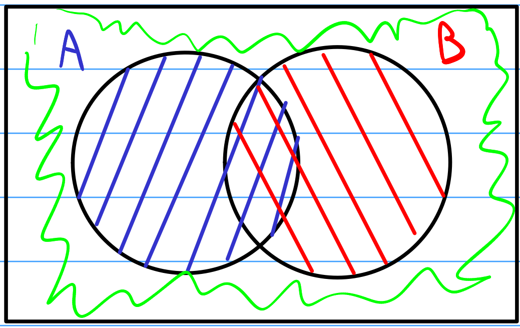
Práctico 3 - Principio de inclusión-exclusión

- Nos permite contar el cardinal de la unión de conjuntos no necesariamente disjuntos.
- Generaliza la regla de la suma

Recordemos: $A \cap B = \emptyset \Rightarrow |A \cup B| = |A| + |B|$

Si $\forall i, j,\ A_i \cap A_j = \emptyset,\ 1 \leq i < j \leq n \Rightarrow \left|\bigcup_{i=1}^{n} A_i\right| = \sum_{i=1}^{n} |A_i|$

U



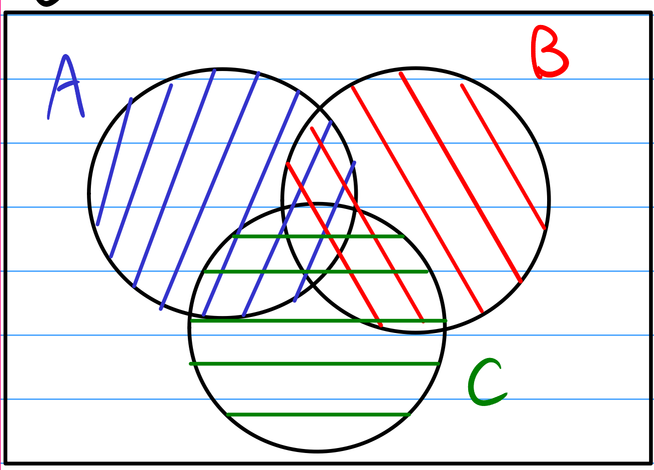
P.I.E. para 2 conjuntos:

$|A \cup B| = |A| + |B| - |A \cap B|$

$|(A \cup B)^c| = |U| - |A \cup B|$

$= |U| - |A| - |B| + |A \cap B|$

U



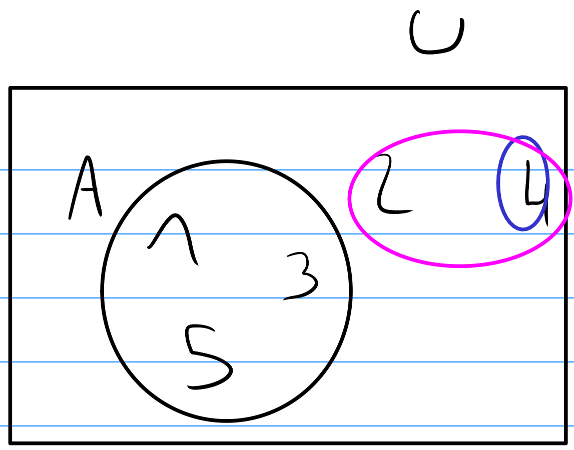
P.I.E. para 3 conjuntos:

$|A \cup B \cup C| = |A| + |B| + |C|$
$- |A \cap B| - |B \cap C| - |A \cap C|$
$+ |A \cap B \cap C|$

$|(A \cup B \cup C)^c| =$

$|U| - (|A| + |B| + |C|) + (|A \cap B| + |B \cap C| + |A \cap C|) - |A \cap B \cap C|$

$\{A, B, C\}$



$A=\{1,3,5\}$

$A^c = U \setminus A = \{2,4\}$

En general:

$$\left|\bigcup_{k=1}^{n} A_k\right| = \sum_{k=1}^{n} (-1)^{k+1} \sum_{1\le i_1<i_2<\dots<i_k\le n} \left|\bigcap_{j=1}^{k} A_{i_j}\right|$$

$$= \sum_{\emptyset \neq J \subset [1..n]} (-1)^{|J|+1} \left|\bigcap_{j\in J} A_j\right| =$$

$$|A_1| + \dots + |A_n| - (|A_1 \cap A_2| + \dots + |A_{n-1} \cap A_n|) + \dots + (-1)^{n} |A_1 \cap \dots \cap A_n|$$

Ejercicio 1

(a) ¿Cuántos naturales entre 1 y 105 inclusive no son múltiplos de 3, 5 ni 7?

$U = [1..105] = \{n \in \mathbb{N} : 1 \le n \le 105\}$ $\quad 105 = 3\cdot 5\cdot 7$

$A := \{n \in U : \exists m \in \mathbb{N}, n = 3\cdot m\} = \{3\cdot m \in U : m \in \mathbb{N}\}$

$= \{3\cdot m : m \in \mathbb{N} \text{ y } 1 \le 3\cdot m \le 105\}$

$1 \leq 3m \leq 105 \Leftrightarrow \frac{1}{3} \leq m \leq 35 \Leftrightarrow m \in [1..35]$

$= \{3 \cdot m : m \in [1..35]\} \Rightarrow |A| = 35$

$B := \{n \in U : \exists q \in \mathbb{N},\ n = 5 \cdot q\} = \{5 \cdot q : q \in [1..21]\}$

$\Rightarrow |B| = 21$

$C := \{n \in U : \exists k \in \mathbb{N},\ n = 7 \cdot k\} = \{7 \cdot k : k \in [1..15]\}$

$\Rightarrow |C| = 15$

$A \cap B = \{n \in A : n \in B\} = \{3 \cdot m : m \in [1..35] \text{ y } 3 \cdot m \in B\}$

$= \{3 \cdot m : m \in [1..35] \text{ y } \exists q \in \mathbb{N},\ 3 \cdot m = 5 \cdot q\}$

$= \{n \in U : \exists r \in \mathbb{N},\ n = 15 \cdot r\}$ $\quad 30 = 2 \cdot 3 \cdot 5$

$1 \leq 15 \cdot r \leq 105 \Leftrightarrow \frac{1}{15} \leq r \leq 7 \Leftrightarrow r \in [1..7]$

$= \{15 \cdot r : r \in [1..7]\} \Rightarrow |A \cap B| = 7$

$|B \cap C| = \{n \in U : \exists \alpha \in \mathbb{N},\ n = 35 \cdot \alpha\}$

$= \{35 \cdot \alpha : [1..3]\} \Rightarrow |B \cap C| = 3$

$|A \cap C| = \{n \in U : \exists \beta \in \mathbb{N},\ n = 21 \cdot \beta\} =$

$\{21 \cdot \beta : \beta \in [1..5]\} \Rightarrow |A \cap C| = 5$

$A \cap B \cap C = \{105\} \Rightarrow |A \cap B \cap C| = 1$

$\Rightarrow |(A \cup B \cup C)^c| = |U| - (|A| + |B| + |C|) + (|A \cap B| + |B \cap C| + |A \cap C|)$
$- |A \cap B \cap C| = 105 - (35 + 21 + 15) + (7 + 3 + 5) - 1 = \boxed{48}$