The Capacity-Coverage Tradeoff in CDMA Systems with Soft Handoff

Andrew Sendonaris
Dept. of Electrical and Computer Engineering
Rice University
Houston, TX 77005
sendos@rice.edu

Venugopal Veeravalli
School of Electrical Engineering
Cornell University
Ithaca, NY 14853
venu@ee.cornell.edu

Abstract

In previous work [7], the tradeoff between coverage and number of users was explicitly characterized for systems without soft handoff. It is well known that CDMA soft handoff can result in gains both in coverage and in number of users supported [3]. In this paper, we extend the analysis of [7] to derive analytical expressions that quantify the gains from soft handoff. Our analysis may be used in cellular planning in the following two ways: first, to calculate the coverage of cells based on the amount of traffic they will support; and second, to set hard limits on the number of users admitted into the cell in order to meet coverage requirements.

1. INTRODUCTION

In cellular CDMA systems with non-orthogonal users and single-user detection (such as the reverse-link of IS-95 [6]) it is well known that the coverage of a cell has an inverse relationship with the user capacity of the cell. An increase in the number of active users in the cell causes the total interference seen at the receiver to increase. This causes an increase in the required received power for each user, due to the fact that each user has to maintain a certain signal-to-interference ratio at the receiver for satisfactory performance. For a maximum allowable transmit power, an increase in the required received power will a result in a decrease in the maximum distance a mobile can be from the base station, thereby reducing coverage.

In CDMA soft handoff, a mobile may be in simultaneous communication with two or more base stations. Previous analyses of CDMA cell coverage have mainly focussed on the extension of cell coverage that results from soft and hard handoff [3, 1, 4, 2]. It has been established that soft handoff can result in gains in coverage as well as capacity of the cell [3]. Our objective is to quantify these gains and thus determine the coverage as a function of the number of users in the cell, when soft handoff is employed. To this end, we extend our previous analysis of coverage in the absence of soft handoff (see [7]) to the case where we allow two and three way soft handoff at the edge of the cell.

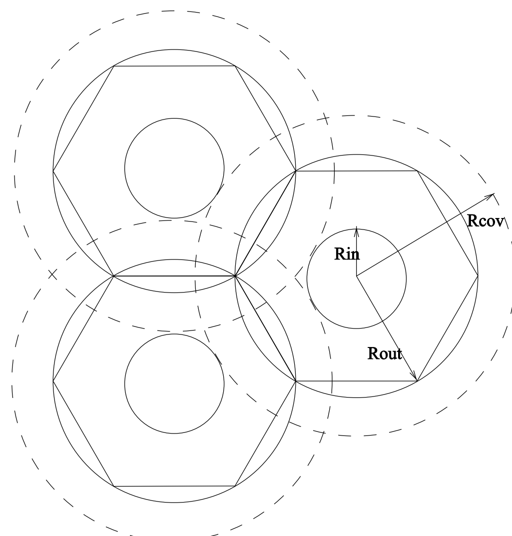

Figure 1. Cell radii and soft handoff regions

Figure 1 describes the various regions of a typical cell. We assume that a requirement on the outage probability P_{out} is specified and that the goal is to ensure that all points in the cell meet this requirement. To this end we design the inner radius of the cell R_{in} so that the outage probability at a distance R_{in} from the BS is P_{out} when the mobile is communicating only with this BS. This will guarantee that the outage probability is smaller than P_{out} over the entire inner circle. The inner circle will then represent the no-soft-handoff region.

In the exterior of the inner circle, the outage probability condition can be met by allowing soft handoff with other base stations. The outer cell radius R_{out} is hence designed such that the maximum outage probability between the edge of the cell and the inner circle (i.e., the soft handoff region) is P_{out}. Note that the maximum outage probability could happen either on the middle of one of the edges of the hexagon or at one of the cell corners. Finally, the actual pilot coverage R_{cov} of a cell is chosen to match the soft handoff regions of its neighboring cells.

2. NO SOFT HANDOFF

For ease of exposition, we will first formulate the problem as it applies to the no-soft-handoff case and provide a summary of the results in [7]. The results will then be extended to the soft handoff case in Section 3.

