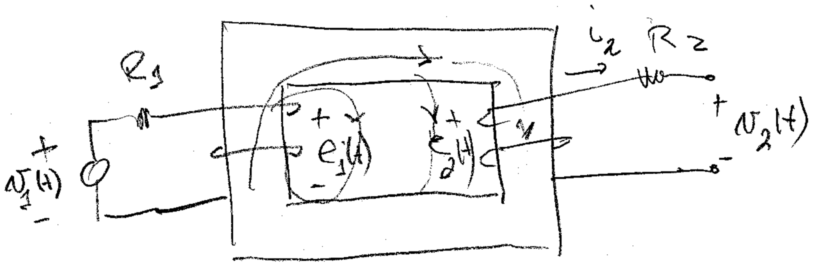
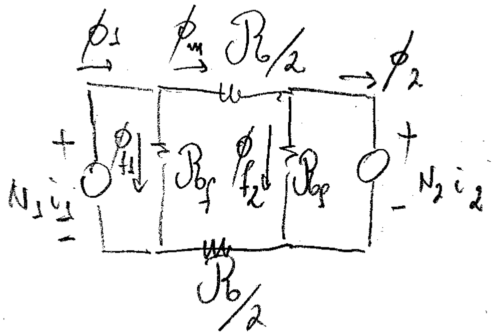
# Transformador Real




$$v_1(t) = R_1 i_1 + e_1(t) \qquad e_1(t) = N_1 \frac{d\phi_1}{dt} \qquad \phi_1 = \phi_{f1} + \phi_m$$

$$e_2(t) = R_2 i_2 + v_2(t) \qquad e_2(t) = N_2 \frac{d\phi_2}{dt} \qquad \phi_2 = \phi_m - \phi_{f2}$$

$$\Rightarrow \quad v_1 = R_1 i_1 + N_1 \frac{d\phi_{f1}}{dt} + N_1 \frac{d\phi_m}{dt} \qquad \phi_{f1} = \frac{N_1 i_1}{\mathcal{R}_{0f}}$$

$$v_2 = -R_2 i_2 - N_2 \frac{d\phi_{f2}}{dt} + N_2 \frac{d\phi_m}{dt} \qquad \phi_{f2} = \frac{N_2 i_2}{\mathcal{R}_{0f}}$$

$$\left.\begin{array}{l} N_1 \dfrac{d\phi_m}{dt} = v_p \\ N_2 \dfrac{d\phi_m}{dt} = v_s \end{array}\right\} \Rightarrow \boxed{\frac{v_p}{N_1} = \frac{v_s}{N_2}}$$

Reluctancia por aire ($\mu_0$)

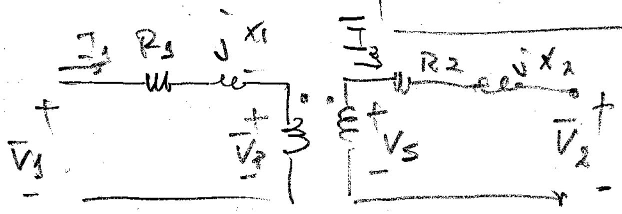
$$\Rightarrow \quad v_1 = R_1 i_1 + \frac{N_1^2}{\mathcal{R}_{0f}} \frac{di_1}{dt} + v_p \qquad \frac{N_1^2}{\mathcal{R}_{0f}} = L_{f1}$$

$$v_2 = -R_2 i_2 - \frac{N_2^2}{\mathcal{R}_{0f}} \frac{di_2}{dt} + v_s \qquad \frac{N_2^2}{\mathcal{R}_{0f}} = L_{f2}$$

Regimen Sinusoidal:

$$\overline{V}_1 = R_1 \overline{I}_1 + jX_1 \overline{I}_1 + V_p \qquad \frac{\overline{V}_p}{N_1} = \frac{\overline{V}_s}{N_2} \qquad X_1 = \omega L_{f1}$$

$$\overline{V}_2 = -R_2 \overline{I}_2 - jX_2 \overline{I}_2 + V_s \qquad X_2 = \omega L_{f2}$$



