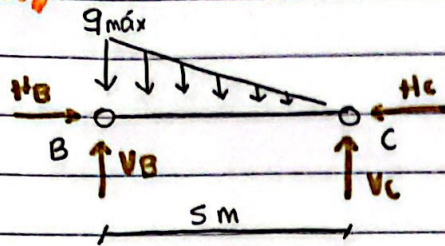


Ejercicio 1

a)



$$\cdot \sum V = 0 \Leftrightarrow V_B + V_C = q_{\max} \cdot \frac{5m}{2}$$

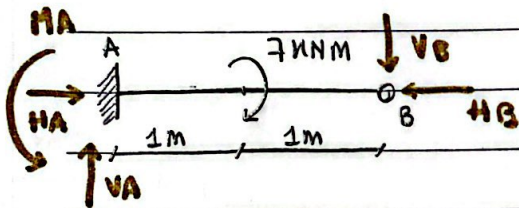
$$\cdot \sum H = 0 \Leftrightarrow H_B = H_C$$

$$\cdot \sum M_C = 0 \Leftrightarrow V_B \cdot 5m = q_{\max} \cdot 2,5m \cdot \frac{2 \cdot 5m}{3}$$

$$V_B = q_{\max} \cdot \frac{5m}{3}$$

$$\Rightarrow V_C = q_{\max} \cdot \frac{5m}{2} - V_B$$

$$V_C = q_{\max} \cdot \frac{5m}{6}$$



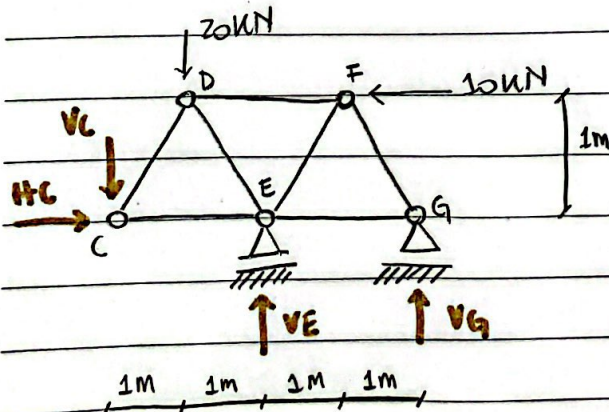
$$\cdot \sum V = 0 \Leftrightarrow V_A = V_B$$

$$V_A = q_{\max} \cdot \frac{5m}{3}$$

$$\cdot \sum H = 0 \Leftrightarrow H_A = H_B$$

$$\cdot \sum M_A = 0 \Leftrightarrow M_A = 7kNm + V_B \cdot 2m = 7kNm + q_{\max} \cdot \frac{10m^2}{3}$$

$$M_A = 7kNm + q_{\max} \cdot \frac{10m^2}{3}$$



$$\cdot \sum V = 0 \Leftrightarrow V_E + V_G = 20kN + V_C$$

$$V_E + V_G = 20kN + q_{\max} \cdot \frac{5m}{6}$$

$$\cdot \sum H = 0 \Leftrightarrow H_C = 10kN \quad (*)$$



$$\cdot \Sigma M_G = 0 \Leftrightarrow 10 \text{ kN} \cdot 1 \text{ m} + 20 \text{ kN} \cdot 3 \text{ m} + V_c \cdot 4 \text{ m} = V_E \cdot 2 \text{ m}$$

$$70 \text{ kNm} + q_{\text{máx}} \cdot \frac{10}{3} \text{ m}^2 = V_E \cdot 2 \text{ m}$$

$$\Rightarrow V_E = 35 \text{ kN} + q_{\text{máx}} \cdot \frac{5}{3} \text{ m}$$

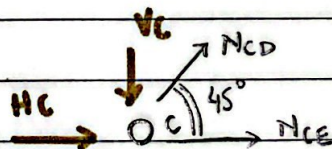
$$\Rightarrow V_G = 20 \text{ kN} + q_{\text{máx}} \cdot \frac{5}{6} \text{ m} - 35 \text{ kN} - q_{\text{máx}} \cdot \frac{5}{3} \text{ m}$$

$$\Rightarrow V_G = -15 \text{ kN} - q_{\text{máx}} \cdot \frac{5}{6} \text{ m}$$

