Física 3 - 1er parcial 2023

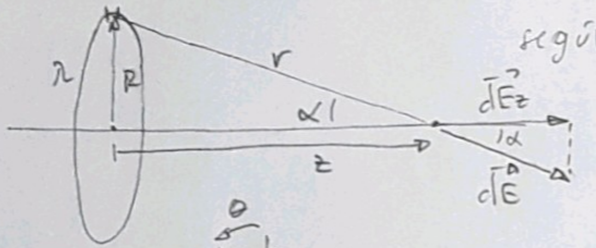
Ejercicio 1

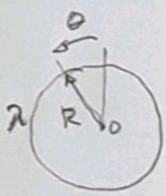
ⓐ

Principio de superposición para el campo eléctrico =

"El campo eléctrico se puede calcular como la suma vectorial de los campos de fuentes individuales"

Campo de una espira:





Por la simetría de la distribución, el campo total es según $\hat{k}$.

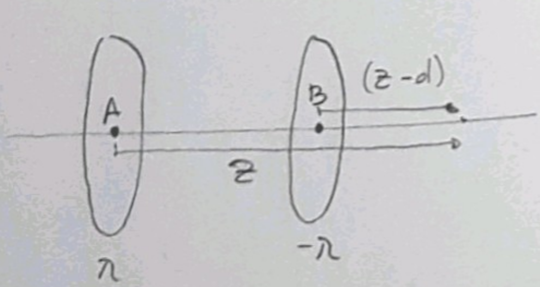
$$\vec{dE}\cdot\hat{k} = \frac{1}{4\pi\varepsilon_0}\,\frac{\lambda R\,d\theta}{r^2}\cdot\cos\alpha$$

$$\vec{dE_z} = \frac{\lambda R\,d\theta}{4\pi\varepsilon_0\,(z^2+R^2)}\cdot\frac{z}{\sqrt{z^2+R^2}}\,\hat{k}$$

$$\vec{E}_\lambda = \int \vec{dE_z} = \int_0^{2\pi}\frac{\lambda R z\,d\theta\,\hat{k}}{4\pi\varepsilon_0\,(z^2+R^2)^{3/2}} = \frac{\lambda R z\,\hat{k}}{4\pi\varepsilon_0\,(z^2+R^2)^{3/2}}\int_0^{2\pi}d\theta$$

$$\vec{E}_\lambda = \frac{\lambda R z}{2\varepsilon_0\,(z^2+R^2)^{3/2}}\,\hat{k}$$

Campo de dos espiras:



$$\vec{E}_z = \vec{E}_\lambda + \vec{E}_{-\lambda}$$

$$\boxed{\vec{E}_z = \left[\frac{\lambda R z}{2\varepsilon_0\,(z^2+R^2)^{3/2}} + \frac{(-\lambda)R(z-d)}{2\varepsilon_0\,((z-d)^2+R^2)^{3/2}}\right]\hat{k}}$$

ⓑ

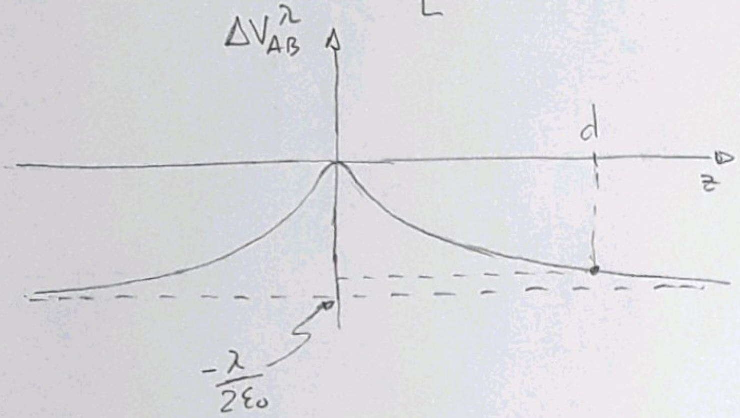
$$\Delta V_{AB} = -\int_A^B \vec{E}_z\cdot dz\,\hat{k} = -\int_A^B |\vec{E}_z|\,dz$$

Para la espira λ

$$\Delta V_{AB}^{\lambda} = -\int_0^d \frac{\lambda R z\, dz}{2\varepsilon_0 (z^2+R^2)^{3/2}} = -\frac{\lambda R}{4\varepsilon_0}\int_{R^2}^{d^2+R^2} \frac{du}{u^{3/2}} = \frac{\lambda R}{2\varepsilon_0}\left(u^{-1/2}\Big|_{R^2}^{d^2+R^2} \right) =$$

Cambio de variable $\left[\begin{array}{l} z^2+R^2 = u \\ 2 z\, dz = du \end{array}\right.$

$$= \frac{\lambda R}{2\varepsilon_0}\left(\frac{1}{\sqrt{d^2+R^2}} - \frac{1}{|R|} \right)$$



