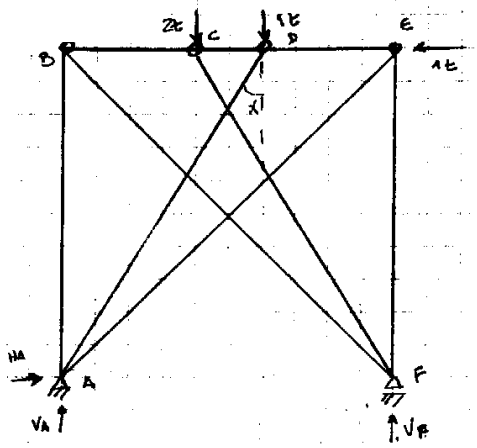
Solución – Examen Febrero 2005 – Resistencia de Materiales 1 y 1N

Ejercicio 1



Reacciones:

$H_A = 1t$

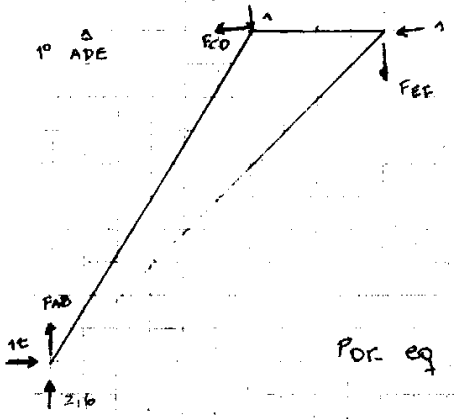
$M_{A=0}: 5V_F + 1\times5 - 1\times3 - 2\times2 = 0$

$V_F = \frac{2}{5}$

$M_F = 0 \quad 5V_A - 2\times3 - 1\times2 - 1\times5 = 0$

$V_A = \frac{13}{5} = 2{,}6 \qquad V_A + V_F = 3t$ ✓

Por el método de las secciones corto: AB, CD y EF

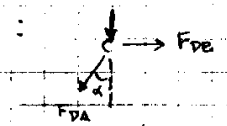


$M_E = 0$

$(F_{AB} + 2{,}6)\cdot5 - 1\times5 - 1\times2 = 0 \qquad F_{AB} = -\frac{6}{5} = -1{,}2\ t$

Eq vertical: $F_{AB} + 2{,}6 - 1 - F_{EF} = 0 \rightarrow F_{EF} = 0{,}4t$

Eq horiz: $F_{CD} = 0$

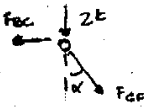
Por eq en el nodo D: 

$F_{DE} + F_{DA}\ \text{sen}\,\alpha = 0$

$1 + F_{DA}\cos\alpha = 0$

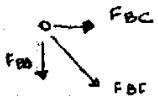
$\text{tg}\,\alpha = \frac{5}{3} \qquad \overline{AD} = 5{,}83 \qquad \text{sen}\,\alpha = \frac{3}{5{,}83} = 0{,}51 \qquad \cos\alpha = \frac{5}{5{,}83} = 0{,}86$

$\rightarrow F_{DA} = \frac{-1}{0{,}86} = -1{,}17 \qquad F_{DE} = -1{,}17\times0{,}51 = -0{,}6$

Eq nudo C: 

$F_{BC} = F_{CF}\ \text{sen}\,\alpha$

$F_{CF}\cos\alpha = -2 \rightarrow F_{CF} = -2{,}3 \rightarrow F_{BC} = -1{,}2$

Eq nudo B: 

$F_{BF} = -(F_{BC} + F_{BA})\sqrt{2}/2 = 1{,}7\ t$

→ Eq E $F_{EA} = -\left((1-0{,}6)^2 + 0{,}4^2\right)^{1/2} = -0{,}57$

$F_{AB} = -1{,}2t$	$F_{CD} = 0$
$F_{AD} = -1{,}17t$	$F_{CF} = -2{,}3\ t$
$F_{AE} = -0{,}57\ t$	$F_{DE} = -0{,}6\ t$
$F_{BC} = -1{,}2\ t$	$F_{EF} = 0{,}4\ t$
$F_{BF} = 1{,}7\ t$	

