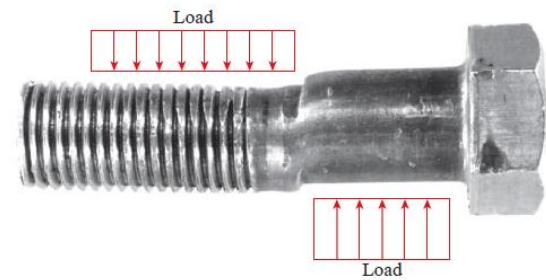
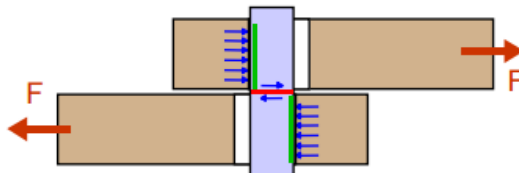
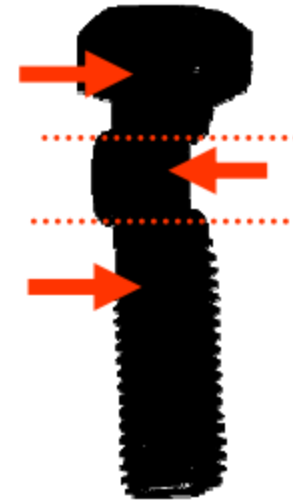
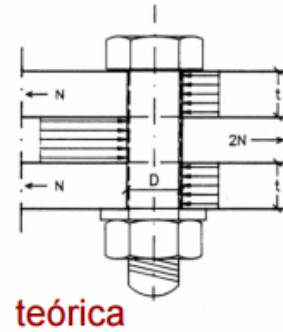
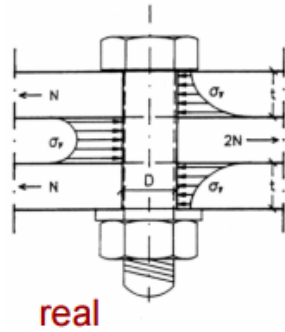
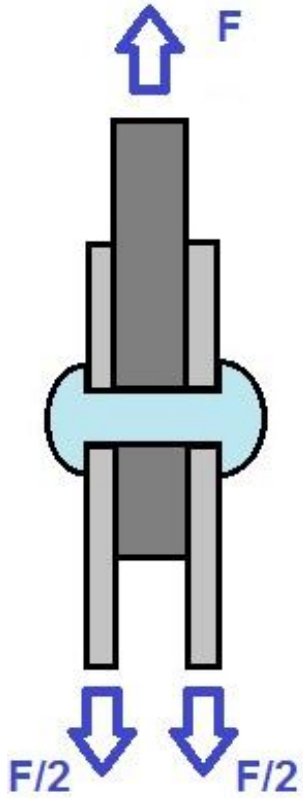
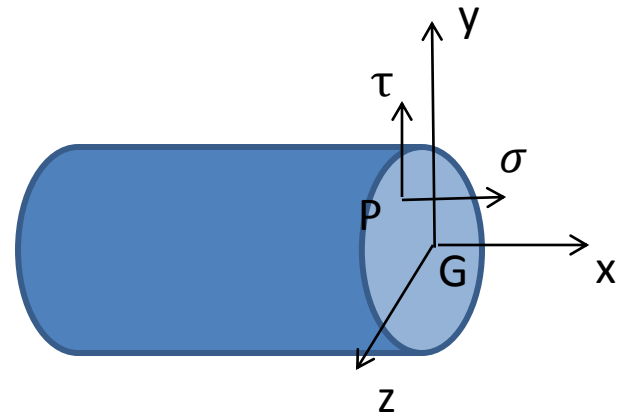
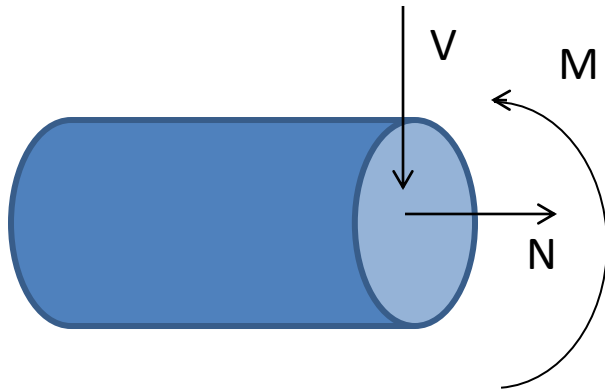
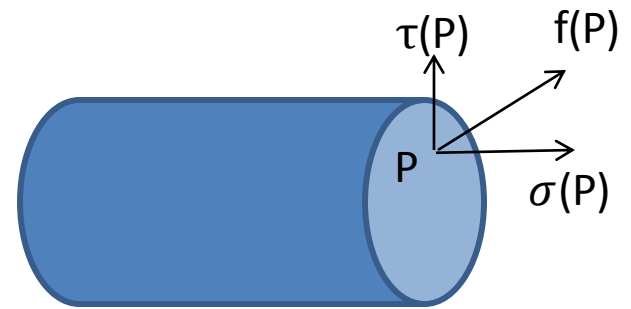
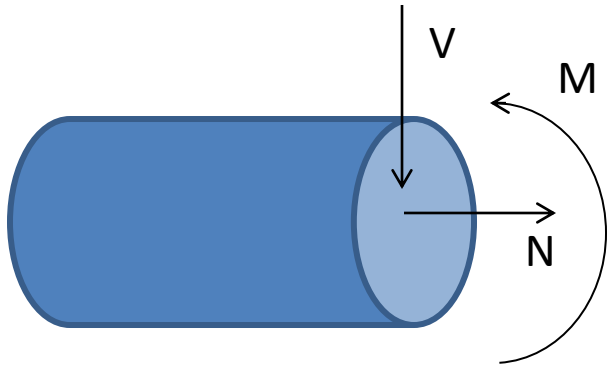


