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CORRELATION EQUATION FOR HOURLY DIFFUSE RADIATION ON A HORIZONTAL SURFACE

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Abstract—This paper presents an analysis of hourly diffuse radiation on a horizontal surface and recommends an equation to determine the hourly ratio of diffuse-to-total radiation received in a horizontal surface. The results of the new correlation equation are compared with earlier equations with recommendations made as to its use with solar energy computer simulation programs.

INTRODUCTION

In simulating solar systems on a digital computer, designers generally use a one-hour time step, this being the minimum interval over which radiation and meteorological data are available. For purposes of calculating the diffuse solar radiation, the correlation developed by Lui and Jordan[1], has been used extensively, even though the correlations is: (i) based on daily values rather than hourly values; and (ii) based on only one location, namely Blue Hill, Mass. Recently Ruth and Chant[2], by analysis of radiation data for four Canadian cities (including Toronto) have shown significant departures from the Liu and Jordan correlation due to change in location. Their study was also based upon daily values. Moreover, a recommended correlation equation was not given.

The present paper presents a new correlation equation for diffuse radiation, based on Toronto data, but based upon hourly, measured values rather than daily totals.

METHOD OF CORRELATION

Environment Canada has made available magnetic tapes containing measured values of both total and diffuse radiation on a horizontal surface at Toronto Airport, over the period Sept. 1967–August 1971. From these data, the ratio, K_T, of hourly total (beam plus diffuse) radiation on a horizontal surface, H_T, to the extra-terrestial radiation on a horizontal surface during the same hour, H_0, were calculated for each daylight hour. Similarly, for each hour, the ratio, K, of diffuse to total radiation on a horizontal surface (H_d/H_T) was also calculated. Then, for each interval in K_T of 0.05, the corresponding values of K were averaged, and these average values plotted against the value of K_T for the mid-point of that interval. Due to instrument error, values of H_T less than 2.0 langleys were not considered in the calculations. In all 12,704 hourly periods were used.

The instantaneous extraterrestial solar irradiation on a horizontal surface, $\dot{H}_0$ is given by

$$\dot{H}_0 = S_c \cdot E \cdot [\cos l \cos d \cos h + \sin l \sin d]. \quad (1)$$

Since the measured terrestial values of solar irradiation are hourly integrated values, the calculation of K_T also requires an hourly integrated value of $\dot{H}_0$, denoted by H_0. H_0 was therefore obtained by integrating eqn (1) between limits h_1 and h_2

$$H_0 = 3600 \frac{12 \cdot S_c \cdot E}{\pi} \int_{h_1}^{h_2} [\cos l \cos d \cos h + \sin l \sin d] dh. \quad (2)$$

Ordinarily $h_2 - h_1 = \pi/12$ (one hr). However, if h_1 was less than the calculated sunrise hour angle, it was set equal to the sunrise hour angle; similarly if h_2 was greater than the sunset hour angle, it was set equal to the sunset hour angle.

RESULTS

Figure 1 shows the plot of K vs K_T obtained. For any interval in K_T the range in observed K values was very wide. Figure 2 shows the maximum and minimum values of K recorded for each interval. Clearly K is dependent upon meteorological parameters other than K_T. For obvious reasons some intervals in K_T contained more K

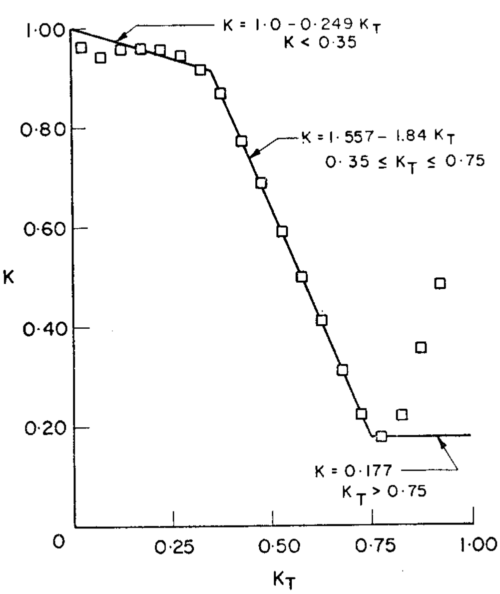

Fig. 1. Plot of K vs K_T, showing recommended correlation equation.

