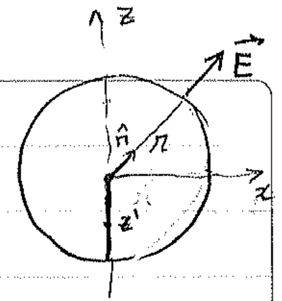


Problema 1

a) Contribución de la esfera

$r<R$, Gauss $\oint_S \vec{E}\,\hat{n}\,dS = 4\pi r^2 E = \dfrac{q_{enc}}{\varepsilon_0}$

$$q_{enc} = \frac{4}{3}\pi r^3 \rho \Rightarrow E = \frac{1}{\varepsilon_0}\frac{\frac{4}{3}\pi r^3 \rho}{4\pi r^2} = \frac{\rho}{3\varepsilon_0} r \rightarrow \vec{E} = \frac{\rho}{3\varepsilon_0} r\,\hat{r}$$

$$r>R,\quad q_{enc} = \frac{4}{3}\pi R^3 \rho \Rightarrow E = \frac{1}{\varepsilon_0}\frac{\frac{4}{3}\pi R^3\rho}{4\pi r^2} = \frac{\rho R^3}{3\varepsilon_0 r^2} \Rightarrow \vec{E} = \frac{\rho R^3}{3\varepsilon_0 r^2}\hat{r}$$

Contribución de la barra (sobre el eje z) $\vec{E} = \dfrac{1}{4\pi\varepsilon_0}\displaystyle\int_{-R}^{0} \frac{-\lambda\, dz'}{(z-z')^2}$

$$= -\frac{\lambda}{4\pi\varepsilon_0}\int_0^R \frac{du}{(u+z)^2}\hat{z} = -\frac{\lambda\hat{z}}{4\pi\varepsilon_0}\left[-\frac{1}{u+z}\right]_{u=0}^{u=R} = -\frac{\lambda\hat{z}}{4\pi\varepsilon_0}\frac{R}{z(z+R)} \qquad z' = -u$$

$\Rightarrow$ sobre el eje z, $z>0$;

$$\boxed{\begin{aligned}\vec{E}(z) &= \left[\frac{\rho}{3\varepsilon_0}z - \frac{\lambda R}{4\pi\varepsilon_0 z(z+R)}\right]\hat{z}, \text{ si } 0<z<R\\ \vec{E}(z) &= \left[\frac{\rho R^3}{3\varepsilon_0 z^2} - \frac{\lambda R}{4\pi\varepsilon_0 z(z+R)}\right]\hat{z}, \text{ si } z>R\end{aligned}}$$

b) $\vec{E}\left(\frac{R}{2}\right) = 0 \Rightarrow \dfrac{\rho}{6\varepsilon_0}R - \dfrac{\lambda R}{4\pi\varepsilon_0 \frac{R}{2}\cdot\frac{3R}{2}} = 0 \Rightarrow \boxed{\rho = \dfrac{2\lambda}{\pi R^2}}$

c) $z>R$, $\underbrace{V(\infty)}_{=0} - V(z) = -\displaystyle\int_z^\infty E(z')\,dz' \Rightarrow \boxed{V(z) = \displaystyle\int_z^\infty\left[\frac{\rho R^3}{3\varepsilon_0 z'^2} - \frac{\lambda R}{4\pi\varepsilon_0 z'(z'+R)}\right]dz'}$

$$= \frac{\lambda}{\pi\varepsilon_0}\int_z^\infty\left[\frac{2R}{3z'^2} - \frac{R}{4z'(z'+R)}\right]dz' = \frac{\lambda}{\pi\varepsilon_0}\int_z^\infty\left[\frac{2R}{3z'^2} - \frac{1}{4z'} + \frac{1}{4(z'+R)}\right]dz'$$

$$= \frac{\lambda}{\pi\varepsilon_0}\left[-\frac{2R}{3}\frac{1}{z'} + \frac{1}{4}\ln\left(\frac{z'+R}{z'}\right)\right]_z^\infty = \boxed{\frac{\lambda}{\pi\varepsilon_0}\left[\frac{2R}{3z} - \frac{1}{4}\ln\left(\frac{z+R}{z}\right)\right]}$$

$$z=R \qquad V(R) = \frac{\lambda}{\pi\varepsilon_0}\left[\frac{2}{3} - \frac{\ln 2}{4}\right]$$

