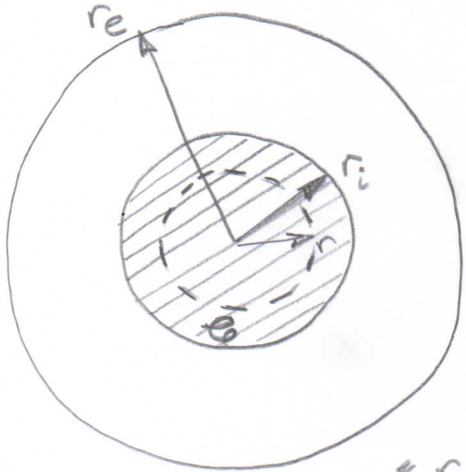
Física 3 – 2º parcial – 27 de Noviembre de 2019.–

(1) Cable coaxial muy largo, despreciamos efectos de borde.
Conductor interno, i distrib. unif., saliente de la hoja
" externo, " " " , entrante a la hoja.

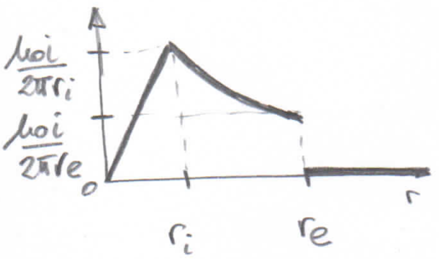


Por simetría $\vec{B}$ tg a $\mathcal{C}$ en sentido antih.

a) Ampère: $\oint \vec{B}\cdot d\vec{l} = \mu_0 i_{\mathcal{C}}$ (*)

$r \le r_i$; $\frac{i}{\pi r_i^2} \overset{j=cte}{=} \frac{i_{\mathcal{C}}}{\pi r^2} \to i_{\mathcal{C}} = \left(\frac{r}{r_i}\right)^2 i$

En (*): $B(2\pi r) = \frac{\mu_0 r^2 i}{r_i^2} \Rightarrow \boxed{B(r) = \left(\frac{\mu_0 i}{2\pi r_i^2}\right) r}$



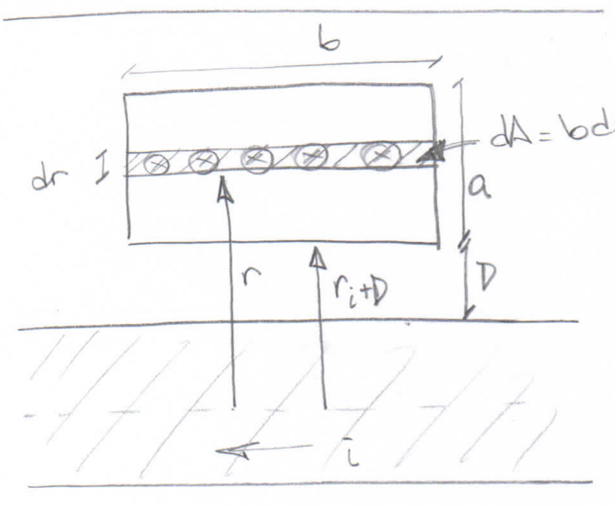
$r_i < r < r_e$; $i_{\mathcal{C}} = i$

En (*): $B(2\pi r) = \mu_0 i \Rightarrow \boxed{B(r) = \frac{\mu_0 i}{2\pi r}}$

$r > r_e$; $i_{\mathcal{C}} = i - i = 0$

En (*): $B(2\pi r) = 0 \to \boxed{B(r) = 0}$

b)



