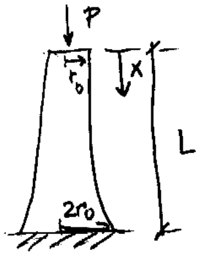
Solución – Examen Diciembre 2004 – Resistencia de Materiales 1 y 1N

Ejercicio 1

a) i)



$$r(x) = a\,e^{bx}$$

$$r(0) = \boxed{a = r_0}$$

$$r(L) = r_0 e^{bL} = 2r_0 \rightarrow \boxed{b = \frac{\log 2}{L}}$$

$$\boxed{A(x) = \pi r^2 = \pi r_0^2 \cdot e^{2\frac{\log 2}{L}x}}$$

$$N(x) = -P - \int_0^x \gamma A(z)\,dz = -P - \int_0^x \gamma \pi r_0^2 e^{2\frac{\log 2}{L}z}\,dz = -P - \gamma r_0^2 \pi \frac{\left(e^{\frac{2\log 2}{L}x} - 1\right)}{2\frac{\log 2}{L}}$$

$$\rightarrow \boxed{N(x) = -P - \gamma L r_0^2 \frac{\pi}{2\log 2}\left(e^{\frac{2\log 2\,x}{L}} - 1\right)}$$

$$N(0) = -P$$

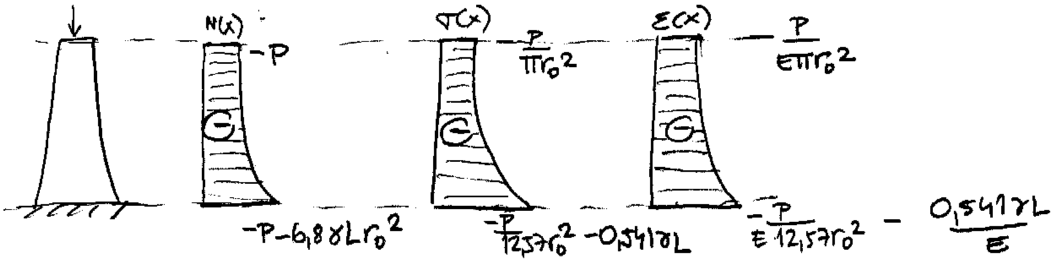
$$N(L) = -P - 6{,}8\,\gamma L r_0^2$$

$$\boxed{\sigma(x) = \frac{N(x)}{A(x)} = -\frac{P e^{-\frac{2\log 2\,x}{L}}}{\pi r_0^2} - \frac{\gamma L}{2\log 2}\left(1 - e^{-\frac{2\log 2\,x}{L}}\right)}$$

$$\sigma(0) = -\frac{P}{\pi r_0^2}$$

$$\sigma(L) = -\frac{P}{\pi 4 r_0^2} - \frac{6{,}80\,\gamma L r_0^2}{\pi 4 r_0^2} = -\frac{P}{12{,}57 r_0^2} - 0{,}541\,\gamma L$$

$$\varepsilon(x) = \sigma(x)/E$$

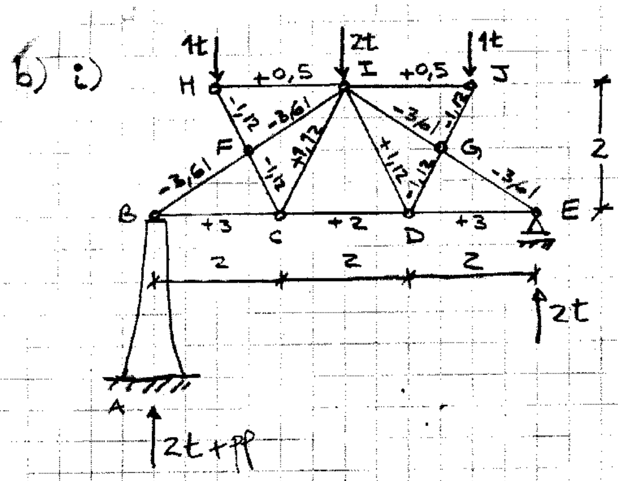


(ii)

