A&P

MM10 : Macro Diversity & interference

FP8-25 : Propagation, Antennas & Diversity

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Synopsis

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- Exercises from MM9
- Single MS,BS - outage
- Single MS – multiple interfering BS
- Single MS – multiple co-BS (Macro diversity)
- Possibly recap of previous MM's

Propagation channel elements

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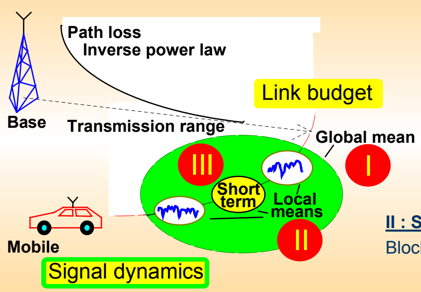
Traditional distinctions for vehicular case. For handsets/nearfield terminals can be near impossible to seperate these effects



I : Path loss
Power decay : global mean, d^{-n}

II : Shadow fading
Blocking : local mean, log-normal

III : Short term fading
'Time sensitive vector interference' : ISI

Systems way of handling shadowing

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- **GSM**
  - Frequency planning and base station location
  - Power control
- **DECT**
  - Base station location
- **IS95 Cellular CDMA**
  - Power control
  - Base station locations
- **Digital Audio Broadcasting**
  - Single frequency networks

Outage - area coverage I

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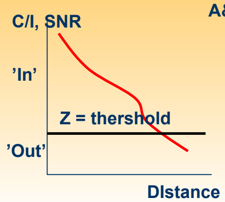
– User power p_0
– Interferer power p_I
– Threshold C/I > z : $p_0 > zp_I$
– Cell radius R_c



$$F_{out} = 1 - F(p_0 > zp_I) = 1 - \int_{0-}^{\infty} f_{pI}(x) \int_{zx}^{\infty} f_{p0}(y)\,dy\,dx$$

$$F_{<out>} = \frac{1}{A}\int_A F_{out}(d)\,dA = \frac{2}{R_c^2}\int_0^{Rc} F_{out}(d)\cdot d \cdot dd$$

Outage - area coverage II

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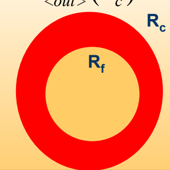
Coverage

– Short term or **long term** fading
– Power law path loss : <snr> = $1/(\kappa\cdot d^{n=4})$
– r_f fraction radius

$$F_{<out>}(R_c,n,\sigma) = F_{<out>}(R_c) - ½\exp\left(2ab+b^2\right)\cdot erfc(a+b)$$

$$a = \frac{z(R_c)}{\sigma\sqrt{2}} \quad b = \frac{\sigma\sqrt{2}\cdot\ln(10)}{10n} \quad F_{<out>}(R_c) = ½ erfc(a)$$

$$z(r_f) = z(R_c) - \frac{10n}{2}\log_{10}(f)$$



Outage - area coverage III

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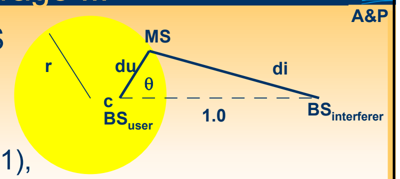
Single interferer BS

BS<-> BS = 1.0

k=di/du

r=k/(k²-1), c=1 /(k²-1),



$$du = \frac{-\cos(\theta) + \sqrt{k^2 - \sin^2(\theta)}}{k^2 - 1}$$

$$F_{<out>} = \frac{2\left(k^2-1\right)^2}{\pi k^2}\int_0^{\pi}\int_0^{dd}\frac{r}{2} erfc\left(\frac{z(r,\theta)}{2\sigma}\right) dr\,d\theta$$

$$z(r,\theta) = 10n\log_{10}\left(\frac{\sqrt{r^2+1-2r\cos(\theta)}}{r}\right) - 10\log_{10}(z_{instant})$$

Outage - area coverage IV

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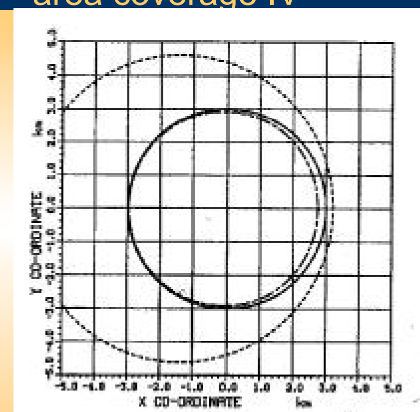