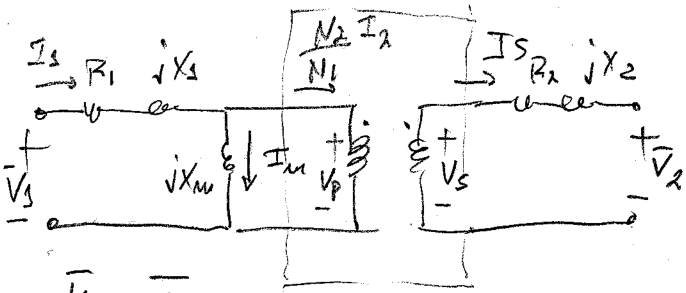

Ademas: $N_1 i_1 - N_2 i_2 = \mathcal{R}_0 \phi_m \Rightarrow N_1 i_m = \mathcal{R}_0 \phi_m$

reluctancia del circ. Magnetico (hierro)

con $i_m = i_1 - \frac{N_2}{N_1} i_2$

$$\Rightarrow \quad v_p = N_1 \frac{d\phi_m}{dt} = \frac{N_1^2}{\mathcal{R}_0} \frac{di_m}{dt} = L_m \frac{di_m}{dt} \Rightarrow \boxed{\begin{array}{l} V_p = jX_m \overline{I}_m \\ \overline{I}_m = \overline{I}_1 - \dfrac{N_2}{N_1} \overline{I}_2 \end{array}}$$

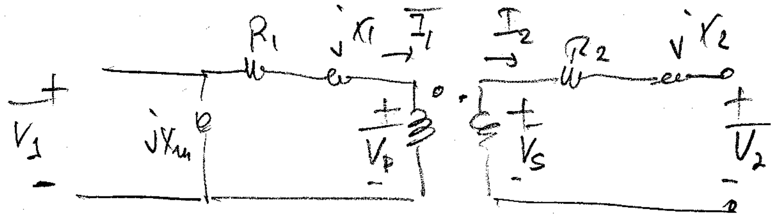
Entonces:




$$\frac{\bar{V}_p}{N_1} = \frac{\bar{V}_S}{N_2}$$

↳ Trafo Ideal.

Trafo Real: Aproximación:

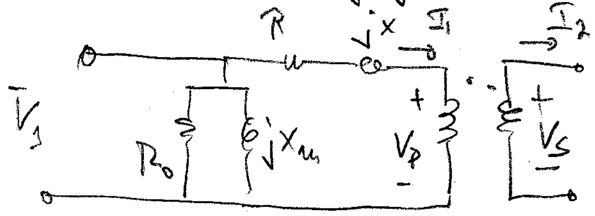



Trafo Real: Perdidas en el hierro.

$$\left.\begin{array}{l} P_{Fe} = (af + bf^2)\phi^2 \\ V = Kf\phi \end{array}\right\} \therefore P_{Fe} = \left(\frac{a_1}{f} + b_1\right) V^2 = \frac{V^2}{R_o}$$

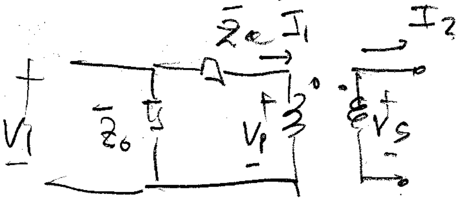
↳ depende de f!!

⟹ Para modelar agrego $R_o$ en paralelo:



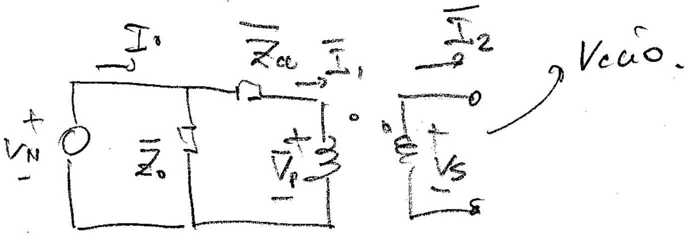

$$\left\{\begin{array}{l} \frac{\bar{V}_p}{N_1} = \frac{\bar{V}_S}{N_2} \\ N_1\bar{I}_1 = N_2\bar{I}_2 \\ R = R_1 + \left(\frac{N_1}{N_2}\right)^2 R_2 \\ X = X_1 + \left(\frac{N_1}{N_2}\right)^2 X_2 \end{array}\right.$$

