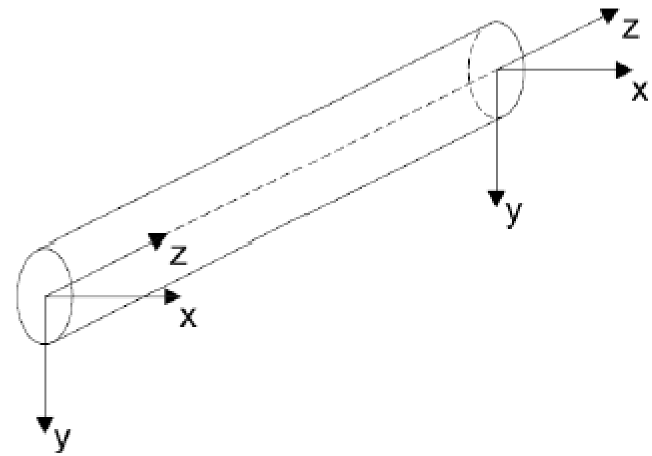
VIGAS RECTAS CONTINUAS, ECUACIONES ANGULARES, ECUACIÓN DE TRES MOMENTOS Y APLICACIONES

Notas realizadas en base a los apuntes de RII (Autores: A. Morquio y L. Delacoste, con colaboración de M. Reboredo, V. Machin y A. Spalvier)

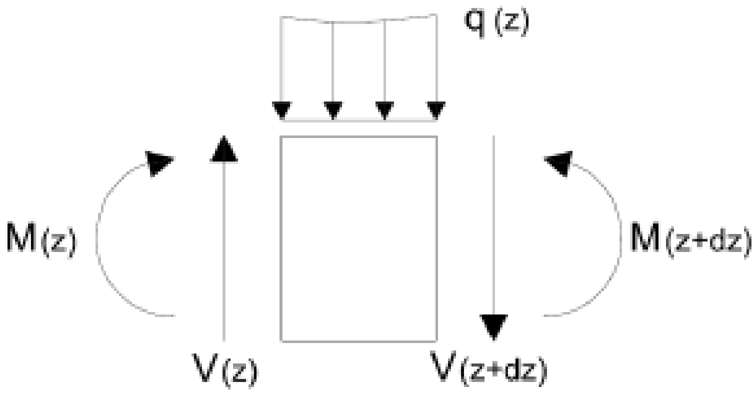
Convenciones e Hipótesis

Estructuras compuestas por **vigas rectas**, para simplificar la representación gráfica que tienen el eje de la viga horizontal

a) Vigas rectas
b) Materiales elásticos y lineales
c) Secciones simétricas respecto del eje y.
d) Cargas distribuidas q o concentradas P según el eje y.



Equilibrio de un tramo



Suma de Fuerzas verticales =0

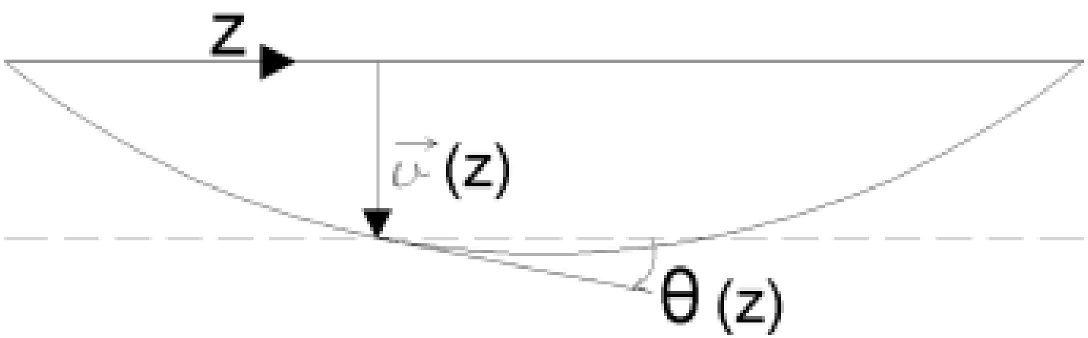
$$\frac{dV}{dz} + q(z) = 0$$

Suma de Momentos en z+dz =0

$$\frac{dM}{dz} = V(z)$$

$$\frac{d^2M}{dz^2} = \frac{dV}{dz} = -q$$

Relación Momento-Curvatura



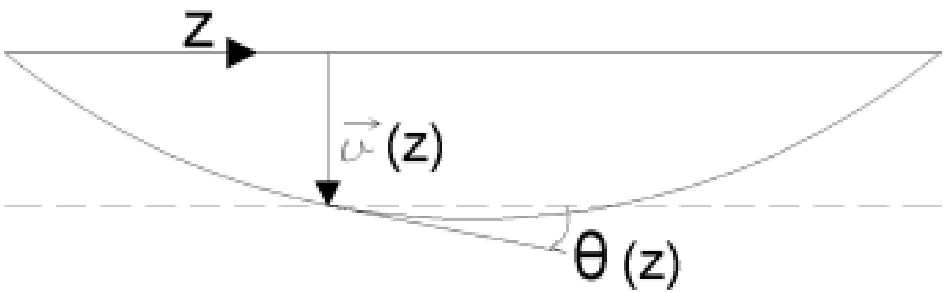
$$\frac{d^2 v}{dz^2} = \frac{d\theta}{dz} = -\frac{M}{EI_x}$$

Analogía de Mohr

$$\nu \leftrightarrow M$$

$$\theta \leftrightarrow V$$

$$\frac{M}{EI_x} \leftrightarrow q$$

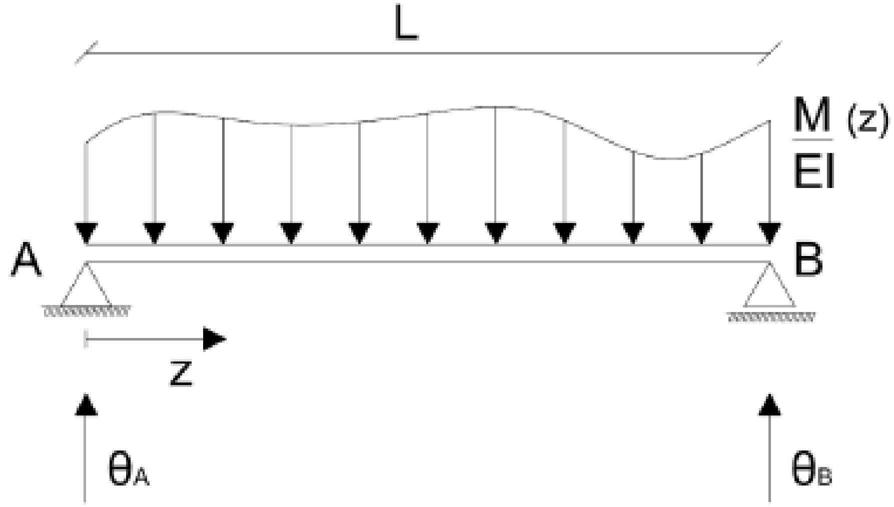


$$\nu(0) = \nu(L) = 0$$

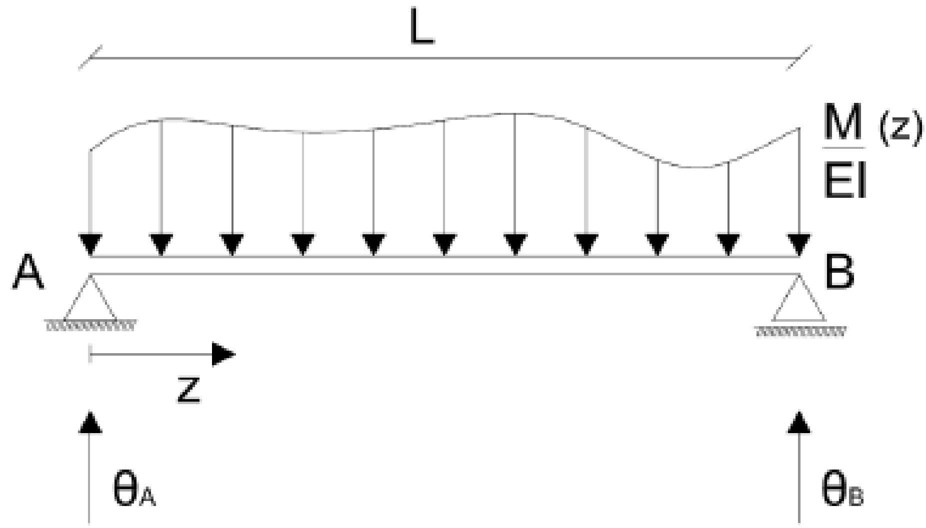
$$M(0) = M(L) = 0$$

Angulos de giro en los apoyos

$$\theta_A = \theta(0)$$
$$\theta_B = -\theta(L)$$
$$v_A = v(0)$$
$$v_B = v(L)$$



Reacción en los apoyos

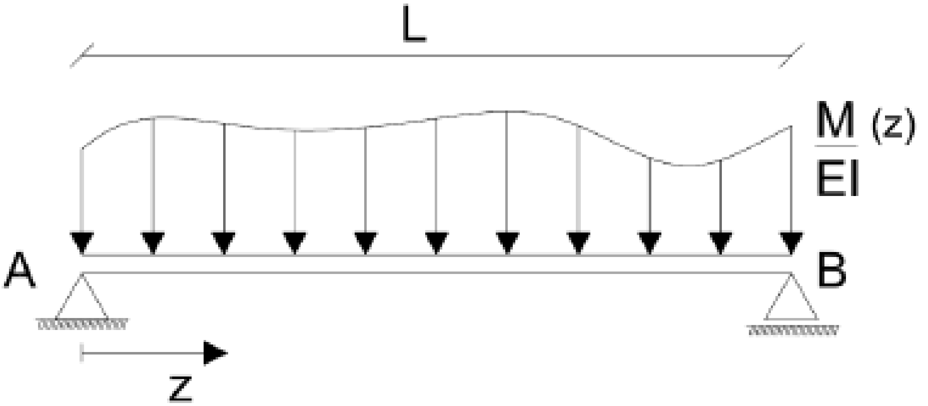


$$\theta_A = \frac{1}{L}\int_0^L \frac{M}{EI_x}(L-z)dz$$

$$\theta_B = \frac{1}{L}\int_0^L \frac{M}{EI_x} z dz$$

Considerando una viga S.A.

