

# Hojas de fórmulas para 1<sup>er</sup> Parcial

(2013)

<b>Transformada de Laplace</b> $F(s)$	<b>Función en el tiempo</b> $f(t)$
$1$	Impulso unitario
$\frac{1}{s}$	Escalón unitario
$\frac{1}{s^2}$	$t$
$\frac{n!}{s^{n+1}}$	$t^n$ ( $n =$ entero positivo)
$\frac{1}{s+a}$	$e^{-at}$
$\frac{1}{(s+a)(s+b)}$	$\frac{e^{-at} - e^{-bt}}{b-a}$
$\frac{\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2}$	$\frac{\omega_n}{\sqrt{1-\zeta^2}} e^{-\zeta\omega_n t} \text{sen}\left(\omega_n \sqrt{1-\zeta^2} t\right)$ $0 < \zeta < 1$
$\frac{s}{s^2 + 2\zeta\omega_n s + \omega_n^2}$	$-\frac{1}{\sqrt{1-\zeta^2}} e^{-\zeta\omega_n t} \text{sen}\left(\omega_n \sqrt{1-\zeta^2} t - \text{Arctg}\left(\frac{\sqrt{1-\zeta^2}}{\zeta}\right)\right)$
$\frac{1}{(1+Ts)^n}$	$\frac{1}{T^n (n-1)!} t^{n-1} e^{-t/T}$
$\frac{\omega_n^2}{(1+Ts)(s^2 + 2\zeta\omega_n s + \omega_n^2)}$	$\frac{T\omega_n^2 e^{-t/T}}{1-2\zeta T\omega_n + T^2\omega_n^2} + \frac{\omega_n e^{-\zeta\omega_n t} \text{sen}\left(\omega_n \sqrt{1-\zeta^2} t - \text{Arctg}\left(\frac{T\omega_n \cdot \sqrt{1-\zeta^2}}{1-T\zeta\omega_n}\right)\right)}{\sqrt{(1-\zeta^2)(1-2\zeta T\omega_n + T^2\omega_n^2)}}$
$\frac{\omega_n}{s^2 + \omega_n^2}$	$\text{sen}(\omega_n t)$
$\frac{\beta}{(s+\alpha)^2 + \beta^2}$	$e^{-\alpha t} \text{sen}(\beta t)$
$\frac{s+\alpha}{(s+\alpha)^2 + \beta^2}$	$e^{-\alpha t} \text{cos}(\beta t)$
$\frac{\omega_n}{(1+Ts)(s^2 + \omega_n^2)}$	$\frac{T\omega_n e^{-t/T}}{1+T^2\omega_n^2} + \frac{\text{sen}(\omega_n t - \text{Arctg}(T\omega_n))}{\sqrt{1+T^2\omega_n^2}}$
$\frac{\omega_n^2}{s(s^2 + 2\zeta\omega_n s + \omega_n^2)}$	$1 - \frac{1}{\sqrt{1-\zeta^2}} e^{-\zeta\omega_n t} \text{sen}\left(\omega_n \sqrt{1-\zeta^2} t + \text{Arctg}\left(\frac{\sqrt{1-\zeta^2}}{\zeta}\right)\right)$
$\frac{\omega_n^2}{s(s^2 + \omega_n^2)}$	$1 - \text{cos}(\omega_n t)$
$\frac{1}{s(1+Ts)}$	$1 - e^{-t/T}$

