



El campo magnético de una corriente

El campo magnético producido por una carga en movimiento



- 1820 Hans Christian Oersted

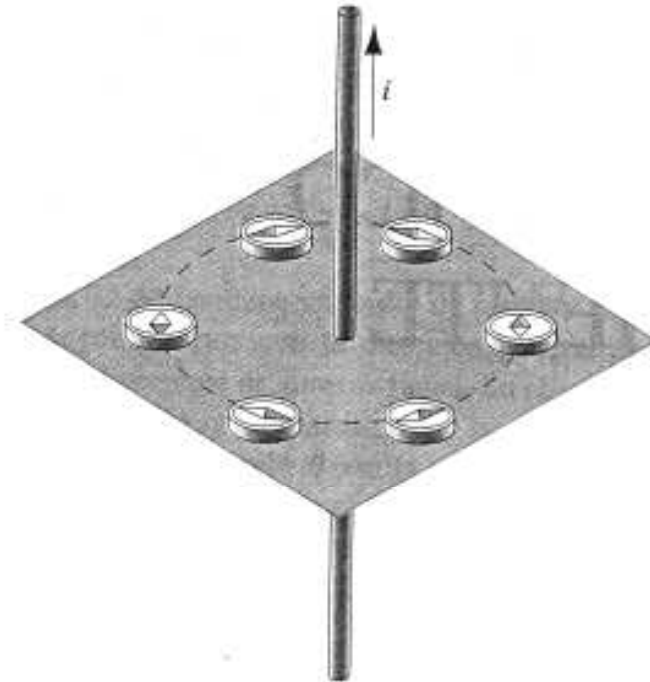


FIGURA 33-1. Experimento de Oersted. La dirección de la aguja de la brújula siempre es perpendicular a la de la corriente del alambre.

El campo magnético producido por una carga en movimiento



- 1876 Henry Rowland
- Campo magnético producido por su disco en rotación fue apenas 0.00001 del campo de la Tierra.

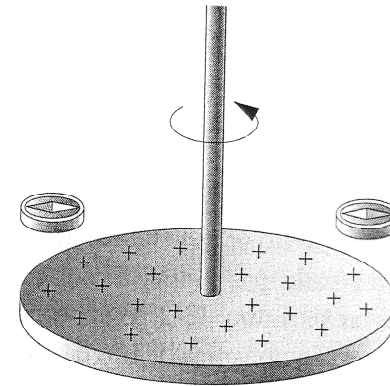


FIGURA 33-2. Diagrama esquemático del experimento de Rowland. Las cargas móviles en la superficie de oro del disco giratorio producen un campo magnético que desvía la aguja de la brújula. En la práctica, la deflexión es muy pequeña y requiere un aparato mucho más sensible que una brújula para ser detectada.

El campo magnético producido por una carga en movimiento

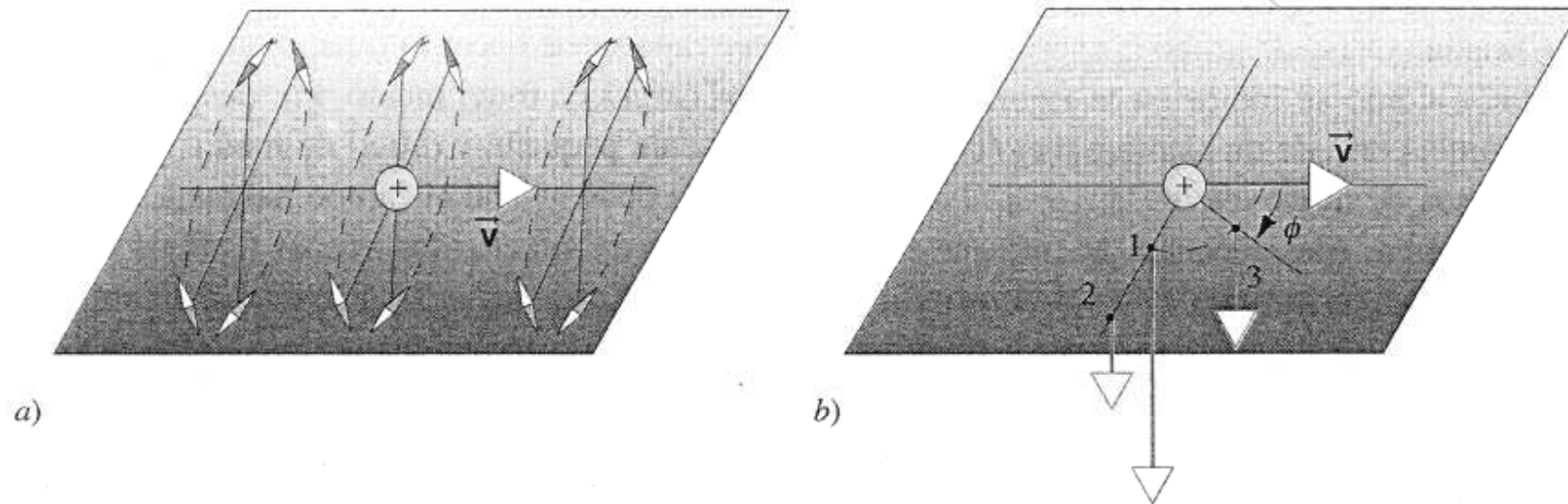


FIGURA 33-3. *a)* Una aguja de compás suspendida libremente indica la dirección del campo magnético en varios lugares, debido a una carga eléctrica en movimiento. *b)* El campo en el punto 2 es $\frac{1}{4}$ del campo en el punto 1, porque el punto 2 está al doble de la distancia de la carga. El punto 3 se halla a la misma distancia de la carga que el punto 1, pero el campo en el punto 3 es más pequeño que en el punto 1 por el factor $\sin \phi$.

El campo magnético producido por una carga en movimiento

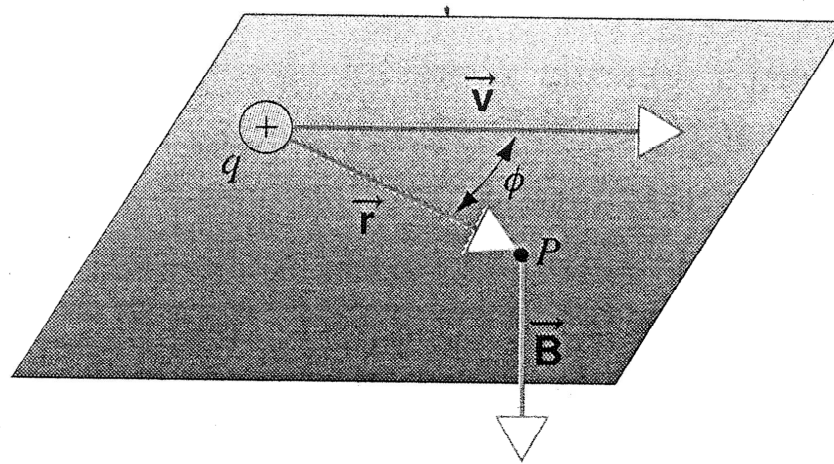


FIGURA 33-4. El campo magnético en el punto P generado por una carga en movimiento es perpendicular al plano que contiene a \vec{v} y \vec{r} .

$$B \propto \frac{qv \sin \phi}{r^2} \quad \vec{B} = K \frac{q \vec{v} \times \hat{r}}{r^2}$$

El campo magnético producido por una carga en movimiento

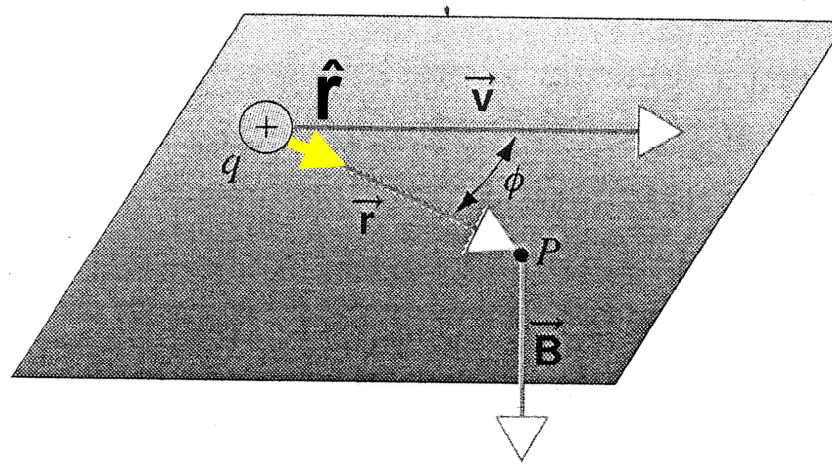
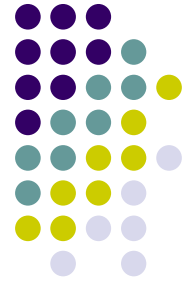