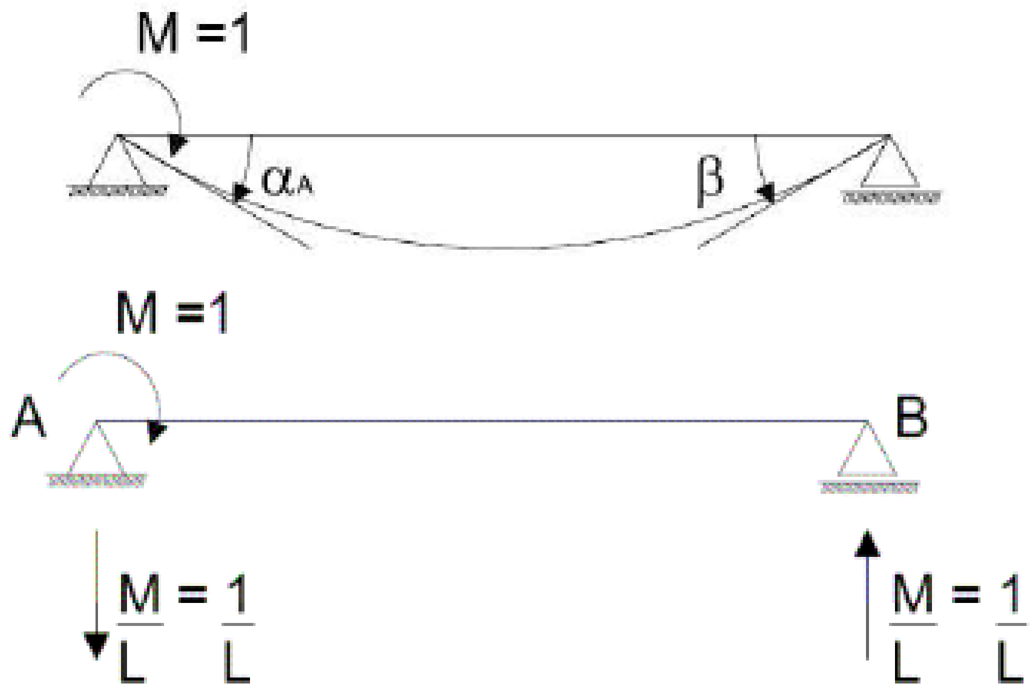
Para las cargas aplicadas en el vano



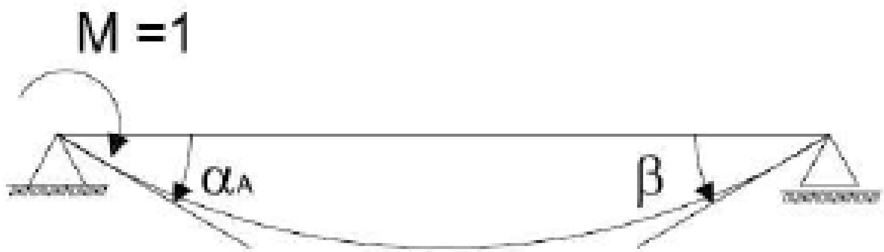
$$\alpha_{0A} = \frac{1}{L}\int_0^L \frac{M}{EI_x}(L-z)dz \qquad \alpha_{0B} = \frac{1}{L}\int_0^L \frac{M}{EI_x} z dz$$

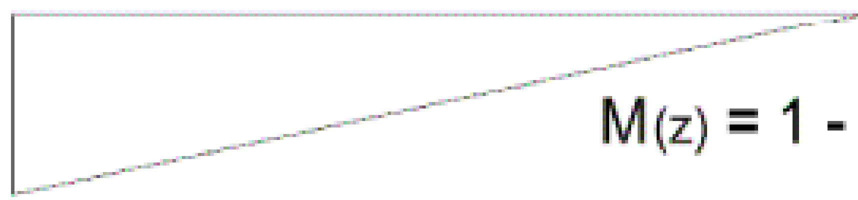
Giros

Considerando un momento unitario aplicado en A:



Giros (cont.)



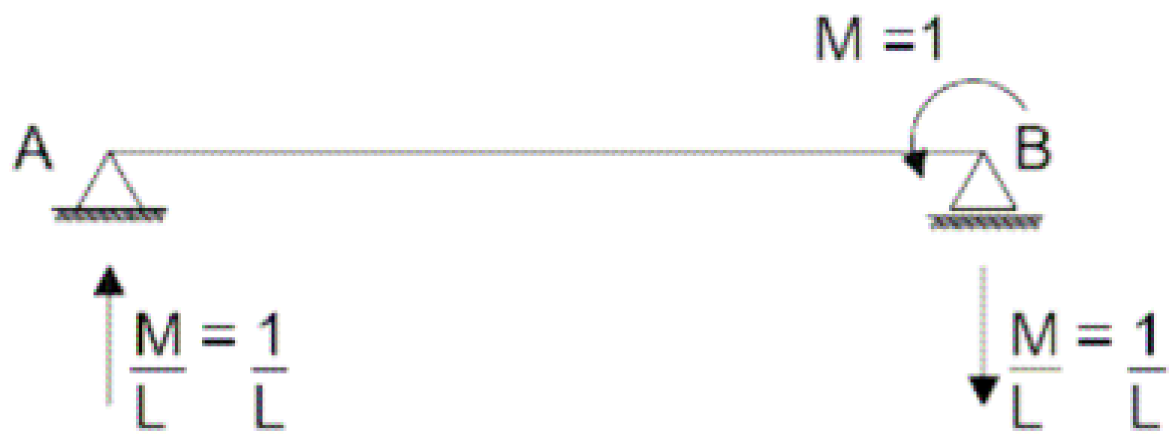


$$M(z) = 1 - \frac{z}{L} = \frac{L - z}{L}$$

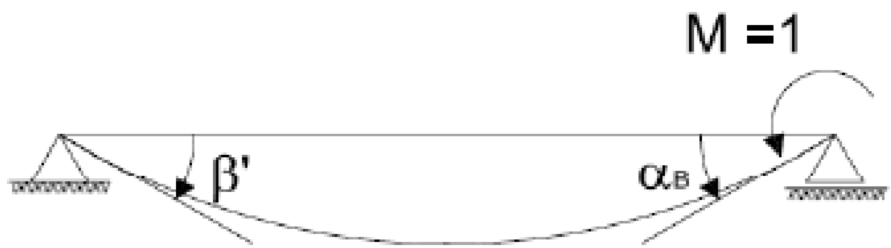
$$\alpha_A = \frac{1}{L}\int_0^L \frac{\left(1 - \frac{z}{L}\right)(L - z)}{EI_x}\,dz$$

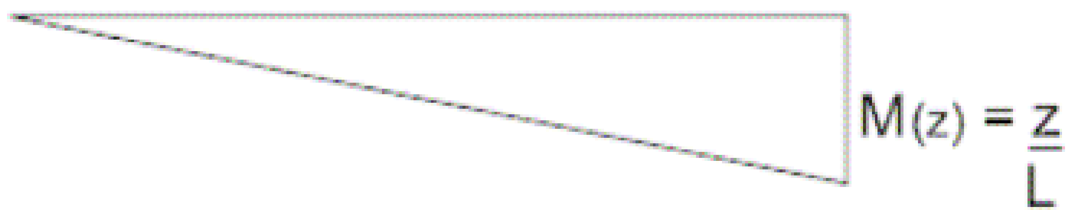
$$\beta = \frac{1}{L}\int_0^L \frac{\left(1 - \frac{z}{L}\right)}{EI_x}\,z\,dz$$

Giros (cont.)



Giros (cont.)

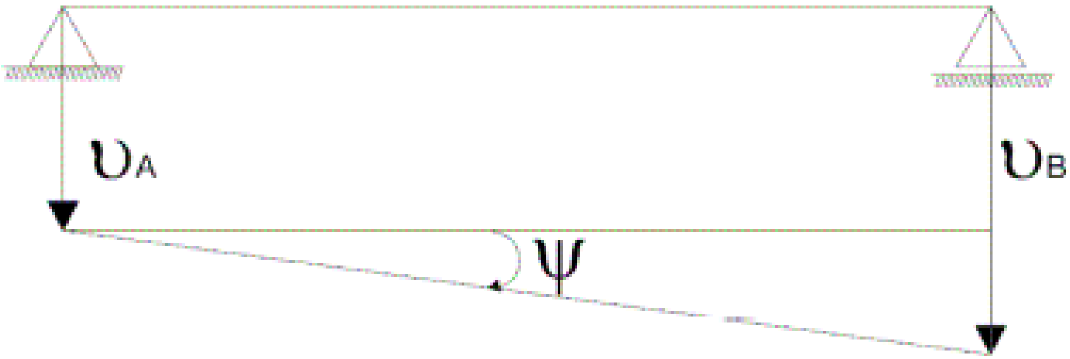




$$\alpha_B = \frac{1}{L}\int_0^L \frac{z}{L} \times \frac{z}{EI_x}\,dz$$

$$\beta' = \frac{1}{L}\int_0^L \frac{\frac{z}{L}(L-z)}{EI_x}\,dz = \beta$$

Descenso de apoyos

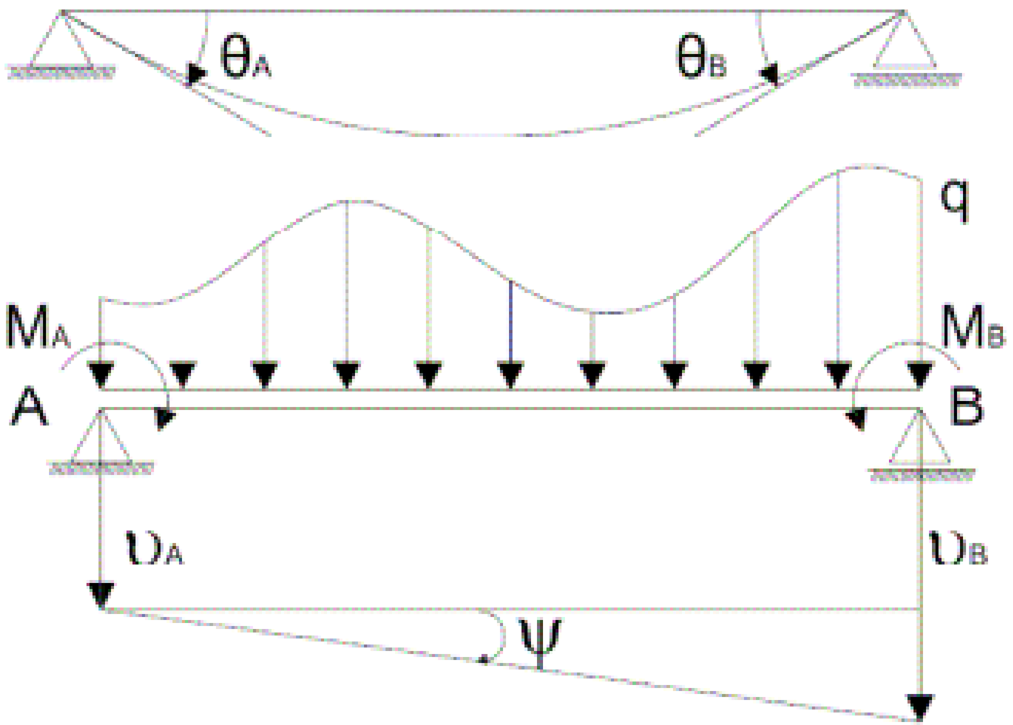


$$\upsilon_A = \upsilon(0)$$
$$\upsilon_B = \upsilon(L)$$

$$\psi \cong tg\,\psi = \frac{\upsilon_B - \upsilon_A}{L}$$

$$\theta_A = \psi$$
$$\theta_B = -\psi$$

Caso General (EI cualquiera)