(+) tracción

(-) compresión

DIMENSIONADO :

Máxima tracción = 1700 kg σ_{adm} = 1400 kg/cm²

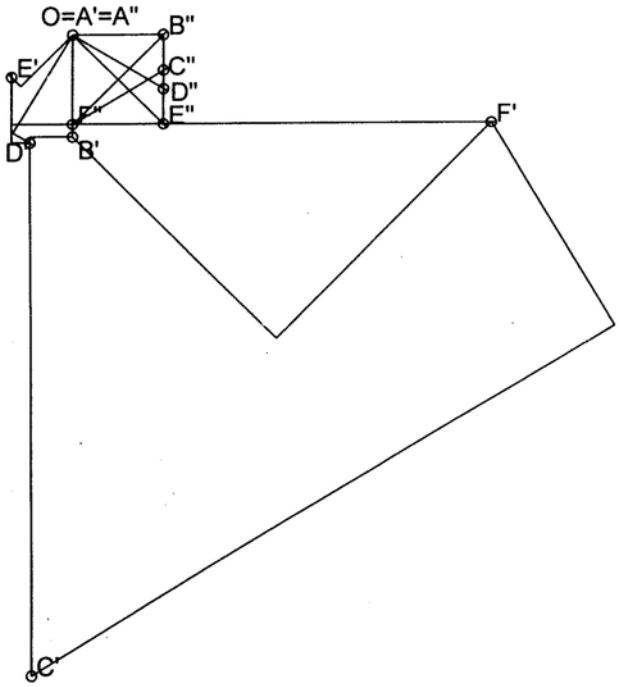
$$\sigma = \frac{F}{A} = \frac{F}{\pi\phi^2/4} \rightarrow \phi = \sqrt{\frac{4F}{\pi\sigma}} = 1{,}24 \text{ cm} \qquad A = 1{,}21 \text{ cm}^2$$

Máxima compresión = 2300 kg σ_{adm} = 1400 kg/cm²

$$\sigma = \frac{F}{\phi^2 - 0{,}8^2\phi^2} = \frac{F}{0{,}36\phi^2} \rightarrow \phi = \sqrt{\frac{F}{0{,}36\sigma}} = 2{,}13 \qquad A = 1{,}64 \text{ cm}^2$$

DESPLAZAMIENTOS : $\Delta L = \frac{PL}{EA}$

BARRA	F	L	A	ΔL (cm)
AB	-1200	500	1,64	-0,17
AD	-1170	583	1,64	-0.19
AE	-570	707	1,64	-0,12
BC	-1200	200	1,64	-0.069
BF	1700	707	1,21	0,47
CD	0	100	-	0
CF	-2300	583	1,64	-0.39
DE	-600	200	1,64	-0.03
EF	400	500	1,21	0.079



	δ_H(cm)	δ_V(cm)
A	**0**	**0**
B	**-0,15**	**+0,17**
C	**-2,19**	**+1,00**
D	**-0,22**	**+0,09**
E	**-0,25**	**-0,08**
F	**+0,69**	**0**

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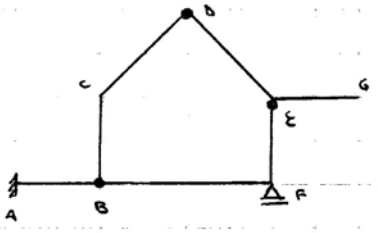
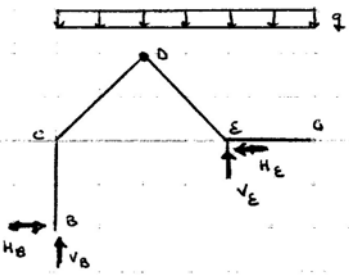
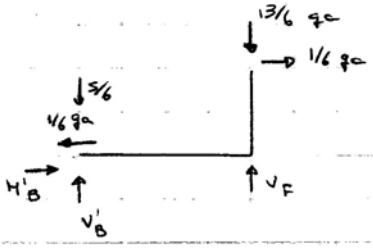
Ejercicio 2



$$\begin{cases} H_B = H_E \\ V_B + V_E = 3qa \\ V_B\,2a + H_B\,a = \frac{3}{2}qa^2 \\ V_B\,a + H_B\,2a = \frac{qa^2}{2} \end{cases}$$