2.1. Problem Formulation

We begin by introducing the relevant variables and the notation required for our analysis. For any power or signal-to-interference ratio variable X expressed in dB, $\hat{X}$ denotes $10^{X/10}$. The number of users in the cell, i.e., those being controlled by the cell's base station (BS), is denoted by k. The power received at the BS from the j-th user in Watts is denoted by $\hat{S}_j$. (The received power in dB-W is, by the above notation, given by $S_j = 10 \log \hat{S}_j$.) Also, $\hat{\varepsilon}_j$ is the signal-to-interference ratio (SIR or E_b/I_0) for the j-th user, and ν_j is the voice activity factor of j-th user. The variables $\{\nu_j\}$ are modeled as independent Bernoulli random variables with parameter ρ. Finally, R denotes the information bit rate in bits per second, W denotes the system bandwidth in Hz, N_0 is the background noise power spectral density, and I is the other-cell interference density.

The signal-to-interference ratio (SIR) for the j-th user at the BS may be expressed in terms of the received powers of the various users as:

$$\hat{\varepsilon}_j = \frac{\dfrac{\hat{S}_j}{R}}{\displaystyle\sum_{i:i\neq j} \frac{\nu_i \hat{S}_i}{W} + N_0 + I}. \qquad (1)$$

The SIR requirements for the various users in the cell vary with time due to changes in the multipath fading environment and imperfections in power control. In particular, let $\hat{\varepsilon}_j^{\text{target}}$ denote the target SIR that is a function of the target FER (Frame Error Rate) and the multipath conditions, and let $\delta_j^{\hat{\varepsilon}}$ denote the error in the power control algorithm. Then, the required SIR for the j-th user is given by

$$\hat{\varepsilon}_j^* = \hat{\varepsilon}_j^{\text{target}} \delta_j^{\hat{\varepsilon}} \qquad (2)$$

That is, $\hat{\varepsilon}_j^*$ is the SIR that the power control algorithm is demanding from the mobile at that particular point in time, even though the target SIR may be slightly different. Field trials reported in [8] have shown that the SIR requirements $\hat{\varepsilon}_j^*$ are well modeled by log-normal random variables. Furthermore, we can assume that the fading processes that cause the fluctuations in SIR requirements for the various users are independent. By the above discussion, we can model $\{\hat{\varepsilon}_j^\star\}$ at any given time by independent and identically distributed (i.i.d.) log-normal random variables.

In order to meet the SIR requirements $\{\hat{\varepsilon}_j^\star\}$, the required received powers $\{\hat{S}_j^\star\}$ must satisfy the power control equations:

$$\hat{\varepsilon}_j^\star = \frac{\dfrac{\hat{S}_j^\star}{R}}{\displaystyle\sum_{i:i\neq j} \frac{\nu_i \hat{S}_i^\star}{W} + N_0 + I} \qquad (3)$$

2.2. Outage Equation

If the SIR of a given user is lower than the desired value for a certain period of time, the call is dropped and we have outage. Ideally, we would like to limit the probability of such an outage to a small number. But since the time correlation of the various random processes that contribute to the SIR cannot be determined *a priori*, obtaining an analytical expression for this outage probability is (in general) intractable. However, if we contain the probability of the event that the SIR falls below the required value at any time to a small value, say p_m, then the probability of outage will necessarily be smaller than p_m. We will hence consider outage for user j to be the event $\{\hat{\varepsilon}_j < \hat{\varepsilon}_j^\star\}$.

There are two ways in which the event $\{\hat{\varepsilon}_j < \hat{\varepsilon}_j^\star\}$ can happen: (i) the power control equations of (3) do not have a feasible solution[1] (call this event A_{out}); and (ii) the power control equations have a feasible solution, but the maximum transmit power $S_{\max}$ at the mobile is exceeded (call this event B_{out}). Thus the probability of outage is given by:

$$P_{\text{out}} = P(A_{\text{out}}) + [1 - P(A_{\text{out}})]\, P(B_{\text{out}} | A_{\text{out}}^c). \qquad (4)$$

We will use the outage equation to characterize the capacity-coverage tradeoff. To begin the analysis, we focus on the feasibility of the power control equations of (3).