10% outage contours, BS_i =13.75km E of BS_{user}
- : coverage, - - : interference, -.- : cov+inteference

Z(C/I) = 6dB
σ = 6dB

Multiple log-normal interferers I

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- many co-channel transmitters
- joint interference power accumulated from several log-normal signals.
  - Non coherent sum $L=\sum L_k=\sum 10^{pk[dB]/10} \approx 10^{pI[dB]/10} = L\hat{}$, i.e. produce approx Log-normal
  - Fenton (most accurate σ<4dB, ok tails of pdf)
  - Schwartz and Yeh (most accurate 4<σ<12dB)
  - If including short term –> dominates = easy without explicitly dealing with joint log-normals

Multiple log-normal interferers II

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- Fenton : μ,σ^2 moment matching (L vs L^), assume identical variance on k interferers

$$\mu_{pI} = \frac{\sigma_{pk}^2 - \sigma_{pI}^2}{2} + \ln\left(\sum_{k=1}^{n} \exp(\mu_{pk})\right)$$

$$\sigma_{pI}^2 = \ln\left(\frac{(\exp(\sigma_{pk}^2)-1)\sum_{k=1}^{n}\exp(2\mu_{pk})}{\left(\sum_{k=1}^{n}\exp(\mu_{pk})\right)^2} + 1\right)$$

- Schwarts & Yeh : exact expression for two 1st moments in sum of two log-normal variable -> more accurate +**complicated**

Short term I :Power (budget parameter)

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- Rayleigh amplitude -> power ($p=r^2$) exponential = Chi-squared X_2^2 (I^2+Q^2)

$$f_p(p) = f_r(r)\left|\frac{dr}{dp}\right| = \frac{2r}{\bar{p}}\exp\left\{-\frac{r^2}{\bar{p}}\right\}\frac{1}{2\sqrt{p}} = \frac{1}{\bar{p}}\exp\left\{-\frac{p}{\bar{p}}\right\}$$

$$\bar{p} = E[r^2] = 2\sigma^2 \quad p = r^2$$

- Exponentials easy to handle mathematically
- Example : Calc. the channel avg. Power behavioir – basically Laplace transform

Linnartz : http://www.wireless.per.nl:202/telelearn/laplace

Short term II : Successfull transmission

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- If C/I large enough
  - User power p_0
  - Interferer power p_I
  - Threshold C/I > z : $p_0 > zp_I$

$$F(p_0 > zp_I) = \int_{0-}^{\infty} f_{pI}(x) \int_{zx}^{\infty} f_{p0}(y)\,dy\,dx$$

$$= \int_{0-}^{\infty} f_{pI}(x)\exp\left(\frac{zx}{p_0}\right)dx$$

← **RAYLEIGH FADING**

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Short term III : Propability of success

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- Rayleigh -> Framework : result is $\mathcal{L}$[pdf(p_0,p_I)]

$$F\left(p_0 > z p_I \mid \overline{p}_0\right) = L\left\{ f_{pI}(x); s = \frac{z}{\overline{p}_0} \right\}$$

- L(f,s) is one side Laplace transform of f at point s
- Interference power = ∑individual interferers
- pdf interference = ⊗ individual pdf's

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Cellular : Outage propability

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- User & interferers : Shadowing + fading

$$L\{f_{pn}(x);s\} = \frac{1}{\sqrt{\pi}} \int_{-\infty}^{\infty} \frac{\exp\left(-x^2\right)}{\left(1+\frac{s}{m}\overline{\overline{q}}\exp\left(\sqrt{2}x\sigma\right)\right)^m} dx$$

- Joint interference pI=p1+p2+p3...pn

$$L\{f_{pI}(x);s\} = \prod_{p1..pn} \left(1 - F(out) + F(out) L\{f_{pn}(x);s\}\right)$$

- Outage propability

Up-link (UL) vs Down-link (DL)

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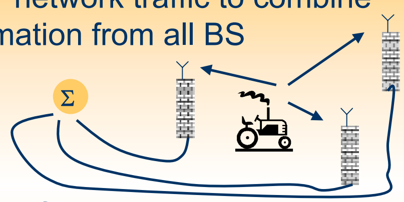
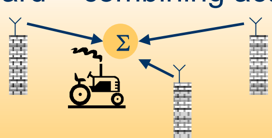
- UL -> network traffic to combine information from all BS



- DL -> BS layout = Diversity structure , but MS/system standard = combining action



