SOLUCIÓN JULIO 2007

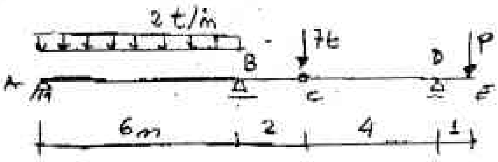


Figura 1

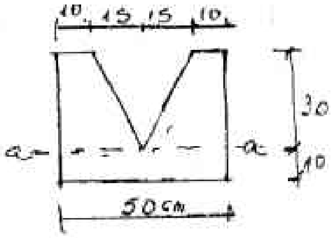
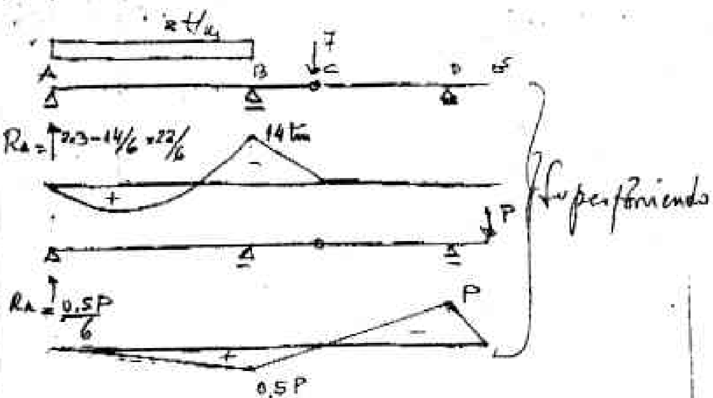
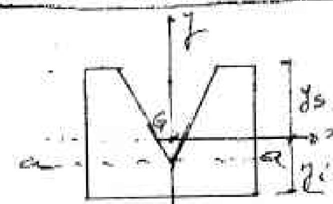


Figura 2

La estructura de la Figura 1 se construye con la sección de la Figura 2. Hallar los valores entre los que debe variar P para que no se superen los 100 kg/cm² como tensión de compresión. Para el mayor valor de P trazar diagramas de solicitación y valor máximo de la tensión rasante en a-a





$$y_i = \frac{50\times 40\times 20 - \frac{30^2}{2}\times 30}{50\times 40 - 30^2/2} = 17{,}1\,cm$$

$(\square - \triangledown)$ $\quad y_s = 22{,}9\,cm$

$$I_x = \frac{50\times 40^3}{12} + 50\times 40(20-17{,}1)^2 - \left[\frac{30^4}{36} + 450\times(22{,}9-10)^2\right] = 186102{,}2\,cm^4$$

$W_{sup} = \frac{I_x}{y_{sup}} = 8126{,}7\,cm^3 \quad W_{inf} = 10.883{,}2\,cm^3$

con $\sigma^{c}_{adm} = 100\,kg/cm^2$

$M^{+}_{adm} = \sigma^c \times W_{sup} = 8{,}13\,tm$ (comp sup)

$M^{-}_{adm} = \sigma^c \times W_{inf} = 10{,}88\,tm$ (comp inf)

en CE:

$M^{-}_{max} = P \leq M_{adm} = 10{,}88\,tm$

en BC

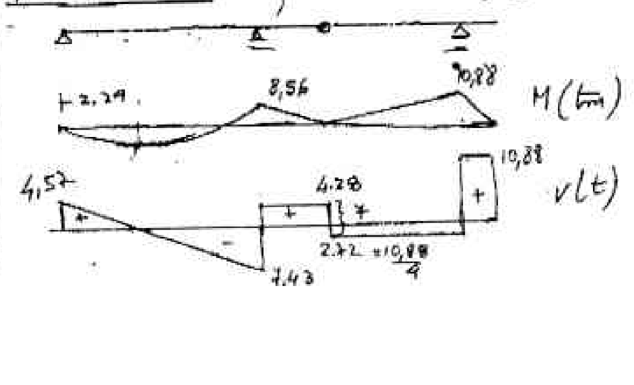
$M^{-}_{max} = 14 - 0.5P \leq 10{,}88\,tm \Rightarrow P \geq 6{,}24\,t$

en AB:

