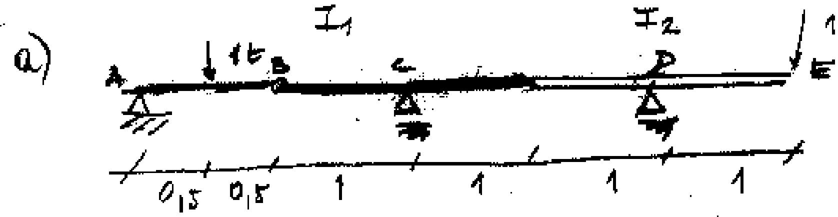
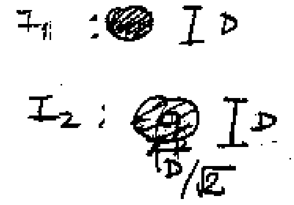
Solución - Examen febrero 2006 – Resistencia de materiales 1 y 1N

Ejercicio 1

a)



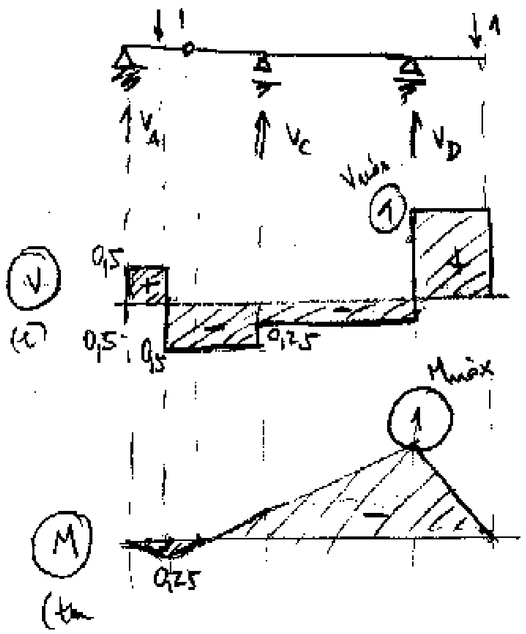


$\delta_E \le 0{,}5\,cm$

$\sigma_{máx} \le 1{,}4\,t/cm^2$

$E = 2{,}1 \times 10^7\ t/m^2$

$$I_1 = \frac{\pi D^4}{64} \qquad I_2 = \frac{\pi}{64}\left(D^4 - \frac{D^4}{4}\right) = \frac{3}{4}\,\frac{\pi D^4}{64} \rightarrow I_2 = \frac{3}{4} I_1$$



$M_B^{iz} = 0 \Rightarrow \boxed{V_A = 0{,}5\,t}$

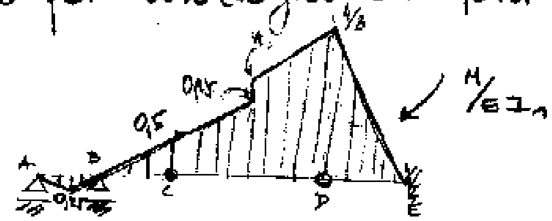
$\left.\begin{array}{l} M_B^{d} = 0 \\ \Sigma V = 0 \end{array}\right\} \Rightarrow \boxed{V_D = 1{,}25}\ \boxed{V_C = 0{,}25\,t}$

$M_{máx} = 1\ tm = 100\ tcm$

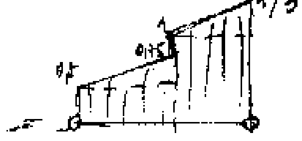
$$\frac{M}{I}\,y \le \sigma \rightarrow \frac{100 \cdot D/2}{\frac{3}{4}\,\frac{\pi D^4}{64}} \le 1{,}4 \rightarrow \boxed{D \ge 9{,}9\,cm}$$

$\delta_E \downarrow$: Resuelvo por analogía de Mohr:

Diagrama de $\frac{M}{EI_1}$ en la viga análoga



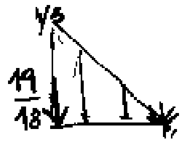
Aislando CD:



$$\Sigma M_C = 2V_D - 0{,}5 \cdot 1 \cdot 0{,}5 - 0{,}25 \cdot \frac{1}{2} \cdot \frac{2}{3} - 1 \cdot 1 \cdot 1{,}5 - \frac{1}{3} \cdot 0{,}5 \cdot \frac{5}{3} = 0$$

$$V_D = 19/18$$

Ahora DE:



$$M_E = \frac{4}{3}\cdot 1 \cdot \frac{1}{2}\cdot\frac{2}{3} + \frac{19}{18}\cdot 1 \rightarrow M_E = 1{,}5$$

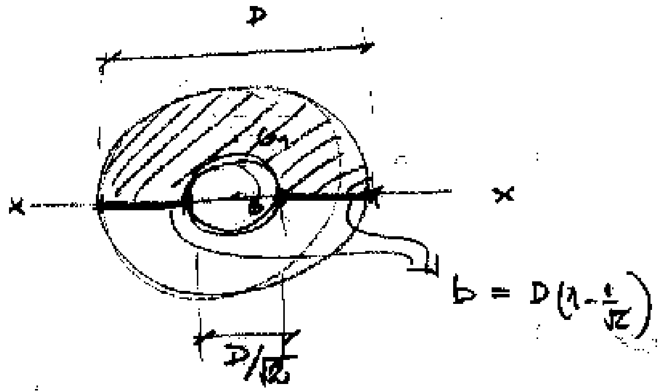
$$\delta_E = \frac{M_E}{EI_2} = \frac{1{,}5}{\frac{3}{4}\frac{\pi D^4}{64}\cdot 2{,}1\times10^7} \leq 0{,}005\,m \rightarrow D \geq 0{,}14\,m$$

$$\boxed{D = 14\ cm}$$

b) $V_{máx} = 1\,t$

El máximo μ se da en en el diámetro:

lo calculo como



$\rightarrow A_1 b_1 - A_2 b_2$

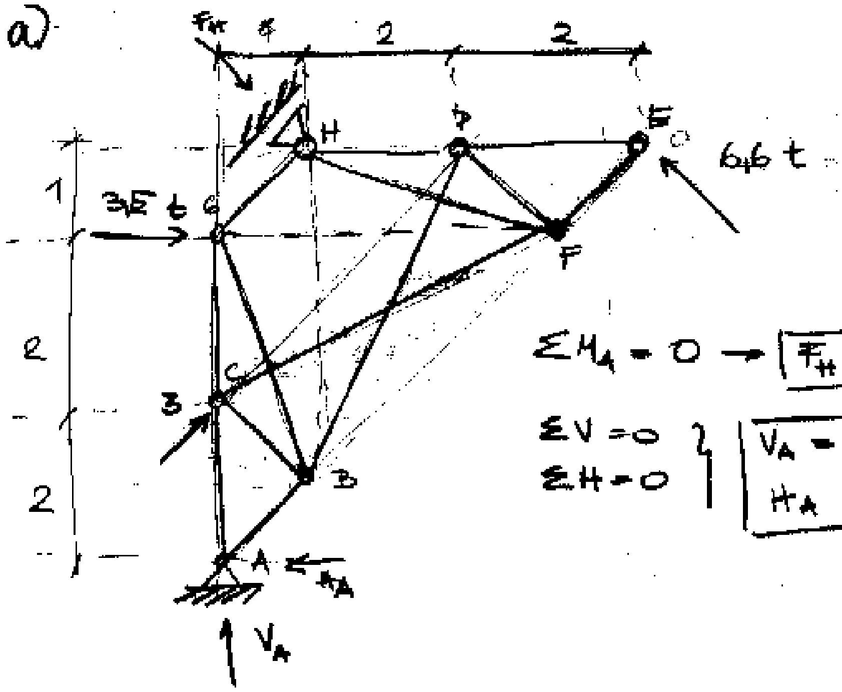
$$\mu_x = \frac{\pi D^2}{2\cdot 4}\cdot\frac{4}{3}\frac{D}{2\pi} - \frac{\pi(D/\sqrt{2})^2}{2\cdot 4}\cdot\frac{4}{3}\frac{D/\sqrt{2}}{2\pi} + \frac{D^3}{12}\left(1-\frac{1}{2\sqrt{2}}\right) = 148{,}94\,cm^3$$