Para la otra espira la situación es análoga, con $\lambda \to -\lambda$

La diferencia en la configuración de 2 espiras es entonces:

$$\Delta V_{AB} = 2\,\Delta V_{AB}^{\lambda} = \frac{\lambda R}{\varepsilon_0}\left(\frac{1}{\sqrt{d^2+R^2}} - \frac{1}{|R|} \right)$$

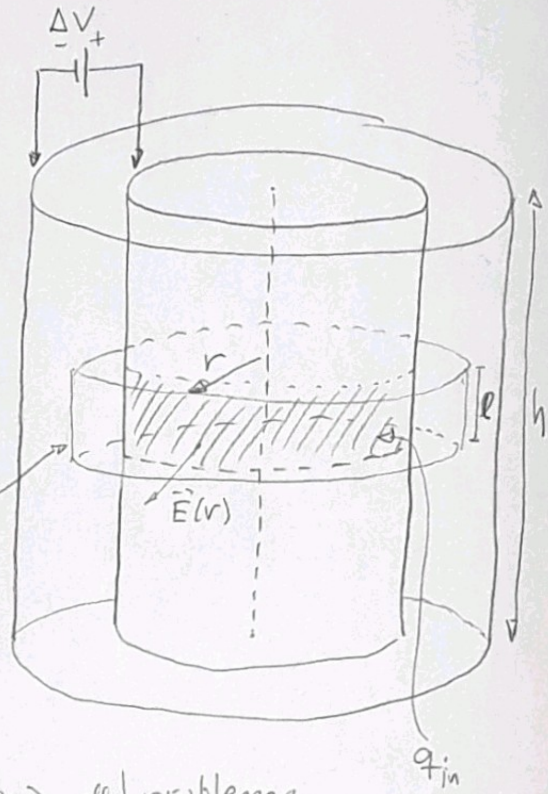
(c) La fuerza eléctrica es conservativa, existe un potencial eléctrico

$U = q_0 V$ (tomando referencia de potencial nulo en A).

<u>Conservación de la energía</u> = $\frac{1}{2} m v_B^2 + q_0 V_B = 0$.

$$v_B = \sqrt{\frac{2 q_0 \lambda R}{m \varepsilon_0}\left(\frac{1}{|R|} - \frac{1}{\sqrt{R^2+d^2}} \right)}$$

Ejercicio (2)



(a) Ley de Gauss. "El flujo de campo eléctrico a través de una superficie cerrada es igual a la carga eléctrica neta interna dividido por la permitividad del material".

$$\Phi_E = \oint \vec{E}(r) \cdot d\vec{A} = \frac{q_{in}}{\varepsilon_0}$$

$$\Phi_E = E(r)\, l\, 2\pi r = \frac{q_{in}}{\varepsilon_0}$$

Por la simetría del problema, asumiremos $\vec{E} = E(r)\hat{e}_r$

$$q_{in} = \sigma_1 2\pi r_1 l$$

$$\vec{E}(r) = \frac{r_1 \sigma_1}{\varepsilon_0 r}\hat{e}_r$$

$$\Delta V = -\int_{r_1}^{r_2} \vec{E}(r)\, d\vec{r} = -\int_{r_1}^{r_2} \frac{\sigma_1 r_1}{\varepsilon_0}\frac{dr}{r} = \frac{\sigma_1 r_1}{\varepsilon_0} \operatorname{Ln}\frac{r_1}{r_2} \quad (\text{Obs.: } \Delta V < 0)$$

$d\vec{r} = dr\,\hat{e}_r$

$$\boxed{\sigma_1 = \frac{\varepsilon_0 \Delta V}{r_1 \operatorname{Ln}(r_2/r_1)}}$$

$$C = \frac{q}{|\Delta V|} = \frac{\sigma_1 2\pi r_1 l}{\frac{\sigma_1 r_1}{\varepsilon_0}\operatorname{Ln}(r_2/r_1)} = \boxed{\frac{2\pi l \varepsilon_0}{\operatorname{Ln}(r_2/r_1)}}$$

(b) Se debe considerar que: a)- la carga en cada placa se conserva
b)- cada placa es una equipotencial

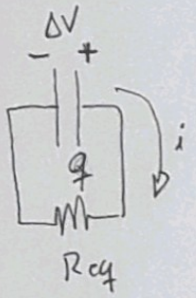
b) $\Delta V' = \frac{\sigma_1' r_1}{\varepsilon_0} \operatorname{Ln}(r_1/r_2) = \frac{\sigma_1'' r_1}{\varepsilon} \operatorname{Ln}(r_1/r_2) \Rightarrow \sigma_1'' = \frac{\varepsilon}{\varepsilon_0}\sigma_1'$