$$\theta_A = \alpha_{0A} + M_A \cdot \alpha_A + M_B \cdot \beta + \psi$$

$$\theta_B = \alpha_{0B} + M_A \cdot \beta + M_B \cdot \alpha_B - \psi$$

Caso EI constante

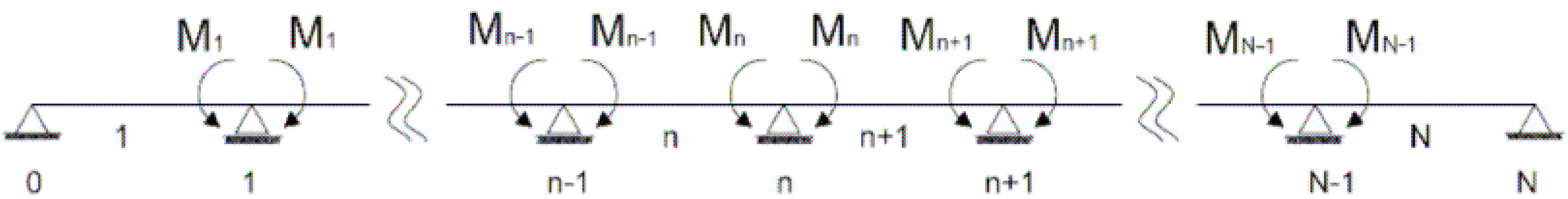
$$\alpha_A = \frac{L}{EI_x}\int_0^1 \left(1 - 2\cdot u + u^2\right) du = \frac{L}{EI_x}\left(u - u^2 + \frac{u^3}{3}\right)\Bigg|_0^1 \Rightarrow \alpha_A = \frac{L}{3EI_x}$$

$$\alpha_B = \frac{L}{EI_x}\int_0^1 u^2 du = \frac{L}{EI_x}\left(\frac{u^3}{3}\right)\Bigg|_0^1 \Rightarrow \alpha_B = \frac{L}{3EI_x}$$

$$\beta = \frac{L}{EI_x}\int_0^1 (1-u)u\,du = \frac{L}{EI_x}\left(\frac{u^2}{2} - \frac{u^3}{3}\right)\Bigg|_0^1 = \frac{L}{EI_x}\left(\frac{1}{2} - \frac{1}{3}\right) \Rightarrow \beta = \frac{L}{6EI_x}$$

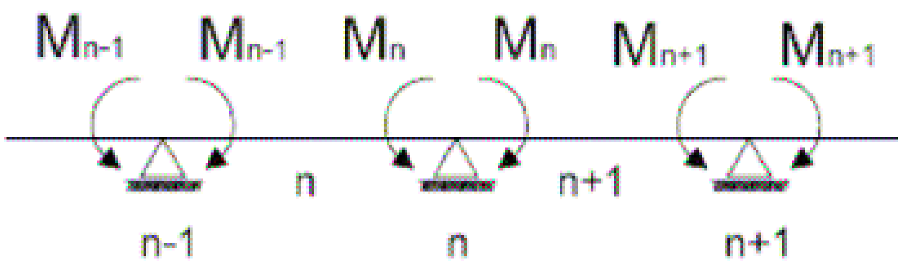
$$\alpha = \alpha_A = \alpha_B = \frac{L}{3EI_x} \qquad \beta = \frac{L}{6EI_x} = \frac{1}{2}\alpha$$

Viga continua de N tramos



En un apoyo genérico n

- confluyen los tramos n y n+1.
- hay trasmisión de momento,
- no hay trasmisión de cortante (ya que cada tramo descarga de forma independiente en el apoyo).



θ_{n-1}^{n} Barra n

Nudo n-1

En el caso anterior era θ_A

θ_{n}^{n} Barra n

Nudo n

En el caso anterior era θ_B

M_n Depende solo del nudo n

ψ^{n} Depende solo de la barra

Ecuaciones angulares

Análoga situación se presenta para el coeficiente β donde para el tramo n es β^n. Los términos α_{0A} y α_{0B} de la barra n se denominan ${\alpha_0}^n_{n-1}$ y ${\alpha_0}^n_n$. Los coeficientes α_A y α_B de la barra n se llaman α^n_{n-1} y α^n_n. En el caso que la sección de la viga sea constante estos coeficientes dependen solo del tramo y se denomina a ambos α^n.

$$\theta^n_{n-1} = {\alpha_0}^n_{n-1} + M_{n-1}\alpha^n_{n-1} + M_n\beta^n + \psi^n$$

$$\theta^n_n = {\alpha_0}^n_n + M_{n-1}\beta^n + M_n\alpha^n_n - \psi^n$$

$$\theta^{n+1}_n = {\alpha_0}^{n+1}_n + M_n\alpha^{n+1}_n + M_{n+1}\beta^{n+1} + \psi^{n+1}$$

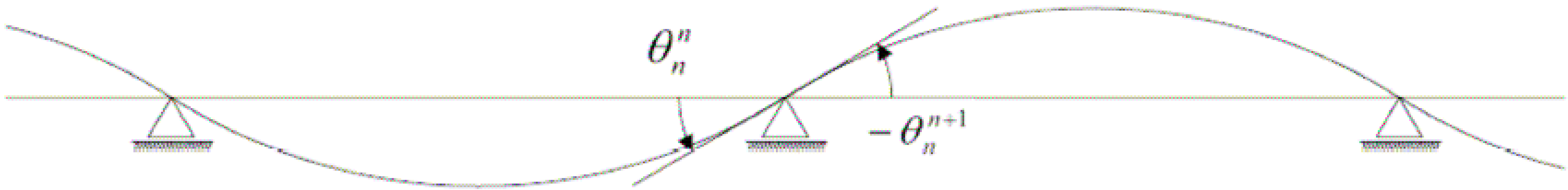
$$\theta^{n+1}_{n+1} = {\alpha_0}^{n+1}_{n+1} + M_n\beta^{n+1} + M_{n+1}\alpha^{n+1}_{n+1} - \psi^{n+1}$$

Donde sabemos que:

$$\psi^n = \frac{\upsilon_n - \upsilon_{n-1}}{L^n}$$

Ecuación de tres momentos

$$\theta_n^n = -\theta_n^{n+1} \qquad (6)$$



$$\theta_n^n + \theta_n^{n+1} = 0$$

$$\theta_n^n = \alpha_{0_n}^{\ n} + M_{n-1}\beta^n + M_n\alpha_n^n - \psi^n$$

$$\theta_n^{n+1} = \alpha_{0_n}^{\ n+1} + M_n\alpha_n^{n+1} + M_{n+1}\beta^{n+1} + \psi^{n+1}$$

$$M_{n-1}\beta^n + M_n\left(\alpha_n^n + \alpha_n^{n+1}\right) + M_{n+1}\beta^{n+1} + \alpha_{0_n}^{\ n} + \alpha_{0_n}^{\ n+1} - \psi^n + \psi^{n+1} = 0$$

$$M_{n-1}\beta^{n} + M_{n}\left(\alpha_{n}^{n} + \alpha_{n}^{n+1}\right) + M_{n+1}\beta^{n+1} + \alpha_{0_n}^{\;n} + \alpha_{0_n}^{\;n+1} - \psi^{n} + \psi^{n+1} = 0$$

$\alpha_{0_n}^{\;n}$ y $\alpha_{0_n}^{\;n+1}$ Dependen de la geometría de la estructura y de las cargas aplicadas

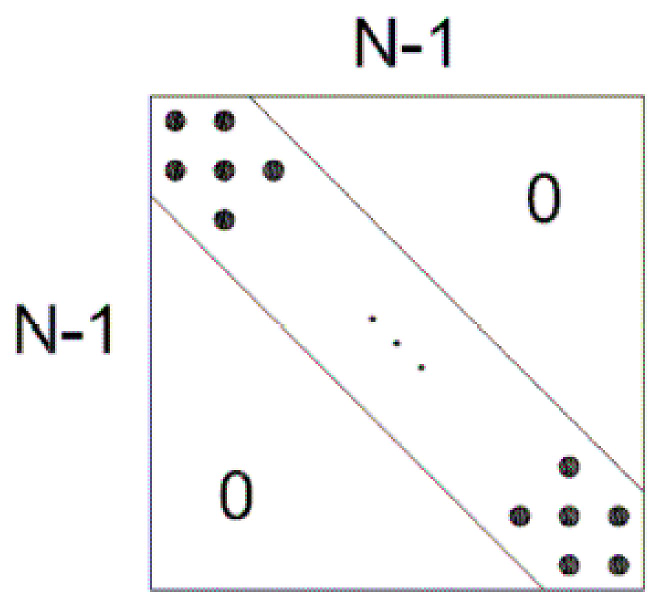
ψ^{n} y ψ^{n+1} Dependen de los descensos de los apoyos y de la geometría de la estructura

$\alpha_{n}^{n},\ \alpha_{n}^{n+1},\ \beta^{n}$ y β^{n+1} Dependen de la geometría y de los Momentos:

$$M_{n-1},\ M_{n}\ y\ M_{n+1}$$

N+1 apoyos, N tramos

$$M_{n-1}\beta^{n} + M_{n}\left(\alpha_{n}^{n} + \alpha_{n}^{n+1}\right) + M_{n+1}\beta^{n+1} + \alpha_{0_n}^{\;n} + \alpha_{0_n}^{\;n+1} - \psi^{n} + \psi^{n+1} = 0$$



Viga continua con empotramientos

Empotramiento en el primer apoyo:

$$\theta_0^1 = 0 = \alpha_{0\,0}^{\;1} + M_0 \alpha_0^1 + M_1 \beta^1 + \psi^1$$

Tenemos una incognita más que sería M_0 y n ecuaciones

$$M_{n-1}\beta^n + M_n\left(\alpha_n^n + \alpha_n^{n+1}\right) + M_{n+1}\beta^{n+1} + \alpha_{0_n}^{\;n} + \alpha_{0_n}^{\;n+1} - \psi^n + \psi^{n+1} = 0$$

Si n=0, entonces no tienen sentido los subindices (apoyos) negativos, ni los tramos igual a cero (superindices).

Viga continua con empotramientos

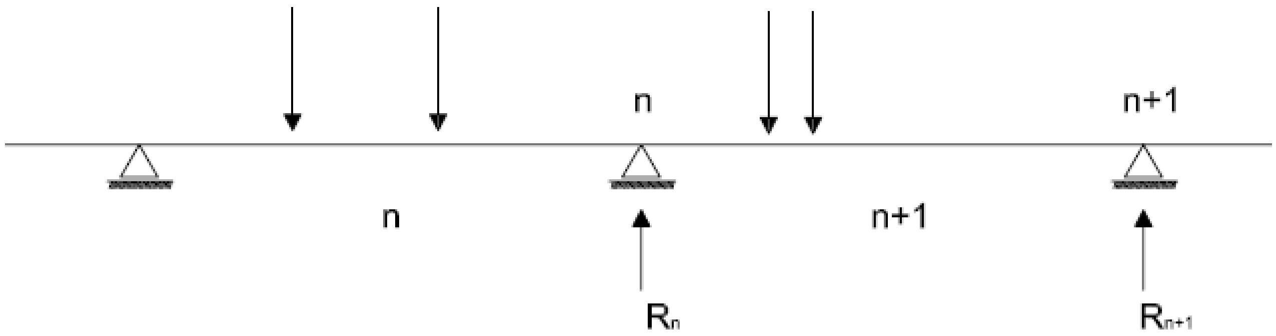
Empotramiento en el apoyo n:

$$\theta_N^N = 0 = \alpha_{0_N}^{\ N} + M_{N-1}\beta^N + M_N\alpha_N^N - \psi^N$$