$$\tau = \frac{V\mu}{Ib} = \frac{1\cdot 148{,}94}{\frac{3}{4}\frac{\pi}{64}14^4\cdot 14\left(1-1/\sqrt{2}\right)} = 0{,}02568\ t/cm^2$$

$$\boxed{\tau_{máx} = 25{,}68\ kg/cm^2}$$

Ejercicio 2

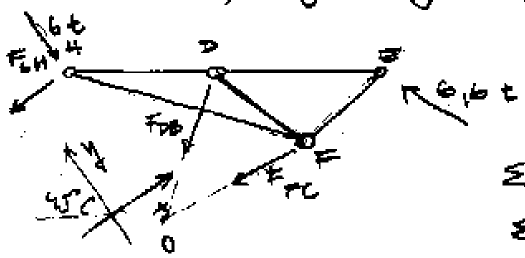
a)



$\Sigma M_A = 0 \rightarrow \boxed{F_H = 6t}$

$\left.\begin{array}{l}\Sigma V = 0 \\ \Sigma H = 0\end{array}\right\} \quad \boxed{\begin{array}{l} V_A = 2{,}4t \\ H_A = 6t \end{array}}$

* Aplico el método de las secciones conclusivas: corto GH, CF y BD y luego equilibrio en (HDEF)



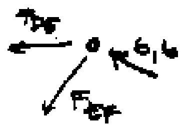
$\Sigma M_O = 0 \Rightarrow \boxed{F_{GH} = -9}$

$3F^y_{DB} = F^x_{DB}$

$3F^y_{FC} = F^x_{FC}$

$\left.\begin{array}{l}\Sigma F_x \rightarrow 3F^y_{FC} + 3F^y_{DB} - 9 = 0 \\ \Sigma F_y \rightarrow 6{,}6 + F^y_{FC} - F^y_{BD} - 6 = 0\end{array}\right\} \Rightarrow \boxed{F_{FC} = 3{,}79} \quad \boxed{F_{BD} = 3{,}69}$

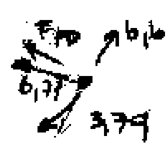
Eq. nodo E



$\boxed{F_{FE} = 6{,}6}$

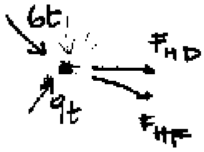
$\boxed{F_{DE} = -9{,}33}$

Eq. nodo F



$\boxed{F_{FD} = -7{,}2}$

Eq. nodo H

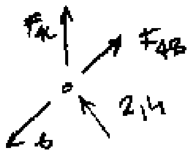


$\boxed{F_{HF} = 6{,}71}$

$\boxed{F_{HD} = -16{,}97}$

Ahora equilibrio en ABCG

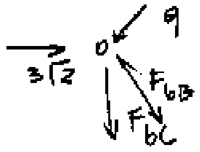
Nudo A.



$\boxed{F_{AC} = -3,39}$

$\boxed{F_{AB} = 8,4}$

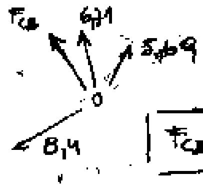
Nudo G



$\boxed{F_{GB} = 6,71}$

$\boxed{F_{GC} = -12,73}$

Nudo



$\boxed{F_{CB} = -7,8}$

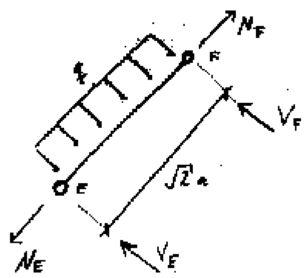
b) MÁX TRACCIÓN: $8,4t \rightarrow \frac{8,4}{1,4} \leq A = b^2 \rightarrow \boxed{b \geq 2,45 cm}$