Tensiones Rasantes

Tensiones Rasantes



Tensiones internas

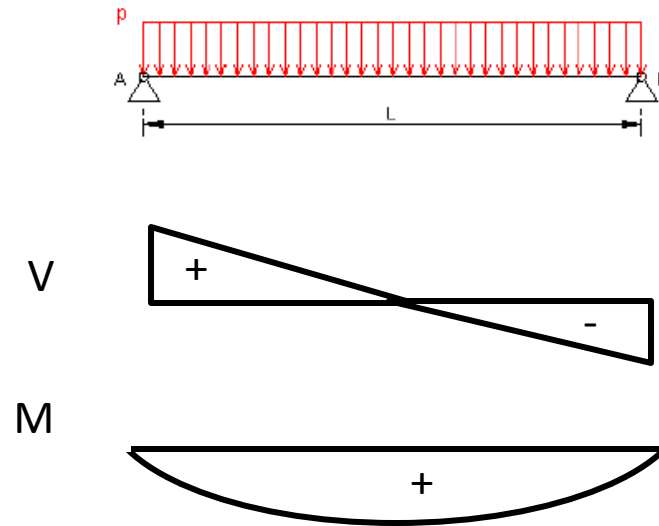
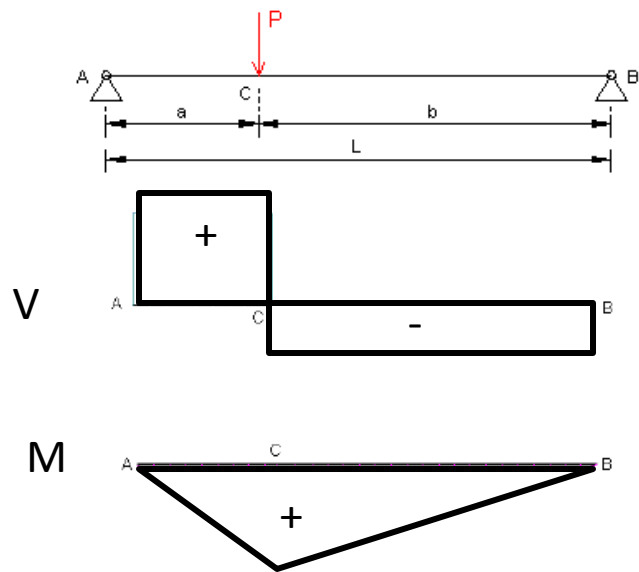


