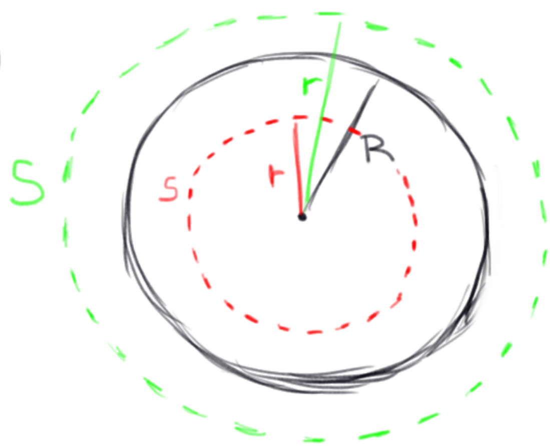
SOLUCIONES PRIMER PARCIAL

4 de mayo de 2021

Ejercicio 1.

Por simetría $\vec{E} = E(r)\,\hat{e}_r$

a)



$r<R$ Tomo esfera gaussiana s.

$$\oint_s \vec{E}\cdot d\vec{a} = \oint_s E(r)\underbrace{\hat{e}_r\cdot\hat{e}_r}_{=1}\,da \quad \text{(cte en } s\text{)}$$

$$= E(r)\oint_s da = E(r)\,4\pi r^2$$

$$Q_{in} = \rho V = \frac{Q}{\cancel{4/3}\,\cancel{\pi} R^3}\,\cancel{4/3}\,\cancel{\pi}\, r^3$$

(ρ: densidad uniforme)

Ley de Gauss:

$$E(r)\,4\pi r^2 = \frac{Qr^3}{\varepsilon_0 R^3} \Rightarrow \boxed{\vec{E}(r) = \frac{Qr}{4\pi\varepsilon_0 R^3}\,\hat{e}_r}$$

$r>R$. Tomo esfera gaussiana S.

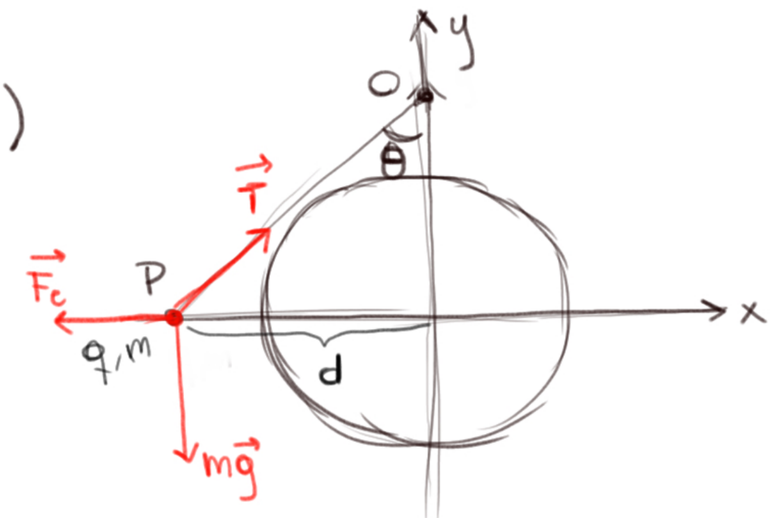
$$\oint_S \vec{E}\cdot d\vec{a} = E(r)\,4\pi r^2 \quad \text{(idem caso anterior)}$$

$$Q_{in} = Q$$

Ley de Gauss:

$$E(r)\,4\pi r^2 = \frac{Q}{\varepsilon_0} \Rightarrow \boxed{\vec{E}(r) = \frac{Q}{4\pi\varepsilon_0 r^2}\,\hat{e}_r}$$

b)



$$\sum \vec{F_i} = 0$$

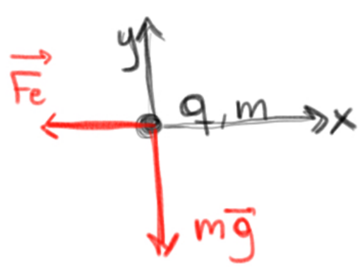
$$\begin{cases} T\sin\theta = qE & ① \\ T\cos\theta = mg & ② \end{cases}$$