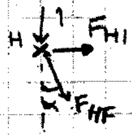
$$\Delta L = \int_0^L \varepsilon(x)\,dx = \int_0^L \frac{1}{E}\left[\frac{-P e^{-\frac{2\log 2\,x}{L}}}{\pi r_0^2} - \frac{\gamma L}{2\log 2}\left(1 - e^{-\frac{2\log 2\,x}{L}}\right)\right]dx$$

$$\boxed{\Delta L = -0{,}172\,\frac{PL}{r_0^2 E} - 0{,}331\,\frac{\gamma L^2}{E}}$$

b) i)

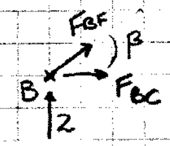


$sen\,\alpha = \frac{1}{\sqrt{5}}$ $\cos\alpha = \frac{2}{\sqrt{5}}$



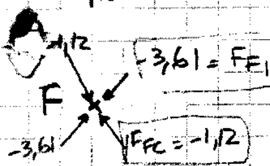
$$F_{HF} = \frac{-1}{\cos\alpha} = -\frac{\sqrt{5}}{2} = -1,12$$

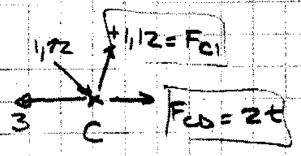
$$F_{HI} = -F_{HF}\,sen\,\alpha = \frac{\sqrt{5}}{2} \times \frac{1}{\sqrt{5}} = 0,5$$



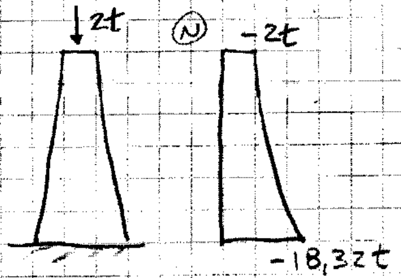
$$F_{BF} = \frac{-2}{sen\,\beta} = \frac{-2}{2/\sqrt{13}} = -\sqrt{13} = -3,616\ F_{BF}$$

$$F_{BC} = -F_{BF}\cos\beta = \sqrt{13} \times \frac{3}{\sqrt{13}} = 3t = F_{BC}$$





$$F_{CD} = 3 - \frac{\sqrt{5}}{2} \times 2 \times \frac{1}{\sqrt{5}} = 2$$



$$P + \gamma L r_0^2 \times 6,80 = 2t + 2,5\,t/m^3 \times 6m \times (0,4m)^2 \times 6,8 =$$
$$= 18,32\ t$$