Cross-correlation vs auto-correlation

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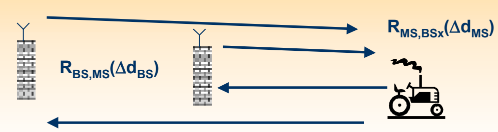
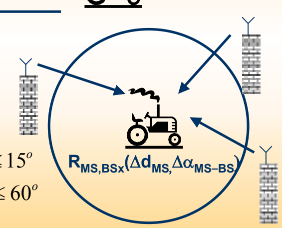
- Radial



- Circumfencial



$$\rho_{BS1,BS2}(\Delta\alpha) = \begin{cases} 0.78 - 0.0056\Delta\alpha & 0^o \le \Delta\alpha \le 15^o \\ 0.48 - 0.0056\Delta\alpha & 15^o \le \Delta\alpha \le 60^o \\ 0 & otherwise \end{cases}$$

Correlated shadowing, 1 interferer

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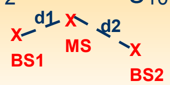
- $P_R = P_T + S - 10n \cdot \log_{10}(d) + \beta$
- DL C/I : $R = P_1 - P_2 = S_1 - S_2 + 10n \cdot \log_{10}(d_2/d_1)$
- $\rho_{12} = E[S_1 S_2]/\sigma_1\sigma_2$



$$\mu_R = E[R] = E[S_1 - S_2] + 10n\log_{10}(d_2/d_1) = 10n\log_{10}(d_2/d_1)$$
$$\sigma_R^2 = E[R^2] - \mu_R^2 = E[S_1^2] + E[S_2^2] - 2\rho_{12}\sigma_1\sigma_2 = \sigma_1^2 + \sigma_2^2 - 2\rho_{12}\sigma_1\sigma_2$$

- New Normal distribution

$$F(R<z) = 1 - erfc\left(\frac{z-\mu_R}{\sigma_R}\right) = erf\left(\frac{z-\mu_R}{\sigma_L\sqrt{2(1-\rho_{12})}}\right)\Bigg|_{\sigma 1=\sigma 2=\sigma L}$$

Correlated shadowing, C/I distribution

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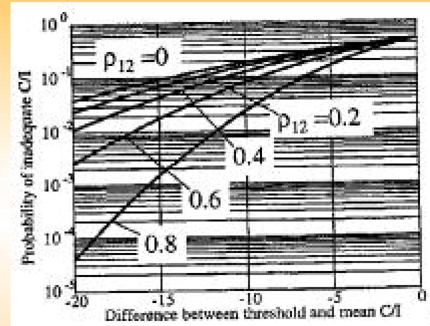

Outage propability with single interferer. Note 7dB for ρ=0->0.8 @ 10% level
With n = 4, equivalently reuse distance d2 -> +50% for same outage
I.e. Shadowing correlation decisive effect on cell design

'Empirical' vs clutter variation

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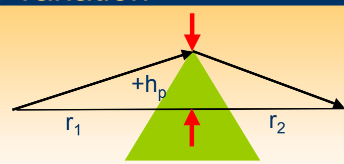
- Diffraction Loss

$$L = 20\log(F)$$
$$F = \frac{S + ½}{\sqrt{2}\sin(\Delta\phi + \pi/4)}$$
$$\Delta\phi = \tan^{-1}\left(\frac{S+½}{C+½}\right) - \frac{\pi}{4}$$
$$C, S = \int_0^{\upsilon} \cos,\sin(\pi/2\, x^2)dx$$
$$\upsilon = -h_p\sqrt{\frac{2}{\lambda}\left(\frac{1}{r_1} + \frac{1}{r_2}\right)}$$



Obstruction -> h_p>0 -> υ<0 -> 0 ≤ F≤ ½. Approx.

L=0dB	1 ≤ υ
L=20log(½+0.62υ)dB	0 ≤ υ ≤ 1
L=20**log**(½**exp**(0.95υ))dB	-1 ≤ υ ≤ 0
L=20log(0.4-√(0.118-(0.1υ+0.38)²))	-2.4 ≤ υ ≤ -1
....	

h_p =N(h,σ^2) -> υ= N(m_υ,σ_υ^2)

$L|_{-1\le\upsilon\le 0} \approx -6 + 20 \cdot 0.95\,\upsilon/LN(10)$ dB

λ, r_1, r_2 fixed (along road), obstruction h_p normal distributed

Multiplicative effects

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Shadow loss L(x)