$$N = \int_{\Omega} \sigma \, d\Omega$$

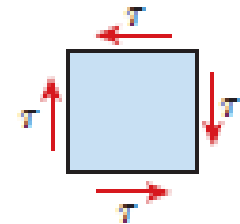
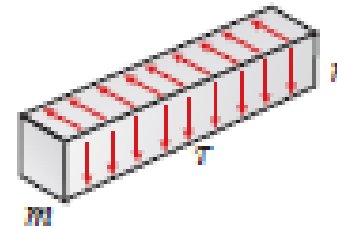
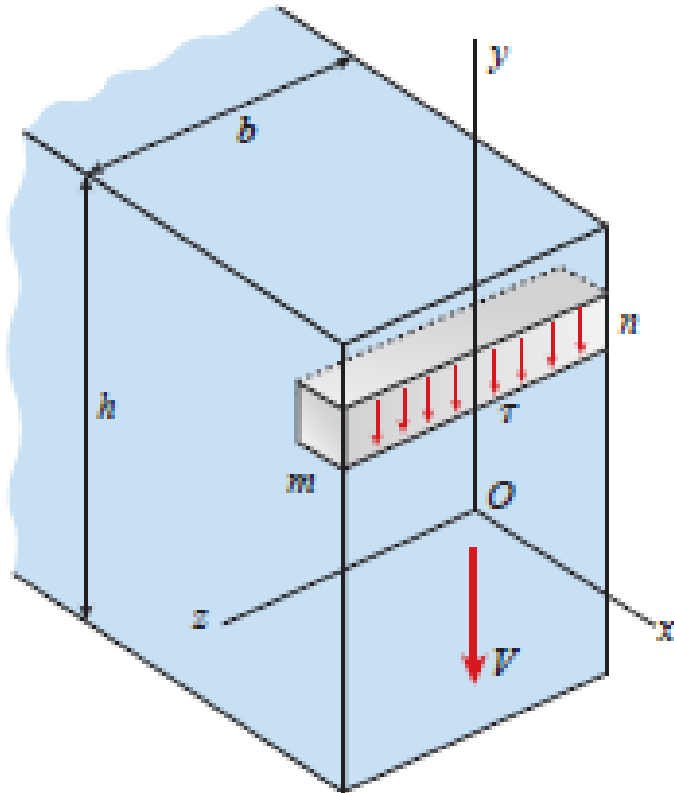
$$V = \int_{\Omega} -\tau \, d\Omega$$