Tenemos una incognita más que sería M_n y n ecuaciones

$$M_{n-1}\beta^n + M_n\left(\alpha_n^n + \alpha_n^{n+1}\right) + M_{n+1}\beta^{n+1} + \alpha_{0_n}^{\ n} + \alpha_{0_n}^{\ n+1} - \psi^n + \psi^{n+1} = 0$$

Reacciones



$$R_n^n = R_{0n}^{\;n} + \frac{M_{n-1}}{L^n} - \frac{M_n}{L^n}$$

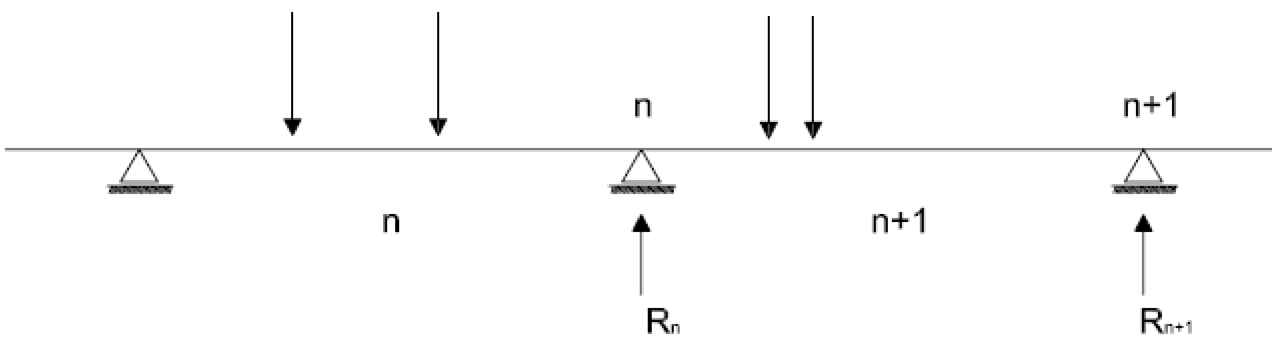
$$R_n = R_n^n + R_n^{n+1}$$

$$R_n^{n+1} = R_{0n}^{\;n+1} - \frac{M_n}{L^{n+1}} + \frac{M_{n+1}}{L^{n+1}}$$

Reacciones

$$R_n = R_{0\,n}^{n} + R_{0n}^{n+1} + \frac{M_{n-1}}{L^n} - \frac{M_n}{L^n} - \frac{M_n}{L^{n+1}} + \frac{M_{n+1}}{L^{n+1}}$$

$$R_{0n} + \frac{1}{L^n}\left(M_{n-1} - M_n\right) + \frac{1}{L^{n+1}}\left(M_{n+1} - M_n\right)$$



Vigas con EI

$$\beta^n = \frac{L^n}{6EI_x}$$

$$\alpha^n = \alpha_n^n = \alpha_{n-1}^n = \frac{L^n}{3EI_x}$$

$$M_{n-1}L^n + 2M_n\left(L^n + L^{n+1}\right) + M_{n+1}L^{n+1} + 6EI_x\left(\alpha_{0_n}^{\ n} + \alpha_{0_n}^{\ n+1}\right) + 6EI_x\left(\psi^{n+1} - \psi^n\right) = 0$$

$$\alpha_{0_n}^{\ n} = \frac{1}{L^n}\int_0^{L^n} \frac{M}{EI_x} z\,dz \qquad 6EI_x\alpha_{0_n}^{\ n} = \frac{6}{L^n}\int_0^{L^n} Mz\,dz$$

$$\alpha_{0_n}^{\ n+1} = \frac{1}{L^{n+1}}\int_0^{L^{n+1}} \frac{M}{EI_x}\left(L^{n+1} - z\right)dz \qquad 6EI_x\alpha_{0_n}^{\ n+1} = \frac{6}{L^{n+1}}\int_0^{L^{n+1}} M\left(L^{n+1} - z\right)dz$$

Términos de Carga

$$M_{n-1}L^n + 2M_n\left(L^n + L^{n+1}\right) + M_{n+1}L^{n+1} + 6EI_x\left(\alpha_{0n}^{\;n} + \alpha_{0n}^{\;n+1}\right) + 6EI_x\left(\psi^{n+1} - \psi^n\right) = 0$$

$$6EI_x\alpha_{0n}^{\;n} = \frac{6}{L^n}\int_0^{L^n} Mzdz \qquad \mathscr{R}^{\text{n}} = \frac{6}{\left(L^n\right)^2}\int_0^{L^n} Mzdz \qquad \mathscr{L}^{\text{n}} = \frac{6}{\left(L^n\right)^2}\int_0^{L^n} M\left(L^n - z\right)dz$$

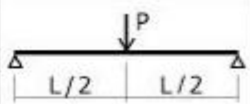
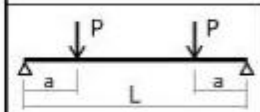
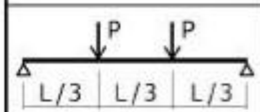
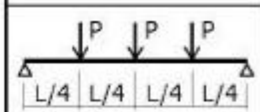
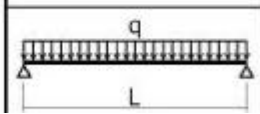
$$6EI_x\alpha_{0n}^{\;n+1} = \frac{6}{L^{n+1}}\int_0^{L^{n+1}} M\left(L^{n+1} - z\right)dz \qquad \mathscr{L}^{\text{n+1}} = \frac{6}{\left(L^{n+1}\right)^2}\int_0^{L^{n+1}} M\left(L^{n+1} - z\right)dz$$

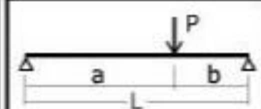
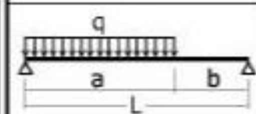
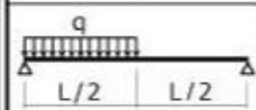
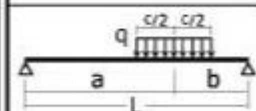
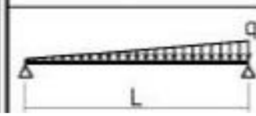
$$M_{n-1}L^n + 2M_n\left(L^n + L^{n+1}\right) + M_{n+1}L^{n+1} + \mathscr{R}^{\text{n}}L^n + \mathscr{L}^{\text{n+1}}L^{n+1} + 6EI_x\left(\psi^{n+1} - \psi^n\right) = 0$$

Términos de Carga

Instituto de Estructuras y Transporte
Prof. Julio Ricaldoni

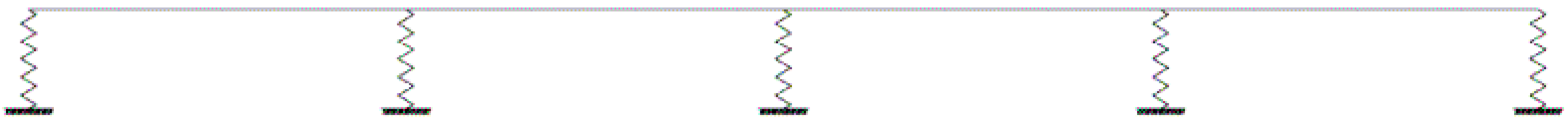
TÉRMINOS DE CARGA

CARGAS SIMÉTRICAS	
ESTADO DE CARGA	$\mathcal{L} = \mathcal{R}$
	$\frac{3}{8} PL$
	$3Pa\left(1-\frac{a}{L}\right)$
	$\frac{2}{3} PL$
	$\frac{15}{16} PL$
	$\frac{1}{4} qL^2$

CARGAS NO SIMÉTRICAS		
ESTADO DE CARGA	$\mathcal{L}$	$\mathcal{R}$
	$\frac{Pab}{L^2}(L+b)$	$\frac{Pab}{L^2}(L+a)$
	$\frac{qa^2}{4}\left(2-\frac{a}{L}\right)^2$	$\frac{qa^2}{4}\left(2-\frac{a^2}{L^2}\right)$
	$\frac{9}{64} qL^2$	$\frac{7}{64} qL^2$
	$\frac{qabc}{L^2}\left(L+b-\frac{c^2}{4a}\right)$	$\frac{qabc}{L^2}\left(L+a-\frac{c^2}{4b}\right)$
	$\frac{7}{60} qL^2$	$\frac{2}{15} qL^2$

Viga sobre apoyos elásticos

$$R_n = k_n \cdot \upsilon_n$$



$$\psi^n = \frac{\upsilon_n - \upsilon_{n-1}}{L^n}$$

$$\psi^{n+1} = \frac{\upsilon_{n+1} - \upsilon_n}{L^{n+1}}$$

$$\psi^n = \frac{\frac{R_n}{k_n} - \frac{R_{n-1}}{k_{n-1}}}{L^n} =$$

Viga sobre apoyos elásticos

$$\psi^{n} = f_1\left(M_{n-2}, M_{n-1}, M_{n}, M_{n+1}, R_{0n}, R_{0n-1}\right)$$
$$\psi^{n+1} = f_2\left(M_{n-1}, M_{n}, M_{n+1}, M_{n+2}, R_{0n}, R_{0n+1}\right)$$

siendo ambas ecuaciones lineales.
Sustituyendo en la ecuación de tres momentos,

$$M_{n-1}\beta^{n} + M_{n}\left(\alpha_n^{n} + \alpha_n^{n+1}\right) + M_{n+1}\beta^{n+1} + {\alpha_0}_n^{n} + {\alpha_0}_n^{n+1} - \psi^{n} + \psi^{n+1} = 0$$

Ecuación de 5 Momentos

Sustituyendo en la Ec. de 3 M

$$A \cdot M_{n-2} + B \cdot M_{n-1} + C \cdot M_n + D \cdot M_{n+1} + E \cdot M_{n+2} + F \cdot R_{0,n-1} + G \cdot R_{0,n} + H \cdot R_{0,n+1} + \alpha_{0n}^{\ n} + \alpha_{0n}^{\ n+1} = 0$$

$$A = \frac{1}{k_{n-1} L^{n-1} L^{n}}$$

$$B = \beta^{n} - \frac{1}{k_n \left(L^n\right)^2} - \frac{1}{k_{n-1} L^{n-1} L^{n}} - \frac{1}{k_{n-1} \left(L^n\right)^2} - \frac{1}{k_n L^n L^{n+1}}$$

$$C = \alpha_n^n + \alpha_n^{n+1} + \frac{1}{k_n \left(L^n\right)^2} + \frac{2}{k_n L^n L^{n+1}} + \frac{1}{k_{n-1} \left(L^n\right)^2} + \frac{1}{k_{n+1} \left(L^{n+1}\right)^2} + \frac{1}{k_n \left(L^{n+1}\right)^2}$$

$$D = \beta^{n+1} - \frac{1}{k_n L^n L^{n+1}} - \frac{1}{k_{n+1} \left(L^{n+1}\right)^2} - \frac{1}{k_{n+1} L^{n+1} L^{n+2}} - \frac{1}{k_n \left(L^{n+1}\right)^2}$$