2.3. Power Control Feasibility And Pole Capacity

It can be shown [7] that

$$P(A_{\text{out}}) = P\left(\sum_{i=1}^{k} \frac{R\, \hat{\varepsilon}_i^\star\, \nu_i}{W + R\, \hat{\varepsilon}_i^\star\, \nu_i} \geq 1 \right). \qquad (5)$$

For a given set of parameter values, $P(A_{\text{out}})$ can be computed as a function of k using Monte Carlo techniques, or

[1] That is, no matter how large the received powers are, the SIR requirements of the users cannot be satisfied

using numerical convolution [7]. Denote this function by $\mathrm{P}_A(k)$.

The probability of event A_{out} is always nonzero as long as $k \geq 2$, and increases with increasing k. This leads to the definition of *pole capacity*, which serves as an upper bound on the maximum number of users that can be accommodated in the cell as the coverage of the cell shrinks to zero.

Definition 1 Let p_m be the maximum allowable outage probability. The pole capacity k_{pole} of a cell is the maximum number of users that can be accommodated in the cell such that $\mathrm{P}_{\text{out}} < p_m$, if there is no constraint on the maximum received power for the various users.

To calculate k_{pole}, we simply evaluate $\mathrm{P}_A(k)$ and pick the largest value of k such that $\mathrm{P}_A(k) < p_m$.

Remark 1 It should be noted that k_{pole} is independent of whether or not the users are in soft handoff.

2.4. Exceeding Maximum Transmit Power

Without loss of generality, we will focus on user 1. The transmit power (in dB-W) of user 1 is given in terms of its received power $S_1^\star$ at the BS by

$$S_{\text{trans}} = S_1^\star + \mathrm{PL}(d) + Z_1, \tag{6}$$

where $\mathrm{PL}(d)$ is the path loss at distance d from the BS (including antenna gains) and Z_1 is a random variable representing shadow fading. The path loss is usually well modeled as (see, e.g., Hata's model [5]): $\mathrm{PL}(d) = K_1 + K_2 \log(d)$. The shadow fading variable Z_1 is well modeled as a zero-mean Gaussian random variable with variance σ_Z^2 [5].

The probability of event B_{out} for user 1 is the probability that S_{trans} exceeds $S_{\max}$, the maximum power available at the mobile. Therefore,

$$\mathrm{P}(B_{\text{out}}|A_{\text{out}}^c) = \mathrm{P}\left(S_1^\star + \mathrm{PL}(d) + Z_1 > S_{\max} \mid A_{\text{out}}^c\right) \tag{7}$$

In [7] it was shown that, conditioned on A_{out}^c, $\hat{S}_1^\star$ is very well approximated by a log-normal random variable for $k < k_{\text{pole}}$, implying that $S_1^\star$ is approximately Gaussian. The mean and variance of $S_1^\star$ can easily be calculated [7] in terms of the system parameters m_ε, σ_ε^2, N_0, I, W, R, ρ, and, more importantly, k. They are henceforth denoted as $m_S(k)$ and $\sigma_S^2(k)$.

Now, in order to evaluate the probability of the event in (7), we need to determine the joint statistics of $S_1^\star$ and Z_1. We can argue that the correlation between $S_1^\star$ and Z_1 is close to zero, since the fluctuations in the required received power $S_1^\star$ are mainly due to multipath fading and imperfections in power control, whereas the fluctuations in Z_1 are due to shadow fading. We can hence compute the conditional probability $\mathrm{P}(B_{\text{out}}|A_{\text{out}}^c)$ as:

$$\begin{aligned}\mathrm{P}\left(S_1^\star + \mathrm{PL}(d) + Z_1 > S_{\max} \mid A_{\text{out}}^c\right) &= \\ Q\left(\frac{S_{\max} - \mathrm{PL}(d) - m_S(k)}{\sqrt{\sigma_S^2(k) + \sigma_Z^2}}\right), \end{aligned} \tag{8}$$

where $Q(\cdot)$ is the complementary c.d.f. of a zero-mean, unit-variance Gaussian random variable. By using $\mathrm{P}_A(k)$, (8) and (4) it is possible to calculate the coverage of the cell as a function of the number of users k [7].