$M^{+}_{max} = \frac{R_A^2}{2p} = \frac{\left(22/6 + 0.5P/6\right)^2}{4}$ — 5,2 tm para P=10,88t; 4,38 tm para P=6,24

⇓ Para la variación posible de P $M^{+}_{max\,AB} \leq M^{+}_{adm}$ ✓

$6{,}24\,t \leq P \leq 10{,}88\,t$; Para $P = 10{,}88\,t$



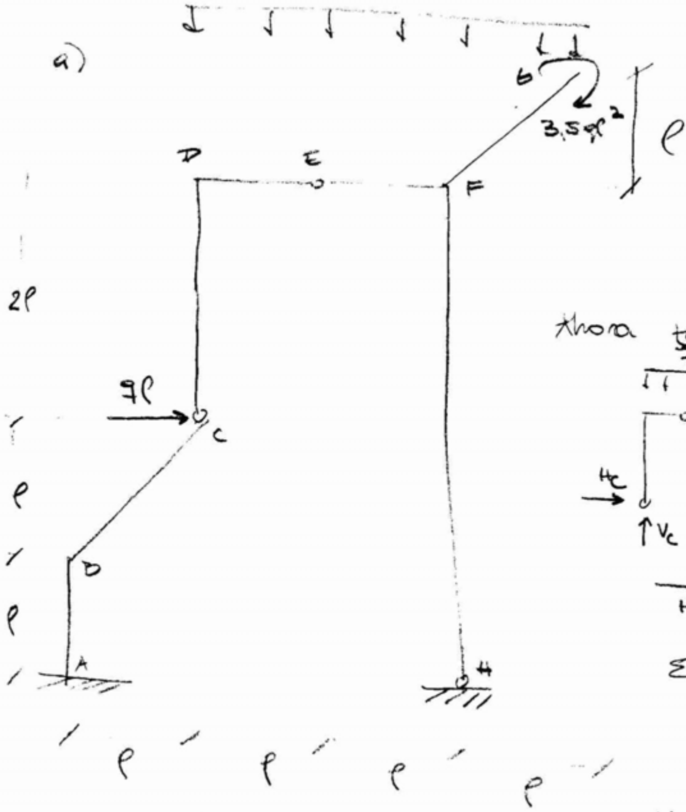
En a-a ; $V_{max} = 10{,}88\,t$

$$\tau_{aa} = \frac{V\cdot S_{aa}}{I_x\cdot b_c} = \frac{10880 \times 50\times 10\times(17{,}1-5)}{186.102{,}2 \times 50}$$

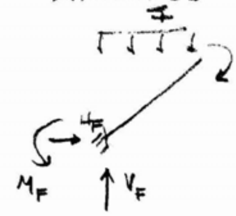
$\tau_{aa} = 7\,kg/cm^2$

Ejercicio 1

a)



Aislando FG

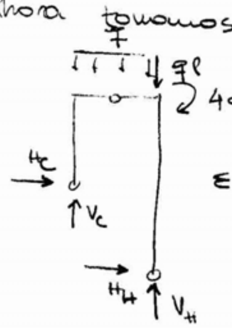


$H_F = 0$

$V_F = q \cdot l$

$M_F = \frac{q \cdot l^2}{2} + 3{,}5ql^2 = 4ql^2$

Ahora tomamos CDEFH



$V_C + V_H = q \cdot 2l + ql - 3ql$ (1)

$H_C + H_H = 0$. (2)

ΣM_E^{der}: $V_H \cdot l + H_H \cdot 4l - ql \cdot l - \frac{ql^2}{2} - 4ql^2 = 0$

$V_H + 4H_H = \frac{11}{2} ql^2$ (3)

$\Sigma M_C^{der} = 2l V_H + 2l H_H - ql \cdot 2l - 3ql \cdot l - 4ql^2 = 0$

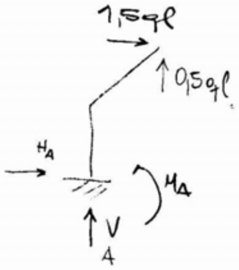
$V_H + H_H = 4ql$ (4)