FIGURA 33-4. El campo magnético en el punto P generado por una carga en movimiento es perpendicular al plano que contiene a \vec{v} y \vec{r} .

$$\vec{B} = K \frac{q\vec{v} \times \hat{r}}{r^2}$$

$$\vec{r} = r\hat{r}$$

$$\vec{B} = K \frac{q\vec{v} \times \vec{r}}{r^3}$$

El campo magnético producido por una carga en movimiento



$$\vec{\mathbf{B}} = \frac{\mu_0}{4\pi} \frac{q\vec{\mathbf{v}} \times \vec{\mathbf{r}}}{r^3}$$

μ_0 : permeabilidad del vacío

$$\mu_0 = 4\pi 10^{-7} \text{ Tm} / \text{A} = 4\pi 10^{-7} \text{ N} / \text{A}^2$$

$$B = \frac{\mu_0}{4\pi} \frac{|q|v \text{sen} \phi}{r^2}$$

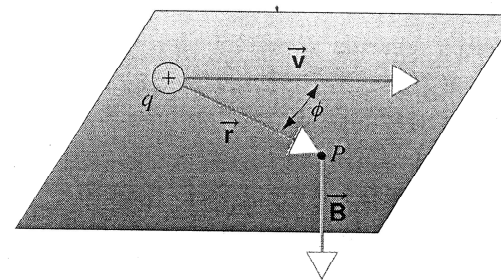


FIGURA 33-4. El campo magnético en el punto P generado por una carga en movimiento es perpendicular al plano que contiene a $\vec{\mathbf{v}}$ y $\vec{\mathbf{r}}$.

El campo magnético producido por una carga en movimiento

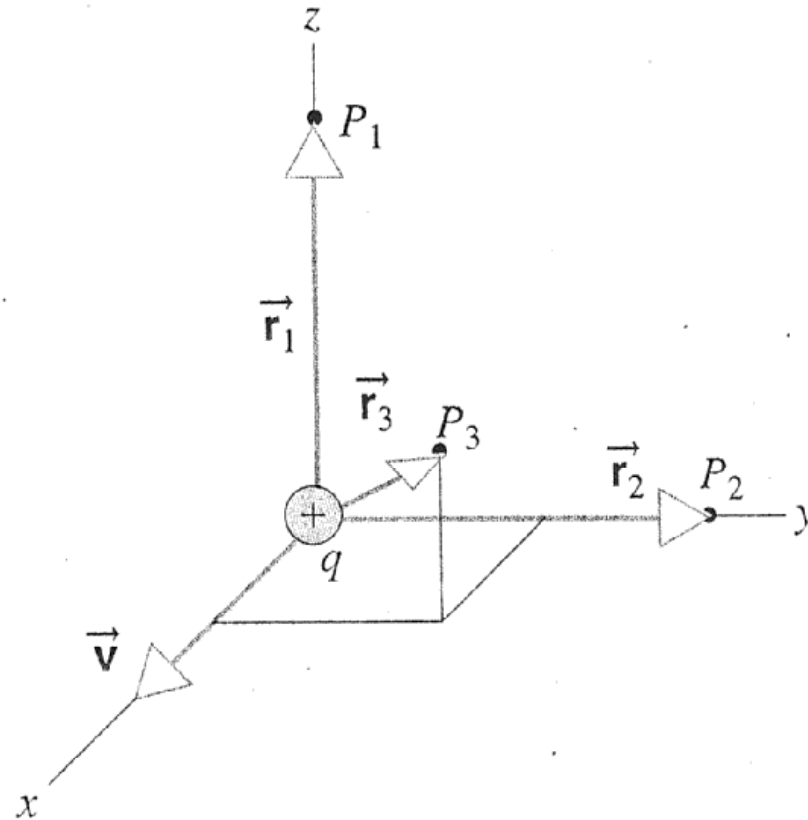


FIGURA 33-5. Problema resuelto 33-1.

El campo magnético de una corriente

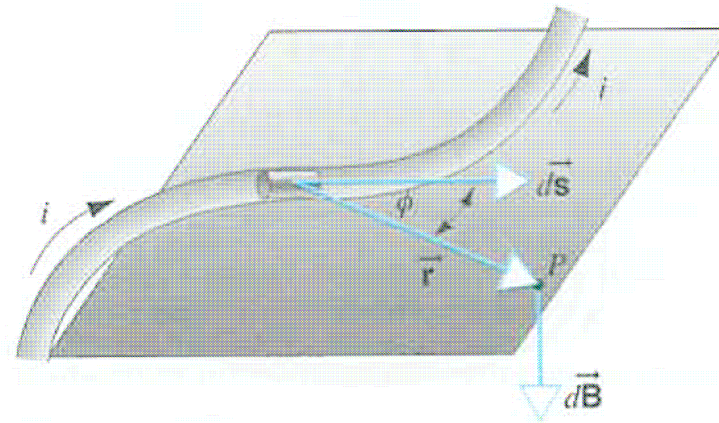


Campo magnético creado por corrientes eléctricas: Ley de Biot y Savart

$$d\vec{B} = \frac{\mu_0}{4\pi} \frac{dq\vec{v} \times \vec{r}}{r^2}$$

.....

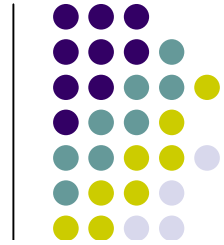
$$d\vec{B} = \frac{\mu_0}{4\pi} \frac{id\vec{s} \times \vec{r}}{r^2}$$



$$d\vec{B} = \frac{\mu_0}{4\pi} \frac{id\vec{s} \times \vec{r}}{r^3}$$

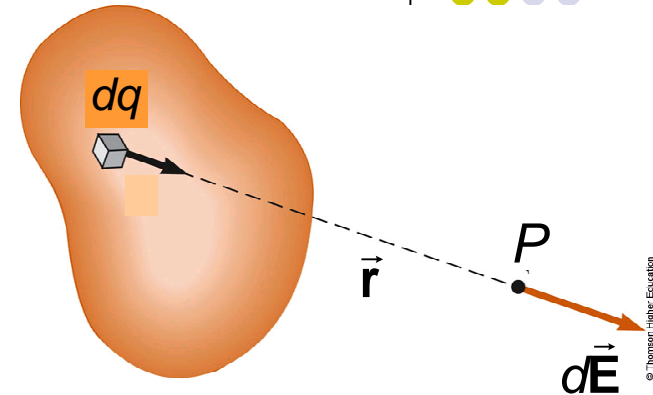
$$dB = \frac{\mu_0}{4\pi} \frac{i(ds)(\text{sen}\phi)}{r^2}$$

Sources of Electric field, magnetic field



From **Coulomb's Law**, a point charge dq generates electric field distance \vec{r} away from the source:

$$d\vec{E} = \frac{1}{4\pi\epsilon_0} \frac{dq}{r^2} \hat{r}$$

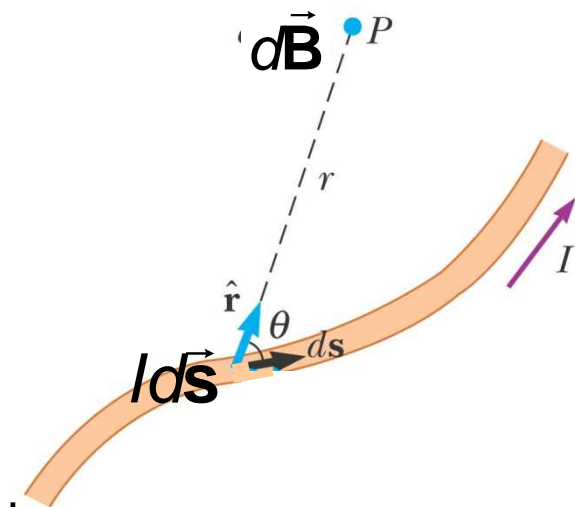


From **Biot-Savart's Law**, a point current $I d\vec{s}$ generates magnetic field distance \vec{r} away from the source:

$$d\vec{B} = \frac{\mu_0}{4\pi} \frac{I d\vec{s} \times \hat{r}}{r^2}$$

Difference:

1. A current segment $I d\vec{s}$, not a point charge dq , hence a vector.
2. Cross product of two vectors, $d\vec{B}$ is determined by the right-hand rule, not \hat{r} .