3. SOFT HANDOFF

We now extend the above results to the case when the mobile is allowed to communicate with 1, 2 or 3 BS's. Denote the number of BS's that a mobile is communicating with at a given point in time by N_{SH}. The expression for the outage probability remains the same as in (4), but now encompasses the other BS's through a re-definition of A_{out} and B_{out}.

A_{out} is now defined as the event that all of the N_{SH} BS's do not have a feasible solution for their power control equations. We assume that the SIR requirements in different cells are independent, and therefore, (5) becomes

$$\mathrm{P}(A_{\text{out}}) = \prod_{n=1}^{N_{\text{SH}}} \mathrm{P}(A_{\text{out}}^{(n)}) \tag{9}$$

where $\mathrm{P}(A_{\text{out}}^{(n)})$ is defined as in (5) and depends on k_n, the number of users that belong to the n-th BS. We denote the resulting $\mathrm{P}(A_{\text{out}})$ by $\mathrm{P}_A(\{k_n\})$.

B_{out} is now defined as the event that the transmit powers required from all BS's currently in communication with the mobile exceed the maximum transmit power $S_{\max}$. Therefore, (7) becomes

$$\begin{aligned}\mathrm{P}(B_{\text{out}}|A_{\text{out}}^c) &= \\ \mathrm{P}\left(\bigcap_{n=1}^{N_{\text{SH}}} \{S_1^{\star(n)} + \mathrm{PL}(d^{(n)}) + Z_1^{(n)} > S_{\max}\} \mid A_{\text{out}}^c\right)\end{aligned} \tag{10}$$

where $d^{(n)}$ is the distance to the n-th BS.

For the link to base station n, the random variable $S_1^{\star(n)} + Z_1^{(n)}$ is Gaussian with mean $m_S(k_n)$ and variance $\sigma_S^2(k_n) + \sigma_Z^2$, where k_n is the number of users in cell n. We assume the following model for the correlation between the $Z_1^{(n)}$'s [3]: $Z_1^{(n)} = a\zeta + \sqrt{1-a^2}\zeta_n$, where ζ and the ζ_n's are i.i.d. and distributed according to $N(0, \sigma_Z^2)$. Using this model, we can write $S_1^{\star(n)} + Z_1^{(n)}$ as $S_1^{\star(n)} + Z_1^{(n)} = m_S(k_n) + X_n$ where $X_n = a\xi + \sqrt{1-a^2}\xi_n$, with $\xi \sim N(0, \sigma_Z^2)$ and $\xi_n \sim N(0, \sigma_Z^2 + \frac{\sigma_S^2(k_n)}{1-a^2})$ being independent.

By using this model it can be shown that

$$\mathrm{P}(B_{\mathrm{out}}|A_{\mathrm{out}}^c) = \int_{-\infty}^{\infty}\left[\prod_{n=1}^{N_{\mathrm{SH}}} Q\left(\phi_n - \frac{a\sigma_Z x}{\sqrt{(1-a^2)\sigma_Z^2+\sigma_S^2(k_n)}}\right)\right]\frac{1}{\sqrt{2\pi}}e^{-\frac{x^2}{2}}dx \quad (11)$$

where

$$\phi_n = \frac{S_{\max} - K_1 - K_2\log_{10} d^{(n)} - m_S(k_n)}{\sqrt{(1-a^2)\sigma_Z^2+\sigma_S^2(k_n)}} \quad (12)$$

We denote the resulting $\mathrm{P}(B_{\mathrm{out}}|A_{\mathrm{out}}^c)$ by $\mathbf{P}_B(\{k_n\},\{d^{(n)}\})$. So, given the number of users in all three cells ($\{k_n\}$), the distance of the mobile from all three BS's ($\{d^{(n)}\}$), and the degree of soft handoff (N_{SH}), the probability of outage can be calculated using

$$\mathbf{P}_{\mathrm{out}}(\{k_n\},\{d^{(n)}\}) = \mathbf{P}_A(\{k_n\}) + [1-\mathbf{P}_A(\{k_n\})]\,\mathbf{P}_B(\{k_n\},\{d^{(n)}\}). \quad (13)$$

4. COVERAGE VERSUS CAPACITY

What is of interest in cellular planning is to design cell coverages and capacities to match projected traffic densities in the network. In this case it may be appropriate to model the number of users in a cell as a random variable (denoted by κ_n). The statistics of κ_n will be a function of the cell admission policy and the offered traffic. We assume that all cells of interest are in the same environment and thus have the same traffic density, implying also that they have the same pole capacity and the same statistics for κ_n. Our goal here is to derive a relationship between the three cell radii and the mean of κ_n, which we denote by c We will refer to c, which represents the *carried traffic* in a cell, as capacity. Clearly, c can be related to the Erlang capacity of the cell through the blocking probability [8].