L(R) = $\overline{L(x)}$ Global loss

Central limit theorem

$$S_n = \sum s_n;\quad s_n : \mu, \sigma^2$$
$$\Downarrow$$
$$\lim_{n\to\infty} P(S_n) = \frac{1}{\sigma_n\sqrt{2\pi}} e^{-\frac{(u-\mu_n)^2}{2\sigma_n^2}}$$
$$\mu_n = n\cdot\mu, \sigma_n = \sigma\sqrt{n}$$

$$L(x) = L_1(x)\cdot L_2(x)\cdot L_3(x)\cdot L_4(x)\cdot\ldots\ldots L_N(x) = \prod_1^N L_n(x)$$

$$\log(L) = \log(L_1) + \log(L_2) + \log(L_3) + \ldots\log(L_N) = \sum_1^N \log(L_n)$$

Normal (Gaussian) probability density function (pdf) $f(u) = N(\mu, \sigma^2)$

Error function, cumulative distribution function (cdf) $F(u) = \int_{-\infty}^{u} f(x)dx = erf(u) = Q(u)$

Log(L) is asymptotically normal distributed. Approx. n=>8 sufficient

Correlated shadowing, model

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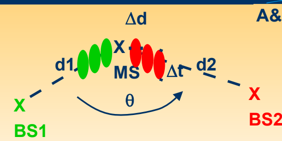
- Simple, physically based
- Good measurement fit
- N=[d/Δd] independent multiplicative sections



$$S = \sum_{n=1}^{N} s\left(\left(n-\tfrac{1}{2}\right)\Delta d\right) \qquad \sigma_S^2 = E\left[S^2\right] = \sum_{n=1}^{N} \sigma_e^2\left(\left(n-\tfrac{1}{2}\right)\Delta d\right)$$

$\sigma^2_e(d)$: variance of element at d -> model
Most effect close to MS

$$Nc = \left[\frac{\Delta t}{\theta \Delta d}\right]\Bigg|_{\theta \geq \Delta t / d_1} \qquad Nc = \left[\frac{d_1}{\Delta d}\right]\Bigg|_{0<\theta<\Delta t / d_1}$$

Assume : elemenst overlap dθ<Δt
-> equal effect on signals

$$S_x = \sum_{n=1}^{Nc} s\left(\left(n-\tfrac{1}{2}\right)\Delta d\right) + \sum_{n=Nc+1}^{Nx} s_x\left(\left(n-\tfrac{1}{2}\right)\Delta d\right)$$

Diffraction effect [dB]
$\Delta P_{dB} \approx \Delta h \; dP/dh \; dP_{dB}/dP$
-> $(\Delta P_{dB})^2 = \sigma^2_e(d)$

$$\rho_{12} = \frac{E\left[S_1 S_2\right]}{\sigma_1 \sigma_2} = \frac{\sum_{n=1}^{Nc} \Delta P_{dB}^2\left(\left(n-\tfrac{1}{2}\right)\Delta d\right)}{\sqrt{\left(\sum_{l=1}^{N1} \Delta P_{dB}^2\left(\left(n-\tfrac{1}{2}\right)\Delta d\right)\right)\cdot\left(\sum_{m=1}^{N2} \Delta P_{dB}^2\left(\left(n-\tfrac{1}{2}\right)\Delta d\right)\right)}}$$

Classic Diversity combiners

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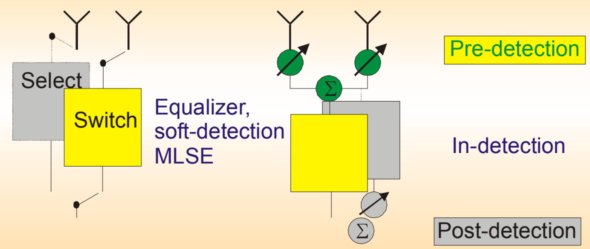
Two major principles : Selection (single branch) or summation (multi branch)



- Noise -> RSSI criteria
- Interference -> RSSI, ISI (CRC) criteria
- Time dispersion -> ISI (CRC) criteria

Macro Diversity combining

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- WB – CDMA – Rake receiver
- Simulcast
- General (NB) – selection, log-normal

$$F\left(p[dB] \leq z\right) = \prod_{k=1}^{n} F\left(p_k \leq z\right)$$

$$= \prod_{k=1}^{n}\left(\tfrac{1}{2} + \tfrac{1}{2} erf\left(\frac{z-\mu_k}{\sigma_k \sqrt{2}}\right)\right)$$

- For either multiple co-BS or interferer BS