MÁX COMPRESIÓN: $16,97 \rightarrow \frac{16,97}{1,4} \leq 12 \leq A \longrightarrow \boxed{PNI 12}$

Solución - Examen febrero 2006 – Resistencia de materiales 1 y 1N

Ejercicio 3

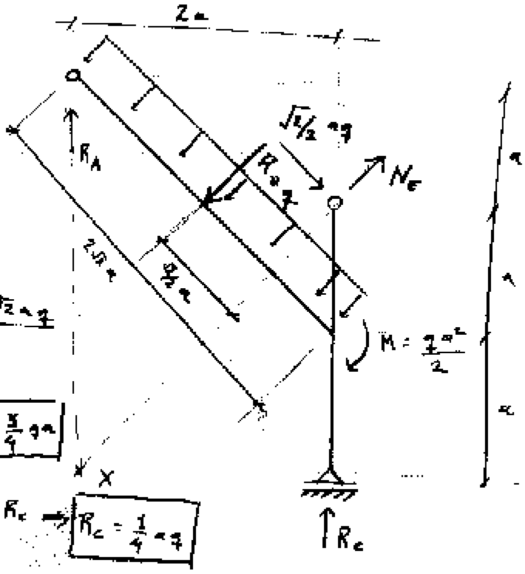
a)



$N_E = N_F$

$V_E = \sqrt{2}/2\ aq$

$V_F = \sqrt{2}/2\ aq$



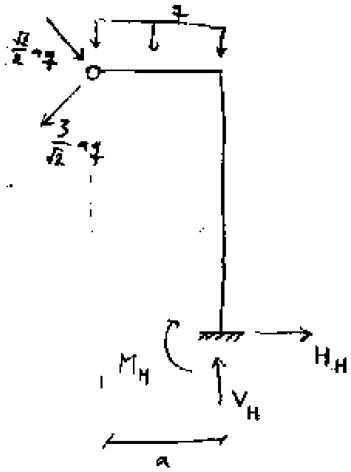
$R: 2\sqrt{2}\, aq$

$M_E = 0 \Rightarrow 2\sqrt{2}\, aq \frac{\sqrt{2}}{2} a - \frac{qa^2}{2} = R_A\, 2a \Rightarrow \boxed{R_A = \frac{3}{4} qa}$

$M_X = 0 \Rightarrow 2\sqrt{2}\, a \cdot \frac{\sqrt{2}}{2} aq - 2\sqrt{2}\, aq \frac{\sqrt{2}}{2} a + \frac{qa^2}{2} = 2a R_C \Rightarrow \boxed{R_C = \frac{1}{4} aq}$

eq. en la dir. de N_E:

$N_E + \frac{3}{4\sqrt{2}} qa + \frac{1}{4\sqrt{2}} aq - 2\sqrt{2}\, qa = 0 \Rightarrow \boxed{N_E = \frac{3}{\sqrt{2}} aq}$

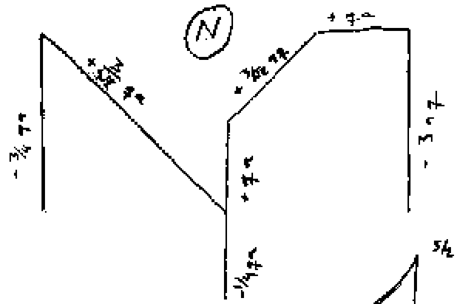


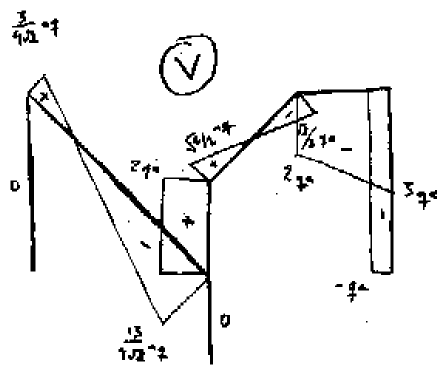
$V_H = \frac{3}{2} aq + \frac{1}{2} aq + aq \Rightarrow \boxed{V_H = 3aq}$