Let $p_\kappa(\cdot)$ denote the probability mass function (p.m.f.) of κ_n. Clearly, any useful admission policy will not allow κ_n to exceed k_{pole}, since power control becomes infeasible with probability greater than p_m when the number of users exceeds k_{pole}.

In the following, we assume that support of p_κ is limited to the set $\{0,1,\ldots,k_{\mathrm{pole}}\}$. Thus, for a given degree of soft handoff, we can compute the average outage probability at distances $\{d^{(n)}\}$ from the base stations by averaging over the distribution of $\{\kappa_n\}$ to get:

$$\overline{\mathbf{P}}_{\mathrm{out}}(N_{\mathrm{SH}}, d^{(1)},\ldots,d^{(N_{\mathrm{SH}})}) = \left(\frac{1}{1-p_\kappa(0)}\right)^{N_{\mathrm{SH}}} \times \sum_{k_1=1}^{k_{\mathrm{pole}}}\cdots\sum_{k_{N_{\mathrm{SH}}}=1}^{k_{\mathrm{pole}}}\left(\prod_{n=1}^{N_{\mathrm{SH}}} p_\kappa(k_n)\right)\mathbf{P}_{\mathrm{out}}(\{k_n\},\{d^{(n)}\}) \quad (14)$$

It can be argued that for a particular degree of soft handoff, there is a location that has the worst outage probability, and for symmetry reasons, that location will be equidistant from all the BS's currently in soft handoff with the mobile. Denote this distance by $d_{N_{\mathrm{SH}}}$. Given a shadow fading correlation a^2 and a degree of soft handoff N_{SH}, equation (14) can be used to obtain the $d_{N_{\mathrm{SH}}}$ that results in the desired outage probability. That is, using

$$\overline{\mathbf{P}}_{\mathrm{out}}(N_{\mathrm{SH}}, d_{N_{\mathrm{SH}}},\ldots,d_{N_{\mathrm{SH}}}) = p_m, \qquad N_{\mathrm{SH}} = 1,2,3. \quad (15)$$

The various cell radii R_{in}, R_{out} and R_{cov} can now be calculated in terms of $\{d_{N_{\mathrm{SH}}}\}$ as follows.

$$R_{\mathrm{in}} = d_1 \quad (16)$$

From Figure 1 we see that the worst point for 2-way soft handoff occurs at a distance which approximately equals $\frac{\sqrt{3}}{2}R_{\mathrm{out}}$, and the worst point for 3-way soft handoff occurs at a distance which equals R_{out}. Therefore, $R_{\mathrm{out}}^{(N_{\mathrm{SH}}=2)} = \frac{2}{\sqrt{3}}d_2$ and $R_{\mathrm{out}}^{(N_{\mathrm{SH}}=3)} = d_3$. One of these two values will be the limiting factor, i.e.

$$R_{\mathrm{out}} = \min\left(\frac{2}{\sqrt{3}}d_2, d_3\right) \quad (17)$$

Simple geometry then gives us the following

$$R_{\mathrm{cov}} = \sqrt{3}R_{\mathrm{out}} - R_{\mathrm{in}} \quad (18)$$

Given a model for the p.m.f. of the number of users, we can use the above method to obtain a plot of coverage versus capacity. As an example, suppose the κ_n have a Poisson distribution[2] with parameter ξ, truncated at k_{pole}, i.e.,

$$p_\kappa(k) = \frac{\xi^k e^{-\xi}/k!}{\sum_{j=0}^{k_{\mathrm{pole}}}\xi^j e^{-\xi}/j!}, \quad k = 0,1,\ldots,k_{\mathrm{pole}}. \quad (19)$$

Then the average number of users in the cell is given by:

$$c = \xi\left[1 - \frac{\xi^{k_{\mathrm{pole}}}\,e^{-\xi}/k_{\mathrm{pole}}!}{\sum_{j=0}^{k_{\mathrm{pole}}}\xi^j e^{-\xi}/j!}\right]. \quad (20)$$

By solving (15) and (20) for various values of ξ, we can obtain tradeoff curves for R_{in}, R_{out} and R_{cov} versus c that are parameterized by ξ. Examples of this calculation are given in Section 5.

