1) a) $r \leq R$ Utilizo Gauss $\oint \vec{E} \cdot d\vec{s} = \frac{q_{enc}}{\varepsilon_0}$

$$2\pi r L E_{in} = \frac{-\lambda L + \rho L \pi r^2}{\varepsilon_0} \quad \therefore \quad \vec{E}_i = \left(\frac{\rho r}{2\varepsilon_0} - \frac{\lambda}{2\pi r \varepsilon_0} \right) \hat{e}_r$$

$r \geq R$

$$E_{out} = \left(\frac{\rho R^2}{2\varepsilon_0 r} - \frac{\lambda}{2\pi\varepsilon_0 r} \right) \hat{e}_r$$

b) $r \leq R$

$$\int_R^r dV = V(r) - \cancel{V(R)}^{0} = -\int_R^r \vec{E}_{in} \cdot d\vec{r} = -\frac{\rho (r^2 - R^2)}{4\varepsilon_0} + \frac{\lambda}{2\pi\varepsilon_0} Ln\left(\frac{r}{R}\right)$$

$r \geq R$

$$\int_R^r dV = V(r) - \cancel{V(R)}^{0} = -\int_R^r \vec{E}_{out} \cdot d\vec{r} = -\left(\frac{\rho R^2 \pi - \lambda}{2\pi\varepsilon_0} \right) Ln\left(\frac{r}{R}\right)$$

c) Esto se cumple sii $\vec{E}_{in} = 0\hat{e}_r$ sii $r^* = \sqrt{\frac{\lambda}{\rho\pi}}$

Además $r^* \leq R \therefore R \geq \sqrt{\frac{\lambda}{\rho\pi}} \therefore \frac{\lambda}{\rho} \leq \pi R^2$

Solución problema 2; Examen F3 Julio 2017

a) Campo eléctrico entre placas de área A (Gauss): $AE = \frac{\pi D^2 E}{4} = q/\epsilon_0 \Rightarrow E = \frac{4q}{\pi\epsilon_0 D^2}$.

Flujo de E a través de un área interior de radio r: $\phi_E(r) = \pi r^2 E = \frac{4qr^2}{\epsilon_0 D^2} \Rightarrow \frac{d\phi_E}{dt} = \frac{4r^2}{\epsilon_0 D^2} i(t)$.

Ley de Ampère en ausencia de corriente: $\oint \vec{B}.d\vec{S} = 2\pi r B = \mu_0\epsilon_0 \frac{d\phi_E}{dt} \Rightarrow \boxed{B(r) = \frac{2\mu_0}{\pi D^2} r i(t)}$.

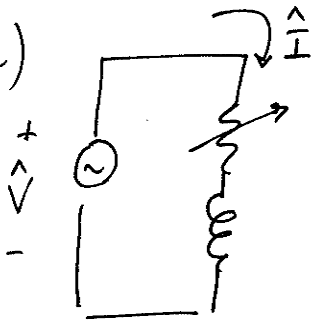
b) Capacitancia del capacitor: $C = \epsilon_0 A/d = \epsilon_0 \frac{\pi D^2}{4d}$.

Descarga de un condensador a través de una resistencia: $q(t) = Q_0 \exp(-t/\tau)$ con $\tau = RC$.

Corriente eléctrica en el circuito: $i(t) = \frac{dq}{dt} = -\frac{Q_0}{\tau}\exp(-t/\tau) \Rightarrow B(r,t) = |-\frac{2\mu_0 Q_0}{\pi D^2 \tau} r \exp(-t/\tau)|$,

explícitamente: $\boxed{B(r,t) = \frac{8\mu_0 Q_0 d}{\pi^2 \epsilon_0 R D^4} r \exp(-\frac{4d}{\pi\epsilon_0 D^2 R} t)}$

③ a)



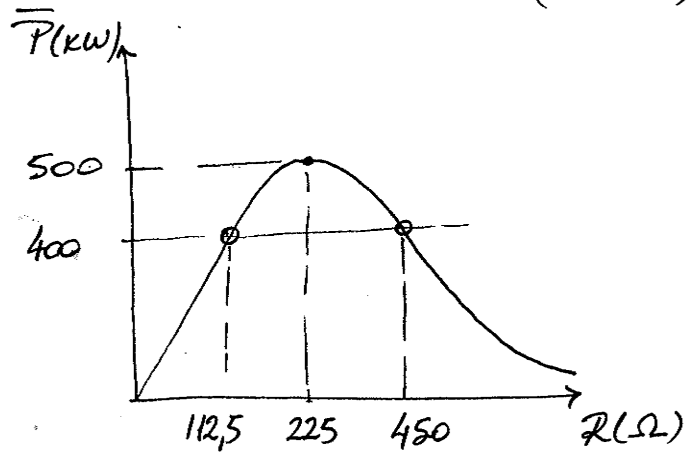
$I_{RMS} = \dfrac{V}{\sqrt{R^2+X^2}}$ $\qquad \bar{P} = R\, I_{RMS}^2 = V^2 \dfrac{R}{R^2+X^2} \Rightarrow$

$$\Rightarrow R^2 - \frac{V^2}{\bar{P}} R + X^2 = 0 \Rightarrow R_{\bar{P}_{min}} = \begin{cases} \boxed{112{,}5\,\Omega} \\ \boxed{450\,\Omega} \end{cases}$$

b) Busco el máximo de $\bar{P}(R) = V^2 \dfrac{R}{R^2+X^2}$:

$$\frac{\partial \bar{P}}{\partial R} = V^2 \cdot \frac{R^2+X^2-R.2R}{(R^2+X^2)^2} = V^2 \frac{X^2-R^2}{(R^2+X^2)^2} = 0 \Leftrightarrow \boxed{R_{\bar{P}_{max}} = X = 225\,\Omega} \Rightarrow$$

$$\Rightarrow \boxed{\bar{P}_{max} = 500\,kW}$$



c) $\hat{V} = (R+jX)\hat{I} \Rightarrow \hat{I} = \dfrac{\hat{V}}{R+jX} = \dfrac{V}{\sqrt{R^2+X^2}} \angle \overbrace{-\text{Atan}\left(\frac{X}{R}\right)}^{=-\varphi}$

$\underline{\bar{P}_{max}}$: $\boxed{\hat{I}_{\bar{P}_{max}} = 47{,}1\,A \angle -45^\circ}$ $\quad (\cos\varphi = 1/\sqrt{2})$

$\underline{\bar{P}_{min}}$: Si $R = 112{,}5\,\Omega \Rightarrow \varphi = 63^\circ \Rightarrow \cos\varphi < 1/\sqrt{2} \Rightarrow$

Descarto esa solución.

$R = 450\,\Omega \Rightarrow \boxed{\hat{I}_{\bar{P}_{min}} = 29{,}8\,A \angle -27^\circ}$

$(\cos\varphi \cong 0{,}9)$