(3) − (4) ⇒ $3H_H = \left(\frac{11}{2} - 4\right) ql = \frac{3}{2} ql$

→ $\boxed{H_H = 0{,}5\,ql}$ → $\boxed{H_C = -0{,}5\,ql}$

→ $\boxed{V_H = 3{,}5\,ql}$ → $\boxed{V_C = -0{,}5\,ql}$

Haciendo equilibrio en ABC:



$V_A = -0{,}5ql$

$H_A = -1{,}5ql$

$M_A + 0{,}5ql^2 - 1{,}5ql \cdot 2l = 0$

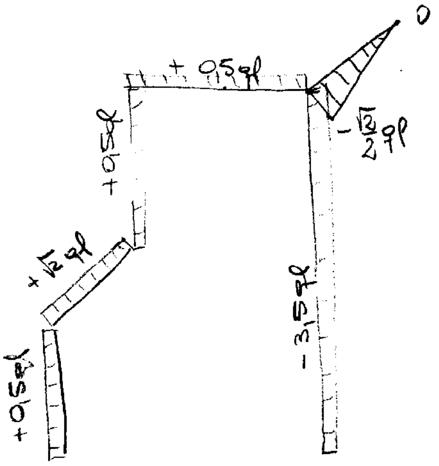
→ $\boxed{M_A = 2{,}5\,ql^2}$

Diagramas de solicitación

En la barra FG. $N_F = -\frac{\sqrt{2}}{2} ql$

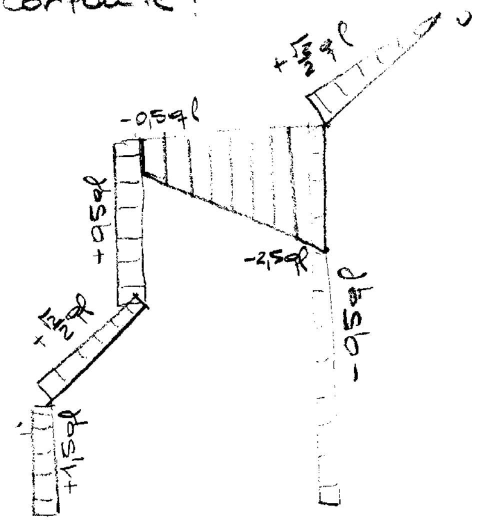
$V_F = +\frac{\sqrt{2}}{2} ql$.

Directa:



$$N_{BC} = \frac{\sqrt{2}}{2}(1{,}5ql + 0{,}5ql) = \sqrt{2}\,ql$$

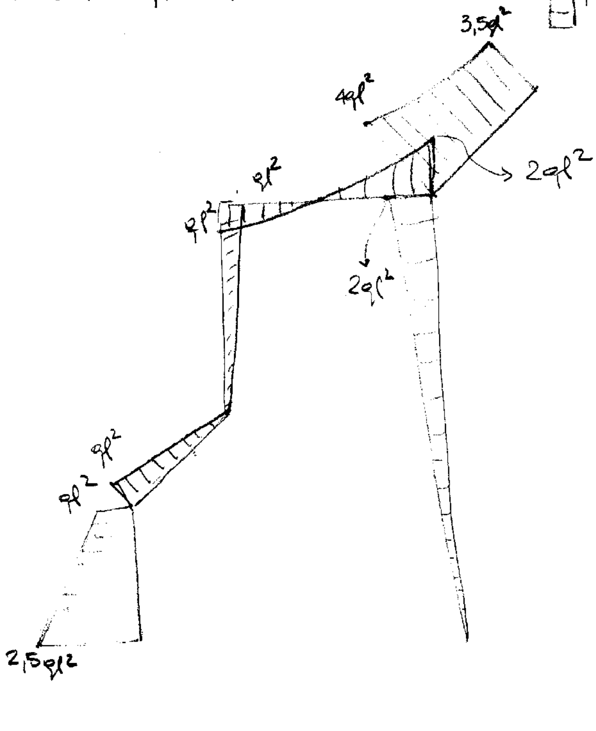
Cortante:



$$V_{BC} = \frac{\sqrt{2}}{2}(1{,}5ql - 0{,}5ql)$$

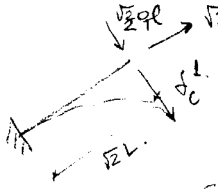
$$V_{BC} = \frac{\sqrt{2}}{2}ql$$

Momento flector.



b) Considero sólo ABC.