Como $H_c = 10 \text{ kN}$ por $\odot \Rightarrow H_A = H_B = H_c = 10 \text{ kN}$

b) Equilibrio en nodo c para obtener N_{ce} :

NODO C:



$$\cdot \Sigma H = 0 \Leftrightarrow H_c + \frac{N_{cd}}{\sqrt{2}} + N_{ce} = 0 \rightarrow N_{ce} = -H_c - \frac{N_{cd}}{\sqrt{2}} \quad (1)$$

$$\cdot \Sigma V = 0 \Leftrightarrow V_c = \frac{N_{cd}}{\sqrt{2}} \quad (2) \rightarrow N_{ce} = -H_c - V_c = -10 \text{ kN} - q_{\text{máx}} \cdot \frac{5}{6} \text{ m}$$

$$N_{cd} = V_c \cdot \sqrt{2} = q_{\text{máx}} \cdot \frac{5}{6} \text{ m} \cdot \sqrt{2}$$

para que $N_{ce} = -10 \text{ kN} - q_{\text{máx}} \cdot \frac{5}{6} \text{ m} = -20 \text{ kN}$

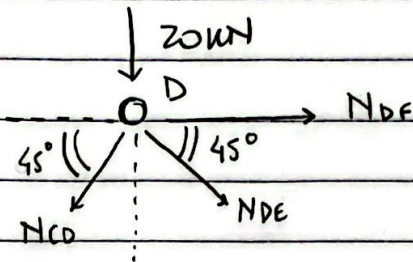
$$\Rightarrow q_{\text{máx}} = 12 \text{ kN/m}$$

tilibra

e) Para diagrama de directa (N): tomando $q_{m\bar{x}} = 12 \text{ kN/m}$

NODO C $\rightarrow N_{CE} = -20 \text{ kN}$
 $\rightarrow N_{CD} = 12 \text{ kN/m} \cdot \frac{5 \text{ m}}{6} \cdot \sqrt{2} \approx 14,14 \text{ kN}$

NODO D:



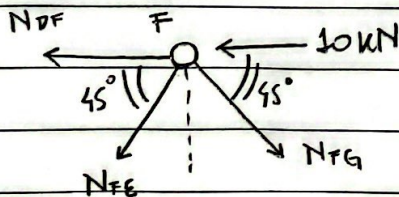
$\cdot \sum V = 0 \Leftrightarrow 20 \text{ kN} + \frac{N_{CD}}{\sqrt{2}} + \frac{N_{DE}}{\sqrt{2}} = 0$

$\hookrightarrow N_{DE} = -20 \text{ kN} \cdot \sqrt{2} - N_{CD} =$
 $= -20 \text{ kN} \cdot \sqrt{2} - 10 \text{ kN} \cdot \sqrt{2} =$
 $= -30 \text{ kN} \cdot \sqrt{2} \approx -42,43 \text{ kN}$

$\cdot \sum H = 0 \Leftrightarrow N_{DF} + \frac{N_{DE}}{\sqrt{2}} = \frac{N_{CD}}{\sqrt{2}} \rightarrow N_{DF} = \frac{N_{CD} - N_{DE}}{\sqrt{2}}$

$\hookrightarrow N_{DF} = \frac{10 \text{ kN} \cdot \sqrt{2}}{\sqrt{2}} - \frac{(-30 \text{ kN}) \cdot \sqrt{2}}{\sqrt{2}} = 40 \text{ kN}$

NODO F:



$\cdot \sum V = 0 \Leftrightarrow \frac{N_{FE}}{\sqrt{2}} + \frac{N_{FG}}{\sqrt{2}} = 0 \rightarrow N_{FE} = -N_{FG}$



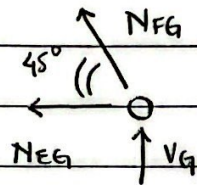
$$\cdot \sum H = 0 \Leftrightarrow N_{DF} + \frac{N_{FE}}{\sqrt{2}} + 10 \text{ kN} = \frac{N_{FG}}{\sqrt{2}}$$

$$40 \text{ kN} + 10 \text{ kN} = \frac{N_{FG}}{\sqrt{2}} + \frac{N_{FG}}{\sqrt{2}}$$

$$\frac{50 \text{ kN}}{\sqrt{2}} = N_{FG} \Rightarrow N_{FE} = -35,36 \text{ kN}$$