$$E = \frac{1}{k_{n+1} L^{n+1} L^{n+2}}$$

$$F = \frac{1}{k_{n-1} L^{n}}$$

$$G = -\frac{1}{k_n L^n} - \frac{1}{k_n L^{n+1}}$$

$$H = \frac{1}{k_{n+1} L^{n+1}}$$

Apoyo

Si un apoyo es fijo (no es elástico) $k_n = \infty$

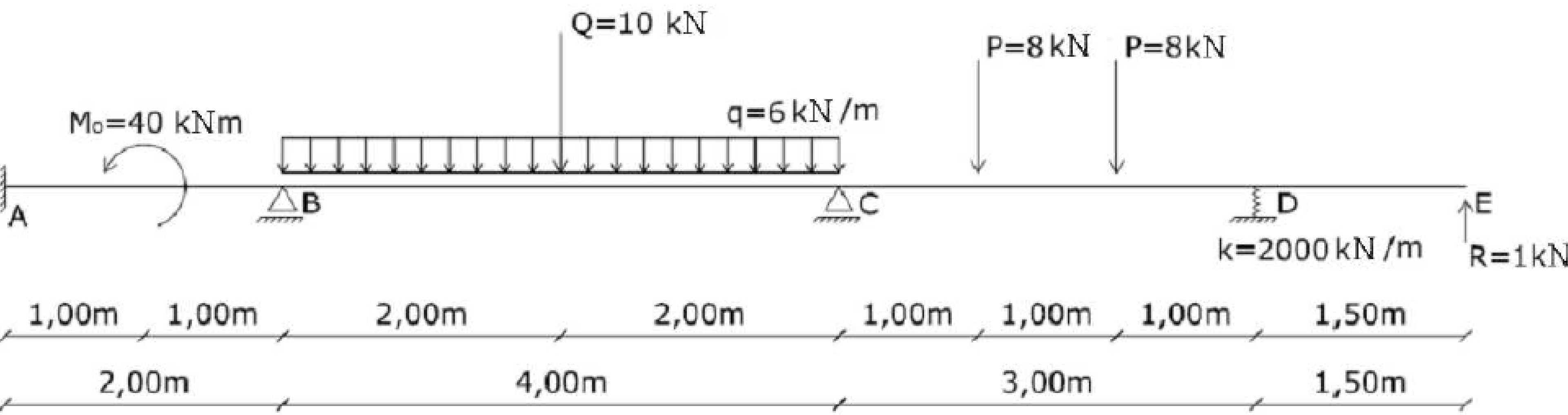
$$R_{n-1} = R_0 = R_{00} + \frac{M_1}{L^1} \qquad n=1$$

$$R_n = R_1 = R_{0,1} - \frac{M_1}{L^1} - \frac{M_1}{L^2} + \frac{M_2}{L^2}$$

no existe el tramo 0, ni el momento M_0, ni tampoco el apoyo –1

Ejemplo

PNI20 material es acero ($E=2.1\times10^{11}\ N/m^2$)

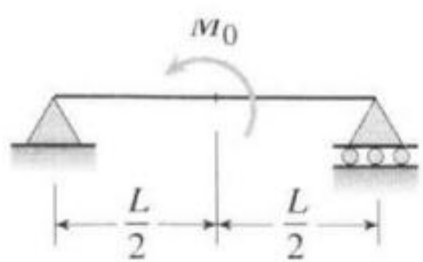


$\theta_A = 0$

Utilizando la ecuación:

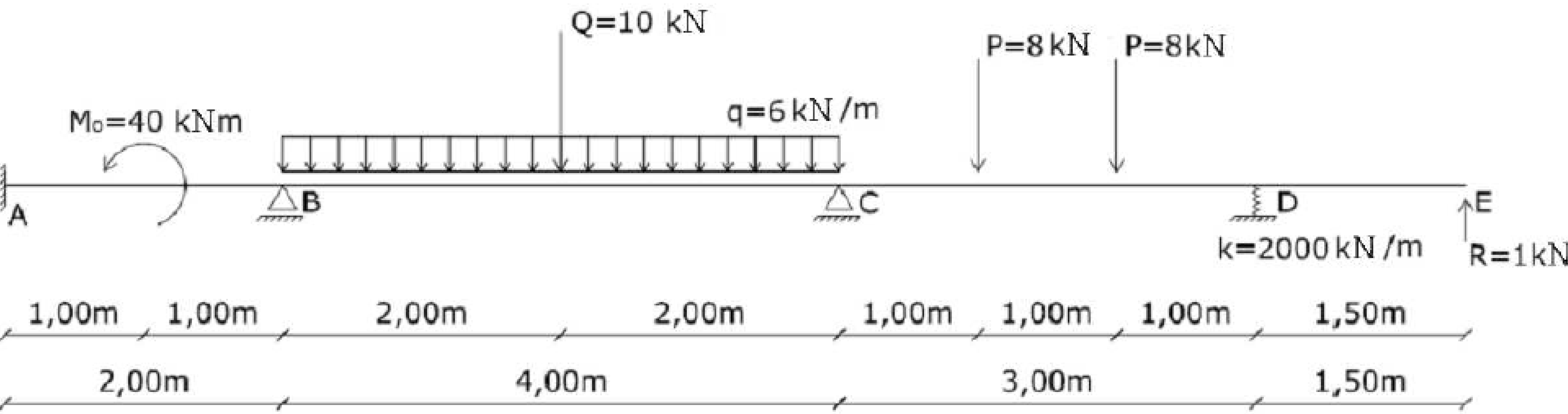
$$\alpha_{0A}^1 + \alpha_A^1 \cdot M_A + \beta^1 \cdot M_B + \psi = 0$$

$$\alpha_{0A}^1 = \frac{M_0 L}{24 \cdot EI} = \frac{40000 \times 2}{24 \cdot EI}$$



$$v = -\frac{M_0 x}{24LEI}(L^2 - 4x^2)$$

$$\delta_C = 0 \qquad \theta_A = \frac{M_0 L}{24EI}$$



$$\theta_A = 0$$

Utilizando la ecuación:

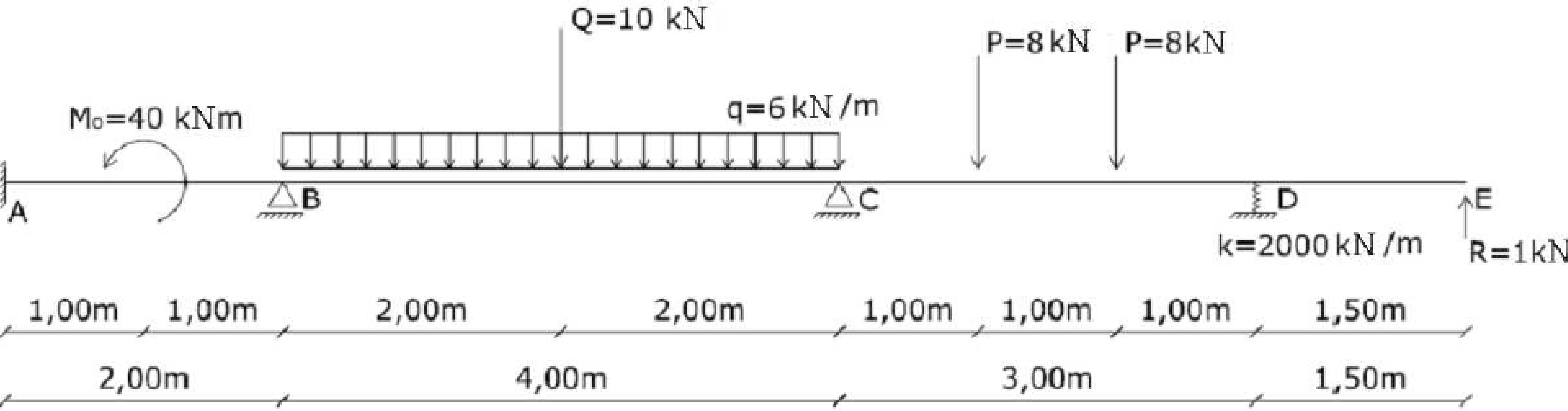
$$\alpha_{0A}^1 + \alpha_A^1 \cdot M_A + \beta^1 \cdot M_B + \psi = 0$$

$$\alpha_A^1 = \frac{L}{3 \cdot EI} = \frac{2}{3 \cdot EI}$$

$$\beta^1 = \frac{L}{6 \cdot EI} = \frac{2}{6 \cdot EI}$$

$$\frac{40000 \times 2}{24 \cdot EI} + \frac{2}{3 \cdot EI} \cdot M_A + \frac{2}{6 \cdot EI} \cdot M_B = 0$$

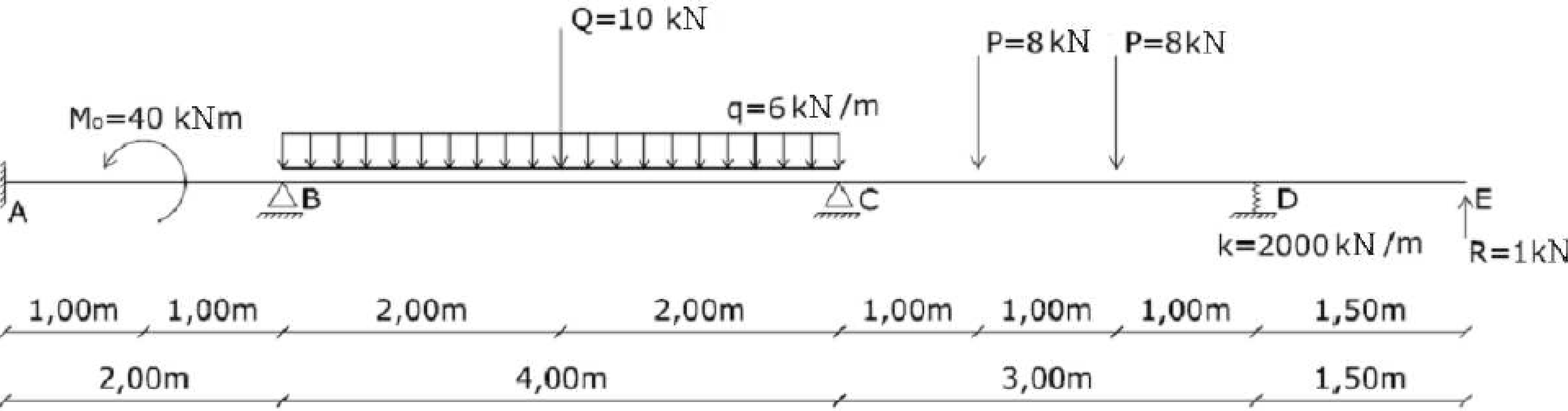
$$\boxed{2 \cdot M_A + M_B = -10000\ Nm}$$



$$M_{n-1}\beta^{n} + M_{n}\left(\alpha_{n}^{n} + \alpha_{n}^{n+1}\right) + M_{n+1}\beta^{n+1} + \alpha_{0n}^{\ n} + \alpha_{0n}^{\ n+1} - \psi^{n} + \psi^{n+1} = 0$$

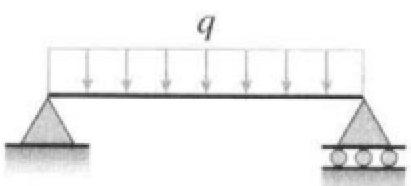
$$M_{A}\beta^{1} + M_{B}\left(\alpha_{B}^{1} + \alpha_{B}^{2}\right) + M_{C}\beta^{2} + \alpha_{0B}^{\ 1} + \alpha_{0B}^{\ 2} - \psi^{1} + \psi^{2} = 0$$

$$\alpha_{0B}^{1} = -\alpha_{0A}^{1} = -\frac{M_{0} \cdot L^{1}}{24 \cdot EI} = -\frac{40000 \times 2}{24 \cdot EI}$$