②/① : $\dfrac{mg}{qE} = \dfrac{1}{\tan\theta} \longrightarrow m = \dfrac{qE}{g\tan\theta}$

por a) : $\boxed{m = \dfrac{qQ}{4\pi\varepsilon_0 d^2 g\tan\theta}}$

c) $m\vec{a} = \sum \vec{F} \longrightarrow \begin{cases} ma_x = -\dfrac{qQ}{4\pi\varepsilon_0 d^2} \rightarrow \text{por b)} \ \boxed{a_x = -g\tan\theta} \\ ma_y = -mg \end{cases}$

$\hookrightarrow \boxed{a_y = -g}$



d) $W_{e_{P\to P'}} = -q\,\Delta V = -q\,(V_{P'} - V_P)$

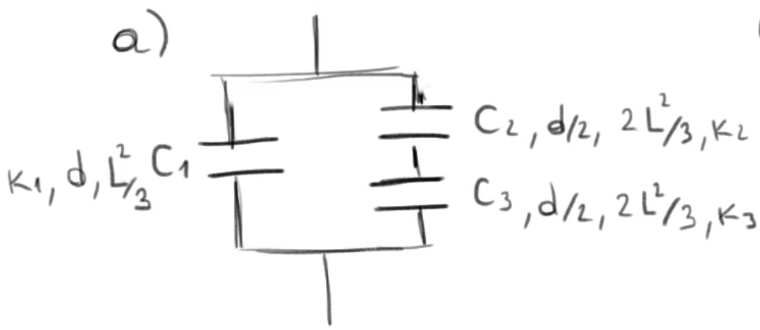
potencial de la esfera:

$$V(r) = -\int_{\infty}^{r} \vec{E}\cdot d\vec{l} = \frac{Q}{4\pi\varepsilon_0 r}$$

$$\rightarrow \boxed{W_{e_{P\to P'}} = -\frac{qQ}{4\pi\varepsilon_0}\left[\frac{1}{D} - \frac{1}{d}\right]}$$

Ejercicio 2.

a)



$$C = C_1 + \left[\frac{1}{C_2} + \frac{1}{C_3}\right]^{-1}$$

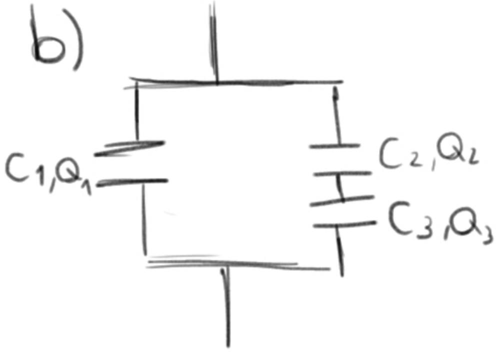
$$C_1 = \frac{K_1\varepsilon_0 L^2}{3d}$$

$$C_2 = \frac{4K_2\varepsilon_0 L^2}{3d}$$

$$C_3 = \frac{4K_3\varepsilon_0 L^2}{3d}$$

$$\rightarrow \boxed{C =} \frac{K_1\varepsilon_0 L^2}{3d} + \frac{(4K_2\varepsilon_0 L^2/3d)(4K_3\varepsilon_0 L^2/3d)}{(4K_2\varepsilon_0 L^2/3d) + (4K_3\varepsilon_0 L^2/3d)} = \boxed{\left[\frac{K_1}{3} + \frac{4}{3}\frac{K_2K_3}{K_2+K_3}\right]\frac{\varepsilon_0 L^2}{d}}$$

b)



$$Q_2 = Q_3 \quad ①$$

$$Q_1 + Q_2 = Q \quad ②$$

$$V_1 = V_2 + V_3 \longrightarrow \frac{Q_1}{C_1} = \frac{Q_2}{C_2} + \frac{Q_3}{C_3}$$

por ① y ②: $\dfrac{Q_1}{C_1} = \dfrac{(Q - Q_1)}{C_2} + \dfrac{(Q - Q_1)}{C_3}$

$$Q_1 \left[\frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}\right] = Q\left[\frac{1}{C_2} + \frac{1}{C_3}\right] \longrightarrow Q_1 = \left[\frac{1/C_2 + 1/C_3}{1/C_1 + 1/C_2 + 1/C_3}\right] Q$$