Modelo a usar en el curso



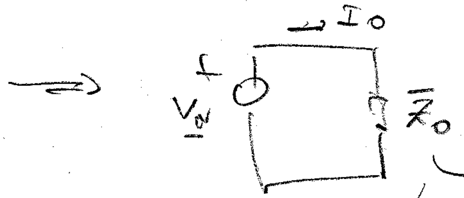
# Ensayos:

## 1) Ensayo de Vacío: Se hace a $V_1 = V_N$ $f = f_N$

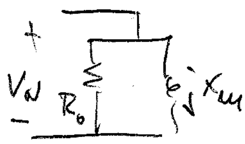



$\bar{I}_2 = 0 \Rightarrow \bar{I}_1 = 0$

$\Rightarrow$




Se Mide $V_0$, $I_0$, $P_0$

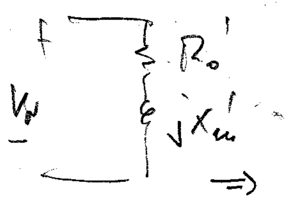



$$P_0 = \frac{V_0^2}{R_0} \Rightarrow R_0$$

$$Q_0 = \sqrt{(V_0 I_0)^2 - P_0^2}$$

$$Q_0 = \frac{V_0^2}{X_m} \Rightarrow X_m.$$

Se puede usar:



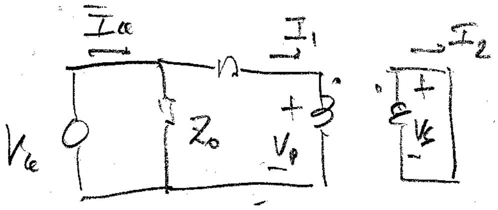

Pero:

$$\Rightarrow P_0 = R_0' I_0^2$$

$$Q_0 = X_m' I_0^2$$

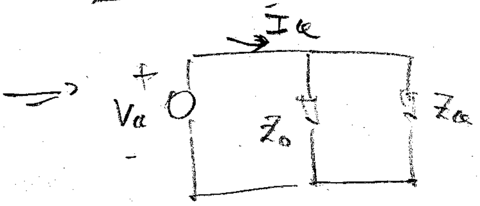
## 2) Ensayo Cortocircuito: @ $V \ll V_N$ / $I_{cc} \leq I_N$

$\hookrightarrow$ 2% - 15%



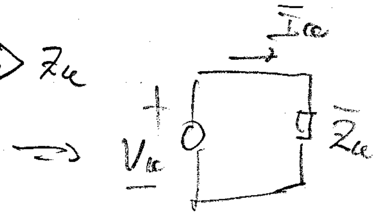

$\bar{V}_s = 0 \Rightarrow \bar{V}_p = 0$

$\Rightarrow$




$Z_0 \gg Z_{cc}$

$\Rightarrow$




Mido: $V_{cc}$, $I_{cc}$, $P_{cc}$

$$\bar{Z}_{cc} = \frac{V_{cc}}{I_{cc}} \angle \operatorname{Arcos}\left(\frac{P_{cc}}{V_{cc} I_{cc}}\right)$$

**Ejemplo:** Trafo $6300V/440V$ $60Hz$ $100 KVA$

1) Ensayo de Vacío: $220V$ $50Hz$ $5A$ $1KW$

@ $220V$ $50Hz$ ← $R_{01} = \frac{220^2}{1000} = 48{,}4\,\Omega$ $Q_0 = \sqrt{(220 \times 5)^2 - 1000^2} = 458{,}3\,VAR$