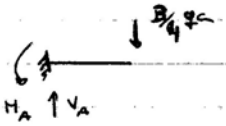


$$2V_B + H_B = 3/2\,qa$$
$$-2V_B + 4H_B = -qa$$
$$3H_B = 1/2\,qa$$
$$H_B = H_E = 1/6\,qa$$
$$V_B = \frac{qa}{2} + 2H_B \Rightarrow V_B = \frac{5}{6}qa$$
$$V_E = 3qa - V_B \Rightarrow V_E = \frac{13}{6}qa$$

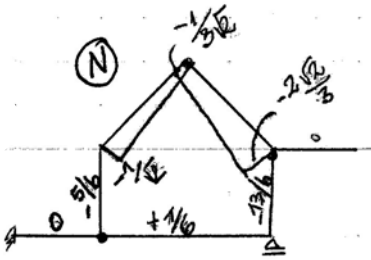


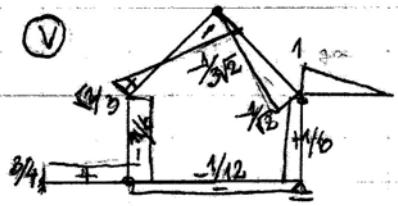
$$\begin{cases} H'_B = 0 \\ V'_A + V_F = 3qa \\ V_F\,2a - \frac{5}{6}qa\,2a - \frac{1}{6}qa\,a = 0 \end{cases} \Rightarrow V_F = \frac{7}{4}qa$$

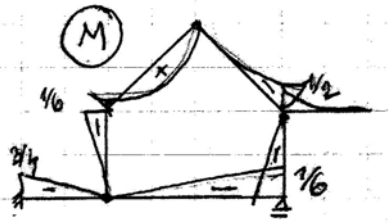
$$V'_B = \left(3 - \frac{7}{4}\right)qa = \frac{5}{4}qa$$



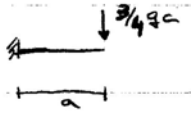
$$\begin{cases} V_A = \frac{3}{4}qa \\ M_A = \frac{3}{4}qa^2 \end{cases}$$