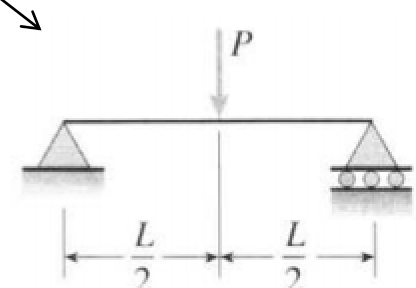


$$M_A\beta^1 + M_B\left(\alpha_B^1 + \alpha_B^2\right) + M_C\beta^2 + \alpha_{0B}^1 + \alpha_{0B}^2 - \psi^1 + \psi^2 = 0$$

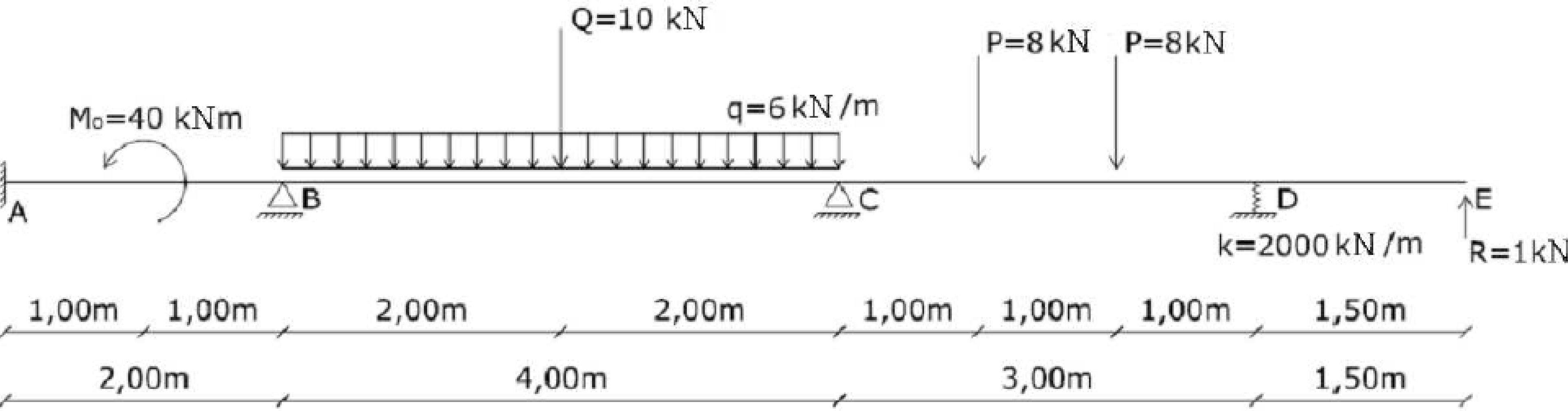
$$\alpha^2_{0\,C} = \alpha_{0B}^2 = \frac{q\cdot\left(L^2\right)^3}{24\cdot EI} + \frac{Q\cdot\left(L^2\right)^2}{16\cdot EI} = \frac{6000\times 4^3}{24\cdot EI} + \frac{10000\times 4^2}{16\cdot EI}$$



$$\theta_A = \theta_B = \frac{qL^3}{24EI}$$



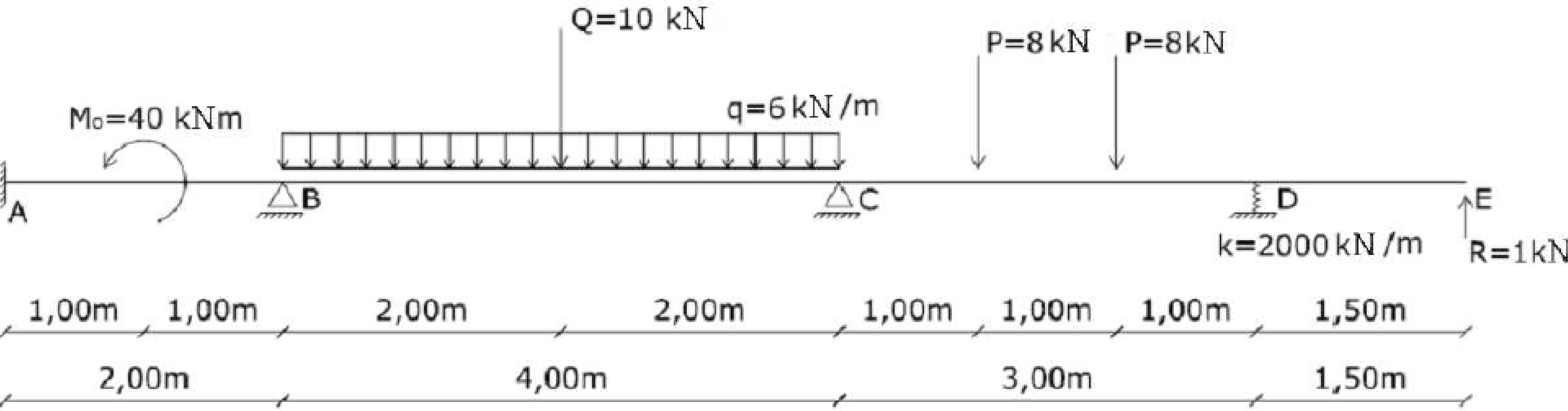
$$\theta_A = \theta_B = \frac{PL^2}{16EI}$$



$$M_A\beta^1 + M_B\left(\alpha_B^1 + \alpha_B^2\right) + M_C\beta^2 + \alpha_{0B}^{\ 1} + \alpha_{0B}^{\ 2} - \psi^1 + \psi^2 = 0$$

$$\alpha_B^1 = \alpha_A^1 = \frac{L^1}{3 \cdot EI} = \frac{2}{3 \cdot EI} \qquad \alpha_B^2 = \frac{L^2}{3 \cdot EI} = \frac{4}{3 \cdot EI}$$

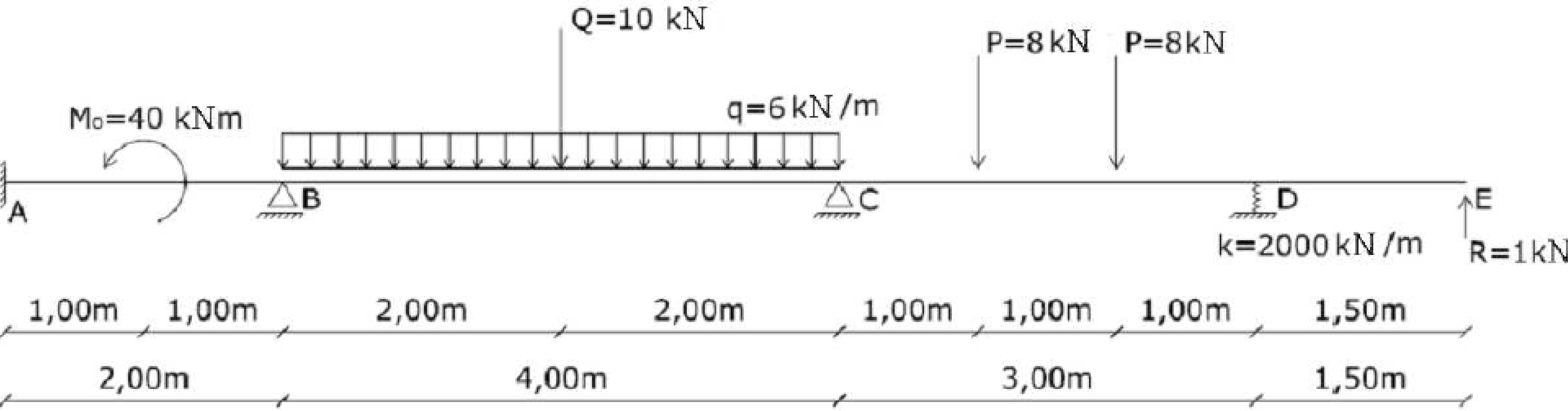
$$\beta^2 = \frac{L^2}{6 \cdot EI} = \frac{4}{6 \cdot EI}$$



$$M_A \beta^1 + M_B\left(\alpha_B^1 + \alpha_B^2\right) + M_C \beta^2 + \alpha_{0B}^1 + \alpha_{0B}^2 - \psi^1 + \psi^2 = 0$$

$$M_A \frac{2}{6 \cdot EI} + M_B\left(\frac{2}{3 \cdot EI} + \frac{4}{3 \cdot EI}\right) + M_C \frac{4}{6 \cdot EI} - \frac{40000 \times 2}{24 \cdot EI} + \frac{6000 \times 4^3}{24 \cdot EI} + \frac{10000 \times 4^2}{16 \cdot EI} = 0$$

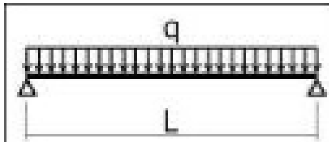
$$M_A + 6 \cdot M_B + 2 \cdot M_C = -68000 Nm$$

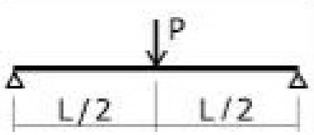


$$M_{n-1}L^{n}+2M_{n}\left(L^{n}+L^{n+1}\right)+M_{n+1}L^{n+1}+\ \mathscr{R}^{n}L^{n}+\ \mathscr{L}^{n+1}L^{n+1}+6EI_{x}\left(\psi^{n+1}-\psi^{n}\right)=0$$

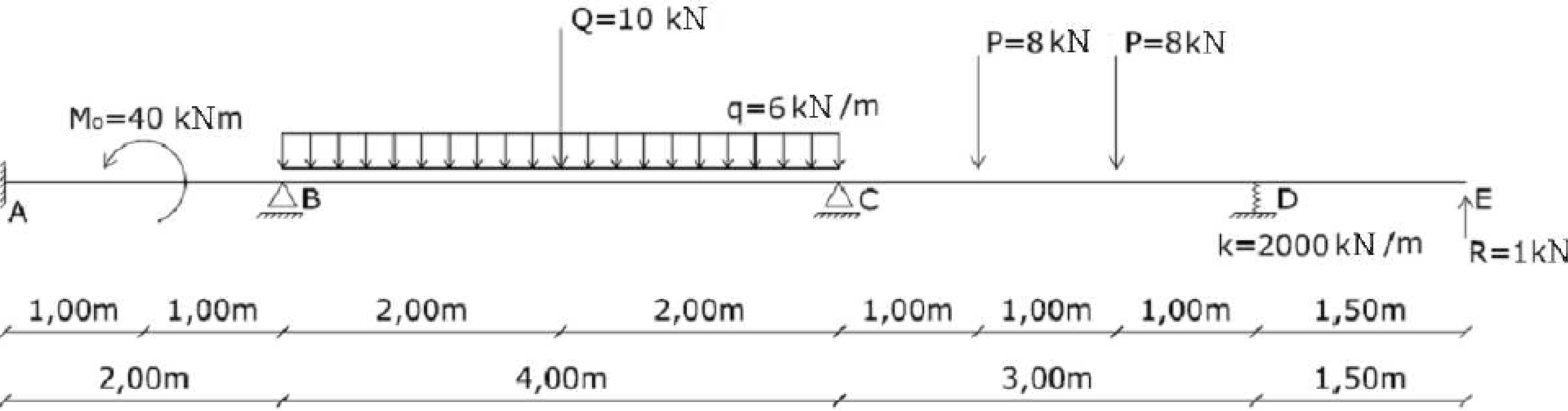
$$M_{B}\cdot L^{2}+2\cdot M_{C}\left(L^{2}+L^{3}\right)+M_{D}\cdot L^{3}+L^{2}\cdot\mathscr{R}^{2}+L^{3}\cdot\mathscr{L}^{3}+6\cdot EI(\psi^{3}-\psi^{2})=0$$

$$\mathscr{R}^{2}=\frac{3}{8}Q\cdot L^{2}+\frac{1}{4}q\cdot\left(L^{2}\right)^{2}=\frac{3}{8}\cdot 10000\times 4+\frac{1}{4}6000\times 4^{2}=39000Nm$$