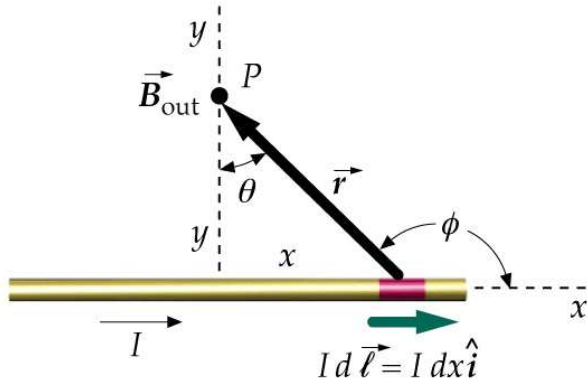
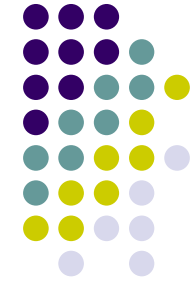
Campo magnético total \vec{B}



$$\vec{B} = \int d\vec{B}$$

$$\vec{B} = \int d\vec{B} = \frac{\mu_0}{4\pi} \int \frac{i d\vec{s} \times \hat{r}}{r^2} = \frac{\mu_0}{4\pi} \int \frac{i d\vec{s} \times \vec{r}}{r^3}$$

Campo magnético debido a una corriente en un conductor rectilíneo



$$d\mathbf{B} = \frac{\mu_0}{4\pi} \frac{I d\mathbf{l} \times \hat{\mathbf{r}}}{r^2}$$

$$dB = \frac{\mu_0}{4\pi} \frac{I dx}{r^2} \sin\phi$$

$$dB = \frac{\mu_0}{4\pi} \frac{I dx}{r^2} \cos\theta$$

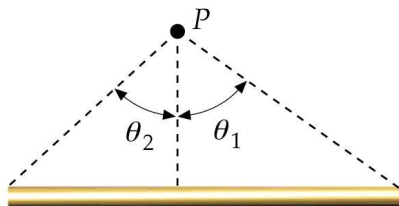
$$\sin\phi = \sin(\pi - \phi)$$

$$y = r \cos\theta$$

$$x = y \tan\theta$$

$$\implies dx = y \sec^2\theta d\theta$$

$$dB = \frac{\mu_0 I}{4\pi y} \cos\theta d\theta$$



$$B = \frac{\mu_0 I}{4\pi y} (\sin\theta_1 + \sin\theta_2)$$

Para un conductor muy largo:

$$\theta_1 = \theta_2 = \frac{\pi}{2}$$

$$B = \frac{\mu_0 I}{2\pi y}$$

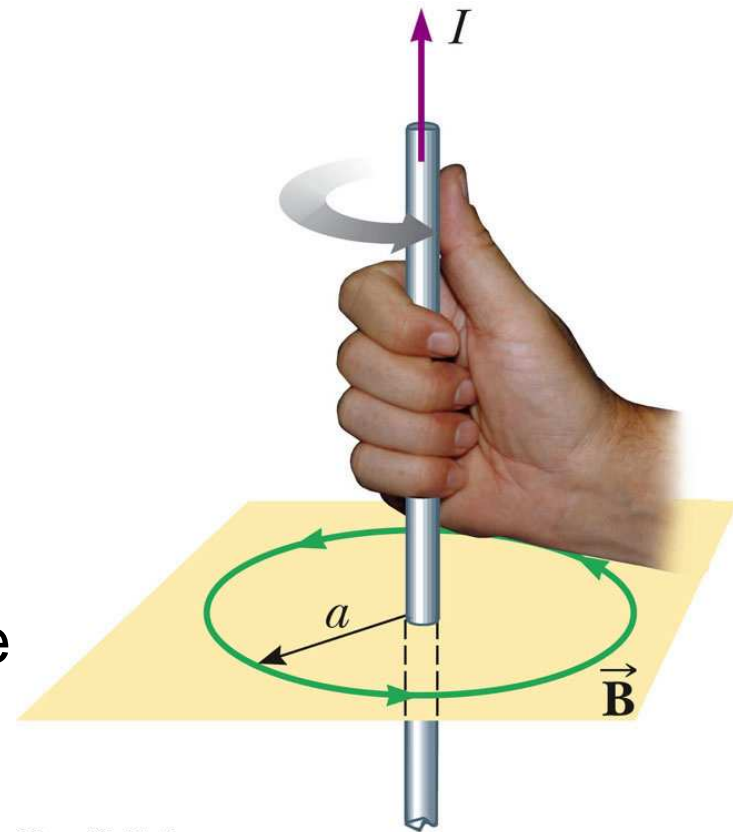


Líneas de campo magnético creadas por un hilo conductor

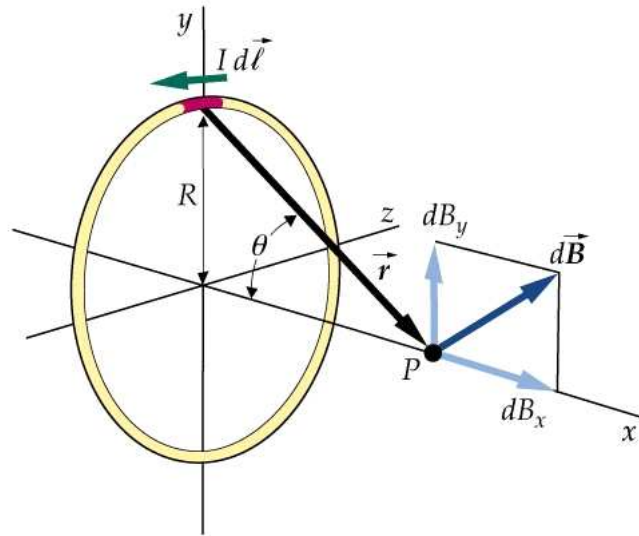
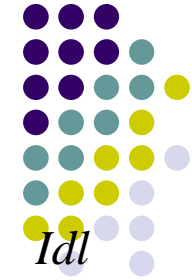
For a long, straight conductor the magnetic field \vec{B} goes in circles



- The magnetic field lines are circles concentric with the wire
- The field lines lie in planes perpendicular to the wire
- The magnitude of the field is constant on any circle of radius a
- A different and more convenient right-hand rule for determining the direction of the field is shown



Campo magnético creado por una espira circular

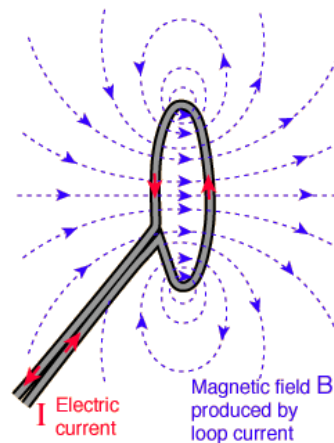


$$d\mathbf{B} = \frac{\mu_0}{4\pi} \frac{I d\mathbf{l} \times \hat{\mathbf{r}}}{r^2} \quad |d\mathbf{B}| = \frac{\mu_0}{4\pi} \frac{|I d\mathbf{l} \times \hat{\mathbf{r}}|}{r^2} = \frac{\mu_0}{4\pi} \frac{I dl}{x^2 + R^2}$$

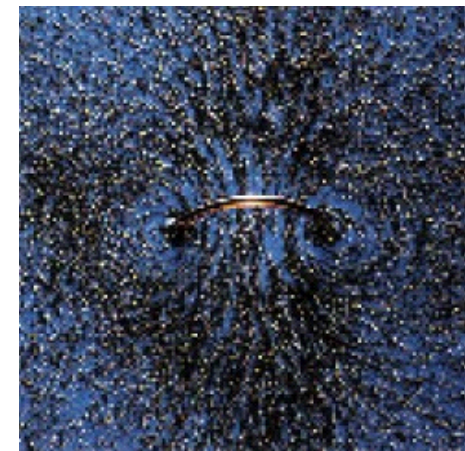
$$dB_x = dB \sin \theta = dB \left(\frac{R}{\sqrt{x^2 + R^2}} \right) = \frac{\mu_0}{4\pi} \frac{I dl}{x^2 + R^2} \frac{R}{\sqrt{x^2 + R^2}}$$