Primero estudio BC.

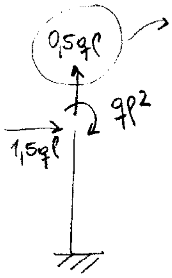


La única carga que genera desplazamiento es el cortante en C.

$$\delta_C^1 = \frac{PL^3}{3EI} = \frac{\frac{\sqrt{2}}{2}qL(\sqrt{2}L)^3}{3EI} = \frac{2}{3}\frac{qL^4}{EI}$$

$$\xrightarrow{\ }\ \delta_{C_H}^1 = \frac{\sqrt{2}}{3}\frac{qL^4}{EI} \qquad \downarrow \delta_{C_V}^1 = \frac{\sqrt{2}}{3}\frac{qL^4}{EI}.$$

Ahora considero AB:



$\rightarrow$ no afecta el desplazamiento.

$$\delta_B^V = 0$$

$$\xrightarrow{\ }\ \delta_B^H = \frac{1{,}5qL \cdot L^3}{3EI} + \frac{qL^2 \cdot L^2}{2EI} = \frac{qL^4}{EI}$$

$$\theta_B = \frac{1{,}5qL \cdot L^2}{2EI} + \frac{qL^2 \cdot L}{EI} = \frac{7}{4}\frac{qL^3}{EI}$$

Desplazamiento de C por giro rígido de BC.

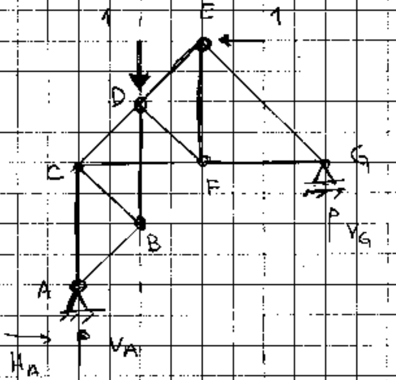
$$\delta_C^2 = \theta_B \sqrt{2}L = \frac{7\sqrt{2}}{4}\frac{qL^4}{EI}$$

$$\xrightarrow{\ }\ \delta_{C_H}^2 = \frac{7}{4}\cdot\frac{qL^4}{EI}.$$

$$\downarrow \delta_{C_V}^2 = \frac{7}{4}\cdot\frac{qL^4}{EI}$$

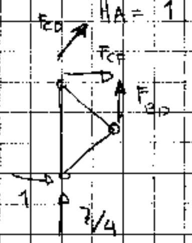
$$\delta_{C_H}^T = \delta_B^H + \delta_{C_H}^1 + \delta_{C_H}^2 = \left(\frac{\sqrt{2}}{3} + 1 + \frac{7}{4}\right)\frac{qL^4}{EI} = \left(\frac{\sqrt{2}}{3} + \frac{11}{4}\right)\frac{qL^4}{EI} \Rightarrow$$

$$\delta_{C_V}^T = \delta_{C_V}^1 + \delta_{C_V}^2 = \left(\frac{\sqrt{2}}{3} + \frac{7}{4}\right)\frac{qL^4}{EI} \Downarrow$$



$M_A: \; 4V_G + 4 = 1 \Rightarrow V_G = -\frac{3}{4} \Rightarrow V_A = \frac{7}{4}$