	$\frac{1}{4}qL^{2}$

ESTADO DE CARGA	$\mathfrak{L}=\mathfrak{R}$
	$\frac{3}{8}PL$

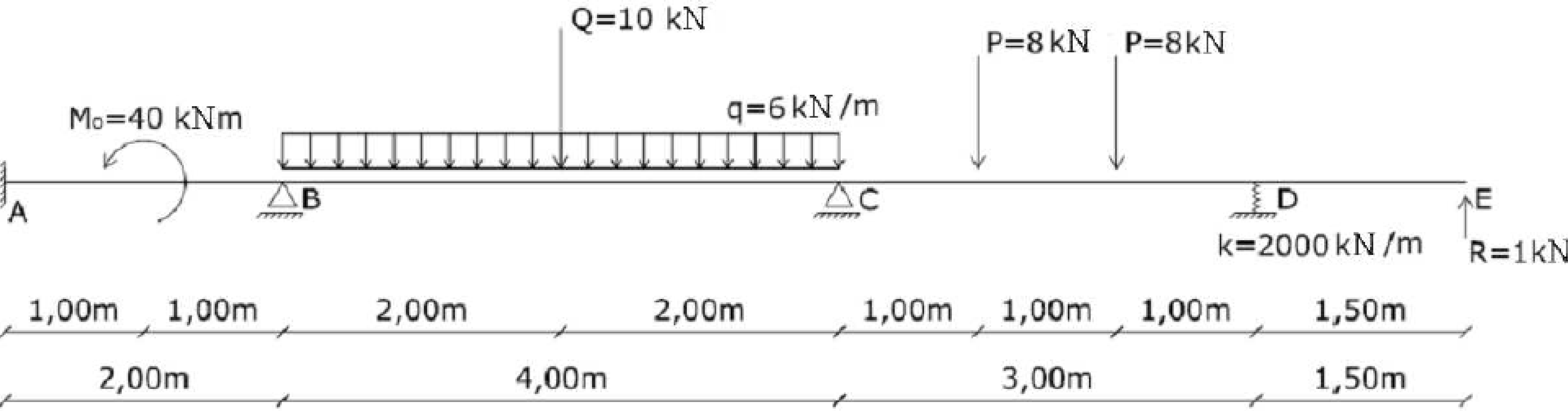
$$\mathscr{L}^{3}=\frac{2}{3}\cdot P\cdot L^{3}=\frac{2}{3}\cdot 8000\times 3=16000Nm$$



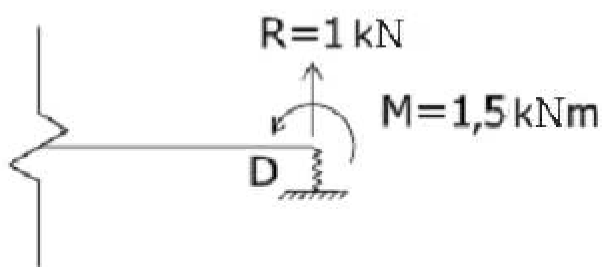
$$M_B \cdot L^2 + 2 \cdot M_C \left(L^2 + L^3\right) + M_D \cdot L^3 + L^2 \cdot \mathscr{R}^2 + L^3 \cdot \mathscr{L}^3 + 6 \cdot EI(\psi^3 - \psi^2) = 0$$

ψ^2 es igual a cero

$$\psi^3 = \frac{\upsilon_D - \upsilon_C}{L^3} = \frac{R_D/k - 0}{3} = \frac{R_D}{2\times10^6\times3}$$

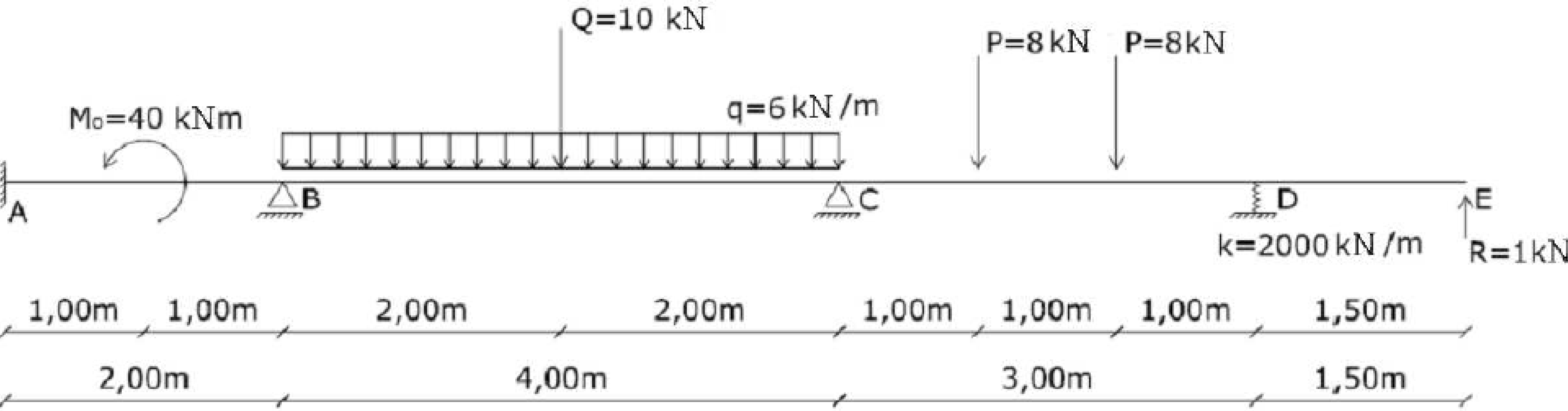


$$M_B \cdot L^2 + 2 \cdot M_C \left(L^2 + L^3\right) + M_D \cdot L^3 + L^2 \cdot \mathscr{R}^2 + L^3 \cdot \mathscr{L}^3 + 6 \cdot EI(\psi^3 - \psi^2) = 0$$



$$R_{0n} + \frac{1}{L^n}\left(M_{n-1} - M_n\right) + \frac{1}{L^{n+1}}\left(M_{n+1} - M_n\right)$$

$$R_D = P - R + \frac{1}{L^3}\left(M_C - M_D\right) = 8000 - 1000 + \frac{M_C}{3} - \frac{1500}{3}$$



$$4 \cdot M_B + 15{,}498 \cdot M_C = -237711 Nm$$

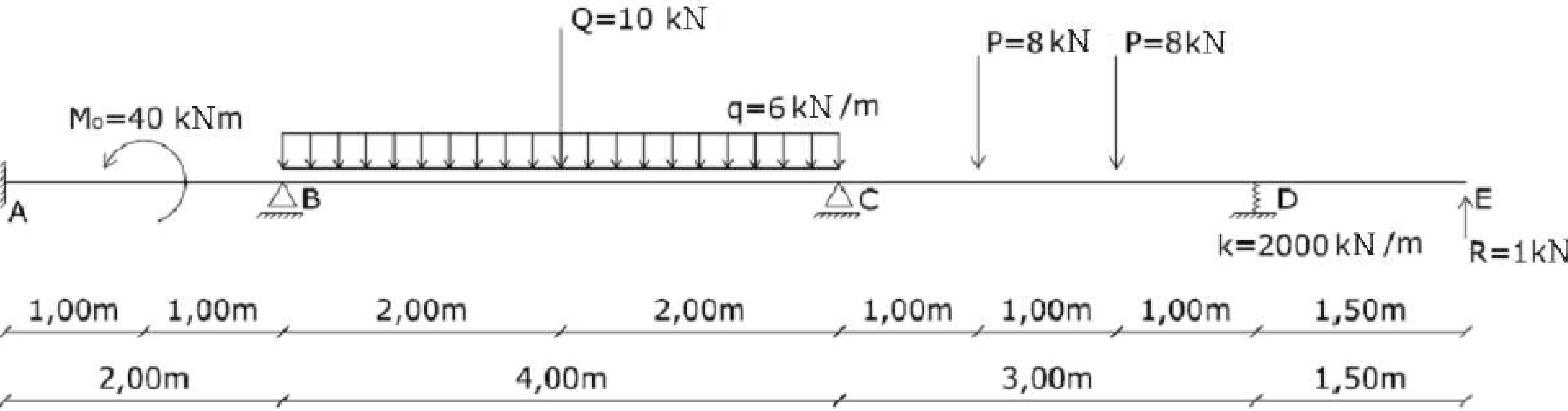
$$M_A + 6 \cdot M_B + 2 \cdot M_C = -68000 Nm$$

$$2 \cdot M_A + M_B = -10000 \, Nm$$

M_A=-1757,13Nm

M_B=-6485,74Nm

M_C=-13664,22Nm



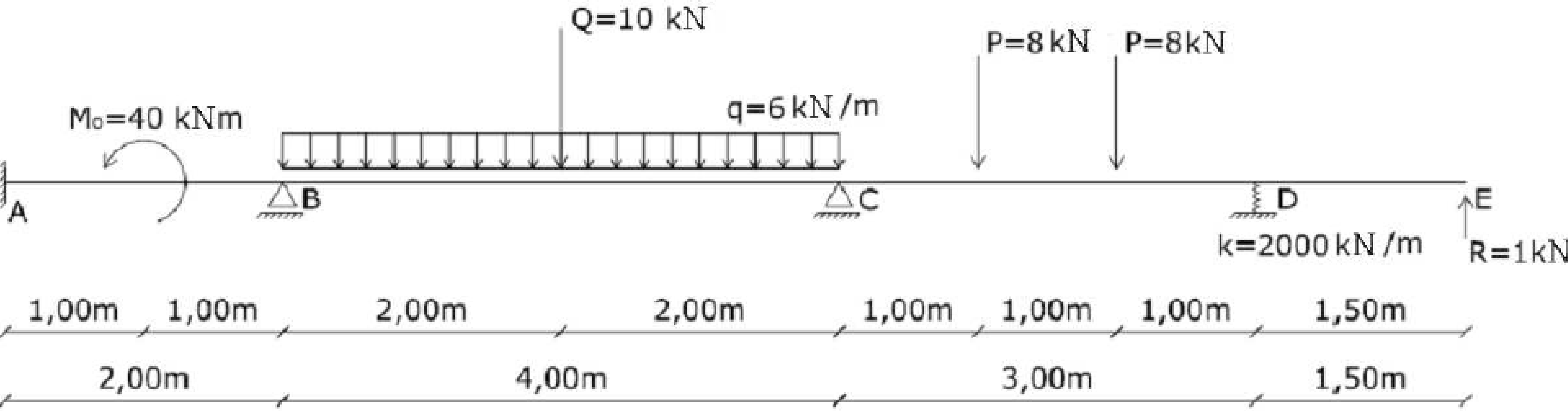
$$\upsilon_D = \frac{R_D}{k} = \left(8000 - 1000 + \frac{-13664{,}22}{3} - \frac{1500}{3}\right)\frac{1}{2{,}0 \times 10^6} = 0{,}000973m = 0.973mm$$