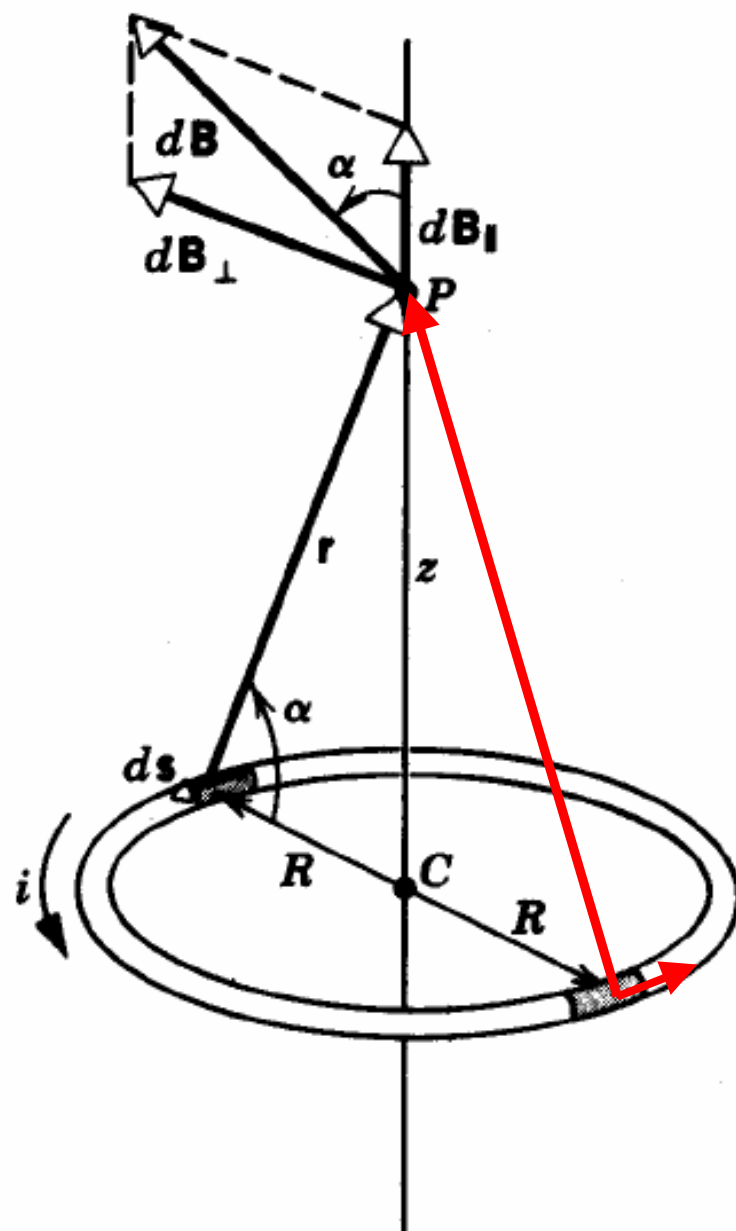
$$B_x = \oint dB_x = \oint \frac{\mu_0}{4\pi} \frac{IR}{(x^2 + R^2)^{3/2}} dl = \frac{\mu_0}{4\pi} \frac{IR}{(x^2 + R^2)^{3/2}} \oint dl$$

$$B_x = \frac{\mu_0}{4\pi} \frac{IR(2\pi R)}{(x^2 + R^2)^{3/2}}$$



Líneas de campo creado por una espira circular





Example: \vec{B} at the center of a circular loop of wire with current I

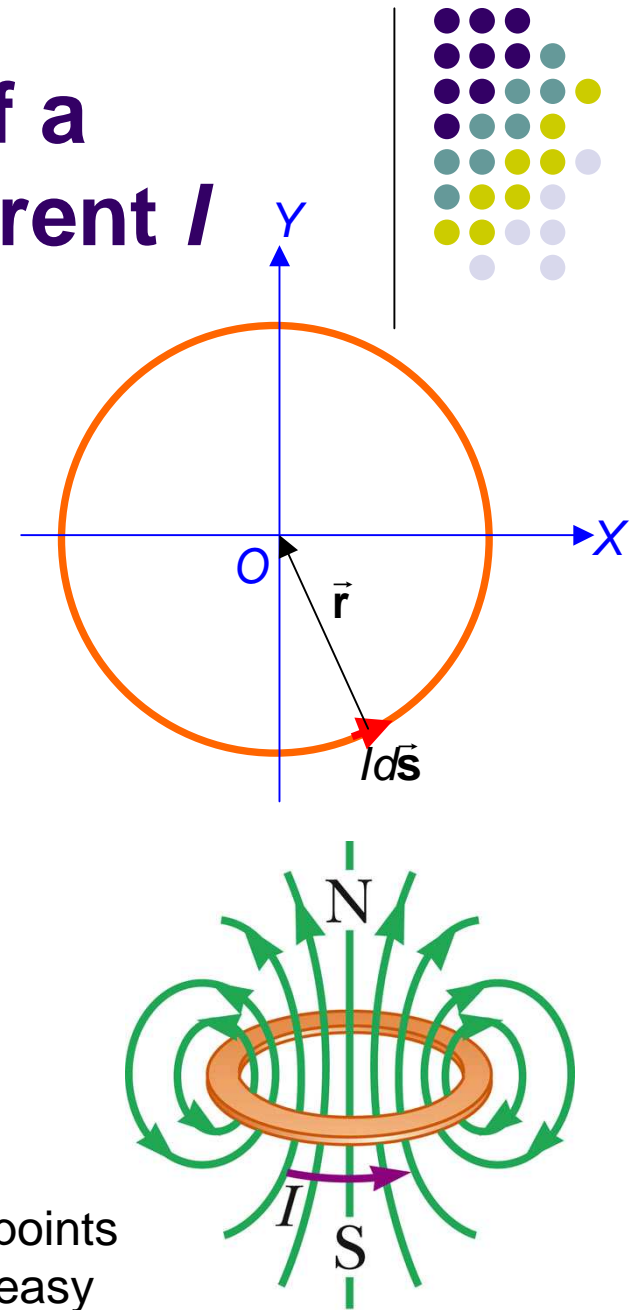
- From Biot-Savart Law, the field at O from $I d\vec{s}$ is

$$d\vec{B} = \frac{\mu_0}{4\pi} \frac{I d\vec{s}}{r^2} \times \hat{r} = \frac{\mu_0}{4\pi} \frac{I ds}{r^2} \hat{k}$$

$$B = \frac{\mu_0 I}{4\pi r^2} \int_{\text{full circle}} ds = \frac{\mu_0 I}{4\pi r^2} 2\pi r = \frac{\mu_0 I}{2r}$$

- This is the field at the *center* of the loop

$$B = \frac{\mu_0 I}{2r} \quad \text{or} \quad \vec{B} = B\hat{k}$$

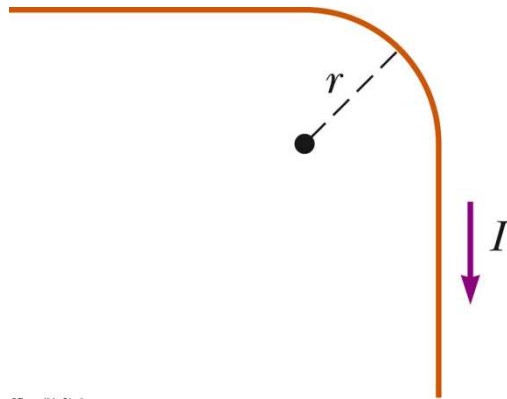


Off center points are not so easy to calculate.

Example problems



A long, straight wire carries current I . A right-angle bend is made in the middle of the wire. The bend forms an arc of a circle of radius r . Determine the magnetic field at the center of the arc.



Formula to use: Biot-Savart's Law, or more specifically the results from the discussed two examples:

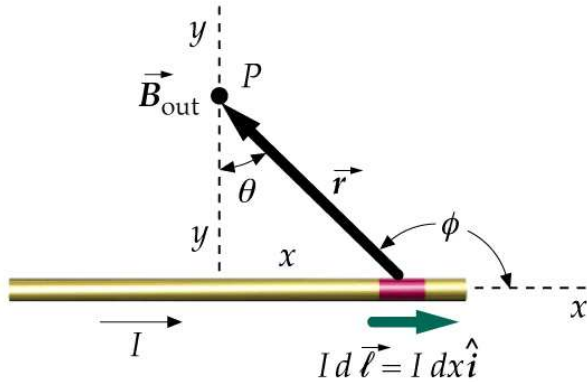
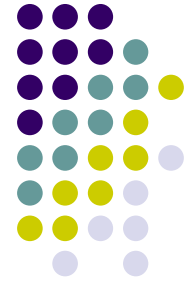
$$\text{For the straight section } B = \frac{\mu_0 I}{4\pi r} \sin \theta \Big|_0^{\frac{\pi}{2}} = \frac{\mu_0 I}{4\pi r}$$

$$\text{For the arc } B = \frac{\mu_0 I}{4\pi r^2} \int_{\frac{1}{4} \text{ circle}} ds = \frac{\mu_0 I}{4\pi r^2} \frac{2\pi r}{4} = \frac{\mu_0 I}{8r}$$

The final answer: magnitude $B = \frac{\mu_0 I}{4\pi r} + \frac{\mu_0 I}{8r} + \frac{\mu_0 I}{4\pi r} = \frac{\mu_0 I}{4r} \left(\frac{2}{\pi} + \frac{1}{2} \right)$

direction pointing into the page.