$B(r) = \frac{\mu_0 i}{2\pi r}$ (tomo $dA = b\,dr$; $d\vec{A} = \hat{n}\,dA$; $\hat{n}$ ⊗)

$$\phi_B = \int_S \vec{B}\cdot d\vec{A} = \frac{b\mu_0 i}{2\pi}\int_{r_i+D}^{r_i+D+a} \frac{dr}{r}$$

$$\boxed{\phi_B = \frac{b\mu_0 i}{2\pi} \ln\left(\frac{r_i+D+a}{r_i+D}\right)}$$ entrante a la hoja

c) Faraday: $\mathcal{E}_{ind} = -\frac{d\phi_B}{dt}$

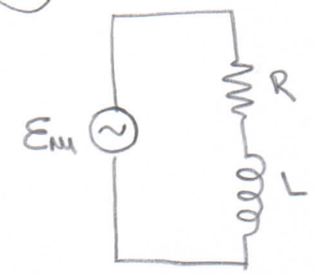
$$\boxed{i_{ind} = \frac{\mathcal{E}_{ind}}{R} = -\frac{1}{R}\left(\frac{d\phi_B}{dt}\right) = -\frac{1}{R}\frac{b\mu_0}{2\pi}\ln\left(\frac{r_i+D+a}{r_i+D}\right)\overbrace{\left(\frac{di}{dt}\right)}^{>0}}$$

El signo de i_{ind} es negativo, es decir que circula en sentido antihorario por la espira, oponiéndose al aumento de flujo a través de ella.

$|i_{ind}| \downarrow$

— x —

②



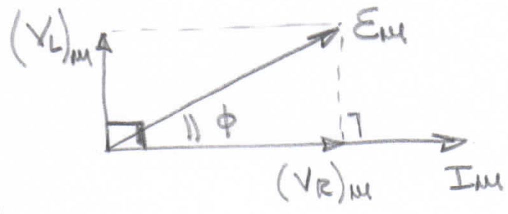
a) RL serie conectado a fuente de alterna, en régimen estacionario.

$$\boxed{I_m = \frac{\mathcal{E}_m}{\sqrt{R^2+(\omega L)^2}}} \quad ; \quad \omega = 2\pi f$$

$$\boxed{(V_R)_m = R\, I_m}$$

$$\boxed{(V_L)_m = X_L I_m = \omega L I_m = 2\pi f L I_m}$$

$$\boxed{F.P. = \cos\phi = \frac{(V_R)_m}{\mathcal{E}_m} = \frac{R\, I_m}{\mathcal{E}_m} = \frac{R}{\sqrt{R^2+(\omega L)^2}}}$$



b) $$\boxed{\overline{P_\mathcal{E}} = \frac{1}{2}\mathcal{E}_m I_m \cos\phi = \frac{1}{2}\mathcal{E}_m^2\left(\frac{R}{R^2+(\omega L)^2}\right)}$$

$$\boxed{\overline{P_L} = \frac{1}{2}(V_L)_m I_m \underbrace{\cos(+\pi/2)}_{=0} = 0}$$ desfasaje entre $(V_L)_m$ e I_m

$$\boxed{\overline{P_R} = \frac{1}{2}(V_R)_m I_m \underbrace{\cos(0)}_{=1}}$$ desfasaje entre $(V_R)_m$ e I_m

$$= \frac{1}{2} R\, I_m^2 = \frac{1}{2} R \frac{\mathcal{E}_m^2}{R^2+(\omega L)^2} = \boxed{\overline{P_\mathcal{E}}}$$

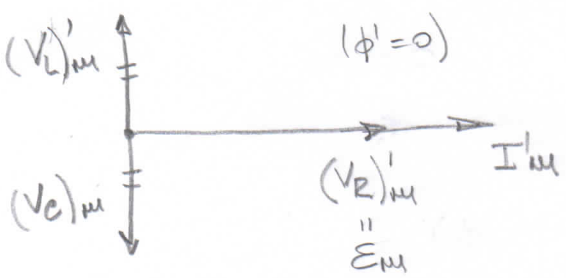
c) $F.P.' = \cos\phi' = 1 \rightarrow$ RLC serie en resonancia $\quad \omega L = \frac{1}{\omega C}$

$$\boxed{C = \frac{1}{\omega^2 L} = \frac{1}{(2\pi f)^2 L}}$$

$$\boxed{I_m' = \frac{\mathcal{E}_m}{\sqrt{R^2+\left(\omega L - \frac{1}{\omega C}\right)^2}} = \boxed{\frac{\mathcal{E}_m}{R}} = \frac{(V_R')_m}{R}}$$