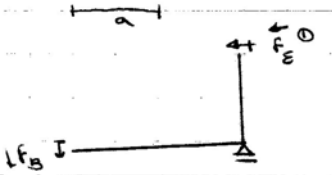




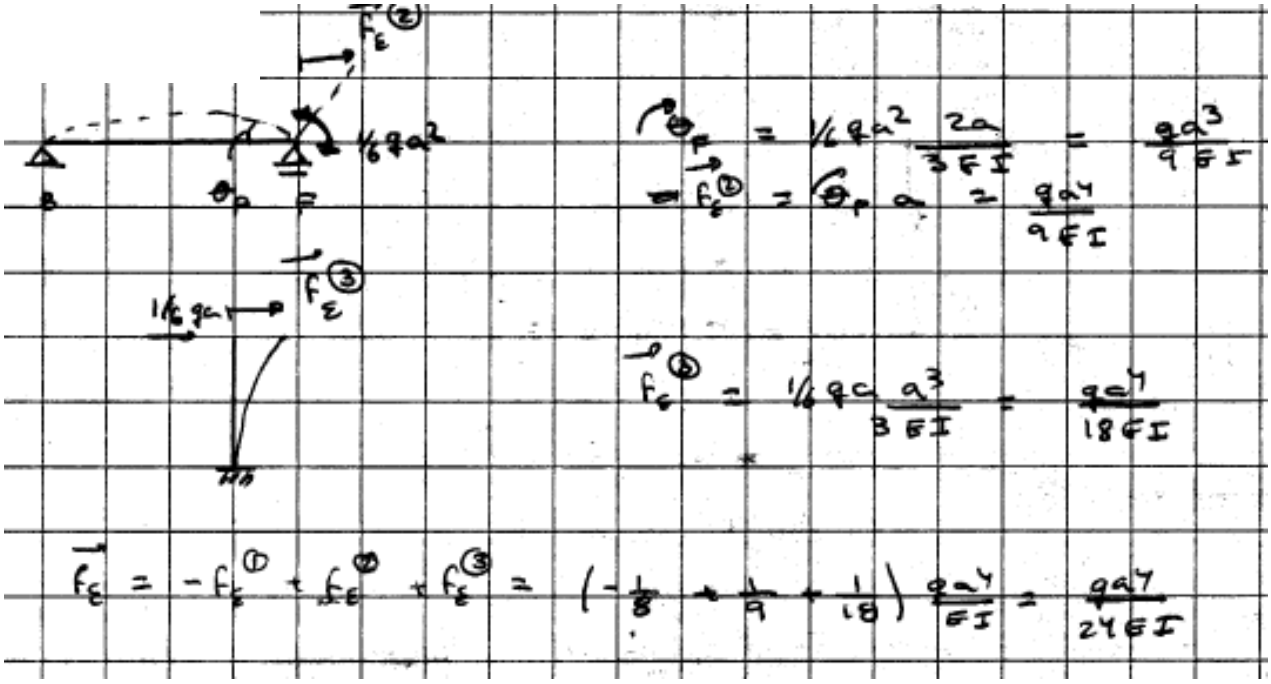
Desplazamiento



$$\downarrow f_B = \frac{\frac{3}{4}qa\,a^3}{3EJ} = \frac{qa^4}{4EI}$$



$$\leftarrow f_E^{①} = \frac{\downarrow f_B}{2} = \frac{qa^4}{8EI}$$



$$\theta_p = \frac{1}{6}qa^2 \frac{2a}{3EI} = \frac{qa^3}{9EI}$$

$$f_\varepsilon^{①} = \theta_p \, a = \frac{qa^4}{9EI}$$

$$f_\varepsilon^{③} = \frac{1}{6}qa \frac{a^3}{3EI} = \frac{qa^4}{18EI}$$

$$\vec{f_\varepsilon} = -f_\varepsilon^{①} + f_\varepsilon^{②} + f_\varepsilon^{③} = \left(-\frac{1}{8} + \frac{1}{9} + \frac{1}{18}\right)\frac{qa^4}{EI} = \frac{qa^4}{24EI}$$

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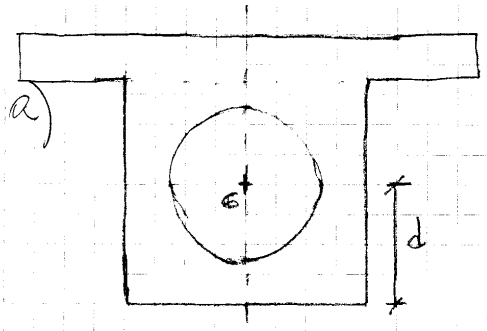
Ejercicio 3.

a)



i) $y_G = \frac{150\times15\times82{,}5 + 80\times75\times37{,}5 - d\,\pi\frac{25^2}{4}}{150\times15 + 80\times75 - \pi\frac{25^2}{4}} = d$

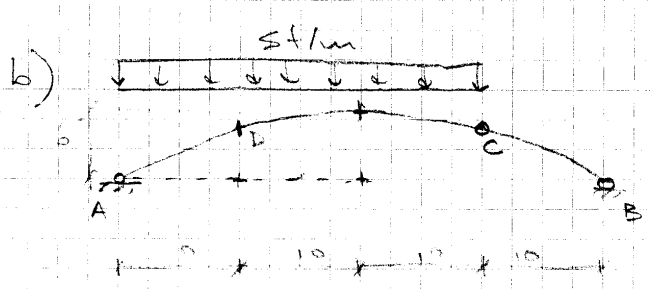
$$\boxed{d = 49{,}77\ \text{cm}}$$

ii) $$I_x = 150\times\frac{15^3}{12} + 150\times15\times32{,}27^2 + 80\times\frac{75^3}{12} + 80\times75\times12{,}27^2 - \frac{\pi 50^4}{64}$$