Campo magnético debido a una corriente en un conductor rectilíneo



$$d\mathbf{B} = \frac{\mu_0}{4\pi} \frac{I d\mathbf{l} \times \hat{\mathbf{r}}}{r^2}$$

$$dB = \frac{\mu_0}{4\pi} \frac{I dx}{r^2} \sin\phi$$

$$dB = \frac{\mu_0}{4\pi} \frac{I dx}{r^2} \cos\theta$$

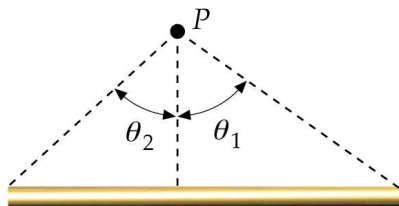
$$\sin\phi = \sin(\pi - \phi)$$

$$y = r \cos\theta$$

$$x = y \tan\theta$$

$$\implies dx = y \sec^2\theta d\theta$$

$$dB = \frac{\mu_0 I}{4\pi y} \cos\theta d\theta$$



$$B = \frac{\mu_0 I}{4\pi y} (\sin\theta_1 + \sin\theta_2)$$

Para un conductor muy largo:

$$\theta_1 = \theta_2 = \frac{\pi}{2}$$

$$B = \frac{\mu_0 I}{2\pi y}$$



Líneas de campo magnético creadas por un hilo conductor

Dos corrientes paralelas



Dos corrientes paralelas

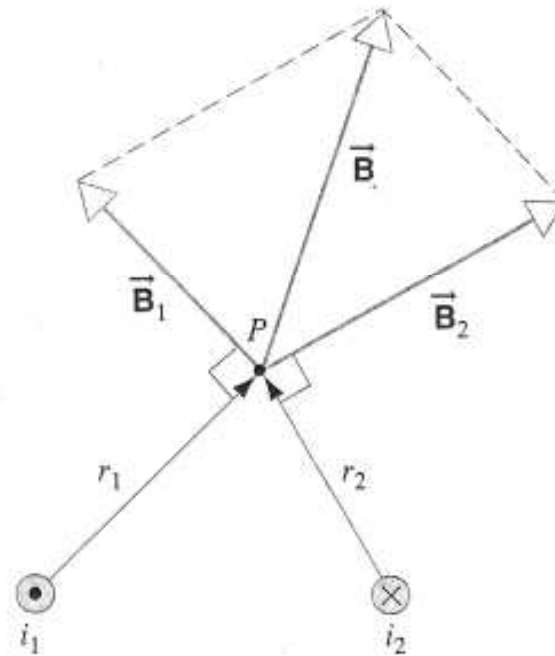
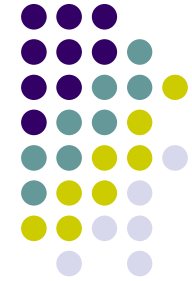


FIGURA 33-11. Dos alambres llevan corrientes perpendicularmente a la página; i_1 está hacia fuera de la página (representada por \odot , lo cual indica la punta de una flecha) e i_2 se dirige al interior de la página (representada por \otimes , lo cual indica las plumas de una flecha). El campo total en el punto P es la suma vectorial de \vec{B}_1 y \vec{B}_2 .

Dos corrientes paralelas

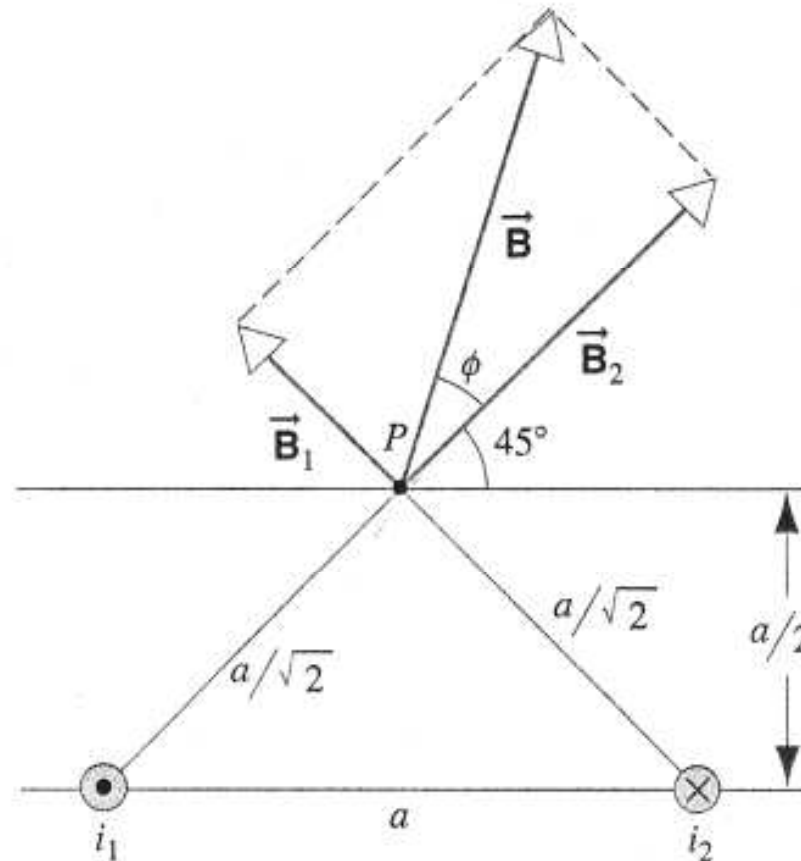
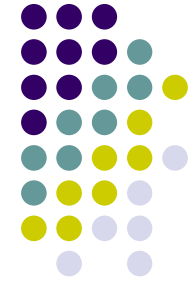


FIGURA 33-12. Problema resuelto 33-3. La corriente i_1 sale de la página, y la corriente i_2 entra en ella.

Magnetic Force Between Two Parallel Conductors



- Two parallel wires each carry steady currents
- The field \vec{B}_2 due to the current in wire 2 exerts a force on wire 1 of $F_1 = I_1 \ell B_2$
- Substituting the equation for B_2 gives

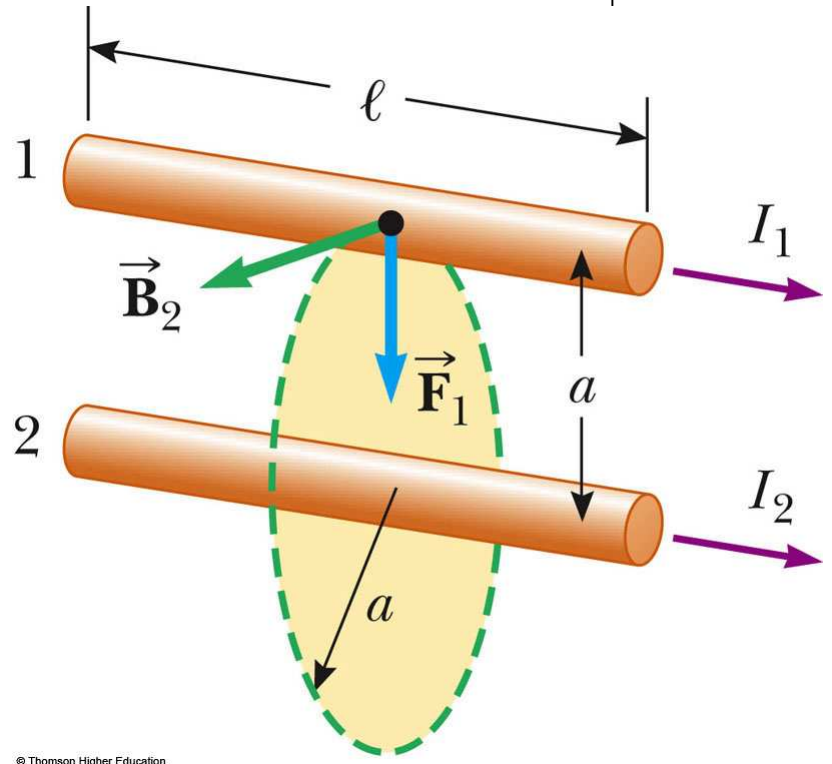
$$F_1 = \frac{\mu_0 I_1 I_2}{2\pi a} \ell$$

- Check with right-hand rule:
 - same direction currents attract each other
 - opposite directions currents repel each other

The force per unit length on the wire is

$$\frac{F_B}{\ell} = \frac{\mu_0 I_1 I_2}{2\pi a}$$

And this formula defines the current unit Ampere.