Transformada de Laplace $F(s)$	Función en el tiempo $f(t)$
$\frac{1}{s(1+Ts)^2}$	$1 - \frac{t+T}{T} e^{-t/T}$
$\frac{\omega_n^2}{s(1+Ts)(s^2 + 2\zeta\omega_n s + \omega_n^2)}$	$1 - \frac{T^2\omega_n^2 e^{-t/T}}{1 - 2T\zeta\omega_n + T^2\omega_n^2} + \frac{e^{-\zeta\omega_n t} \text{sen}(\omega_n \sqrt{1-\zeta^2} t - \Phi)}{\sqrt{(1-\zeta^2)(1-2\zeta T\omega_n + T^2\omega_n^2)}}$  <i>donde</i> $\Phi = \tan^{-1}\left(\frac{\sqrt{1-\zeta^2}}{-\zeta}\right) + \tan^{-1}\left(\frac{T\omega_n \sqrt{1-\zeta^2}}{1-T\zeta\omega_n}\right)$
$\frac{\omega_n^2}{s^2(s^2 + 2\zeta\omega_n s + \omega_n^2)}$	$t - \frac{2\zeta}{\omega_n} + \frac{1}{\omega_n \sqrt{1-\zeta^2}} e^{-\zeta\omega_n t} \text{sen}\left(\omega_n \sqrt{1-\zeta^2} t - 2 \cdot \text{Arctg}\left(\frac{\sqrt{1-\zeta^2}}{-\zeta}\right)\right)$
$\frac{\omega_n^2}{s^2(1+Ts)(s^2 + 2\zeta\omega_n s + \omega_n^2)}$	$t - T - \frac{2\zeta}{\omega_n} + \frac{T^3\omega_n^2 e^{-t/T}}{1 - 2T\zeta\omega_n + T^2\omega_n^2} + \frac{e^{-\zeta\omega_n t} \text{sen}(\omega_n \sqrt{1-\zeta^2} t - \Phi)}{\omega_n \sqrt{(1-\zeta^2)(1-2\zeta T\omega_n + T^2\omega_n^2)}}$  <i>donde</i> $\Phi = 2 \tan^{-1}\left(\frac{\sqrt{1-\zeta^2}}{-\zeta}\right) + \tan^{-1}\left(\frac{T\omega_n \sqrt{1-\zeta^2}}{1-T\zeta\omega_n}\right)$
$\frac{1}{s^2(1+Ts)^2}$	$t - 2T + (t + 2T)e^{-t/T}$
$\frac{\omega_n^2(1+as)}{s^2 + 2\zeta\omega_n s + \omega_n^2}$	$\omega_n \sqrt{\frac{1+2a\zeta\omega_n + a^2\omega_n^2}{1-\zeta^2}} e^{-\zeta\omega_n t} \text{sen}\left(\omega_n \sqrt{1-\zeta^2} t + \text{Arctg}\left(\frac{a\omega_n \sqrt{1-\zeta^2}}{1-a\zeta\omega_n}\right)\right)$
$\frac{\omega_n^2(1+as)}{s^2 + \omega_n^2}$	$\omega_n \sqrt{1+a^2\omega_n^2} \text{sen}(\omega_n t + \text{Arctg}(a\omega_n))$
$\frac{\omega_n^2(1+as)}{(1+Ts)(s^2 + 2\zeta\omega_n s + \omega_n^2)}$	$\frac{\omega_n \sqrt{(1-2\zeta a\omega_n + a^2\omega_n^2)} e^{-\zeta\omega_n t} \text{sen}(\omega_n \sqrt{1-\zeta^2} t + \Phi)}{\sqrt{(1-\zeta^2)(1-2\zeta T\omega_n + T^2\omega_n^2)}}$  $+ \frac{(T-a)\omega_n^2 e^{-t/T}}{1-2T\zeta\omega_n + T^2\omega_n^2}$  <i>donde</i> $\Phi = \tan^{-1}\left(\frac{a\omega_n \sqrt{1-\zeta^2}}{1-a\zeta\omega_n}\right) - \tan^{-1}\left(\frac{T\omega_n \sqrt{1-\zeta^2}}{1-T\zeta\omega_n}\right)$
$\frac{\omega_n^2(1+as)}{(1+Ts)(s^2 + \omega_n^2)}$	$\frac{\omega_n^2(T-a)}{1+T^2\omega_n^2} e^{-t/T} + \frac{\omega_n \sqrt{1+a^2\omega_n^2}}{\sqrt{1+T^2\omega_n^2}} \text{sen}(\omega_n t + \Phi)$  <i>donde</i> $\Phi = \tan^{-1}(a\omega_n) - \tan^{-1}(T\omega_n)$
$\frac{\omega_n^2(1+as)}{s(s^2 + 2\zeta\omega_n s + \omega_n^2)}$	$1 + \frac{\sqrt{(1-2\zeta a\omega_n + a^2\omega_n^2)}}{\sqrt{(1-\zeta^2)}} e^{-\zeta\omega_n t} \text{sen}(\omega_n \sqrt{1-\zeta^2} t + \Phi)$  <i>donde</i> $\Phi = \tan^{-1}\left(\frac{a\omega_n \sqrt{1-\zeta^2}}{1-a\zeta\omega_n}\right) - \tan^{-1}\left(\frac{\sqrt{1-\zeta^2}}{-\zeta}\right)$