$0<z<R$ $\quad V(R) - V(z) = -\displaystyle\int_z^R E(z)\,dz' \Rightarrow \boxed{V(z)} = V(R) + \displaystyle\int_z^R E(z')\,dz' =$

$$= V(R) + \int_z^R\left[\frac{\rho z'}{3\varepsilon_0} - \frac{\lambda R}{4\pi\varepsilon_0 z'(z'+R)}\right]dz' = V(R) + \frac{\lambda}{\pi\varepsilon_0}\int_z^R\left[\frac{2}{3}\frac{z'}{R^2} - \frac{1}{4z'} + \frac{1}{4(z'+R)}\right]dz'$$

$$= V(R) + \frac{\lambda}{\pi\varepsilon_0}\left[\frac{1}{3}\frac{z'^2}{R^2} + \frac{1}{4}\ln\left(\frac{z'+R}{z'}\right)\right]_z^R =$$

$$= \frac{\lambda}{\pi\varepsilon_0}\left[\frac{2}{3} - \cancel{\frac{\ln 2}{4}}\right] + \frac{\lambda}{\pi\varepsilon_0}\left[\frac{1}{3} - \frac{1}{3}\frac{z^2}{R^2} + \cancel{\frac{1}{4}\ln 2} - \frac{1}{4}\ln\left(\frac{z+R}{z}\right)\right]$$

$$= \boxed{\frac{\lambda}{\pi\varepsilon_0}\left[1 - \frac{1}{3}\frac{z^2}{R^2} - \frac{1}{4}\ln\left(\frac{z+R}{z}\right)\right]}$$

Problema 2

a) Por Gauss, $r < r_1$ carga encerrada 0, $\Rightarrow \vec{E} = 0$

$r > r_2$ " " $= Q - Q = 0 \Rightarrow \vec{E} = 0$

$r_1 < r < r_2$ " " $= Q \Rightarrow 2\pi r L E = \dfrac{Q}{\varepsilon_0}$

$$\Rightarrow \boxed{\vec{E} = \frac{Q}{2\pi\varepsilon_0 L r}\,\hat{r}}$$ Coordenadas cilíndricas.

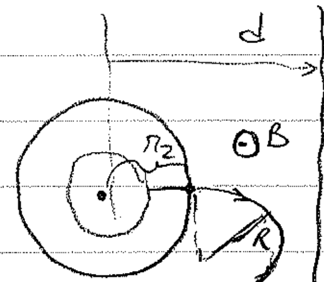
b) $$\boxed{V(r_2) - V(r_1) = -\int_{r_1}^{r_2} \vec{E}(\vec{r})\cdot d\vec{r} = -\int_{r_1}^{r_2} \frac{Q\,dr}{2\pi\varepsilon_0 L r} = -\frac{Q}{2\pi\varepsilon_0 L}\ln\left(\frac{r_2}{r_1}\right)} \ (<0)$$

$\Rightarrow V(r_2) < V(r_1)$; la placa interior, positiva, estará a un potencial mayor

c) $$\frac{1}{2} m \underset{\overset{\parallel}{0}}{v_{(r_1)}^2} + q V(r_1) = \frac{1}{2} m v_{(r_2)}^2 + q V(r_2)$$

$$\Rightarrow \frac{1}{2} m v_{(r_2)}^2 = q\left[V(r_1) - V(r_2)\right] = \frac{qQ}{2\pi\varepsilon_0 L}\ln\left(\frac{r_2}{r_1}\right)$$

$$\Rightarrow v_{(r_2)} = \left[\frac{qQ}{\pi\varepsilon_0 L m}\ln\left(\frac{r_2}{r_1}\right)\right]^{1/2}$$



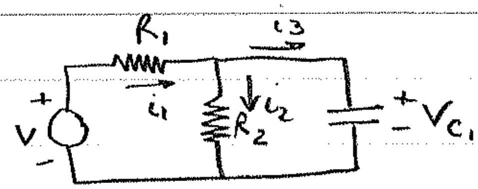
Después de salir del cilindro exterior interacciona con $\vec{B}$ cte. $\Rightarrow$ trayectoria circular de radio R tal que $F_B = qvB = m\cdot\dfrac{v^2}{R}$