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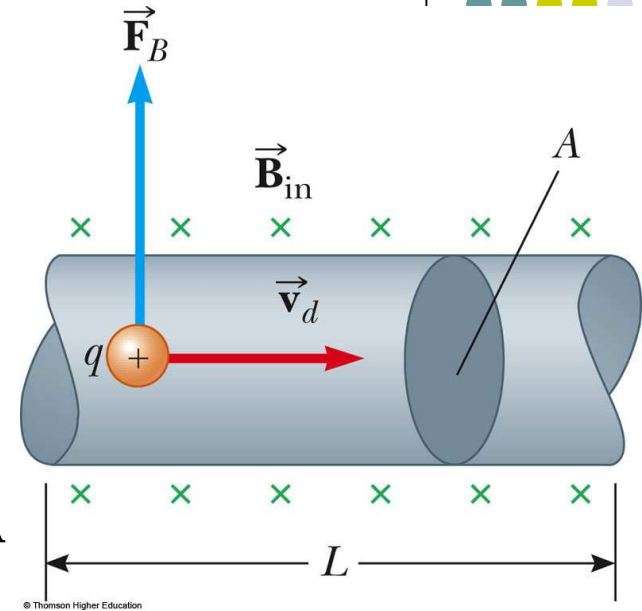
Force on a Wire, the formula



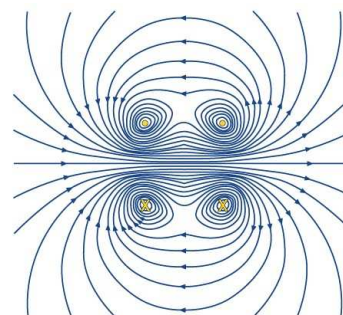
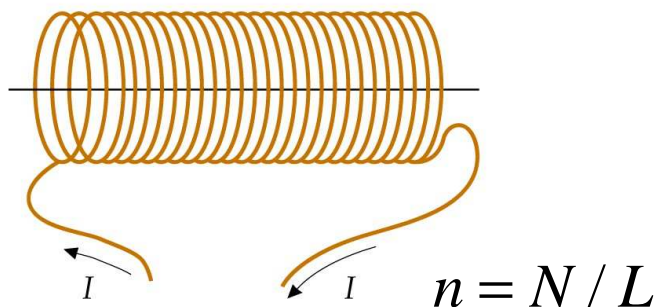
- The magnetic force is exerted on each moving charge in the wire
 - $\vec{F} = q\vec{v}_d \times \vec{B}$
- The total force is the product of the force on one charge and the number of charges in the wire
 - $\vec{F} = (q\vec{v}_d \times \vec{B})nAL = qnA\vec{v}_dL \times \vec{B} \quad \because I = nq\vec{v}_d \cdot \vec{A}$
- In terms of the current, this becomes

$$\vec{F}_B = I\vec{L} \times \vec{B}$$

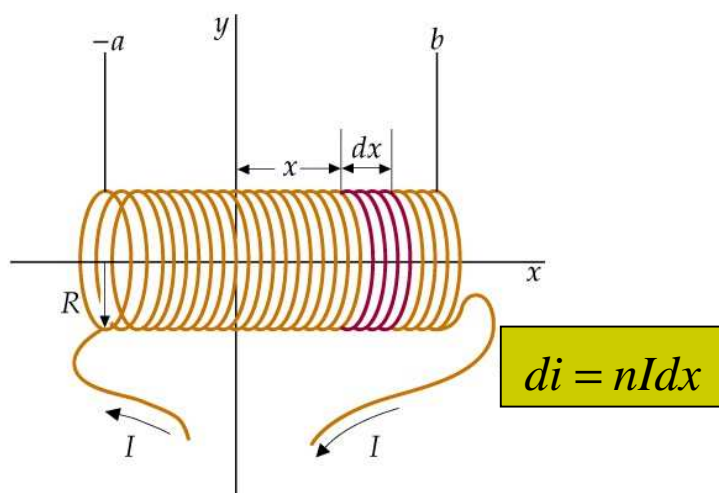
- I is the current
- \vec{L} is a vector that points in the direction of the current
 - Its magnitude is the length L of the segment



Campo magnético creado por un solenoide



Líneas de campo debidas a dos espiras que transportan la misma corriente en el mismo sentido

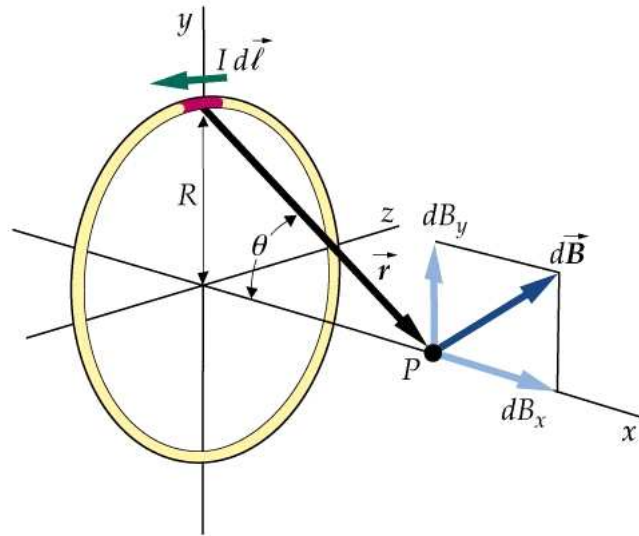
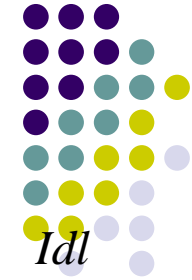


$$dB_x = \frac{\mu_0}{4\pi} \frac{di(2\pi R^2)}{(x^2 + R^2)^{3/2}} = \frac{\mu_0}{4\pi} \frac{2\pi R^2 n I dx}{(x^2 + R^2)^{3/2}}$$

$$B_x = \frac{\mu_0}{4\pi} 2\pi R^2 n I \int_{-a}^b \frac{dx}{(x^2 + R^2)^{3/2}}$$

$$B_x = \frac{1}{2} \mu_0 n I \left(\frac{b}{\sqrt{b^2 + R^2}} + \frac{a}{\sqrt{a^2 + R^2}} \right)$$

Campo magnético creado por una espira circular

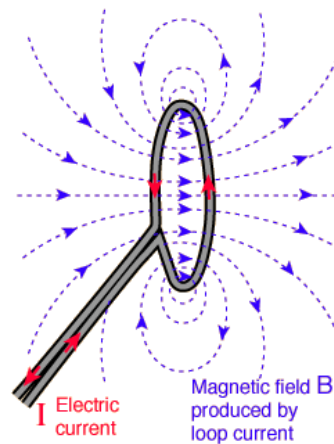


$$d\mathbf{B} = \frac{\mu_0}{4\pi} \frac{I d\mathbf{l} \times \hat{\mathbf{r}}}{r^2} \quad |d\mathbf{B}| = \frac{\mu_0}{4\pi} \frac{|I d\mathbf{l} \times \hat{\mathbf{r}}|}{r^2} = \frac{\mu_0}{4\pi} \frac{I dl}{x^2 + R^2}$$

$$dB_x = dB \sin \theta = dB \left(\frac{R}{\sqrt{x^2 + R^2}} \right) = \frac{\mu_0}{4\pi} \frac{I dl}{x^2 + R^2} \frac{R}{\sqrt{x^2 + R^2}}$$

$$B_x = \oint dB_x = \oint \frac{\mu_0}{4\pi} \frac{IR}{(x^2 + R^2)^{3/2}} dl = \frac{\mu_0}{4\pi} \frac{IR}{(x^2 + R^2)^{3/2}} \oint dl$$

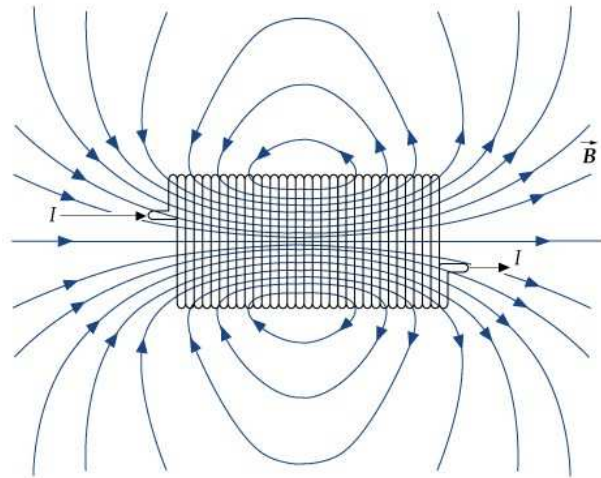
$$B_x = \frac{\mu_0}{4\pi} \frac{IR(2\pi R)}{(x^2 + R^2)^{3/2}}$$



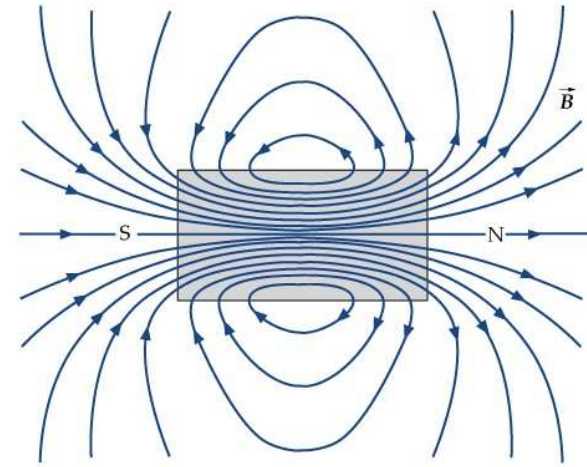
Líneas de campo creado por una espira circular