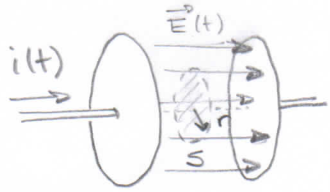
$\frac{1}{\omega C} = \omega L$

$$(V_C)_m = X_C I_m' = \frac{1}{\omega C} I_m' = \underbrace{\omega L}_{= X_L} I_m' = (V_L)_m'$$



— x —

(3) $i(t) = I_m \cos(\omega t)$



a) $i_d = \varepsilon_0 \dfrac{d\phi_E}{dt}$

E uniforme entre las placas

$$\phi_E = \int_S \vec{E}\cdot d\vec{A} = E(\pi r^2)$$

$E(t) = \dfrac{V(t)}{d} = \dfrac{q(t)}{Cd}$ $\qquad C = \dfrac{\varepsilon_0(\pi R^2)}{d}$

$$\Rightarrow \boxed{i_d = \varepsilon_0 \frac{d\phi_E}{dt} = \varepsilon_0(\pi r^2)\frac{dE}{dt} = \varepsilon_0(\pi r^2)\frac{1}{Cd}\underbrace{\frac{dq}{dt}}_{=i(t)} = \frac{\cancel{\varepsilon_0}(\cancel{\pi} r^2)\cancel{d}}{\cancel{\varepsilon_0}(\cancel{\pi} R^2)\cancel{d}}\, i(t) = \left(\frac{r}{R}\right)^2 i(t)}$$

b) Ampère-Maxwell: $\displaystyle\oint_{\mathcal{C}} \vec{B}\cdot d\vec{l} = \underbrace{\mu_0 i}_{=0} + \mu_0 \underbrace{i_d}_{=\left(\frac{r}{R}\right)^2 i(t)}$

curva $\mathcal{C}$: cfa. radio r entre las placas

$$B(2\pi r) = \mu_0 i_d = \mu_0\left(\frac{r}{R}\right)^2 i(t) \Rightarrow \boxed{B(r,t) = \frac{\mu_0}{2\pi R^2}\, r\, i(t)}$$

sentido de $\vec{B}$, tg a $\mathcal{C}$ según regla de la mano dcha.

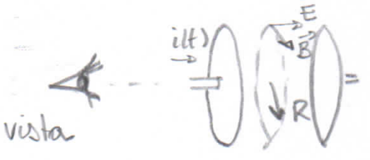
c) $\vec{S}(r,t) \equiv \dfrac{1}{\mu_0}\vec{E}(r,t) \times \vec{B}(r,t)$ (*)

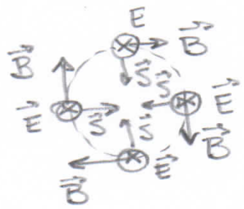
diferencia de potencial en el C, $V_c(t)$, atrasa en $\pi/2$ a la corriente por él.

$V_C(t) = \underbrace{(V_c)_m}_{= X_C I_m = \frac{1}{\omega C} I_m} \cos(\omega t - \pi/2)$

$$E(t) = \frac{V_c(t)}{d} \stackrel{\triangle}{=} \frac{I_m}{d\omega C}\cos(\omega t - \pi/2)$$

$$= \frac{I_m}{\cancel{d}\omega}\frac{\cancel{d}}{\varepsilon_0 \pi R^2}\cos(\omega t - \pi/2)$$





dirección radial entrante

En (*) para r=R:

$$\Rightarrow \vec{S}(R,t) = \frac{1}{\cancel{\mu_0}}\frac{I_m}{\omega\varepsilon_0(\pi R^2)}\frac{\cancel{\mu_0}\cancel{R}\, I_m}{2\pi R^{\cancel{2}}}\underbrace{\cos(\omega t - \pi/2)}_{=\operatorname{sen}(\omega t)}\cos(\omega t)\,(-\hat{e}_r)$$

$$\vec{S}(R,t) = \frac{I_m^2}{2\omega\varepsilon_0\pi^2 R^3}\operatorname{sen}(\omega t)\cos(\omega t)\,(-\hat{e}_r)$$