(1)

BINOMIO DE NEWTON

$$(a+b)^n = \sum_{i=0}^{n} a^i \cdot b^{n-i} \cdot f(n,i)$$

$f(n,i)$ se definió en el práctico mismo!

* Se prueba por inducción

Paso Base: $n=0$ La expresión será válida evaluada en $n=0$?

$$(a+b)^n = \sum_{i=0}^{n} a^i b^{n-i} f(n,i) \underset{\substack{\text{evalúo} \\ \text{en } n=0}}{\Longleftrightarrow} (a+b)^0 = \sum_{i=0}^{0} a^i b^{0-i} f(0,i)$$

$\Rightarrow (a+b)^0 = 1$

$$\sum_{i=0}^{0} a^i b^{0-i} f(0,i) = \underbrace{a^0}_{=1} \cdot \underbrace{b^{0-0}}_{=1} \cdot \underbrace{f(0,0)}_{=1} = 1$$ por definición!

$\Rightarrow$ Se cumple el paso base para $n=0$!

(2)

Paso Inductivo: (H) Vale la expresión para un "n" genérico: $(a+b)^n = \sum_{i=0}^{n} a^i b^{n-i} f(n,i)$

(T) Vale para el natural siguiente "$n+1$": $(a+b)^{n+1} = \sum_{i=0}^{n+1} a^i \cdot b^{(n+1)-i} \cdot f(n+1,i)$

Dem

$$(a+b)^{n+1} = (a+b)^n \cdot (a+b) \overset{(H)}{=} \left[\sum_{i=0}^{n} a^i \cdot b^{n-i} f(n,i)\right](a+b) =$$

$$\underset{\text{Distributiva}}{=} a \cdot \left[\sum_{i=0}^{n} a^i b^{n-i} f(n,i)\right] + b \left[\sum_{i=0}^{n} a^i b^{n-i} \cdot f(n,i)\right] =$$

$$\overset{\text{Prop. sumatoria}}{=} \sum_{i=0}^{n} a \cdot a^i b^{n-i} f(n,i) + \sum_{i=0}^{n} b \cdot a^i \cdot b^{n-i} \cdot f(n,i) =$$

$$= \underbrace{\sum_{i=0}^{n} a^{i+1} \cdot b^{n-i} \cdot f(n,i)}_{(A)} + \underbrace{\sum_{i=0}^{n} a^i \cdot b^{(n+1)-i} \cdot f(n,i)}_{(B)} \overset{(*)}{=}$$

Paréntesis (1)

$$\left[\begin{array}{l} (A) \sum_{i=0}^{n} a^{i+1} \cdot b^{n-i} \cdot f(n,i) \underset{\substack{\text{Cambio de variable} \\ i+1=j}}{=} \sum_{j=1}^{n+1} a^j \cdot b^{(n+1)-j} \cdot f(n,j-1) \\ (B) \sum_{i=0}^{n} a^i b^{(n+1)-i} f(n,i) \underset{\substack{\text{Cambio de variable} \\ i=j}}{=} \sum_{j=0}^{n} a^j b^{n-j} f(n,j) \end{array}\right]$$

$$\overset{(*)}{=} \underbrace{\sum_{j=1}^{n+1} a^j \cdot b^{(n+1)-j} \cdot f(n,j-1)}_{(A2)} + \underbrace{\sum_{j=0}^{n} a^j \cdot b^{n-j} f(n,j)}_{(B2)} \overset{(**)}{=}$$

(3)

Paréntesis (2)

(1) $$\sum_{j=1}^{n+1} a^j \cdot b^{(n+1)-j} \cdot f(n,j-1) = \sum_{j=1}^{n} a^j \cdot b^{(n+1)-j} \cdot f(n,j-1) +$$

$$+ a^{(n+1)} \cdot \underbrace{b^{(n+1)-(n+1)}}_{b^0 = 1} \cdot \underbrace{f(n,n+1-1)}_{f(n,n)=1}$$

$\uparrow$ definición

$$\Rightarrow \sum_{j=1}^{n+1} a^j \cdot b^{(n+1)-j} \cdot f(n,j-1) = \left[\sum_{j=1}^{n} a^j \cdot b^{(n+1)-j} \cdot f(n,j-1)\right] + a^{(n+1)}$$

(2) $$\sum_{j=0}^{n} a^j \cdot b^{(n+1)-j} \cdot f(n,j) = \sum_{j=1}^{n} a^j \cdot b^{(n+1)-j} \cdot f(n,j) + \underbrace{a^0}_{1} \cdot \underbrace{b^{(n+1)-0}}_{b^{n+1}} \cdot \underbrace{f(n,0)}_{1 (def)}$$

$$\Rightarrow \sum_{j=0}^{n} a^j \cdot b^{(n+1)-j} \cdot f(n,j) = \left[\sum_{j=1}^{n} a^j \cdot b^{(n+1)-j} f(n,j)\right] + b^{(n+1)}$$

(1)+(2) $$= \sum_{j=1}^{n} a^j \cdot b^{(n+1)-j} \cdot f(n,j-1) + a^{n+1} + \sum_{j=1}^{n} a^j b^{(n+1)-j} \cdot f(n,j) + b^{n+1} =$$

$$= a^{n+1} + b^{n+1} + \sum_{j=1}^{n} \left[a^j \cdot b^{(n+1)-j} \cdot f(n,j-1) + a^j \cdot b^{(n+1)-j} \cdot f(n,j)\right] =$$

$$\underset{\uparrow}{=} a^{n+1} + b^{n+1} + \sum_{j=1}^{n} \left(a^j \cdot b^{(n+1)-j} \left[f(n,j-1) + f(n,j)\right]\right) =$$

Distributiva

$$\underset{\uparrow}{=} a^{n+1} + b^{n+1} + \sum_{j=1}^{n} a^j \cdot b^{(n+1)-j} \cdot f(n+1,j) \Longrightarrow$$

$f(n,j-1) + f(n,j) = f(n+1,j)$ POR DEFINICIÓN!!

(4)

$$\Rightarrow a^{n+1}+b^{n+1}+\sum_{j=1}^{n} a^j \cdot b^{(n+1)-j} f(n+1,j) \overset{?}{=} \sum_{j=0}^{n+1} a^j \cdot b^{(n+1)-j} f(n+1,j)$$

SI SE DÁ ESA IGUALDAD

ENTONCES EL EJERCICIO ESTÁ

PRONTO!!

Observemos, descomponiendo la sumatoria de la tesis en 3 miembros:

$$\sum_{j=0}^{n+1} a^j \cdot b^{(n+1)-j} f(n+1,j) = \sum_{j=1}^{n} a^j \cdot b^{(n+1)-j} \cdot f(n+1,j) + \overbrace{a^0}^{1} \cdot \overbrace{b^{(n+1)-0}}^{b^{n+1}} \overbrace{f(n+1,0)}^{1} + \underbrace{a^{(n+1)}}_{a^{n+1}} \cdot \underbrace{b^{(n+1)-(n+1)}}_{b^0=1} \cdot \underbrace{f(n+1,n+1)}_{1} =$$

$$\Rightarrow \sum_{j=0}^{n+1} a^j b^{(n+1)-j} f(n+1,j) = a^{n+1}+b^{n+1}+\sum_{j=1}^{n} a^j b^{(n+1)-j} \cdot f(n+1,j) =$$

$$=(a+b)^{n+1}$$

Fernando