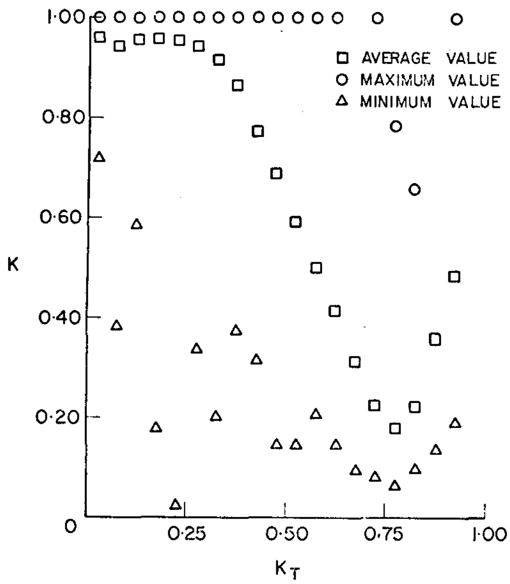

Fig. 2. Plot showing range of observed K values.

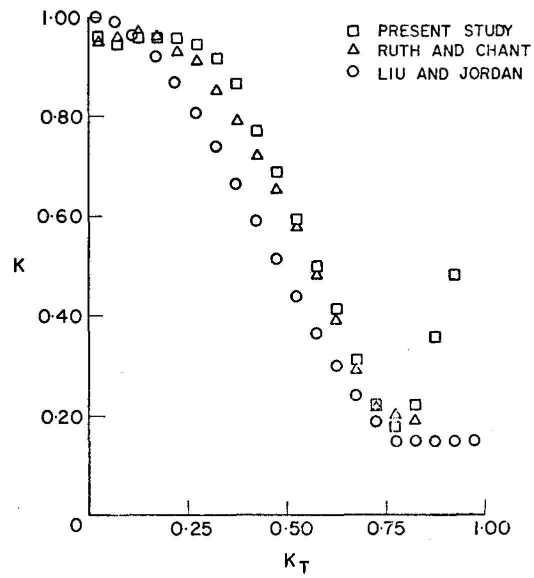

Fig. 4. Comparison of present correlation with that of Jordan and Liu, and of Ruth and Chant.

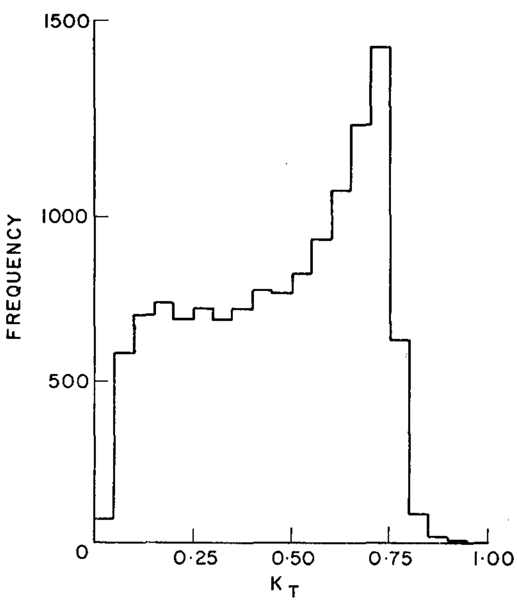

Fig. 3. Histogram of observed K values.

values than others; Fig. 3 shows a frequency histogram of the data.

The above correlation was repeated, using as an approximation to H_0, the value of $\dot{H}_0$ calculated at the mid-point of the hour. No significant departure in the correlation shown in Fig. 1 was observed.