$$M = \int_{\Omega} -\sigma * y \, d\Omega$$

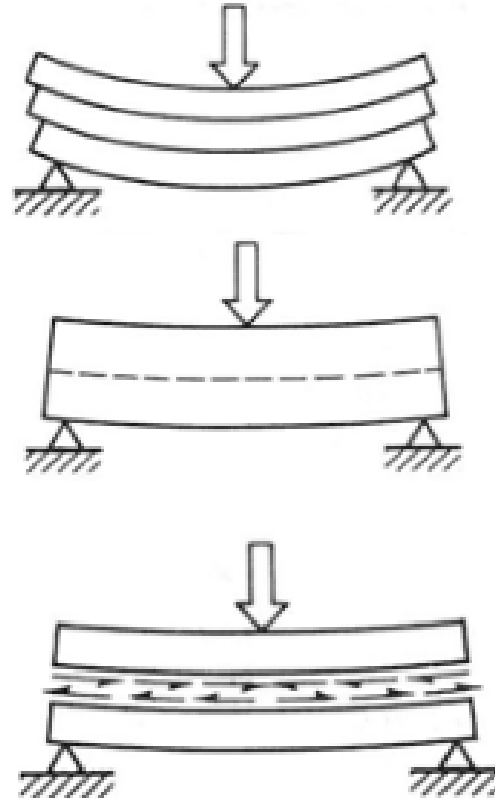
Tensiones internas



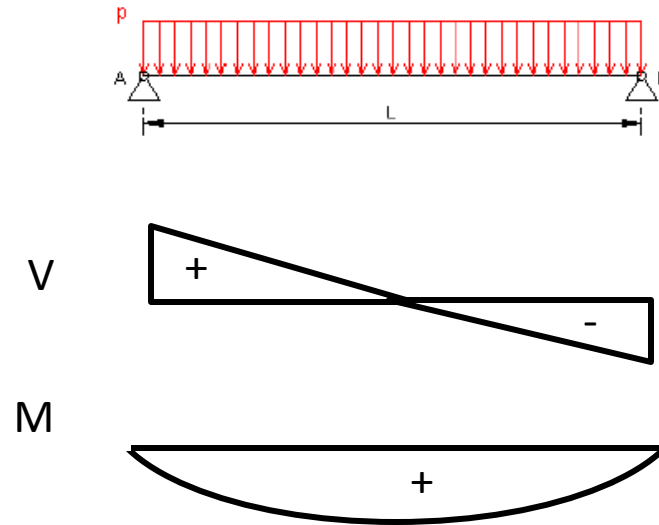
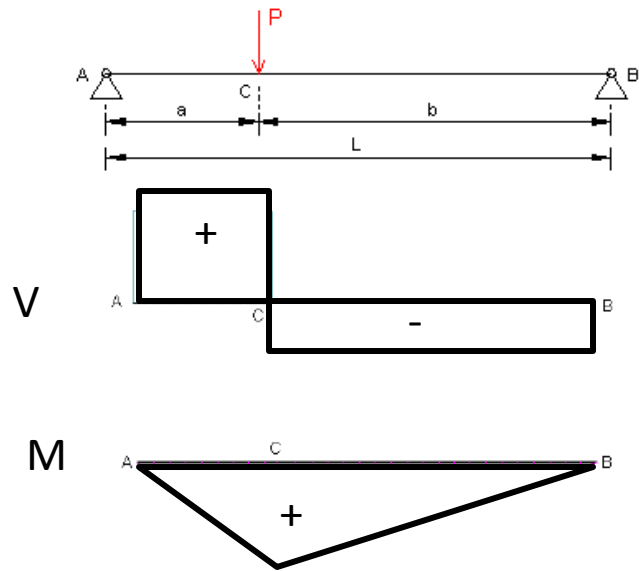
Tensiones rasantes en Vigas



Rasantes en vigas



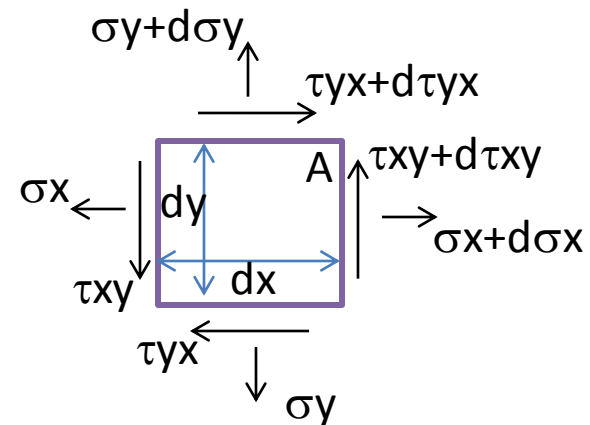
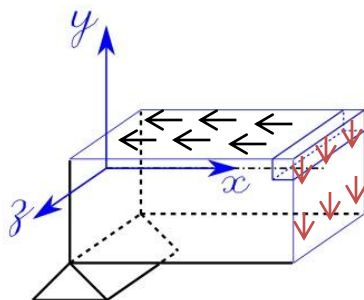
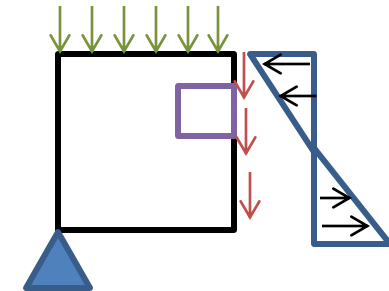
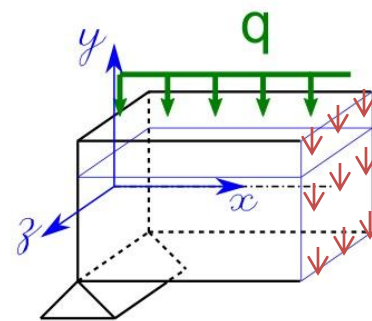
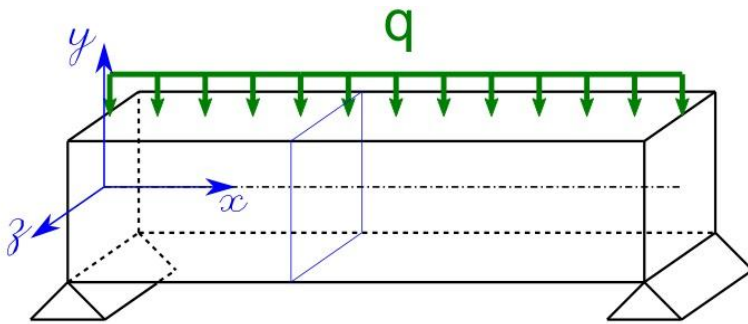
Tensiones internas