35,36 kN

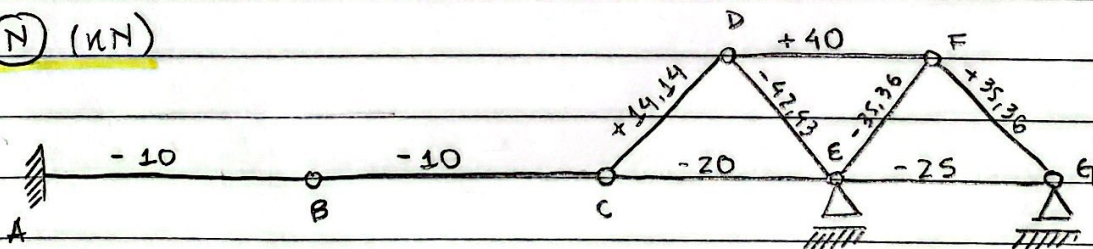
NODO G:



$$\cdot \sum H = 0 \Leftrightarrow \frac{N_{FG}}{\sqrt{2}} + N_{EG} = 0 \longrightarrow N_{EG} = -\frac{N_{FG}}{\sqrt{2}}$$

$$N_{EG} = -25 \text{ kN}$$

(N) (kN)



para diagrama de cortante (V) obtengo el valor de las reacciones para $q_{\text{máx}} = 12 \text{ kN/m}$:

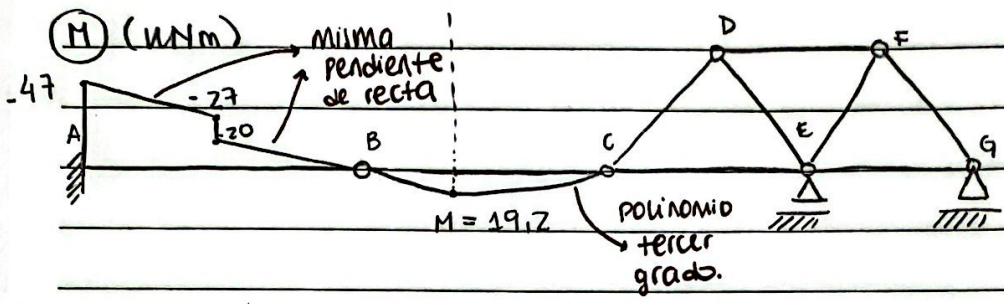
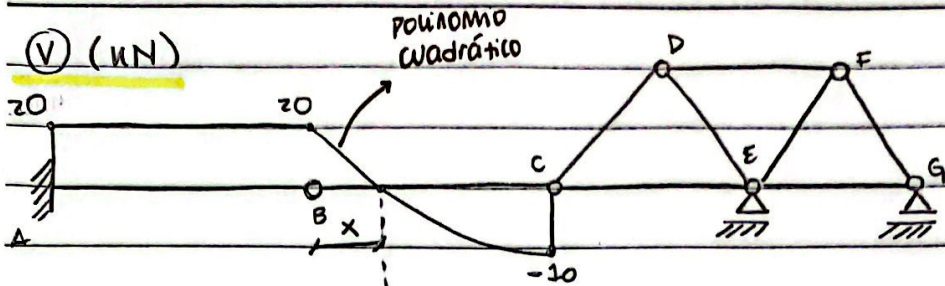
$$\cdot V_A = 20 \text{ kN} \uparrow$$

$$\cdot V_E = 55 \text{ kN} \uparrow$$

$$\cdot H_A = 10 \text{ kN} \rightarrow$$

$$\cdot V_G = 25 \text{ kN} \downarrow$$

$$\cdot M_A = 47 \text{ kN} \cdot \text{m}$$



para obtener x: $20 \text{ kN} - \left(12 \text{ kN/m} + 12 \text{ kN/m} - \frac{12x \text{ kN}}{5} \right) \cdot \frac{x}{2} = 0$

$$20 - 12x + \frac{6}{5}x^2 = 0$$

↳ Bhāskara:

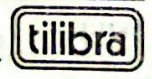
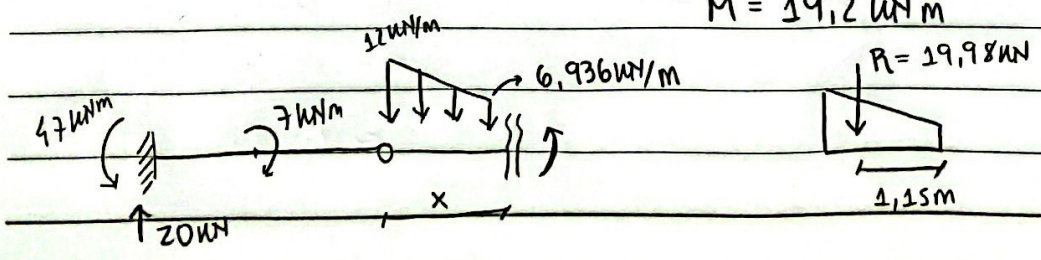
$$x = \frac{12 \pm \sqrt{144 - 4 \cdot \frac{6}{5} \cdot 20}}{2 \cdot \frac{6}{5}}$$

x = $\begin{cases} 7,89 \text{ m} \\ 2,11 \text{ m} \end{cases}$ descartado por ser superior a 5m.

$x = 2,11 \text{ m}$

para obtener M: $M + 47 \text{ kN} - 20 \text{ kN} \cdot 4,11 \text{ m} - 7 \text{ kNm} + 19,98 \cdot 1,15 \text{ m} = 0$

↳ $M \approx 19,2 \text{ kNm}$



Ejercicio 2

Parte A

- Reacciones en apoyo:

$$\sum F_H = 0 \Rightarrow H_F = 100kN \leftarrow$$

$$\sum M_F = 0 \Rightarrow 8m \times V_A + 2m \times 100kN = (4m + 2m) \times 100kN + 8m \times 50kN \Rightarrow V_A = 100kN \uparrow$$

$$\sum F_V = 0 \Rightarrow V_F = 250kN - V_A \Rightarrow V_F = 150kN \uparrow$$

- Por condiciones geométricas

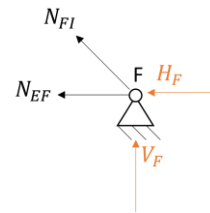
$$N_{DH} = 0kN$$

$$N_{AB} = -V_A = -100kN$$

- Nodo F

$$\sum F_V = 0 \Rightarrow V_F + \frac{\sqrt{2}}{2} N_{FI} = 0 \Rightarrow N_{FI} = -212kN$$

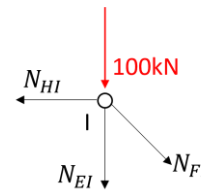
$$\sum F_H = 0 \Rightarrow N_{EF} + 100kN + \frac{\sqrt{2}}{2} N_{FI} = 0 \Rightarrow N_{EF} = 50kN$$



- Nodo I

$$\sum F_H = 0 \Rightarrow N_{HI} = \frac{\sqrt{2}}{2} N_{FI} = -150kN$$

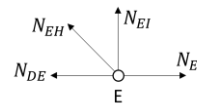
$$\sum F_V = 0 \Rightarrow 100kN + N_{EI} + \frac{\sqrt{2}}{2} N_{FI} = 0 \Rightarrow N_{EI} = 50kN$$



- Nodo E

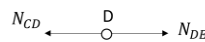
$$\sum F_V = 0 \Rightarrow N_{EI} + \frac{\sqrt{2}}{2} N_{EH} = 0 \Rightarrow N_{EH} = -71kN$$

$$\sum F_H = 0 \Rightarrow N_{EF} = N_{DE} + \frac{\sqrt{2}}{2} N_{EH} \Rightarrow N_{DE} = 100kN$$



- Nodo D

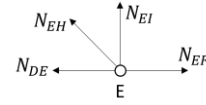
$$\sum F_H = 0 \Rightarrow N_{CD} = N_{DE} = 100kN$$