(ii) 2PNC 10 ([ ]) : $A = 2 \times 13,50 = 27\,cm^2$

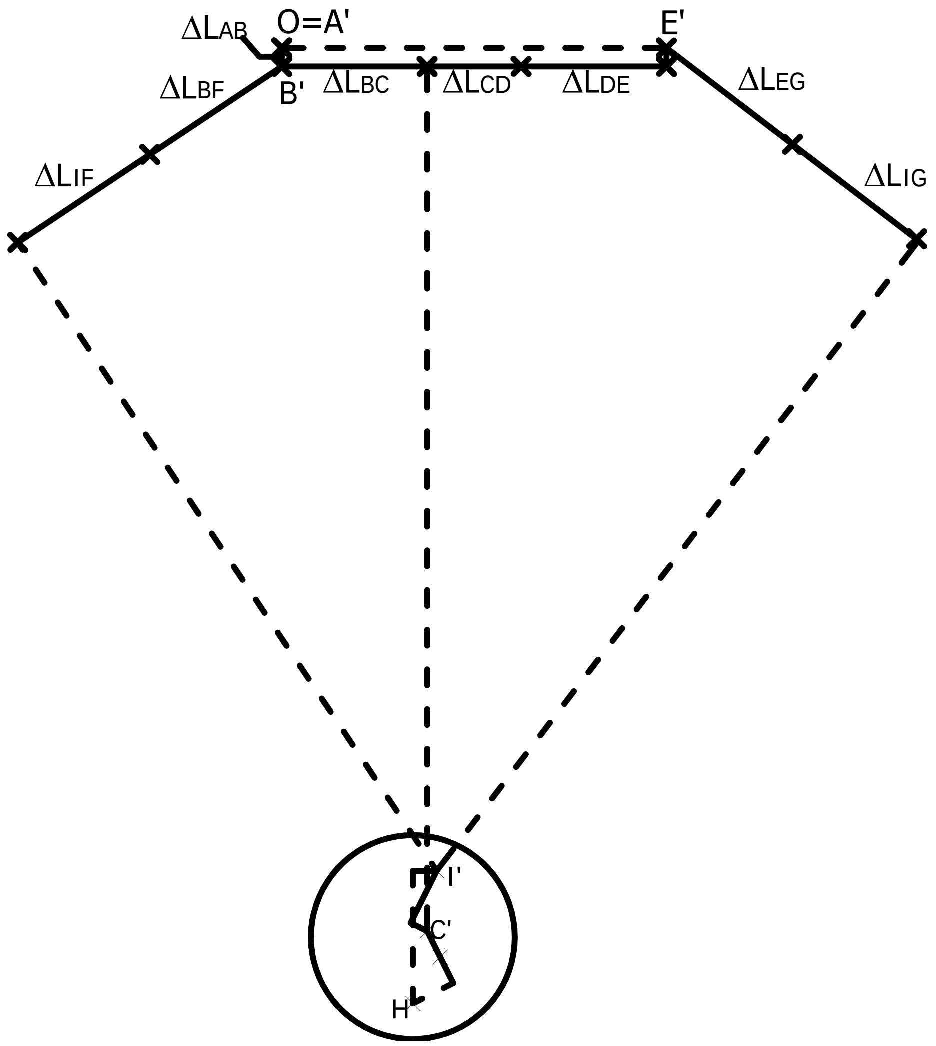
$EA = 2100\,t/cm^2 \times 27\,cm^2 = 56700\,t$

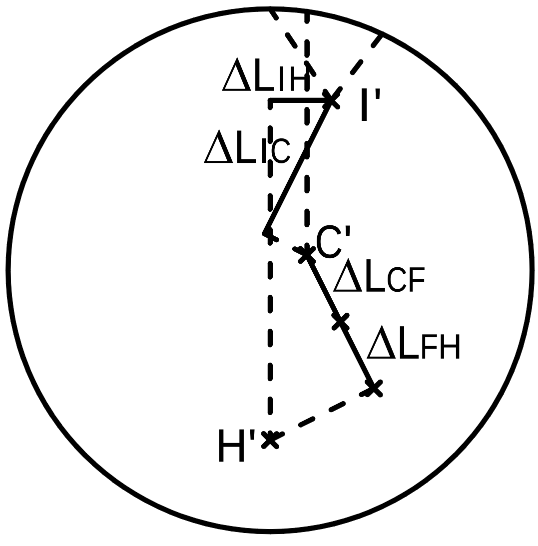
barra	F(t)	ℓ(cm)	EA(t)	Δℓ(cm)
BC	+3	200	56700	+0,011
BF=FI	-3,61	180	"	-0,012
HF=FC	-1,12	112	"	-0,0022
HI	+0,5	200	"	+0,0018
CD	+2	200	"	+0,0071
CI	+1,12	224	"	+0,0044

$$\Delta L_{AB} = \frac{-0,172 \times 2t \times 6m}{40\,cm^2 \times 300\,t/cm^2} - 0,331 \times \frac{2,5 \times 10^{-6}\,t/cm^3 \times (600\,cm)^2}{300\,t/cm^2} = -4,3 \times 10^{-4} - 9,93 \times 10^{-4}$$

$$\Delta L_{AB} = 0,0014\ cm$$

b) ii)