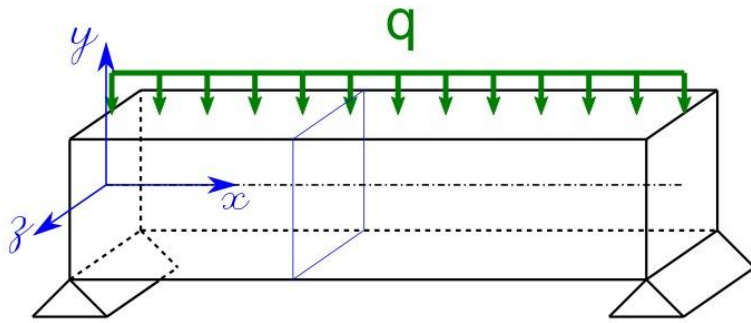


Reciprocidad de las Tensiones Rasantes

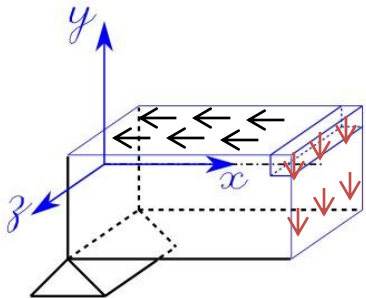
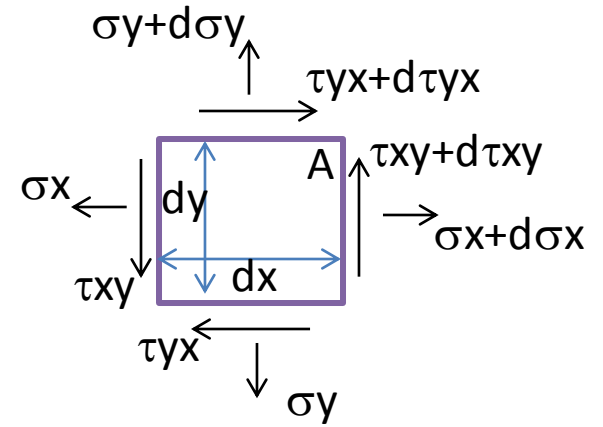
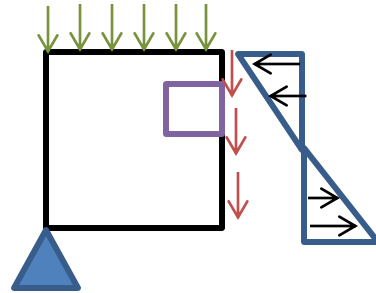
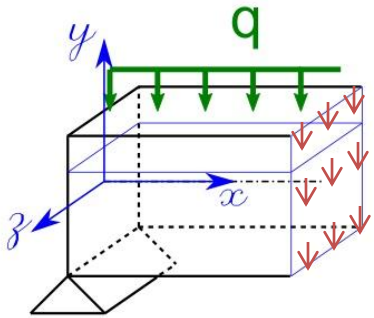
Hipótesis :

Las tensiones son uniformes en todo el ancho de la viga.