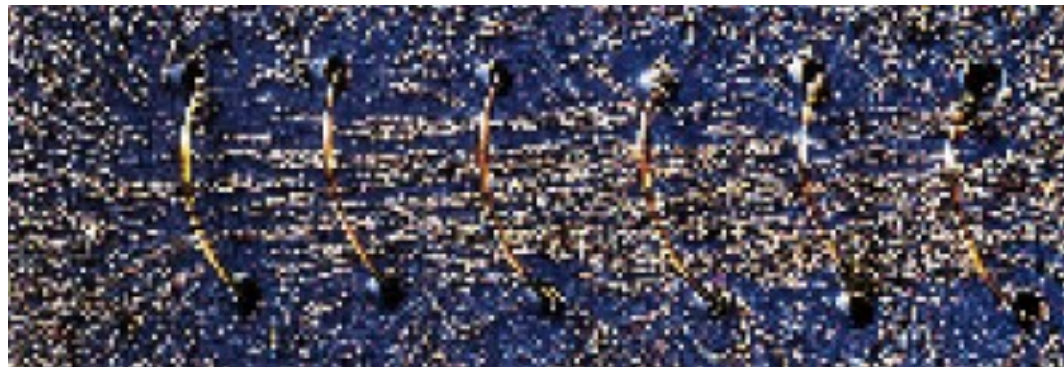
Campo magnético creado por un solenoide



(a)



(b)



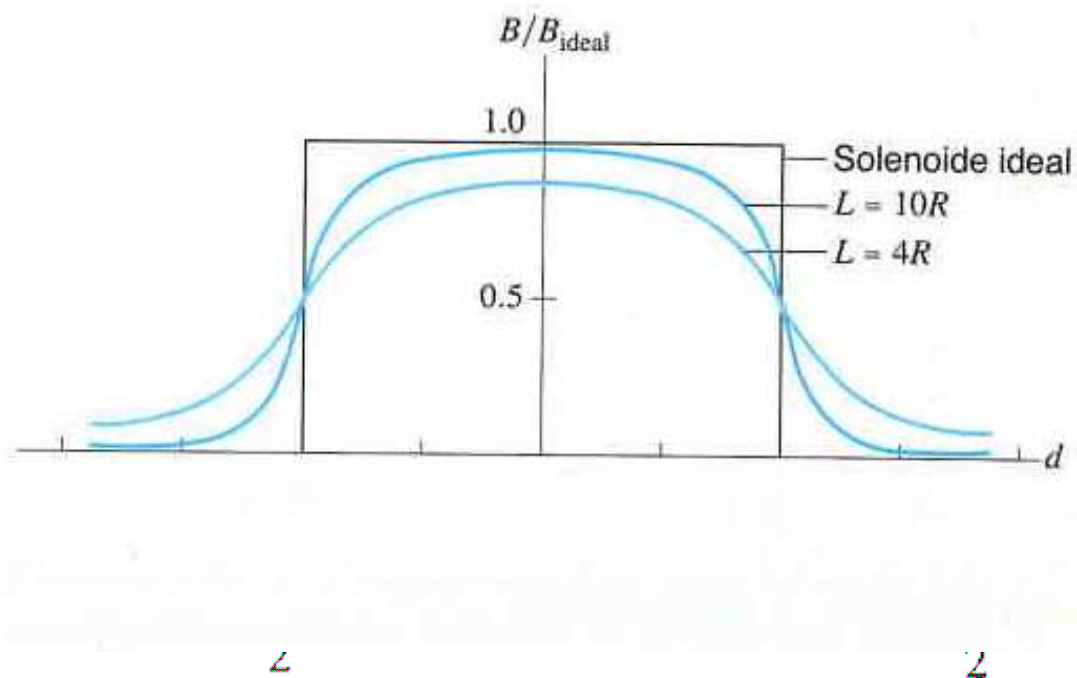


FIGURA 33-17. Campos magnéticos de un solenoide ideal y dos solenoides no ideales en función de la distancia d respecto al centro.

Ley de Ampère



$$\oint \vec{\mathbf{B}} \cdot d\vec{\mathbf{s}} = \mu_0 i.$$

Ley de Ampère

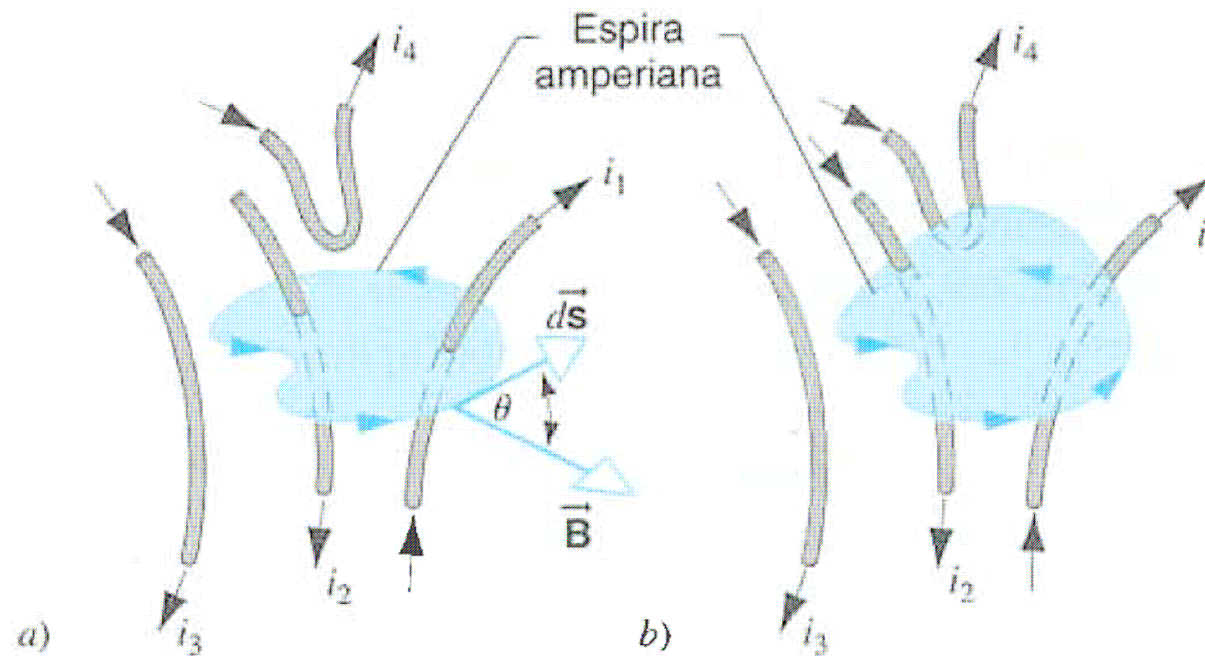


FIGURA 33-20. *a)* Al aplicar la ley de Ampère integramos alrededor una espira cerrada. La integral está determinada por la corriente neta que atraviesa la superficie acotada por la espira. *b)* La superficie acotada ha sido estirada hacia arriba.

\vec{B} from a long, straight conductor re-calculated using Ampere's Law



Choose the Gauss's Surface,
oops, not again!

Choose the Ampere's loop, as a
circle with radius a .

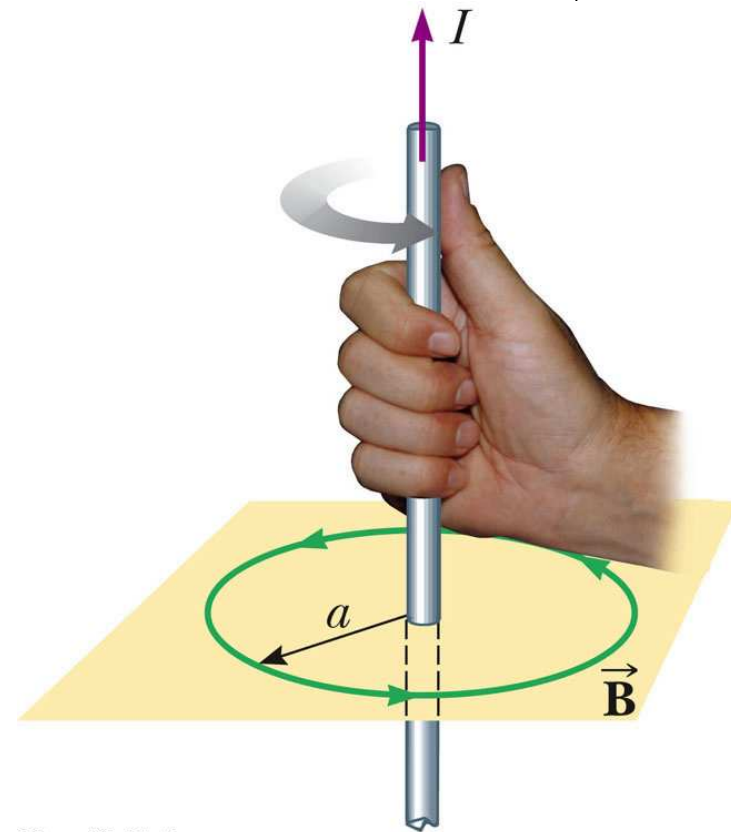
Ampere's Law says

$$\oint \vec{B} \cdot d\vec{s} = \mu_0 I$$

\vec{B} is parallel with $d\vec{s}$, so

$$B \oint ds = B 2\pi a = \mu_0 I$$

$$B = \frac{\mu_0 I}{2\pi a}$$



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When the wire has a size: with radius R