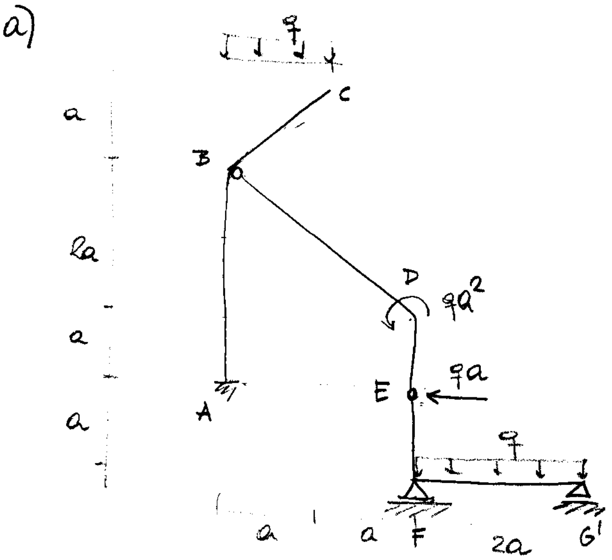
Si el punto B no bajara, los descensos de H y J serían iguales.
Como debo sumar un giro rígido a los descensos, el punto H bajará más que el J por lo que no necesito hallarlo.

$\delta_{máx}=\delta_H=0.072cm$

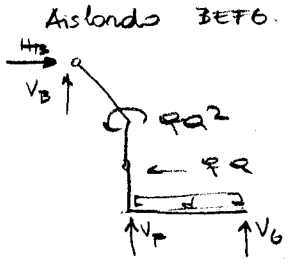
Solución - Examen diciembre 2004 – Resistencia de materiales 1 y 1N

Ejercicio 2

a)



Aislando BEFG.



$$M_E^d = 2a\cdot V_G - q\,2a\cdot a = 0$$

$$\boxed{V_G = qa}$$

$$M_B^d = 4a\cdot qa + qa^2 + 2a V_F - 3a\cdot qa - 2qa\cdot 3a = 0$$

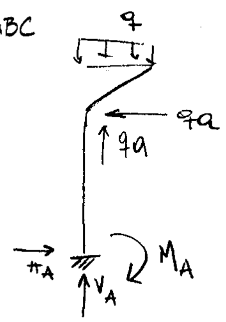
$$2aV_F = 4a^2 q \rightarrow \boxed{V_F = 2aq}$$