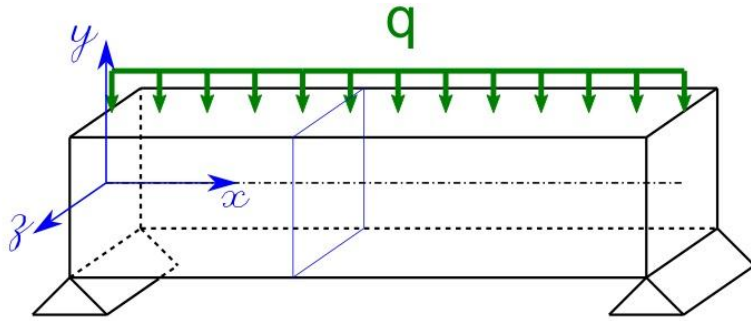




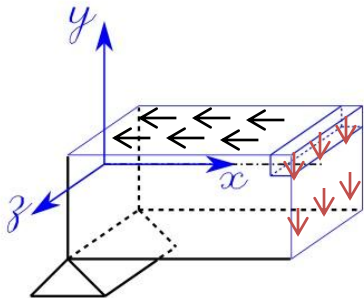
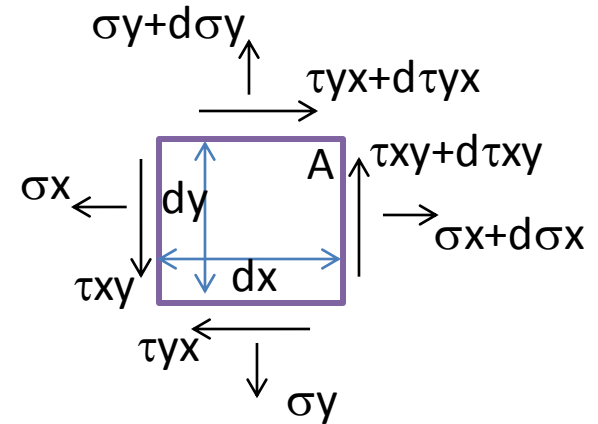
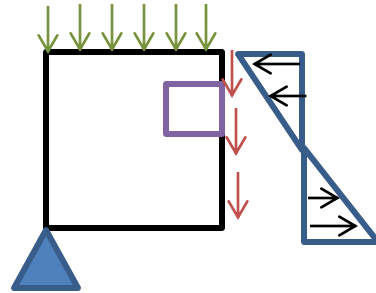
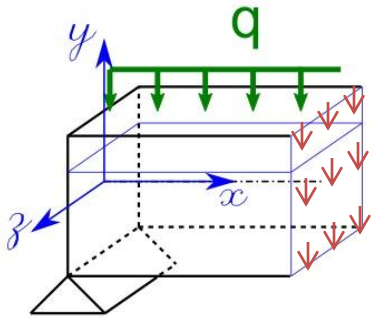
hipótesis :
Las tensiones son uniformes en todo el ancho de la viga.



Sum(MA)=0
 $\sigma_x \cdot dy \cdot dy / 2 - (\sigma_x + d\sigma_x) \cdot dy \cdot dy / 2 + \dots$



hipótesis :
Las tensiones son uniformes en todo el ancho de la viga.