- **Nodo H**

$$\sum F_V = 0 \Rightarrow 100kN + \frac{\sqrt{2}}{2} N_{EH} + \frac{\sqrt{2}}{2} N_{CH} = 0 \Rightarrow N_{CH} = -71kN$$

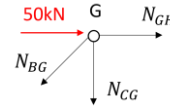
$$\sum F_H = 0 \Rightarrow N_{HI} + \frac{\sqrt{2}}{2} N_{EH} = N_{GH} + \frac{\sqrt{2}}{2} N_{CH} \Rightarrow N_{GH} = -150kN$$



- **Nodo G**

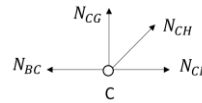
$$\sum F_H = 0 \Rightarrow 100kN + N_{GH} = \frac{\sqrt{2}}{2} N_{BG} \Rightarrow N_{BG} = -71kN$$

$$\sum F_V = 0 \Rightarrow \frac{\sqrt{2}}{2} N_{BG} + N_{CG} = 0 \Rightarrow N_{CG} = 50kN$$



- **Nodo C**

$$\sum F_H = 0 \Rightarrow N_{CD} + \frac{\sqrt{2}}{2} N_{CH} = N_{BC} \Rightarrow N_{BC} = 50kN$$



Parte B

$$N_{AB} = -100kN$$

$$\sigma_{adm} = 20MPa; E = 11GPa; u_{adm} = 1,5mm$$

$$\sigma_{max} = \frac{100kN}{\Omega}; \sigma_{min} = \frac{100kN}{2\Omega} \Rightarrow l_{min} = 8cm$$

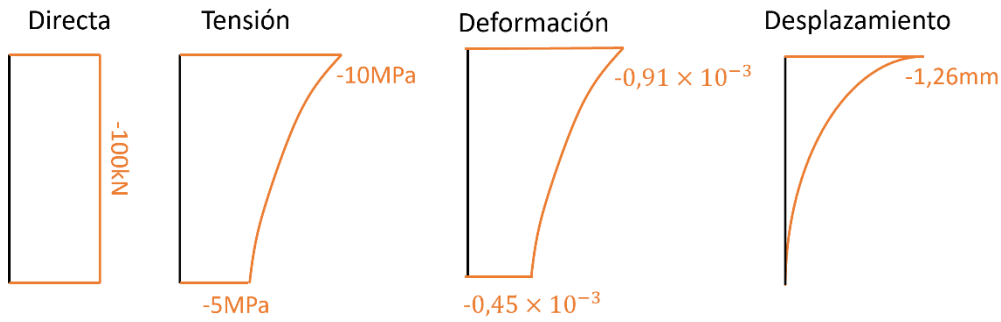
$$\varepsilon_{max} = \frac{100kN}{\Omega \times 11GPa}; \varepsilon_{min} = \frac{100kN}{2A \times 11GPa}$$

$$u = \int_0^l \frac{N}{E(\Omega + \frac{\Omega z}{l})} dz = \frac{Nl}{E\Omega} \int_0^1 \frac{1}{(1+z)} dz = \frac{Nl}{E\Omega} \ln(1+z) \Big|_0^1 = \frac{Nl}{E\Omega} \ln(2) \Rightarrow l_{min} = 10cm$$

Entonces $l = 10cm$

$$\sigma_{max} = 10MPa; \quad \sigma_{min} = 5MPa; \quad \varepsilon_{max} = 0,91 \times 10^{-3}; \quad \varepsilon_{min} = 0,45 \times 10^{-3}; \quad u = 1,26mm$$

Parte C



Parte D

Compresión máxima: 212 kN

$$\text{Área necesaria: } \frac{N}{\sigma_{adm}} = 17,68\text{ cm}^2$$

$$A = r^2\pi - (r - e)^2\pi \Rightarrow r_{min} = 6\text{ cm} (A = 18,06\text{ cm}^2); e = 0,5\text{ cm}$$

Parte E

Tracción máxima: 100 kN

$$\text{Área necesaria: } \frac{N}{\sigma_{adm}} = 7,14\text{ cm}^2$$

$$A = l^2 - (l - 2e)^2 \Rightarrow l_{min} = 3\text{ cm} (A = 8,00\text{ cm}^2); e = 1\text{ cm}$$