$$\Sigma H = 0 \rightarrow \boxed{H_B = qa}$$

$$\Sigma V = 2aq + qa + V_B - 2qa = 0$$

$$\boxed{V_B = -qa}$$

En ABC

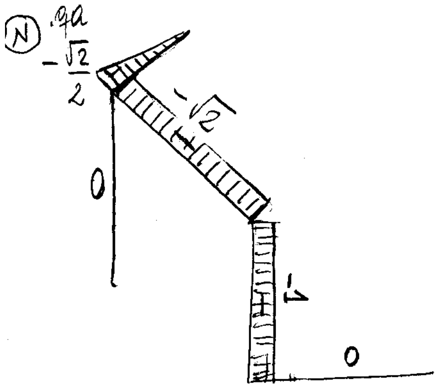


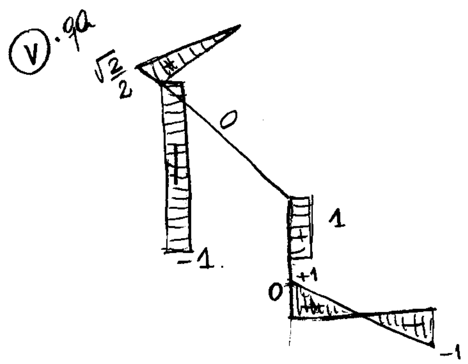
$$V_A = -qa + qa = 0$$

$$\boxed{H_A = qa}$$

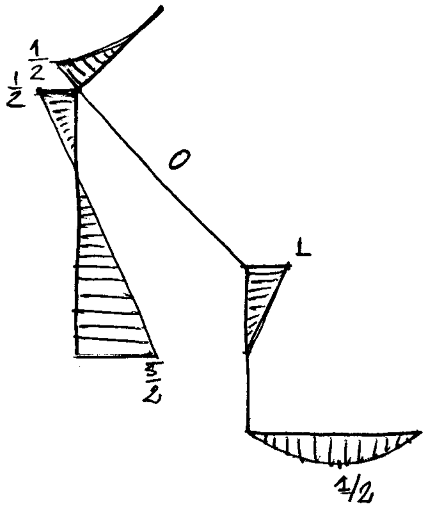
$$M_A = 3a\cdot qa - qa\cdot\frac{a}{2} = \frac{5}{2}qa^2$$

Diagramas

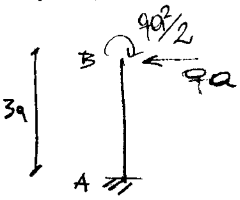




(M) · qa^2



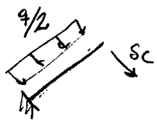
b) E, I



$$\vec{\delta}_B = \frac{qa^2}{2}\cdot\frac{(3a)^2}{2EI} - \frac{qa(3a)^3}{3EI} = \frac{qa^4}{EI}\left(\frac{9}{4}-9\right) = -\frac{27qa^4}{4EI}$$

$$\theta_B = \frac{qa^2}{2}\cdot\frac{3a}{EI} - \frac{qa(3a)^2}{2EI} = \frac{qa^3}{EI}\left(\frac{3}{2}-\frac{9}{2}\right) = -\frac{3qa^3}{EI}$$

Ahora BC



$$\downarrow\delta_C = \frac{\frac{q}{2}(\sqrt{2}a)^4}{8EI} = \frac{qa^4}{4EI}$$

$$\vec{\delta}'_C = \frac{\sqrt{2}qa^4}{8EI}$$

$$\downarrow\delta'_C = \frac{\sqrt{2}qa^4}{8EI}$$

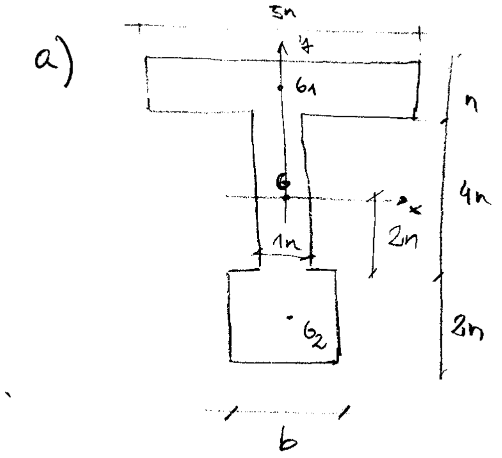
$$\boxed{\vec{\delta}_C^{TOT} = \vec{\delta}_B + \vec{\delta}'_C + \theta_B\cdot a = \frac{qa^4}{EI}\left(-\frac{27}{4} - 3 + \frac{\sqrt{2}}{8}\right) = -\frac{9{,}57qa^4}{EI}}$$

$$\boxed{\downarrow\delta_C^{TOT} = \downarrow\delta'_C + \theta_B a = \frac{qa^4}{EI}\left(-3 + \frac{\sqrt{2}}{8}\right) = -\frac{2{,}82qa^4}{EI}}$$

Solución - Examen diciembre 2004 – Resistencia de materiales 1 y 1N

Ejercicio 3 (R1)

a)

