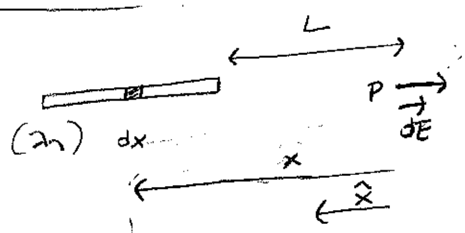
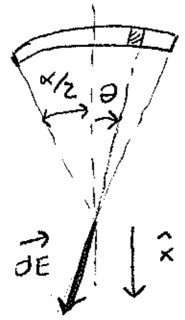
Ejercicio 1

1)



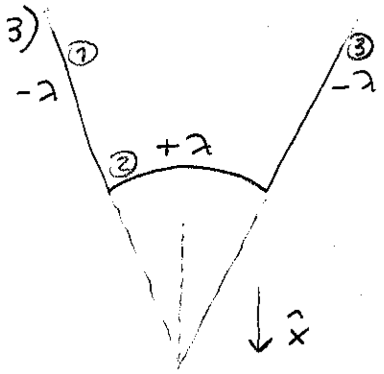
$$\vec{E} = \frac{1}{4\pi\epsilon_0}\int_L^{2L} \frac{\lambda_1 dx}{x^2}(-\hat{x}) = \frac{1}{4\pi\epsilon_0}\frac{\lambda_1}{2L}(-\hat{x})$$

2)



$$\vec{E} = \frac{1}{4\pi\epsilon_0}\int_{-\frac{\alpha}{2}}^{\frac{\alpha}{2}} \frac{(\lambda_2 L\, d\theta)}{L^2}\cos\theta\, \hat{x} = \frac{1}{4\pi\epsilon_0}\frac{2\lambda_2 \operatorname{sen}\alpha/2}{L}\hat{x}$$

3)



$$\vec{E} = \vec{E}_1 + \vec{E}_2 + \vec{E}_3 \, ;$$

$$\vec{E}_1 + \vec{E}_3 = 2E_1 \cos\frac{\alpha}{2}\,\hat{x}\,, \quad E_1 = \frac{1}{4\pi\epsilon_0}\frac{(-\lambda)}{2L}$$

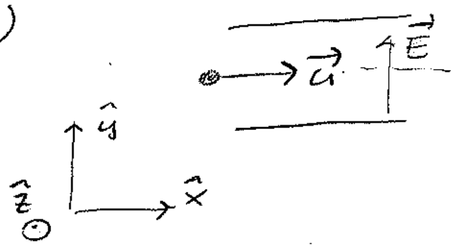
$$\vec{E}_2 = \frac{1}{4\pi\epsilon_0}\frac{2\lambda \operatorname{sen}\alpha/2}{L}\hat{x}$$

$$\Rightarrow \vec{E} = \frac{1}{4\pi\epsilon_0}\frac{2\lambda}{L}\left[-\frac{1}{2}\cos\frac{\alpha}{2} + \operatorname{sen}\frac{\alpha}{2}\right]\hat{x} = 0 : \quad \operatorname{tg}\frac{\alpha}{2} = \frac{1}{2} \Rightarrow$$

$$\Rightarrow \alpha = 2\operatorname{arctg}\left(\frac{1}{2}\right) \sim 53^\circ$$

Ejercicio 2

1)



A partir de la conservación de la energía, la velocidad $\vec{u}$ con que la partícula entra a la región entre placas verifica:

$$\frac{1}{2}mu^2 = qV \quad : \quad u = \sqrt{\frac{2qV}{m}}$$

La fuerza que experimenta la partícula es:

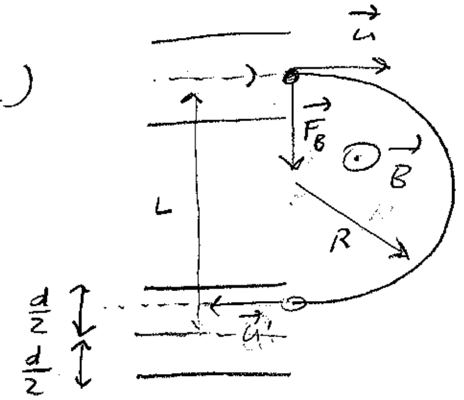
$$\vec{F} = q(\vec{E} + \vec{u}\times\vec{B}) \quad ; \quad \vec{E} = E\hat{y} \quad , \vec{B} = B_y\hat{y} + B_z\hat{z}, \quad \vec{u} = u\hat{x}$$