$$\text{Sum}(MA)=0$$

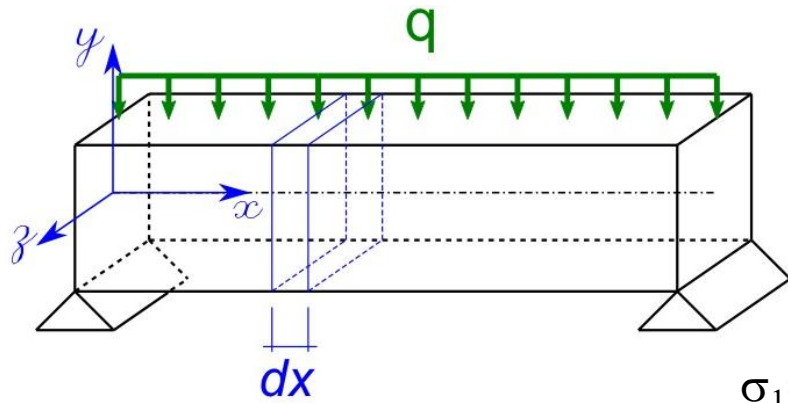
$$\sigma_x \cdot dy \cdot dy / 2 - (\sigma_x + d\sigma_x) \cdot dy \cdot dy / 2 + (\sigma_y + d\sigma_y) \cdot dx \cdot dx / 2 - \sigma_y \cdot dx \cdot dx / 2 + \tau_{yx} \cdot dx \cdot dy - \tau_{xy} \cdot dx \cdot dy = 0 \text{ despreciando terminos de 3er orden}$$

$$\tau_{yx} \cdot dx \cdot dy - \tau_{xy} \cdot dx \cdot dy = 0$$

$$\rightarrow \tau_{yx} = \tau_{xy}$$

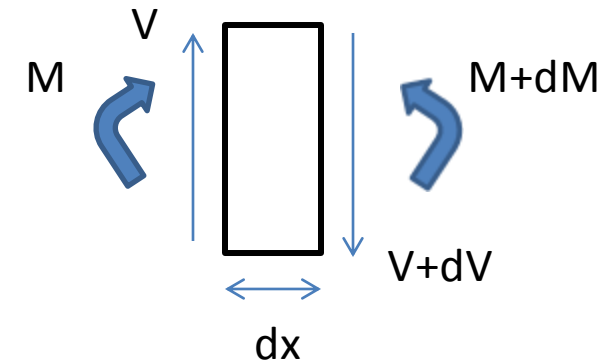
Se concluye que las **tensiones rasantes** sobre **caras adyacentes** de un elemento infinitesimal **son iguales en magnitud**, y tienen sentidos tales que **ambos** esfuerzos se dirigen **hacia el vértice común**, o **ambos esfuerzos se alejan de él**.

Jourawski

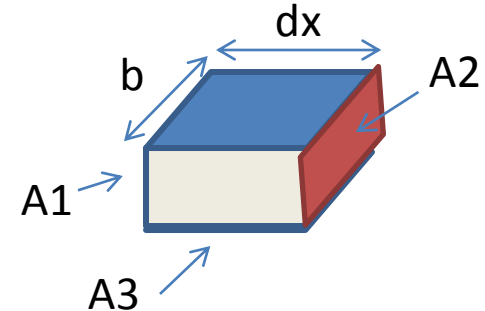
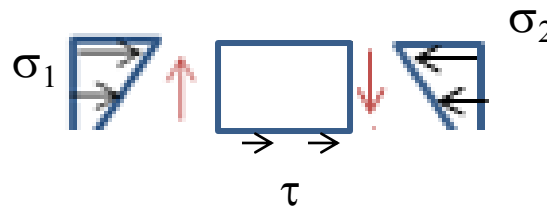
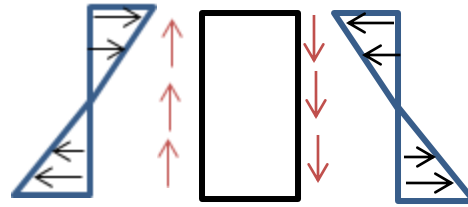