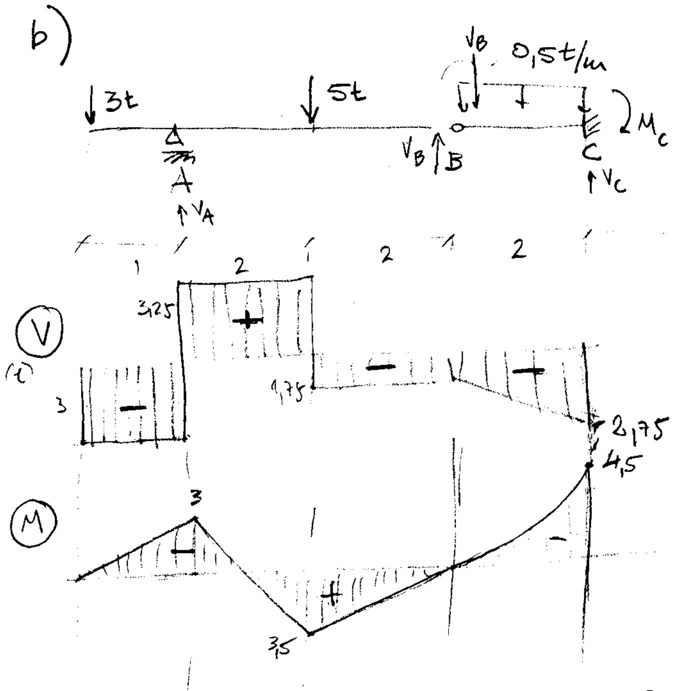


$$y_G = A_1 y_{G_1} + A_2 y_{G_2} = 0$$

$$y_G = 5n \cdot n \cdot \left(2n + \frac{n}{2}\right) - 2n\, b \cdot (2n + n) = 0$$

$$\Rightarrow \frac{25}{2} n^3 - 6n^2 b = 0 \rightarrow \boxed{b = \frac{25}{12} n}$$

b)



$$M_B^{iz} = 4 \cdot V_4 - 5 \cdot 2 - 5 \cdot 3 = 0$$

$$V_4 = 6{,}25t$$

$$V_B = 1{,}75\,t$$

En BC:

$$V_C = 0{,}5 \cdot 2 + 1{,}75 = 2{,}75t$$

$$M_C = 2 \cdot 0{,}5 \cdot 1 + 1{,}75 \cdot 2 = 4{,}5\, tm$$

$$M^{\oplus}_{máx} = 3{,}5tm = 350000 kgcm$$

$$M^{\ominus}_{máx} = 4{,}5tm = 450000 kgcm$$

$$I = \frac{5n\, n^3}{12} + 5n \cdot n\,(2{,}5n)^2 + \frac{n\,(4n)^3}{12} + \frac{25}{12} n \cdot \frac{(2n)^3}{12} + \frac{25}{12} n\, 2n\,(3n)^2$$

$$I = 75{,}9\, n^4$$

$$y_{sup} = 3n$$

$$y_{inf} = 4n$$

$$\sigma_{adm}^{tracción} = 80 kg/cm^2$$

$$\sigma_{adm}^{compr} = 100 kg/cm^2$$

TRACCIÓN - no sé cuál es más crítica

Sup: $\frac{M^{\oplus} \cdot y_{sup}}{I} \leq \sigma_{adm}^{trac} \rightarrow n \geq 6{,}06\ cm$

inf: $\frac{M^{\ominus} \cdot y_{inf}}{I} \leq \sigma_{adm}^{trac} \rightarrow n \geq 6{,}13\ cm$

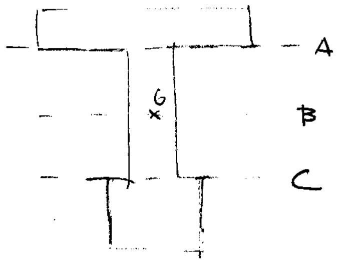
COMPRESIÓN - la inf. es más crítica

inf: $\frac{M^{\oplus} y_{inf}}{I} \leq \sigma_{adm}^{comp} \rightarrow n \geq 6{,}2\ cm$