$X_{01} = \frac{220^2}{458{,}3} = 105{,}6\,\Omega$

El ensayo fue realizado alimentando el transformador desde el bobinado de B.T. pero a una tensión menor a la nominal

$$\phi_{440,60} = \frac{440}{K\,60} = \frac{7{,}3}{K}$$
$$\phi_{220,50} = \frac{220}{K\,50} = \frac{4{,}4}{K}$$

$\Rightarrow \phi_{220,50} < \phi_{440,60} \Rightarrow$ No satura

$\Rightarrow L_m = \frac{N_1^2}{R_0} = cte$ $X_0 = \omega L_m$

@ 440, 60

$\Rightarrow X_{02} = \frac{6}{5} X_{01} = 126{,}7\,\Omega$

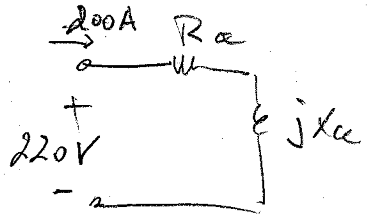
$$P_0 = (af + bf^2)\phi^2 = \left(\underbrace{\frac{a_1}{f}}_{\text{histeresis}} + \underbrace{b_1}_{F}\right)V^2 = P_H + P_F$$

Supongo $P_H = P_F = 500W$ @ $220V, 50Hz$

$$\Rightarrow \begin{cases} \frac{a_1}{50} \times 220^2 = 500 \Rightarrow a_1 = 0{,}52 \\ b_1 = \frac{500}{220^2} \Rightarrow b_1 = 0{,}01 \end{cases}$$

$$P_{0\,440,60} = \left(\frac{0{,}52}{60} + 0{,}01\right) 440^2 \simeq 3614W = \frac{440^2}{R_{02}} \Rightarrow R_{02} = 53{,}6\,\Omega$$

2) Ensayo Cortocircuito: $220V$ $50Hz$ $15A$ $8KW$




obs. $220V$ es ~ $3{,}5\%$ de $6300V$ y $50\%$ de $440V$ ⇒ el ensayo fue hecho alimentando el trafo desde el bobinado de A.T.

$\Rightarrow \bar{Z}_{cc} = \frac{220}{15} \angle \text{Arcos} \frac{1500}{220 \times 15} = 14{,}7 \angle 63° = (6{,}7 + j\,13{,}1)\,\Omega$

($6{,}7 \rightarrow R_{cc_1}$, $13{,}1 \rightarrow X_{cc_1}$) @ 220 V, 50 Hz

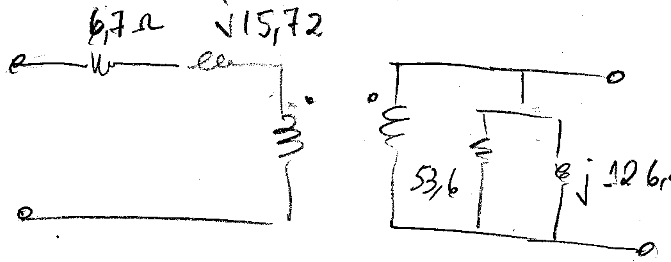
$R_{cc} = R_{cu} + R_{ad} \simeq R_{cu} \Rightarrow R_{cc_2} = R_{cc_1}$ (@ 440 V, 60 Hz)

- $R_{ad}$ (efecto pelicular) → depende de f
- $R_{cu}$ → No depende de V y f

$X_{cc_1} = 2\pi \times 50\, L_f = \frac{N^2}{\mathcal{R}_f} \Rightarrow X_{cc_2} = \frac{6}{5} X_{cc_1} \simeq 15{,}72\,\Omega$

$\mathcal{R}_f$ → $\mu_0$ ⇒ No satura

Modelo:




Pasando todo @ BT.

$R_{cc\,BT} = \left(\frac{440}{6300}\right)^2 \times 6{,}7 = 0{,}033\,\Omega$

$X_{cc\,BT} = \left(\frac{440}{6300}\right)^2 \times 15{,}72 = 0{,}077\,\Omega$