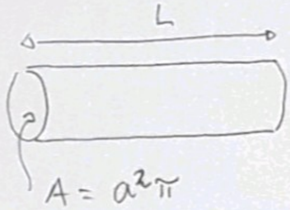
a) $\sigma_1 2\pi r_1 l = \sigma_1' 2\pi r_1 (l-d) + \sigma_1'' 2\pi r_1 d \Rightarrow \sigma_1 l = \sigma_1'(l-d) + \sigma_1'' d$

$$\sigma_1 l = \sigma_1'(l-d) + \frac{\varepsilon}{\varepsilon_0}\sigma_1' d \Longrightarrow \boxed{\sigma_1' = \left[\frac{\varepsilon_0 l}{\varepsilon d + \varepsilon_0 (l-d)}\right]\sigma_1}$$

$$\boxed{\sigma_1'' = \left[\frac{\varepsilon l}{\varepsilon d + \varepsilon_0 (l-d)}\right]\sigma_1}$$

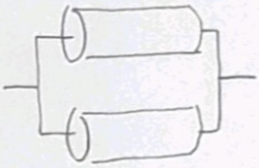
(c)





$A = a^2\pi$

$$R = \frac{\rho L}{a^2\pi}$$



$$R_{eq} = \frac{1}{\frac{1}{R} + \frac{1}{R}} = \frac{R}{2}$$

Kirchhoff:

$$\frac{q}{C} + R_{eq}\, i = 0$$

$$\frac{q}{C} + R_{eq}\frac{dq}{dt} = 0$$

$$\frac{1}{q}\frac{dq}{dt} = -\frac{1}{R_{eq}C} \longrightarrow \int_{t=0}^{t} \frac{1}{q}\frac{dq}{dt} = -\frac{t}{R_{eq}C}$$

$$\ln\left(\frac{q}{q_{in}}\right) = -\frac{t}{R_{eq}C}$$

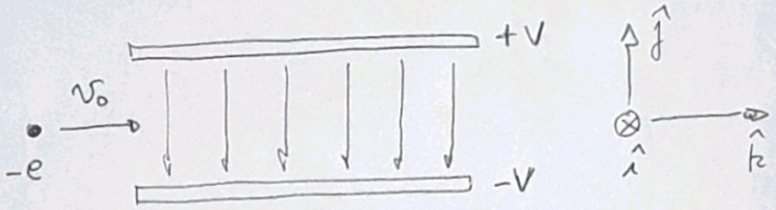
$$q(t) = q_{in} e^{-t/R_{eq}C}$$

$$q(t_1) \,/\, q(t_1) = q_{in}/2 \Rightarrow e^{-t_1/R_{eq}C} = \frac{1}{2}$$

$$\boxed{t_1 = R_{eq} C \ln(2)}$$

Ejercicio 3

a)



$$\vec{F}_m = -\vec{F}_e \Rightarrow \vec{F}_m = -\frac{2Ve}{d}\,\hat{j}$$

$$\vec{E} = -\frac{2V}{d}\,\hat{j}$$

$$\boxed{\vec{F}_e = \frac{2Ve}{d}\,\hat{j}}$$

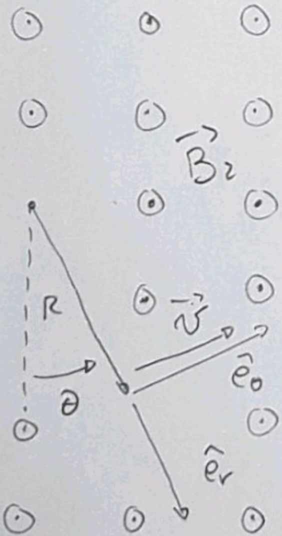
$$\vec{F}_m = -e\,v_0\,\hat{k} \times \vec{B}_1 = -\frac{2Ve}{d}\,\hat{j}$$

$$\Downarrow$$

$$\vec{B}_1 = B_1\,\hat{i}$$

$$\cancel{e}\,v_0\,B_1 = \frac{2V\cancel{e}}{d} \longrightarrow \boxed{\vec{B}_1 = \frac{2V}{d\,v_0}\,\hat{i}}$$

b)



$$m\,\vec{a}\cdot\hat{e}_r = (-e\,v_0\,\hat{e}_\theta \times \vec{B}_2)\cdot\hat{e}_r$$

$$\boxed{\vec{B}_2 = B_2(-\hat{i})}$$

$$-mR\dot{\theta}^2 = -eR\dot{\theta}\,B_2$$

$$m\,\frac{v_0^2}{R} = e\,v_0\,B_2$$

$$\boxed{\vec{B}_2 = \frac{m\,v_0}{e\,R}\,(-\hat{i})}$$