Figure 4 shows a comparison of the results of the present study with the results of Ruth and Chant *for Toronto only*, and with the Liu and Jordan correlation. It should be underlined that in both of these latter studies the values of K and K_T were calculated from daily values rather than hourly values of radiation. A small but significant difference between the present study and that of Ruth and Chant is observed, indicating that a new correlation, based on the present hourly values is justified.

CORRELATION EQUATION

For the range $0.35 \leq K_T \leq 0.74$, the interval which included 62 per cent of the measured data over a 4-year period, the correlation can accurately be represented by the linear expression

$$K = 1.557 - 1.84K_T \qquad 0.35 \leq K_T \leq 0.75. \qquad 3(a)$$

The range $K_T > 0.75$ included only 5.6 per cent of the measured data over a four year period. Values in this range represent relatively clear periods with some cloud cover but the sun itself not shaded by a cloud. In this case solar radiation is reflected off clouds in substantial quantities. In effect, the clouds act as diffuse concentrators of solar radiation. (This effect can be so strong that instantaneous solar irradiation values in excess of the solar constant have been observed[3]). The direction of this reflected radiation is off-beam and therefore recorded as diffuse. However, it is far from being uniformly distributed across the sky. Due to the limited frequency of data in this range and the unpredictable nature of cloud reflection, it is recommended that a constant K value be used in this range

$$K = 0.177 \qquad K_T > 0.75. \qquad 3(b)$$

The range $0 \leq K_T < 0.35$ represented 32.4% of the measured data. Values in this range represent extremely overcast days with over 90 per cent of the total insolation being diffuse. Because of the high percentage of diffuse radiation, measured values of the total radiation tend to be small and consequently affected by instrument sensitivity and accuracy. For the sake of simplicity a linear correlation

$$K = 1.0 - 0.249K_T \qquad 0 \leq K_T < 0.35 \qquad (3(c)$$

is recommended. Higher order correlations could be fitted to the data in this range; however, the increased accuracy cannot be justified in view of the uncertainty associated with data in this range and in view of the increased computation costs when used in computer simulation programs. Equations 3(a), (b) and (c) are plotted in Fig. 1.

CONCLUSIONS

Equation 3 is recommended for determining the hourly ratio of diffuse-to-total radiation received on a horizontal surface. Although the correlation is based on four years of data from Toronto, Ontario it is expected, based on the study by Ruth and Chant, that the correlation will accurately represent radiation insolation between the latitudes of 43°N and 54°N.

For the purposes of computer simulation programs, which handle vast amounts of data, it is recommended that the hourly midpoint method (eqn 1) be used for calculating the extraterrestial radiation in order to reduce the computation time.

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NOMENCLATURE

- d declination angle, rad
- E the eccentricity correction factor for the solar constant
- h hour angle, rad
- H_T total solar irradiation received on a horizontal surface, over 1 hr, J/m^2
- H_d diffuse solar irradiation received on a horizontal surface over 1 hr J/m^2
- H_0 extraterrestial solar irradiation received on a horizontal surface over 1 hr J/m^2
- $\dot{H}_0$ instantaneous extraterrestial irradiation on a horizontal, w/m^2
- K the ratio H_d/H_T
- K_T the ratio H_T/H_0
- l latitude, rad
- S_c solar constant, w/m^2

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Resumen—Este artículo presenta un análisis de la radiación difusa horaria sobre una superficie horizontal y recomienda una ecuación para determinar la relación horaria de la radiación total y difusa recibida sobre la misma. Los resultados de la nueva ecuación son comparados con los de las anteriores con recomendaciones hechas como si se usaran en programas de simulación helioenergética por computación.

Résumé—Cet article présente une analyse du rayonnement horaire diffusé sur une surface horizontale et conseille une équation pour déterminer le taux horaire du rayonnement diffus/global reçu sur une surface horizontale. Les résultats de la nouvelle équation de corrélation sont comparés avec les équations établies précédemment avec des avis au sujet de leurs utilisations avec des programmes de simulation d'énergie solaire.