Para que se mueva con velocidad constante:

$$\vec{F} = 0 : \quad \vec{E} + \vec{u}\times\vec{B} = 0$$

$$E\hat{y} + \sqrt{\frac{2qV}{m}}\,(B_y\hat{z} - B_z\hat{y}) = 0 \quad : \quad \boxed{\begin{array}{l} B_y = 0 \\ B_z = \sqrt{\dfrac{m}{2qV}}\,E \end{array}}$$

2)



En la región sin campo eléctrico la partícula tiene un movimiento circular uniforme de radio R:

$$ma_c = F_B \quad : \quad \frac{mu^2}{R} = quB \quad : \quad \boxed{R = \frac{mu}{qB}}$$

Para que la partícula entre a la región entre el segundo par de placas el radio R debe verificar:

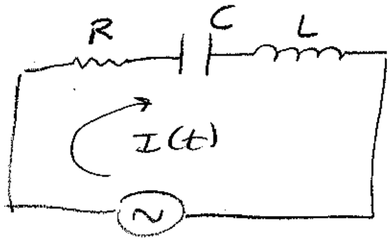
$$L - \frac{d}{2} < 2R < L + \frac{d}{2} \quad \leftrightarrow \quad \boxed{2R - \frac{d}{2} < L < 2R + \frac{d}{2}}$$

3) Nuevamente $\vec{F} = 0 \quad : \quad \vec{E}' + \vec{u}' \times \vec{B} = 0$

como $\vec{u}' = -\vec{u}$ $\qquad \boxed{\vec{E}' = -\vec{E}}$

Ejercicio 3

1)

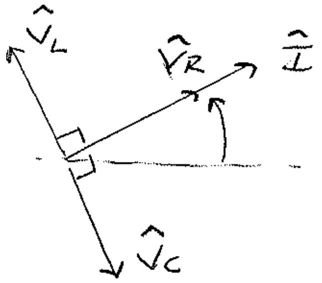


$$\hat{V}_R = R\hat{I} \qquad \left(I(t) = \mathrm{Re}\{\hat{I}\} \right)$$

$$\hat{V}_L = j\omega L \hat{I} = e^{j\pi/2}\,\omega L\,\hat{I}$$

$$\hat{V}_C = \frac{1}{j\omega C}\hat{I} = e^{-j\pi/2}\,\frac{1}{\omega C}\hat{I}$$

Representación fasorial:



2) $$Z_{eq} = R + j\omega L + \frac{1}{j\omega C} \Rightarrow \hat{I} = \frac{V_0 e^{j\omega t}}{Z_{eq}}$$

$$Z_{eq} = \frac{e^{j\phi}}{\sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}} \qquad \text{con} \quad \mathrm{tg}\,\phi = \frac{\omega L - \frac{1}{\omega C}}{R}$$

$$\Rightarrow \begin{cases} I(t) = \underbrace{\dfrac{V_0}{\sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}}}_{I_0} \cos(\omega t - \phi) \\ v_R(t) = \mathrm{Re}\{R\hat{I}\} = R\,I(t) \\ v_L(t) = \mathrm{Re}\left\{e^{j\pi/2}\,\omega L\,\hat{I}\right\} = \omega L\, I_0 \cos\left(\omega t - \phi + \frac{\pi}{2}\right) \\ v_C(t) = \mathrm{Re}\left\{e^{-j\pi/2}\,\frac{1}{\omega C}\hat{I}\right\} = \frac{I_0}{\omega C}\cos(\omega t - \phi - \pi/2) \end{cases}$$

3) $$P(t) = R I^2(t) \; : \; \langle P \rangle = \frac{1}{2} R I_0^2$$

$$\langle P \rangle = \frac{1}{2} R \frac{V_0^2}{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2},$$

el valor máximo se obtiene para $\omega L = 1/\omega C$; de acuerdo a los datos numéricos:

$\omega L < \frac{1}{\omega C}$ (circuito capacitivo)

Para maximizar $\langle P \rangle$ se puede colocar un capacitor en paralelo con C: $C_{eq} = C + C_1$, de manera que $\omega L = \frac{1}{\omega C_{eq}}$: $C_{eq} = C + C_1 = \frac{1}{\omega^2 L}$: $C_1 = \frac{1}{\omega^2 L} - C$, $\omega = 2\pi f$

$$\Rightarrow \boxed{C_1 = 275\,\mu F} \; ; \text{ luego } \langle P \rangle_{máx} = \frac{1}{2}\frac{V_0^2}{R} = \boxed{25\,mW}$$