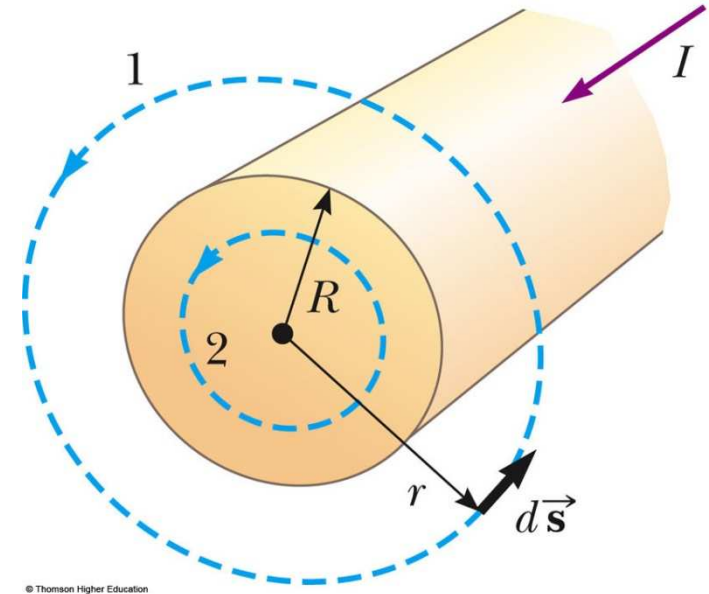
- Outside of the wire, $r > R$

$$\oint \vec{\mathbf{B}} \cdot d\vec{\mathbf{s}} = B(2\pi r) = \mu_0 I \quad \rightarrow \quad B = \frac{\mu_0 I}{2\pi r}$$

- Inside the wire, we need I' , the current inside the ampere's circle

$$\oint \vec{\mathbf{B}} \cdot d\vec{\mathbf{s}} = B(2\pi r) = \mu_0 I' \quad \rightarrow \quad I' = \frac{r^2}{R^2} I$$

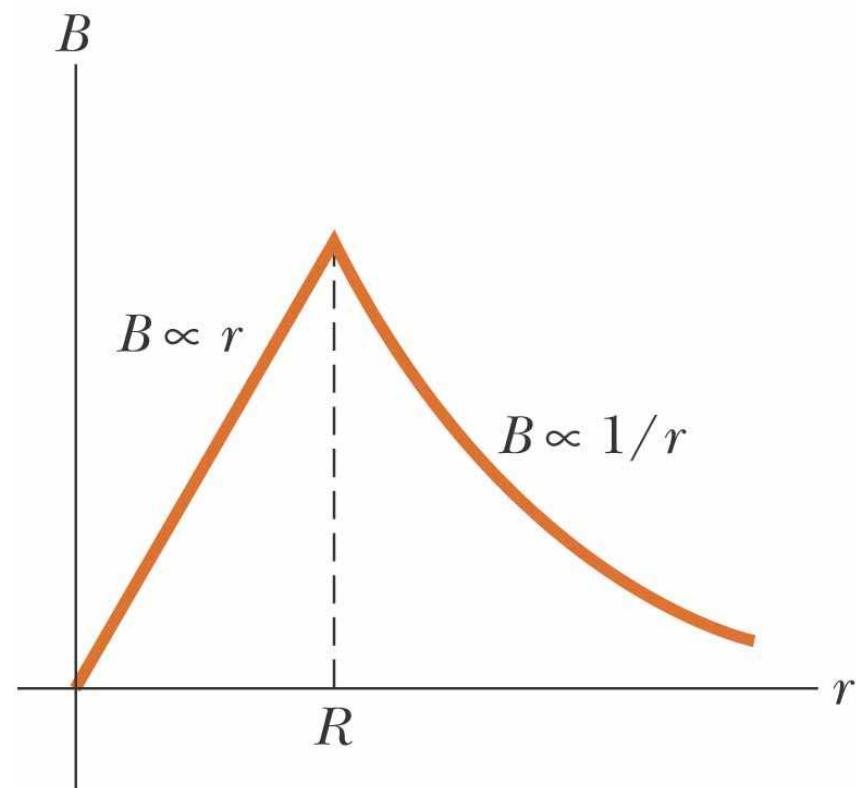
$$B = \left(\frac{\mu_0 I}{2\pi R^2} \right) r$$





Plot the results

- The field is proportional to r inside the wire
- The field varies as $1/r$ outside the wire
- Both equations are equal at $r = R$



Ideal (infinitely long) Solenoid



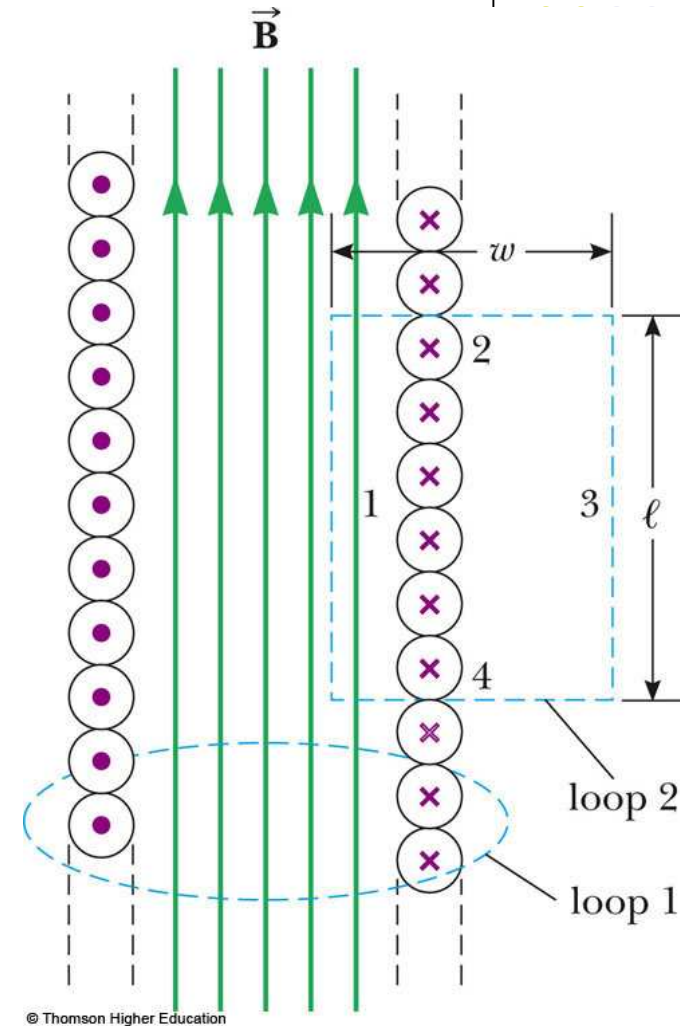
- An *ideal solenoid* is approached when:
 - the turns are closely spaced
 - the length is much greater than the radius of the turns
- Apply Ampere's Law to loop 2:

$$\oint \vec{\mathbf{B}} \cdot d\vec{\mathbf{s}} = \int_{\text{path1}} \vec{\mathbf{B}} \cdot d\vec{\mathbf{s}} = B \int_{\text{path1}} ds = B\ell$$

$$\oint \vec{\mathbf{B}} \cdot d\vec{\mathbf{s}} = B\ell = \mu_0 NI$$

$$B = \mu_0 \frac{N}{\ell} I = \mu_0 n I$$

$n = N / \ell$ is the number of turns per unit length



Magnetic Field of a Toroid



- The toroid has N turns of wire
- Find the field at a point at distance r from the center of the toroid (loop 1)

$$\oint \vec{\mathbf{B}} \cdot d\vec{\mathbf{s}} = B(2\pi r) = \mu_0 N I$$

$$B = \frac{\mu_0 N I}{2\pi r}$$

- There is no field outside the coil (see loop 2)

