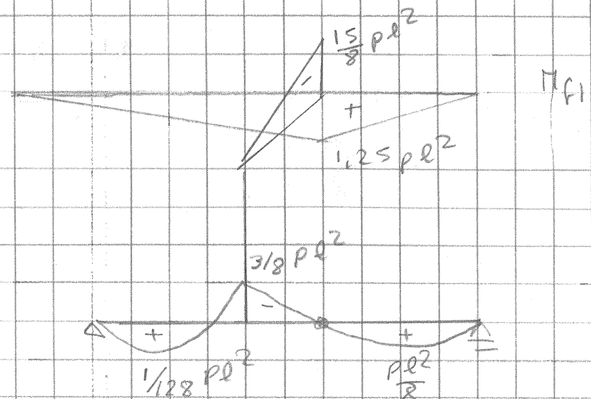
N



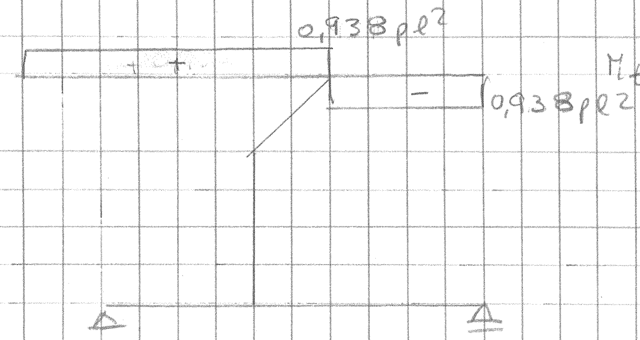
V



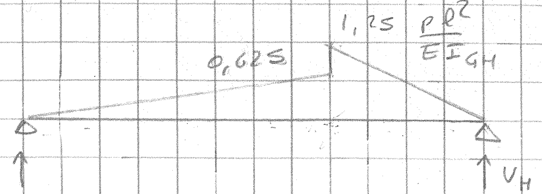
M_f



M_t



Viga análoga:

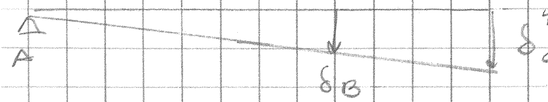


$$V_H = \frac{1}{3l}\left(\frac{0,625 \times 2l}{2}\,\frac{2}{3}2l + \frac{1,25 \times l}{2}\left(2+\frac{1}{3}\right)l\right)\frac{pl^2}{EI_{GH}} = 0,76\,\frac{pl^3}{EI_{GH}}$$

$$\downarrow\delta_G = M_G = V_H\, l - 1,25\,\frac{pl^2}{EI_{GH}}\,\frac{l}{2}\,\frac{1}{3}l = 0,76\,\frac{pl^4}{EI_{GH}} - 0,21\,\frac{pl^4}{EI_{GH}} = 0,55\,\frac{pl^4}{EI_{GH}}$$

$$\Delta l_{EB} = \frac{V_B\, l}{E\Omega} = \frac{15/8\, pl^2}{E\Omega}$$

$$\downarrow\delta_B = \downarrow\delta_G + \downarrow\delta_E' + \Delta l_{EB} = 0,55\,\frac{pl^4}{E\underbrace{I_{GH}}_{=J_{GH}/2}} + 2,3\,\frac{pl^4}{EJ_{GH}} + \frac{15pl^2}{8E\Omega} = \boxed{3,4\,\frac{pl^4}{EJ_{GH}} + \frac{15pl^2}{8E\Omega} = \downarrow\delta_B}$$



$$\downarrow\delta_C^4 = 1,5\downarrow\delta_B = 5,1\,\frac{pl^4}{EI} + 2,81\,\frac{pl^2}{E\Omega}$$

$$\downarrow\delta_C = \delta_C^1 + \delta_C^2 + \delta_C^3 + \delta_C^4 = \left(\frac{1}{48} + \frac{1}{128} + \frac{1}{24} + \frac{3,4}{4}\right)\frac{pl^4}{EI} + 2,81\,\frac{pl^2}{E\Omega} = \boxed{0,92\,\frac{pl^4}{EI} + 2,81\,\frac{pl^2}{G\Omega}}$$