<b>Transformada de Laplace</b>	<b>Función en el tiempo</b>
$F(s)$	$f(t)$
$\frac{\omega_n^2(1+as)}{s(1+Ts)(s^2+\omega_n^2)}$	$1 + \frac{T\omega_n^2(a-T)}{1+T^2\omega_n^2} e^{-t/T} - \frac{\sqrt{1+a^2\omega_n^2}}{\sqrt{1+T^2\omega_n^2}} \cos(\omega_n t + \Phi)$ donde $\Phi = \tan^{-1}(a\omega_n) - \tan^{-1}(T\omega_n)$
$\frac{\omega_n^2(1+as)}{s(1+Ts)(s^2+2\zeta\omega_n s+\omega_n^2)}$	$1 + \frac{\sqrt{(1-2\zeta a\omega_n+a^2\omega_n^2)}}{\sqrt{(1-\zeta^2)(1-2\zeta T\omega_n+T^2\omega_n^2)}} e^{-\zeta\omega_n t} \text{sen}(\omega_n \sqrt{1-\zeta^2} t + \Phi)$ $+ \frac{T(a-T)\omega_n^2 e^{-t/T}}{1-2T\zeta\omega_n+T^2\omega_n^2}$ donde $\Phi = \tan^{-1}\left(\frac{a\omega_n\sqrt{1-\zeta^2}}{1-a\zeta\omega_n}\right) - \tan^{-1}\left(\frac{T\omega_n\sqrt{1-\zeta^2}}{1-T\zeta\omega_n}\right) - \tan^{-1}\left(\frac{\sqrt{1-\zeta^2}}{-\zeta}\right)$
$\frac{1+as}{s^2(1+Ts)}$	$t + (a-T)(1-e^{-t/T})$
$\frac{s}{s^2+\omega_n^2}$	$\cos(\omega_n t)$
$\frac{s}{(s^2+\omega_n^2)^2}$	$\frac{1}{2\omega_n} t \text{sen}(\omega_n t)$
$\frac{s}{(s^2+\omega_{n1}^2)(s^2+\omega_{n2}^2)}$	$\frac{1}{\omega_{n1}^2-\omega_{n2}^2} (\cos(\omega_{n1} t) - \cos(\omega_{n2} t))$
$\frac{s}{(1+Ts)(s^2+\omega_n^2)}$	$\frac{-1}{1+T^2\omega_n^2} e^{-t/T} + \frac{1}{\sqrt{1+T^2\omega_n^2}} \cos(\omega_n t - \text{Arctg}(T\omega_n))$
$\frac{1+as+bs^2}{s^2(1+T_1s)(1+T_2s)}$	$t + (a-T_1-T_2) + \frac{b-aT_1+T_1^2}{T_1-T_2} e^{-t/T_1} - \frac{b-aT_2+T_2^2}{T_1-T_2} e^{-t/T_2}$
$\frac{\omega_n^2(1+as+bs^2)}{s(s^2+2\zeta\omega_n s+\omega_n^2)}$	$1 + \frac{\sqrt{(1-a\zeta\omega_n-b\omega_n^2+2b\zeta^2\omega_n^2)^2+\omega_n^2(1-\zeta^2)(a-2b\zeta\omega_n)^2}}{\sqrt{(1-\zeta^2)}} x e^{-\zeta\omega_n t} \text{sen}(\omega_n \sqrt{1-\zeta^2} t + \Phi)$ donde $\Phi = \tan^{-1}\left(\frac{\omega_n\sqrt{1-\zeta^2}(a-2b\zeta\omega_n)}{b\omega_n^2(2\zeta^2-1)+1-a\zeta\omega_n}\right) - \tan^{-1}\left(\frac{\sqrt{1-\zeta^2}}{-\zeta}\right)$
$\frac{s^2}{(s^2+\omega_n^2)^2}$	$\frac{1}{2\omega_n} (\text{sen}(\omega_n t) + \omega_n t \cos(\omega_n t))$