$$\boxed{n = 6{,}2 cm}$$

c) $V_{max} = 3{,}25\ t$

$I = 112\,152{,}4\ cm^4$

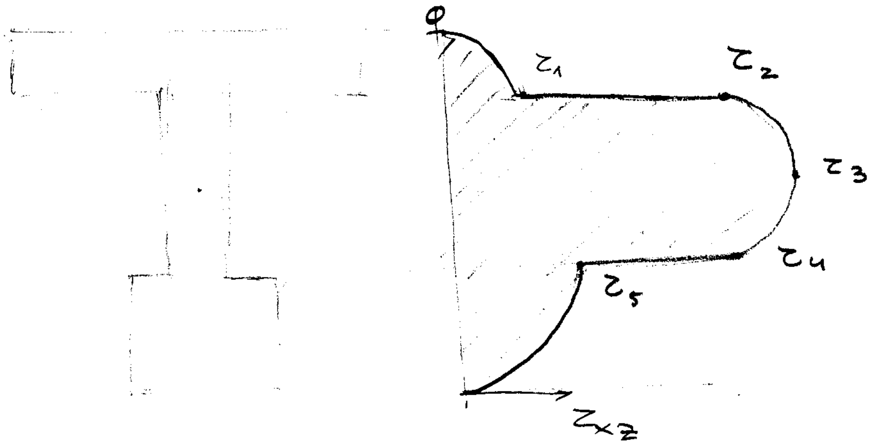


$\mu_A = 5 \cdot 6{,}2 \cdot 6{,}2 \cdot 2{,}5 \cdot 6{,}2 = 2979{,}1\ cm^3$

$\mu_B = 6{,}2^3 [5 \cdot 1 \cdot 2{,}5 + 2 \cdot 1 \cdot 1] = 3455{,}76\ cm^3$

$\mu_C = 6{,}2^3 [5 \cdot 1 \cdot 2{,}5 + 4 \cdot 1 \cdot 0] = 2979{,}1\ cm^3$

$\tau = \frac{V\mu}{Ib}$



$\tau_1 = \frac{3250 \cdot 2979{,}1}{112152{,}4 \cdot 5 \cdot 6{,}2} = 2{,}78\ kg/cm^2$

$\tau_2 = \frac{3250 \cdot 2979{,}1}{112152{,}4 \cdot 6{,}2} = 13{,}92\ kg/cm^2$

$\tau_3 = \frac{3250 \cdot 3455{,}76}{112152{,}4 \cdot 6{,}2} = 16{,}15\ kg/cm^2$

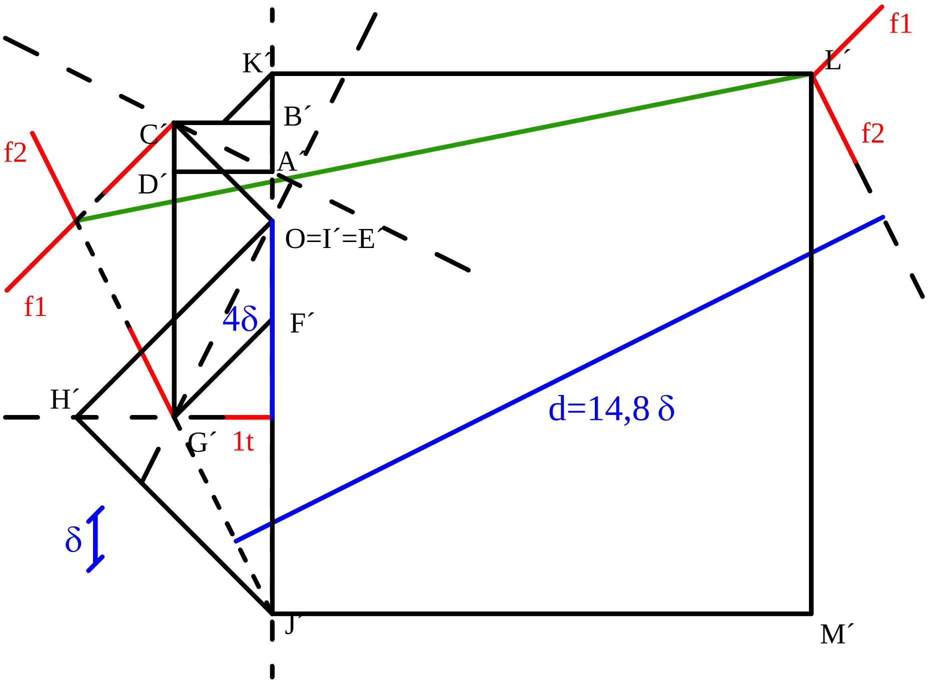
$\tau_4 = \tau_2 = 13{,}92\ kg/cm^2$

$\tau_5 = \frac{3250 \cdot 2979{,}1}{112152{,}4 \cdot \frac{25}{12} \cdot 6{,}2} = 6{,}68\ kg/cm^2$

