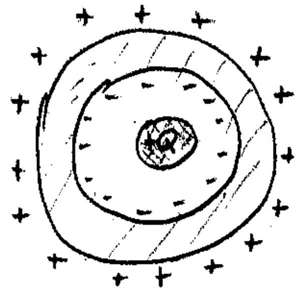
Física 3 – 1º semestre – 2016

Examen julio (26/07/2016)

Solución

Problema 1:

a)



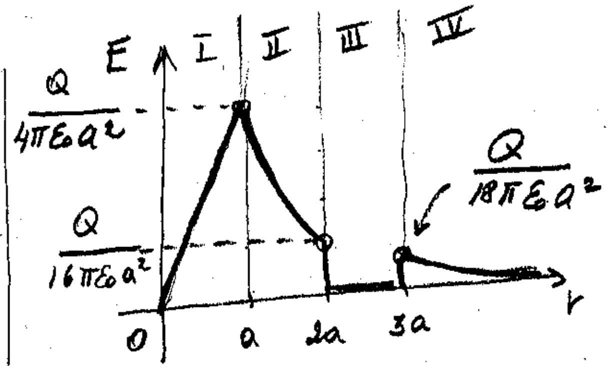
$r < a$: densidad uniforme $\rho = \frac{3Q}{4\pi a^3}$

$r = 2a$: carga de superficie $-Q$

$r = 3a$: " " " $+2Q$

b)

- $r \leq a$: $E_I = \frac{Q}{4\pi\varepsilon_0 a^3} r$ (a^3)
- $a \leq r \leq 2a$: $E_{II} = \frac{Q}{4\pi\varepsilon_0} \frac{1}{r^2}$
- $2a \leq r \leq 3a$: $E_{III} = 0$
- $r \geqslant 3a$: $E_{IV} = \frac{Q}{2\pi\varepsilon_0} \frac{1}{r^2}$

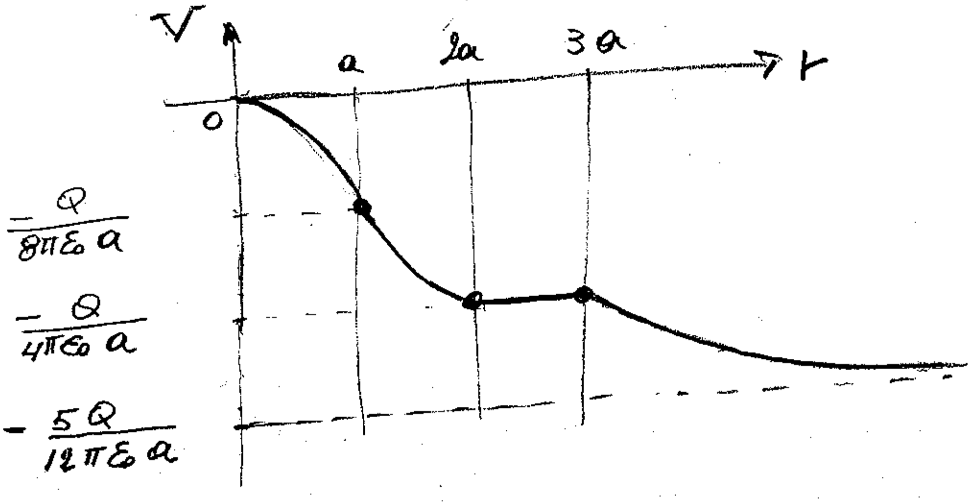


c) en (I) $V_I = -\int_0^r \frac{Q}{4\pi\varepsilon_0 a^3} r' dr' = \frac{-Q}{8\pi\varepsilon_0 a^3} r^2$

en (II) $V_{II} = \frac{-Q}{8\pi\varepsilon_0 a} - \frac{Q}{4\pi\varepsilon_0} \int_a^r \frac{dr'}{r'^2} = -\frac{Q}{4\pi\varepsilon_0 a}\left[\frac{3}{2} - \frac{a}{r}\right]$

en (III) $V_{III} = V_{II}(a) + 0 = -\frac{Q}{4\pi\varepsilon_0 a}$

en IV $V_{IV} = -\frac{Q}{4\pi\varepsilon_0 a} - \frac{Q}{2\pi\varepsilon_0} \int_{3a}^r \frac{dr'}{r'^2} = \frac{-5Q}{12\pi\varepsilon_0 a} + \frac{Q}{2\pi\varepsilon_0} \frac{1}{r}$



d) $\Delta V = \frac{Q}{2\pi\varepsilon_0}\left(\frac{1}{3a} - \frac{1}{d}\right) = \frac{Q}{12\pi\varepsilon_0 a}$, with $d = 6a$

$\frac{1}{2} m v_{MAX}^2 = q\Delta V = \frac{qQ}{12\pi\varepsilon_0 a} \Rightarrow \boxed{v_{MAX} = \sqrt{\frac{qQ}{6\pi\varepsilon_0 m a}}}$

Problema 2 :

a) $d \underbrace{\sin\theta}_{y/D} = n\lambda \Rightarrow \boxed{\frac{d}{D}\Delta y = \lambda} \Rightarrow \boxed{\lambda = 460\,nm}$

b) $n = \frac{c}{v}$; $\left.\begin{array}{l} v = \omega/k' \\ c = \omega/k \end{array}\right\} \Rightarrow \frac{k'}{k} = \frac{c}{v} = n \Rightarrow \frac{\lambda}{\lambda'} = n \Rightarrow \boxed{\lambda' = \frac{\lambda}{n}}$

$\boxed{\Delta' y = \frac{D}{d}\lambda' = \frac{\Delta y}{n} \simeq 346\,\mu m}$

c) $\frac{D'}{d}\frac{\lambda}{n} = \frac{D}{d}\lambda \Rightarrow \boxed{D' = nD = 1.33\,m}$

Problema 3 :

$E(r) = \frac{q}{4\pi\varepsilon_0}\frac{1}{r^2} \Rightarrow V = -\int_R^{R+d} E(r)\,dr = -\frac{q}{4\pi\varepsilon_0}\frac{d}{(R+d)R}$

a) $R \gg d \Rightarrow |V| \simeq \frac{q}{4\pi\varepsilon_0}\frac{d}{R^2} \Rightarrow \boxed{C = \frac{q}{V} = \frac{4\pi\varepsilon_0 R^2}{d} = 10.04\,\mu F}$

b) 

$I = I_C + I_{RL} = V\left[i\omega C + \frac{1}{R + i\omega L}\right] = V\,\frac{R - i\omega\left[L - C(R^2 + (L\omega)^2)\right]}{R^2 + (\omega L)^2}$

$\Rightarrow I \simeq V\,\frac{3 - 2i}{69} \Rightarrow \boxed{I_{RMS} = \underbrace{230}_{V_{RMS}}\,\frac{\sqrt{13}}{69} \simeq 12\,A}$

$\tan\phi = -2/3 \Rightarrow \boxed{\phi = -33.7^\circ}$

c) $\boxed{FP = \cos\phi = 0.83}$

d) $\cos\phi = 1 \Leftrightarrow L = C(R^2 + (L\omega)^2) \Rightarrow C = \frac{L}{R^2 + (L\omega)^2} = 101\,mF$

$\Rightarrow \boxed{\kappa_E = \frac{C}{C_0} = 10.01}$

e) $\overline{P} = V_{RMS} I_{RMS} = V_{RMS}^2 \frac{R}{\ldots}$