### Respuesta al escalón de sistemas de 2º orden sin ceros

Tiempo de levantamiento 10%-90% :

$$t_R = \frac{0,366.e^{2-\zeta} + 0,6536}{\omega_n} \quad 0 < \zeta \leq 1$$

$$t_R = \frac{2 \cdot \ln(9) \cdot \zeta - 1,0364 / \zeta}{\omega_n} \quad 1 < \zeta$$

Tiempo de establecimiento n% :

$$t_s = \frac{-\ln\left(\frac{n\%}{100} \sqrt{1-\zeta^2}\right)}{\zeta \omega_n}; \quad t_s^{5\%} = \frac{3}{\zeta \omega_n} \quad 0 < \zeta < 0,6; \quad t_s^{2\%} = \frac{4}{\zeta \omega_n} \quad 0 < \zeta < 0,8$$

Máximo sobretiro y tiempo de máximo sobretiro:

$$M_p(\%) = 100e^{-\frac{\pi\zeta}{\sqrt{1-\zeta^2}}} \quad \zeta^2 = \frac{\left[\ln\left(\frac{M_p(\%)}{100}\right)\right]^2}{\pi^2 + \left[\ln\left(\frac{M_p(\%)}{100}\right)\right]^2} \quad t_p = \frac{\pi}{\omega_n \sqrt{1-\zeta^2}}$$

# Transformadas Z

$\{e(k)\}$	$E(z)$
{1}	$\frac{z}{z-1}$
{k}	$\frac{z}{(z-1)^2}$
{k <sup>2</sup> }	$\frac{z(z+1)}{(z-1)^3}$
{a <sup>k</sup> }	$\frac{z}{z-a}$
{ka <sup>k</sup> }	$\frac{az}{(z-a)^2}$
{sin ak}	$\frac{z \sin a}{z^2 - 2z \cos a + 1}$
{cos ak}	$\frac{z(z - \cos a)}{z^2 - 2z \cos a + 1}$
{a <sup>k</sup> sin bk}	$\frac{az \sin b}{z^2 - 2az \cos b + a^2}$
{a <sup>k</sup> cos bk}	$\frac{z^2 - az \cos b}{z^2 - 2az \cos b + a^2}$

**TABLE 2-3 TABLE OF COMMONLY USED z-TRANSFORMS**

Laplace transform, $E(s)$	Time function, $e(t)$	z-transform, $E(z)$
$\frac{1}{s}$	$u(t)$	$\frac{z}{z-1}$
$\frac{1}{s^2}$	$t$	$\frac{Tz}{(z-1)^2}$
$\frac{1}{s+a}$	$e^{-at}$	$\frac{z}{z - e^{-aT}}$
$\frac{a}{s(s+a)}$	$1 - e^{-at}$	$\frac{z(1 - e^{-aT})}{(z-1)(z - e^{-aT})}$
$\frac{1}{(s+a)^2}$	$te^{-at}$	$\frac{Tze^{-aT}}{(z - e^{-aT})^2}$
$\frac{a}{s^2(s+a)}$	$t - \frac{1 - e^{-at}}{a}$	$\frac{Tz}{(z-1)^2} - \frac{(1 - e^{-aT})z}{a(z-1)(z - e^{-aT})}$
$\frac{a}{s^2 + a^2}$	$\sin(at)$	$\frac{z \sin(aT)}{z^2 - 2z \cos(aT) + 1}$
$\frac{s}{s^2 + a^2}$	$\cos(at)$	$\frac{z(z - \cos(aT))}{z^2 - 2z \cos aT + 1}$
$\frac{1}{(s+a)^2 + b^2}$	$\frac{1}{b} e^{-at} \sin bt$	$\frac{1}{b} \left[ \frac{ze^{-aT} \sin bT}{z^2 - 2ze^{-aT} \cos(bT) + e^{-2aT}} \right]$
$\frac{s+a}{(s+a)^2 + b^2}$	$e^{-at} \cos bt$	$\frac{z^2 - ze^{-aT} \cos bT}{z^2 - 2ze^{-aT} \cos bT + e^{-2aT}}$