<u>**Solución - Examen diciembre 2004 – Resistencia de materiales 1 y 1N**</u>

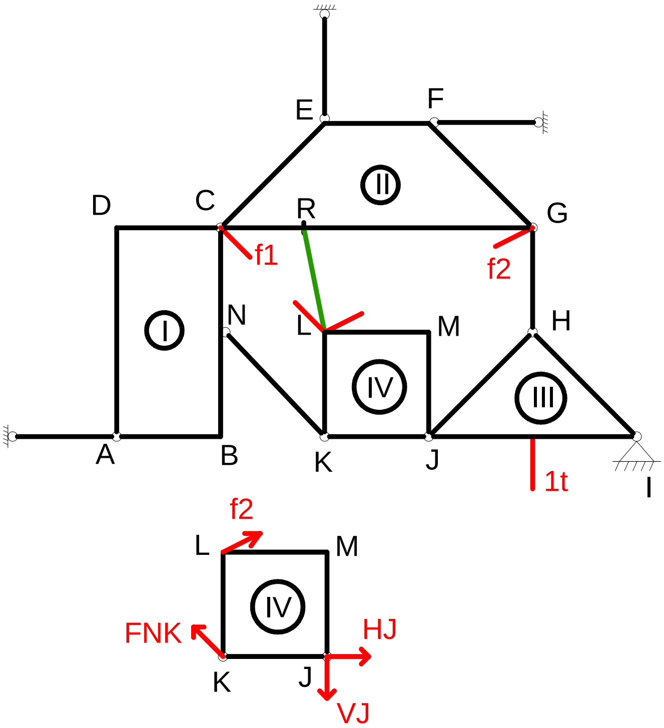
Ejercicio 3 (R1)

a) Cond. necesaria: $3D=1B+2A+3S+V_{AT}$
$3.4=1.2+2.2+0+5$
$12>11$ => el sist. no es invariante.



Palanca de Yucovski
Impongo un pequeño giro al disco III
$\delta_J=8$.
El orden en que determino los puntos es el siguiente: K´,A´,L´,M´,J´,E´,B´,F´, G´,H´,I´,C´,D.
Ubico los puntos donde giran las fuerzas.
b1) Una biela en LR hace al sistema invariante
CR= 0.8a
b2) La biela en LG realizaría el menor esfuerzo

diagrama de desplazamientos



c) PTV d=14.8 δ
$4\delta \,.\, 1t. + f2 \,.\, 14{,}8\delta = 0$
f2 = -0,27t

$\Sigma M_J=0$ =>
$-0{,}27t.0{,}45.a - 0{,}27t.0{,}89.a + F_{NK}.\frac{a}{\sqrt{2}} = 0$

$\mathbf{F_{NK}=0{,}5t}$
$\Sigma M_I=0$ =>
$-0{,}27t.0{,}45.3a - 0{,}27t.0{,}89.a +$
$+ 0{,}5t.\frac{3a}{\sqrt{2}} + F_{HG}.a - 1t.a=0$
$\mathbf{F_{HG}= 0{,}54t}$
$\Sigma H^{IV}=0$
$-0{,}27t.0{,}89 - 0{,}5t/\sqrt{2} + H_J=0$
$H_J=0.59t$
$\Sigma V^{IV}=0$
$0{,}27t.0{,}45 - 0{,}5t/\sqrt{2} + V_J=0$
$V_J=-0{,}23t$
$\mathbf{R_J=0{,}63t}$