Fórmula de Jourawski:

$$\tau = \frac{V \cdot \mu}{I_{zG} \cdot b}$$



$$\sigma_1 = M \cdot y / I_{zG} \quad \sigma_2 = (M + dM) \cdot y / I_{zG}$$



$$\text{Suma(FH)} = \int_{A1} \sigma_1 dA + \int_{A3} \tau dA - \int_{A2} \sigma_2 dA = 0$$

$$\int_{A1} \frac{M y}{I_{zG}} dA + \tau \cdot b \cdot dx - \int_{A2} \frac{(M + dM) y}{I_{zG}} dA = 0$$

$$\int_{A1} \frac{My}{I} dA + \tau \cdot b \cdot dx - \int_{A2} \frac{(M + dM)y}{I} dA = 0$$

$$\tau \cdot b \cdot dx - \int_{A2} \frac{(dM)y}{I} dA = 0$$

$$\tau \cdot b \cdot dx - \frac{(dM)}{I} \int_{A2} y dA = 0 \quad \tau = \frac{dM}{dx \cdot b \cdot I} \int_{A2} y dA$$

$$\frac{dM}{dx} = V \quad \mu = \int_{A2} y dA$$

$$\tau = \frac{V \cdot \mu}{b \cdot I_{ZG}}$$

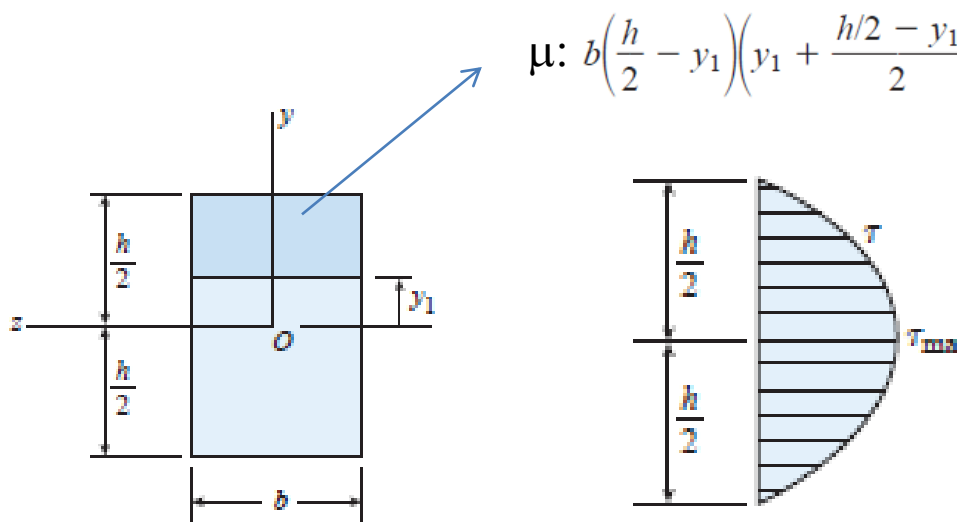
Ejemplo

Definición:
Flujo cortante

$$q = \frac{V \cdot \mu}{I}$$

El flujo cortante es la fuerza cortante horizontal por unidad de longitud en la dirección del eje de la barra.

Esta definición es más general. Para determinar q no es necesario imponer que las tensiones rasantes sean uniformes en el ancho de la sección



The diagram shows a rectangular cross-section of height h and width b . The vertical axis is y and the horizontal axis is z . The origin O is at the center. The top half of the section is shaded light blue. A horizontal line is drawn at a distance y_1 from the neutral axis. An arrow points from this line to the shear stress distribution diagram on the right. The shear stress distribution is a parabolic curve that is zero at the top and bottom surfaces and reaches its maximum value τ_{\max} at the neutral axis. The total height of the section is h , with $h/2$ above and below the neutral axis.

$$\mu: b \left(\frac{h}{2} - y_1 \right) \left(y_1 + \frac{h/2 - y_1}{2} \right) = \frac{b}{2} \left(\frac{h^2}{4} - y_1^2 \right) \quad \therefore \int y \, dA = \int_{y_1}^{h/2} y b \, dy = \frac{b}{2} \left(\frac{h^2}{4} - y_1^2 \right)$$
$$\tau_{\max} = \frac{V h^2}{8I} = \frac{3V}{2A}$$

Ancho variable / Ej.: PNI