$H_A = 1$

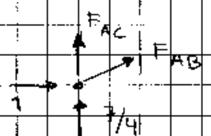


$M_C = 0 \Rightarrow \boxed{F_{BD} = -2}$

$\Sigma V = 0 \Rightarrow -\frac{1}{4} + \frac{F_{CD}}{\sqrt2} = 0 \Rightarrow \boxed{F_{CD} = +\frac{\sqrt2}{4} = \frac{+1}{2\sqrt2}}$

$\Sigma H = 0 \Rightarrow -1 = \underbrace{\frac{F_{CD}}{\sqrt2}}_{+\frac{1}{4}} + F_{CF} \Rightarrow \boxed{F_{CF} = -\frac{5}{4}}$

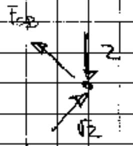
Nudo A:



$-\frac{F_{AB}}{\sqrt2} = 1 \Rightarrow \boxed{F_{AB} = -\sqrt2}$

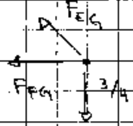
$\frac{F_{AB}}{\sqrt2} + F_{AC} + \frac{7}{4} = 0 \Rightarrow \boxed{F_{AC} = -\frac{3}{4}}$

Nudo B



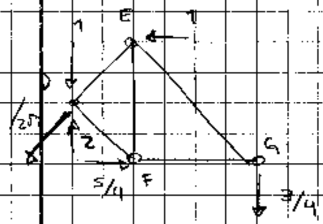
$\frac{F_{CB}}{\sqrt3} = \frac{\sqrt2}{\sqrt2} \Rightarrow \boxed{F_{CB} = \sqrt2}$

Nudo G

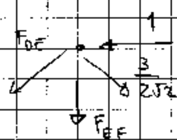


$\frac{F_{EG}}{\sqrt2} = \frac{3}{4} \Rightarrow \boxed{F_{EG} = \frac{3}{2\sqrt2}}$

$\frac{F_{EG}}{\sqrt2} = -F_{FG} \Rightarrow \boxed{F_{FG} = -\frac{3}{4}}$



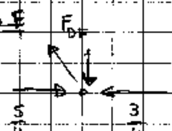
Nudo E



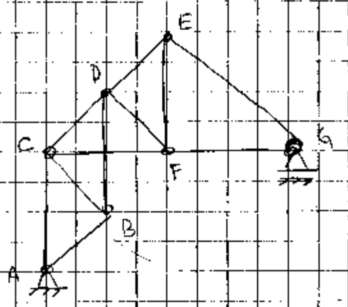
$\frac{F_{DE}}{\sqrt2} + 1 = \frac{3}{2 \cdot \sqrt2 \sqrt2} \Rightarrow \boxed{F_{DE} = -\frac{1}{2\sqrt2}}$

$-F_{EF} = \frac{F_{DE}}{\sqrt2} + \frac{3}{4} = \frac{1}{2} \Rightarrow \boxed{F_{EF} = -\frac{1}{2}}$

Nudo F



$\frac{F_{DF}}{\sqrt2} = \frac{1}{2} \Rightarrow \boxed{F_{DF} = \frac{+1}{\sqrt2}}$



Barra	F	L	A	$\Delta L \cdot E\Omega$
AB	$-\sqrt{2}$	$\sqrt{2}$	Ω	-2
BC	$\sqrt{2}$	$\sqrt{2}$	Ω	$+2$
AC	$-3/4$	2	Ω	$-3/2$
BD	-2	2	2Ω	-2
CD	$1/2\sqrt{2}$	$\sqrt{2}$	2Ω	$+1/4$
CF	$-5/4$	2	2Ω	$-5/4$
DE	$-1/2\sqrt{2}$	$\sqrt{2}$	Ω	$-1/2$
DF	$1/\sqrt{2}$	$\sqrt{2}$	Ω	$+1$
EF	$-1/2$	2	Ω	-1
EG	$3/2\sqrt{2}$	$2\sqrt{2}$	Ω	3
FG	$+3/4$	2	Ω	$-3/2$

$$\vec{\Delta}_B = -\frac{7{,}4}{EA}$$

$$^{A}\Delta_B = \frac{4{,}5}{E\Omega}$$