$\Rightarrow R = \dfrac{mv}{qB}$; v es la velocidad con que sale del cilindro exterior $\Rightarrow R = \dfrac{m}{qB}\cdot\left[\dfrac{qQ}{\pi\varepsilon_0 L m}\ln\left(\dfrac{r_2}{r_1}\right)\right]^{1/2}$

Va a chocar con la pantalla si $r_2 + R > d$, o sea

si $$\boxed{d < r_2 + \frac{1}{B}\left[\frac{m}{\pi\varepsilon_0 L}\,\frac{Q}{q}\ln\left(\frac{r_2}{r_1}\right)\right]^{1/2}}$$

Problema 3



a) Kirchhoff $\quad i_1 = i_2 + i_3$

$$V - R_1 i_1 - V_{C_1} = 0$$

$$V_{C_1} = R_2 i_2$$

$$\boxed{V_{C_1} = \frac{Q_1}{C}} \quad , \quad \boxed{i_3 = \frac{dQ_1}{dt}} = C_1 \frac{dV_{C_1}}{dt}$$

$$\Rightarrow i_1 = \frac{V - V_{C_1}}{R_1} \quad , \quad i_2 = \frac{V_{C_1}}{R_2} \quad \Rightarrow \quad \frac{V - V_{C_1}}{R_1} = \frac{V_{C_1}}{R_2} + C_1 \frac{dV_{C_1}}{dt}$$

$$\boxed{\frac{dV_{C_1}}{dt} + \frac{R_1 + R_2}{R_1 R_2 C_1} V_{C_1} = \frac{V}{R_1 C_1}}$$

Homogénea $\quad V_{C_1}(t) = A \cdot e^{-\frac{(R_1+R_2)t}{R_1 R_2 C_1}}$

Particular $\quad V_{C_1}(t) = \dfrac{R_2 V}{R_1 + R_2}$

Como inicialmente $V_{C_1}(0) = 0$

$$\longrightarrow \boxed{V_{C_1}(t) = \frac{R_2 V}{R_1 + R_2}\left(1 - e^{-\frac{(R_1+R_2)t}{R_1 R_2 C_1}}\right)}$$

b) $t = \infty \quad V_{C_1}(\infty) = \dfrac{R_2 V}{R_1 + R_2}$ ✓

c) La carga del condensador C_1, $Q_{1\,in} = C_1 \cdot \dfrac{R_2 V}{R_1 + R_2}$ se distribuye entre los dos condensadores, que estarán al mismo potencial después de mucho tiempo.

$$\rightarrow \frac{C_1 R_2 V}{R_1 + R_2} = (C_1 + C_2) V_f \quad \Rightarrow \quad \boxed{V_f = \frac{C_1}{C_1 + C_2} \cdot \frac{R_2}{R_1 + R_2} \cdot V}$$

d) La energía disipada en la resistencia será igual a la disminución de la energía electrostática en los condensadores,

$$\boxed{E_{dis}} = \frac{1}{2} C_1 \cdot \left(\frac{R_2 V}{R_1 + R_2}\right)^2 - \frac{1}{2}(C_1 + C_2)\left[\frac{C_1}{C_1 + C_2} \cdot \frac{R_2 V}{R_1 + R_2}\right]^2 =$$

$$= \boxed{\frac{1}{2} \frac{C_1 C_2}{C_1 + C_2} \left(\frac{R_2 V}{R_1 + R_2}\right)^2}$$

Con los datos dados,

a) $V_{C_1}(t) = 4\left(1 - e^{-\frac{t}{1,6}}\right) V$ voltios

c) $V_f = \frac{1}{1,1} \cdot \frac{2}{10} \cdot 20 = 3,64\ V$

c) $E_{dis} = \frac{1}{2} \frac{10^{-6} \times 10^{-7}}{1,1 \times 10^{-6}} \cdot 4^2 = \frac{0,8}{1,1} \times 10^{-6} = 0,727 \times 10^{-6}\ J$