**TABLA 3.1** Muestreo de un sistema continuo  $G(s)$ .

La tabla da el equivalente con mantenedor de orden cero del sistema continuo,  $G(s)$ , precedido por el mantenedor de orden cero. El sistema muestreado se describe por el operador de transferencia discreta. Para sistemas de segundo orden el operador de transferencia discreta viene dado en términos de los coeficientes de los

$$H(q) = \frac{b_1 q + b_2}{q^2 + a_1 q + a_2}$$

$G(s)$	$H(q)$ o los coeficientes de $H(q)$
$\frac{1}{s}$	$\frac{h}{q-1}$
$\frac{1}{s^2}$	$\frac{h^2(q+1)}{2(q-1)^2}$
$e^{-sh}$	$q^{-1}$
$\frac{a}{s+a}$	$\frac{1 - \exp(-ah)}{q - \exp(-ah)}$
$\frac{a}{s(s+a)}$	$b_1 = \frac{1}{a}(ah - 1 + e^{-ah})$ $b_2 = \frac{1}{a}(1 - e^{-ah} - ahe^{-ah})$ $a_1 = -(1 + e^{-ah})$ $a_2 = e^{-ah}$
$\frac{a^2}{(s+a)^2}$	$b_1 = 1 - e^{-ah}(1 + ah)$ $b_2 = e^{-ah}(e^{-ah} + ah - 1)$ $a_1 = -2e^{-ah}$ $a_2 = e^{-2ah}$
$\frac{ab}{(s+a)(s+b)}$	$b_1 = \frac{b(1 - e^{-ah}) - a(1 - e^{-bh})}{b-a}$ $b_2 = \frac{a(1 - e^{-bh})e^{-ah} - b(1 - e^{-ah})e^{-bh}}{b-a}$ $a_1 = -(e^{-ah} + e^{-bh})$ $a_2 = e^{-(a+b)h}$
$\frac{(s+c)}{(s+a)(s+b)}$	$b_1 = \frac{e^{-bh} - e^{-ah} + (1 - e^{-bh})c/b - (1 - e^{-ah})c/a}{b-a}$ $b_2 = \frac{c}{ab}e^{-(a+b)h} + \frac{b-c}{b(a-b)}e^{-ah} + \frac{c-a}{a(a-b)}e^{-bh}$ $a_1 = -e^{-ah} - e^{-bh}$ $a_2 = e^{-(a+b)h}$
$\frac{\omega_0^2}{s^2 + 2\zeta\omega_0 s + \omega_0^2}$	$b_1 = 1 - \alpha\left(\beta + \frac{\zeta\omega_0}{\omega}\gamma\right)$ $\omega = \omega_0\sqrt{1-\zeta^2}$ $\zeta < 1$ $b_2 = \alpha^2 + \alpha\left(\frac{\zeta\omega_0}{\omega}\gamma - \beta\right)$ $\alpha = e^{-\zeta\omega_0 h}$ $a_1 = -2\alpha\beta$ $\beta = \cos(\omega h)$ $a_2 = \alpha^2$ $\gamma = \sin(\omega h)$
$\frac{s}{s^2 + 2\zeta\omega_0 s + \omega_0^2}$	$b_1 = \frac{1}{\omega}e^{-\zeta\omega_0 h} \sin(\omega h)$ $b_2 = -b_1$ $\omega = \omega_0\sqrt{1-\zeta^2}$ $a_1 = -2e^{-\zeta\omega_0 h} \cos(\omega h)$ $a_2 = e^{-2\zeta\omega_0 h}$

$q$  = variable compleja ( $z$ )  
 $h$  = período de muestreo (T)