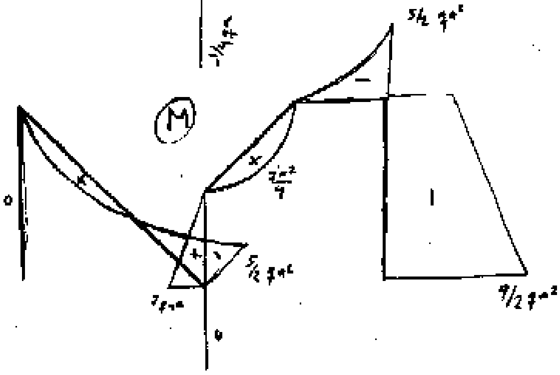
$H_H = \frac{3}{2} aq - \frac{1}{2} aq \Rightarrow \boxed{H_H = aq}$

$M_H = aq \cdot \frac{a}{2} + \frac{3}{\sqrt{2}} aq \frac{3}{\sqrt{2}} a - \frac{\sqrt{2}}{2} aq \frac{a}{\sqrt{2}}$

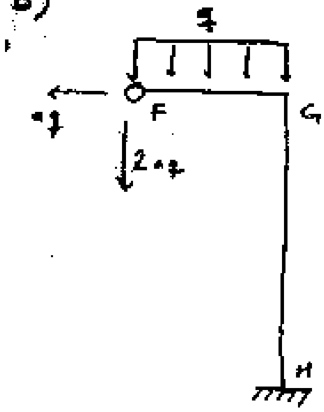
$\Rightarrow \boxed{M_H = \frac{7}{2} a^2 q}$

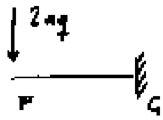




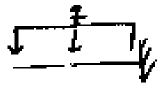


b)

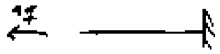




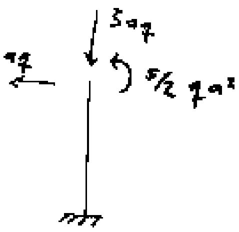
$$\delta_F^{I} = \frac{2aq\,a^3}{3EI} \; (\downarrow)$$

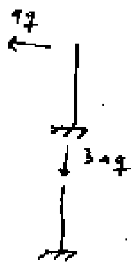


$$\delta_F^{II} = \frac{q a^4}{8EI} \; (\downarrow)$$



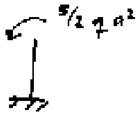
$$\delta_F^{III} = \frac{aqa}{AE} \; (\leftarrow)$$





$$\delta_G^{I} = \frac{aq(2a)^3}{3EI} \; (\leftarrow) \qquad \theta_G^{I} = \frac{aq(2a)^2}{2EI} \; (\circlearrowright)$$

$$\delta_G^{II} = \frac{3aq\,2a}{AE} \; (\downarrow)$$



$$\delta_G^{III} = \frac{5qa^2(2a)^2}{2\;2EI} \; (\leftarrow) \qquad \theta_G^{III} = \frac{5\,qa^2\,2a}{2\;EI} \; (\circlearrowright)$$

$$\Rightarrow \theta_G = \theta_G^{I} + \theta_G^{III} = 7\frac{qa^3}{EI} \Rightarrow \delta_F^{IV} = \theta_G \cdot a = 7\frac{qa^4}{EI} \; (\downarrow)$$

$$\delta_F(\leftarrow) = \delta_F^{III} + \delta_G^{I} + \delta_G^{III} = \boxed{\frac{qa^2}{AE} + \frac{23}{3}\frac{qa^4}{EI}}$$

$$\delta_F(\downarrow) = \delta_F^{I} + \delta_F^{II} + \delta_G^{II} + \delta_F^{IV} = \boxed{6\frac{qa^2}{AE} + \frac{187}{24}\frac{qa^4}{EI}}$$