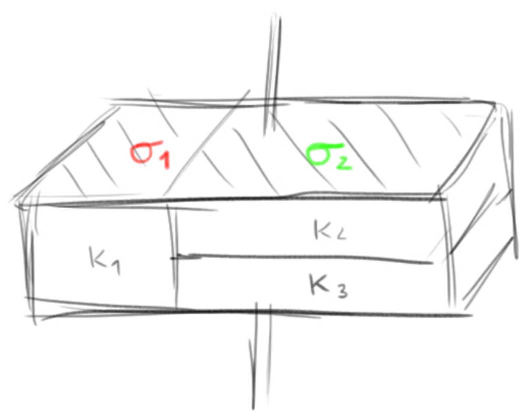
usando las capacitancias calculadas en a)

$$Q_1 = \left[\frac{3/4k_2 + 3/4k_3}{3/k_1 + 3/4k_2 + 3/4k_3}\right] Q = \left[\frac{(k_2 + k_3)/k_2 k_3}{(4k_2k_3 + k_2k_1 + k_3k_1)/k_1k_2k_3}\right] Q$$

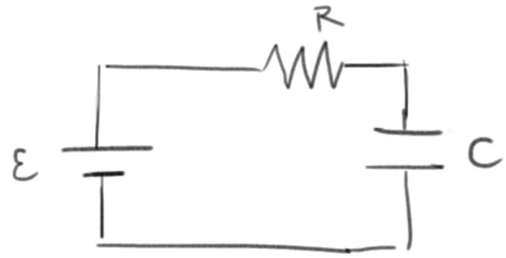
$$\longrightarrow Q_1 = \left[\frac{k_1(k_2 + k_3)}{k_1k_2 + 4k_2k_3 + k_3k_1}\right] Q$$

$$Q_2 = \left[1 - \frac{k_1(k_2 + k_3)}{k_1k_2 + 4k_2k_3 + k_3k_1}\right] Q = \left[\frac{4k_2k_3}{k_1k_2 + 4k_2k_3 + k_3k_1}\right] Q$$

$$\boxed{\sigma_1 = \frac{Q_1}{L^2/3} \qquad \sigma_2 = \frac{Q_2}{2L^2/3}}$$



c) en $t \to \infty$: $i = 0$, $V_C = \varepsilon$

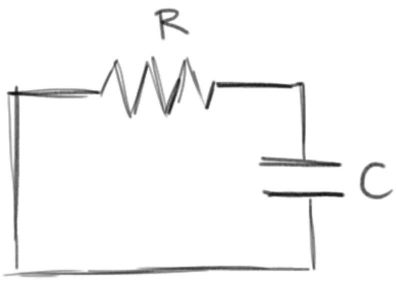


$\hookrightarrow \frac{Q}{C} = \varepsilon \to \boxed{Q = \varepsilon C}$

energía almacenada:

$$\boxed{U =} \frac{Q^2}{2C} = \boxed{\frac{\varepsilon^2 C}{2}}$$

d)



Ley de mallas: $iR + \frac{q}{C} = 0$

$$\to \frac{dq}{dt} = -\frac{q}{RC} \to q(t) = q_0 e^{-t/RC}$$

$$q_0 = \varepsilon C \to q(t) = \varepsilon C e^{-t/RC}$$

derivando:

$$i(t) = -\frac{\varepsilon}{R} e^{-t/RC}$$

potencia disipada:

$$P(t) = R i^2(t) \longrightarrow \boxed{P(t) = \frac{\varepsilon^2}{R} e^{-2t/RC}}$$

d) $$\boxed{U =} \int_0^\infty P(t)\,dt = \frac{\varepsilon^2}{R} \int_0^\infty e^{-2t/RC} dt = \boxed{\frac{\varepsilon^2 C}{2}}$$

igual a la energía almacenada inicialmente en C (conservación de la energía)