$$R_n = R_1 = R_{0,1} - \frac{M_1}{L^1} - \frac{M_1}{L^2} + \frac{M_2}{L^2}$$

$$R_A = \frac{M_0}{L^1} + \frac{1}{L^1}(M_B - M_A) = 17635{,}70N$$

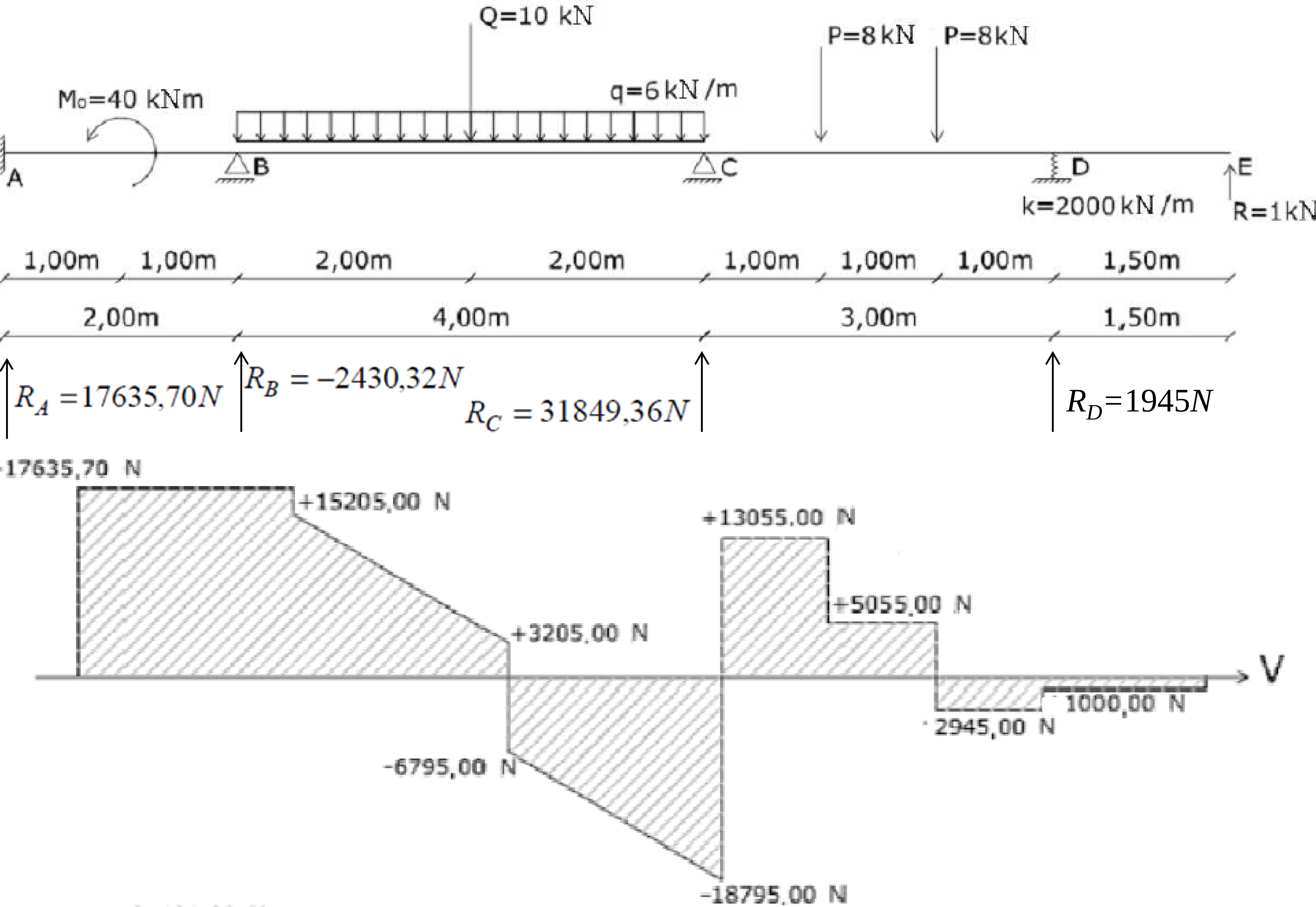
$$R_n = R_{0\,n}^{n} + R_{0n}^{n+1} + \frac{M_{n-1}}{L^n} - \frac{M_n}{L^n} - \frac{M_n}{L^{n+1}} + \frac{M_{n+1}}{L^{n+1}} =$$

$$R_B = -\frac{M_0}{L^1} + \quad Q/2 + q\,L^2/2 \quad + \frac{1}{L^1}(M_A - M_B) + \frac{1}{L^2}(M_C - M_B) = -2430{,}32N$$

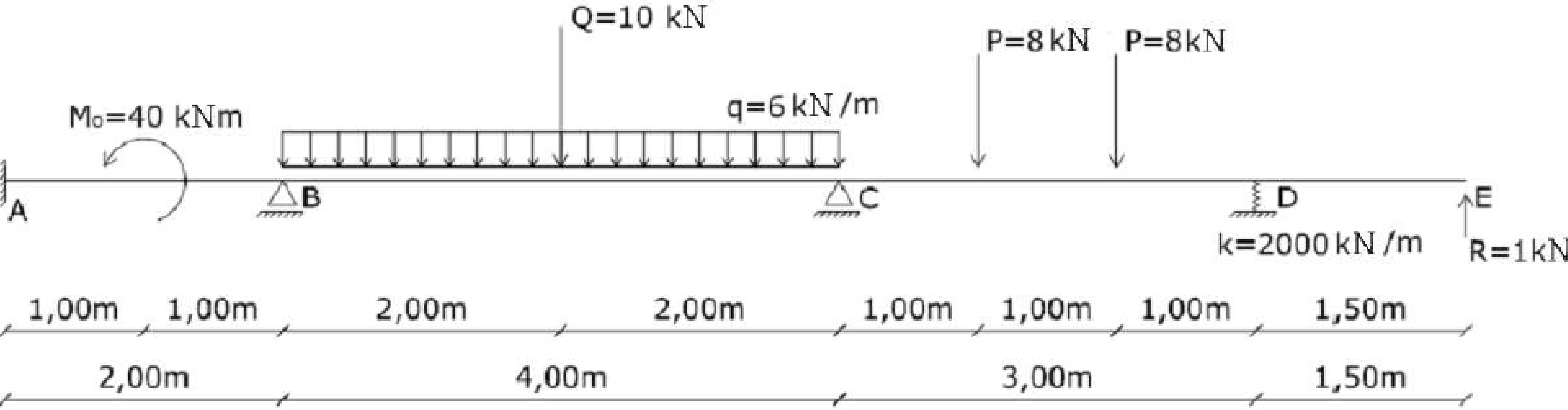


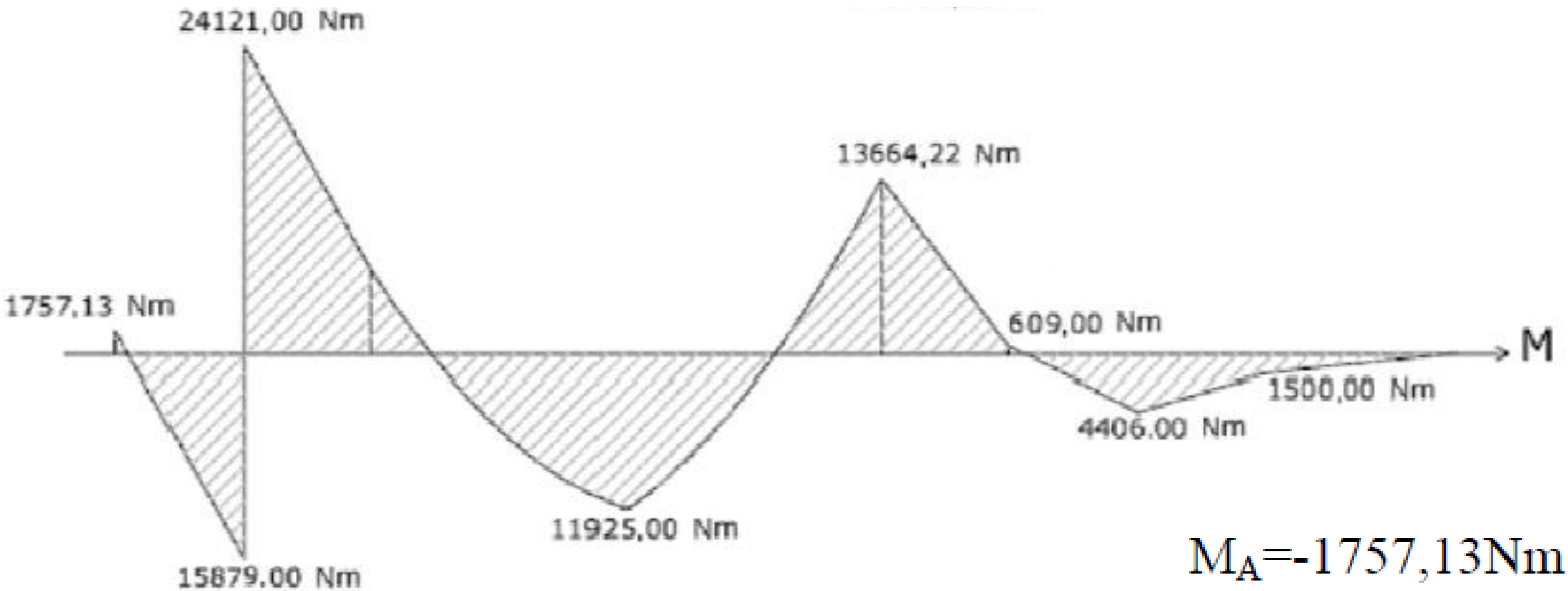
$$R_n = R_{0\,n}^{n} + R_{0n}^{n+1} + \frac{M_{n-1}}{L^n} - \frac{M_n}{L^n} - \frac{M_n}{L^{n+1}} + \frac{M_{n+1}}{L^{n+1}} =$$

$$R_C = \quad Q/2 + q\,L^2/2 \quad + P + \frac{1}{L^2}(M_B - M_C) + \frac{1}{L^3}(M_D - M_C) = 31849{,}36N$$



Q=10 kN
P=8 kN
P=8 kN
M₀=40 kNm
q=6 kN /m
A
B
C
D
E
k=2000 kN /m
R=1 kN
1,00m
1,00m
2,00m
2,00m
1,00m
1,00m
1,00m
1,50m
2,00m
4,00m
3,00m
1,50m
R_A =17635,70N
R_B = −2430,32N
R_C = 31849,36N
R_D=1945N
17635,70 N
+15205,00 N
+3205,00 N
-6795,00 N
+13055,00 N
+5055,00 N
2945,00 N
1000,00 N
-18795,00 N
V





M_A=-1757,13Nm
M_B=-6485,74Nm
M_C=-13664,22Nm