5. Numerical Results

The parameter values used in our numerical results are as follows. W = 1.25 MHz, R = 14.4 kbps, ρ = 0.45, m_ϵ = 7 dB, σ_ϵ = 2.5 dB, K_1 = 17.3 dB, K_2 = 33.8 dB, $S_{\max}$ =

[2] For an admission policy based on interference levels, the p.m.f. is well modeled as Poisson [8].

23 dBm, N_0 = -169 dBm/Hz, and p_m = 0.05, where K_1 and K_2 are obtained using Hata's model (see page 119 of [5]) for a medium sized city with carrier frequency of 900 MHz, transmit antenna height of 50 m, receive antenna height of 1 m, and a net antenna gain of 6 dB.

Without loss of generality, we will assume that the other-cell interference density I is a multiple η of the background thermal noise density N_0. In the following simulation, η = 2 was used.

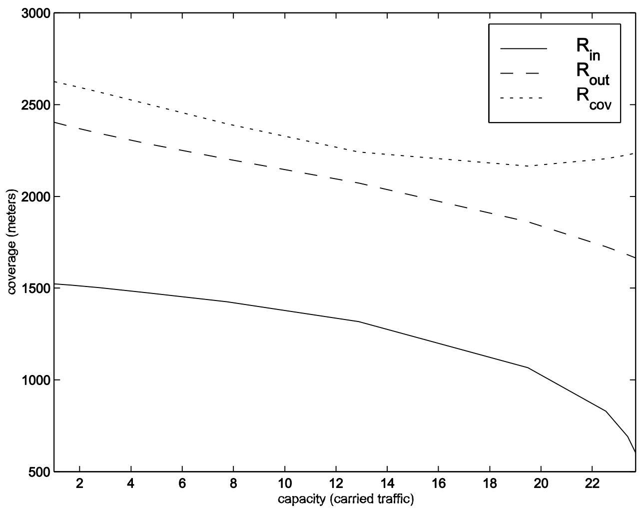

Figure 2. Cell radii versus capacity

Figure 2 shows the three cell radii (R_{in}, R_{out} and R_{cov}) as a function of the capacity of the cell. Notice how the no-soft-handoff radius (R_{in}) approaches zero as the capacity reaches k_{pole}, whereas the outer radii do not. In fact, because of the geometry of the problem, and since R_{in} is going to zero, R_{cov} starts increasing for values of capacity near k_{pole}. This does not mean that the coverage increases near k_{pole} (since R_{out} is indeed decreasing), it only reflects on the fact that the pilot of the BS has to extend further due to the fact that R_{in} is going to zero. Notice also that, as expected, soft handoff results in gains both in capacity and coverage of the cell, and that these gains are illustrated quantitatively in terms of the tradeoff between coverage and capacity.

6. CONCLUSIONS

We have presented a way to determine the tradeoff between coverage and capacity in a cell whose mobiles are allowed to be in soft handoff with more than one BS. This technique may be used to characterize the capacity-coverage tradeoff for an arbitrary admission policy, and thus may be used for cellular planning.

Avenues for further research include extending the capacity-coverage analysis to incorporate the effects of using sectorized cells. Extension to sectorized cells should be straightforward, with the understanding that the sectors in a given cell could, in general, have different capacity-coverage operating points. Also of interest is the incorporation of a soft-handoff gain in the required SIR of each user in soft handoff. This gain is due to the fact that the multipath environments at different BS's vary independently of each other, and thus a lower SIR is required in order to meet probability of error requirements. It is thus expected that the capacity and coverage gains from soft handoff will be greater than what is reported in this paper.

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