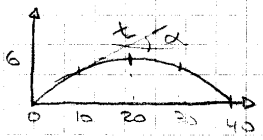
$$\boxed{I_x = 5.861.683\ \text{cm}^4}$$

$$I_y = 15\times\frac{150^3}{12} + 75\times\frac{80^3}{12} - \frac{\pi 50^4}{64} = \boxed{7.112.104\ \text{cm}^4 = I_G}$$

b)



Parábola:



$$M_G(x) = ax^2 + bx \qquad M_G(20) = 400a + 20b = 6$$

$$M_G'(x) = 2ax + b \qquad M_G'(20) = 40a + b = 0$$

$$\Rightarrow \boxed{a = -\frac{3}{200}} \qquad \boxed{b = 3/5}$$

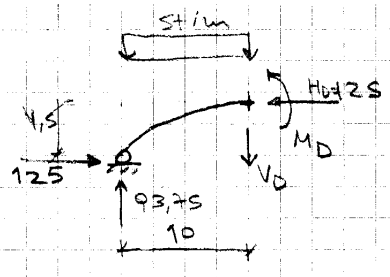
$$M_G(x) = -\frac{3}{200}x^2 + \frac{3}{5}x \Rightarrow \boxed{M_G(10) = M_G(30) = 4{,}5\ \text{m}}$$

$$M_G'(x) = -\frac{3}{100}x + \frac{3}{5} \Rightarrow M_G'(10) = 0{,}3 = \text{tg}\,\alpha \Rightarrow \boxed{\alpha \simeq 16{,}7^\circ}$$

Reacciones: $M_A = 5\times30\times15 - V_B\cdot30 = 0 \Rightarrow \boxed{V_B = 56{,}25\ \text{t}}$

$$V_A = 5\times30 - 56{,}25 = \boxed{93{,}75\ \text{t} = V_A}$$

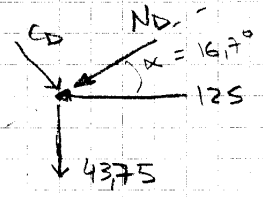
$$M_C^d = 4{,}5\times H - 56{,}25\times10 = 0 \Rightarrow \boxed{H = 125\ \text{t}}$$



$\downarrow V_D = 93{,}75 - 5 \times 10 = 43{,}75\,t$

$\curvearrowleft M_D = -125 \times 4{,}5 - 5 \times 10 \times 5 + 93{,}75 \times 10$

$\curvearrowleft M_D = 125\,t\,m$



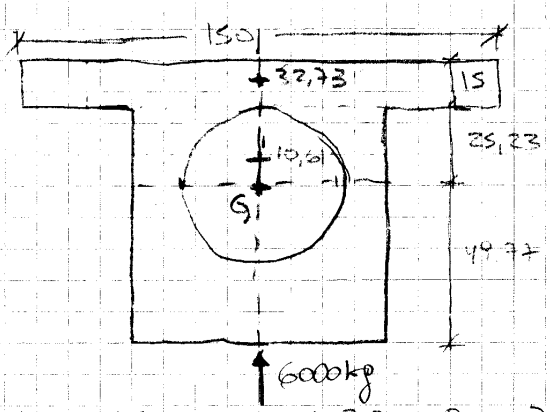
$N_D = 125 \cos 16{,}7° + 43{,}75 \operatorname{sen} 16{,}7°$

$\boxed{N_D = 132{,}3\,t}$ (comp.)

$C_D = -125 \operatorname{sen} 16{,}7° + 43{,}75 \cos 16{,}7°$

$\boxed{C_D = 6{,}0\,t}$

c)



$M_6 = 15 \times 150 \times 32{,}73 + 80 \times 25{,}23 \times 12{,}62 - \pi \frac{25^2}{2} \times 10{,}61$

$M_6 = 887038{,}6\ cm^3$

$b = 80 - 50 = 30$

$\tau_6 = \frac{V_M}{b\,I} = \frac{6000kg \times 887038{,}6}{30 \times 5.861.683} = 3{,}0\ kp/cm^2$