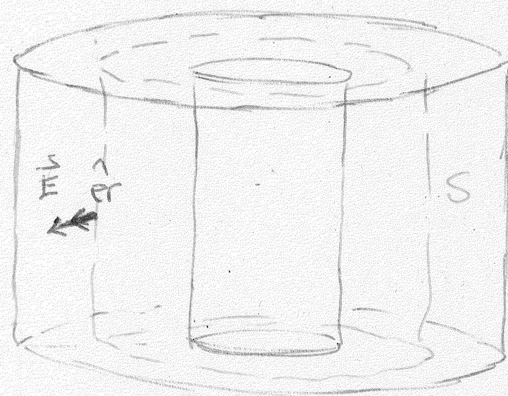
EJERCICIO 1

a) Sea S la superficie cilíndrica de la figura:



Gauss

$$\Rightarrow \quad \underbrace{\oint_S \vec{E}\cdot\hat{n}\, ds}\ = \frac{Q}{\varepsilon_0}$$

$$\cancelto{0}{\int_{TAPA1} \vec{E}\cdot\hat{n}\, ds} + \cancelto{0}{\int_{TAPA2} \vec{E}\cdot\hat{n}\, ds} + \underbrace{\int_{sup.\ lat} \vec{E}\cdot\hat{n}\, ds} = \frac{Q}{\varepsilon_0}$$

$$\int |\vec{E}|\overbrace{\hat{e}_r\cdot\hat{e}_r}^{=1}\, ds = \frac{Q}{\varepsilon_0} \Rightarrow |\vec{E}|\cdot\int_{sup.\ lat} ds = \frac{Q}{\varepsilon}$$

$$\Rightarrow |\vec{E}|\, 2\pi r l = \frac{Q}{\varepsilon} \Rightarrow \boxed{|\vec{E}| = \frac{Q}{2\pi r l \varepsilon_0}}$$

$$\Rightarrow \boxed{\vec{E} = \frac{Q}{2\pi r l \varepsilon}\hat{e}_r}$$

$$\Delta V = -\int_d^D \vec{E}\cdot\underbrace{d\vec{r}}_{= dr\,\hat{e}_r} = -\frac{Q}{2\pi l \varepsilon_0}\int_d^D \frac{1}{r}\cdot\overbrace{\hat{e}_r\, dr\,\hat{e}_r}^{=1} = -\frac{Q}{2\pi l \varepsilon}\ln\left(\frac{D}{d}\right)$$

$$\Rightarrow \underbrace{|\Delta V|}_{= \mathcal{E}} = \frac{Q\ln(D/d)}{2\pi l \varepsilon_0} \Rightarrow \boxed{Q_0 = \frac{2\pi l \varepsilon_0 \mathcal{E}}{\ln(D/d)}}$$

b)

$$C_{eq} \overset{\text{paralelo}}{=} kC_1 + C_2 = k\frac{2\pi l\varepsilon_0}{4\ln(D/d)} + \frac{2\pi\, 3l/4\, \varepsilon_0}{\ln(D/d)} = \frac{2\pi l \varepsilon_0}{\ln(D/d)}\left(\frac{k}{4} + \frac{3}{4}\right)$$

$$= \frac{\pi l \varepsilon_0}{\ln(D/d)}\left(\frac{3+k}{2}\right)$$

Luego la carga acumulada será: $\boxed{Q_f = C_{eq}\mathcal{E} = \frac{\pi l \varepsilon}{\ln(D/d)}\left(\frac{3+k}{2}\right)\mathcal{E}}$

c)

$$W_F = \int_{Q_0}^{Q_f} V\, dq = \int_{Q_0}^{Q_f} V_0\, dq = V_0\int_{Q_0}^{Q_f} dq = \overbrace{V_0}^{=\mathcal{E}}(Q_f - Q_0)$$

$$= \frac{\pi l \varepsilon}{\ln(D/d)}\left(\frac{3+k}{2} - 2\right)\mathcal{E}^2 = \frac{\pi l \varepsilon}{\ln(D/d)}\left(\frac{k-1}{2}\right)\mathcal{E}^2$$