The Interrelationship and Characteristic Distribution of Direct, Diffuse and Total Solar Radiation*

By Benjamin Y. H. Liu† and Richard C. Jordan‡

Based upon the data now available, this paper presents relationships permitting the determination on a horizontal surface of the instantaneous intensity of diffuse radiation on clear days, the long term average hourly and daily sums of diffuse radiation, and the daily sums of diffuse radiation for various categories of days of differing degrees of cloudiness. For these determinations, it is necessary to have, either from actual measurements or estimates, a knowledge of the total (direct plus diffuse) radiation on a horizontal surface—its measurement is now regularly made at 98 localities in the United States and Canada. For localities where only an estimate of the long term average total radiation is available, relationships presented in this paper can be utilized to determine the statistical distribution of the daily total radiation at these localities.

NOMENCLATURE

D and $\bar{D}$ = daily and monthly average daily diffuse radiation received on a horizontal surface, Btu/day-sq ft
f = fractional time during which the daily total radiation received on a horizontal surface is less than or equal to a certain value, dimensionless
H and $\bar{H}$ = daily and monthly average daily total (direct plus diffuse) radiation received on a horizontal surface, Btu/day-sq ft
H_o = extraterrestrial daily insolation received on a horizontal surface, Btu/day-sq ft
I_{Dh} = intensity of direct radiation incident upon a horizontal surface, Btu/hr-sq ft
I_{Dn} = intensity of direct radiation at normal incidence, Btu/hr-sq ft
I_{dh} = intensity of diffuse radiation on a horizontal surface, Btu/hr-sq ft
$\bar{I}_{dh}$ = long term average of the hourly diffuse radiation received on a horizontal surface = long term hourly average of the intensity of diffuse radiation on a horizontal surface, Btu/hr-sq ft
I_{oh} = intensity of solar radiation incident upon a horizontal surface outside the atmosphere of the earth, Btu/hr-sq ft
I_{on} = intensity of solar radiation at normal incidence outside the atmosphere of the earth = rI_{sc}, Btu/hr-sq ft
I_{sc} = solar constant = 442 Btu/hr-sq ft = 2 ly/min
I_{Th} = intensity of total (direct plus diffuse) radiation incident upon a horizontal surface, Btu/hr-sq ft
$\bar{I}_{Th}$ = long term average of the hourly total radiation received on a horizontal surface = long term hourly average of the intensity of total radiation on a horizontal surface, Btu/hr-sq ft
K_d and $\bar{K}_d$ = D/H_o and $\bar{D}/H_o$, dimensionless
K_D = $(H - D)/H_o$, dimensionless
K_T and $\bar{K}_T$ = H/H_o and $\bar{H}/H_o$, dimensionless
L = latitude, degrees
m = air mass = $csc\alpha$ except at low altitude, dimensionless
r = ratio of solar radiation intensity at normal incidence outside the atmosphere of the earth to solar constant, dimensionless
r_d = $\bar{I}_{dh}/\bar{D}$ = ratio of hourly to daily diffuse radiation, dimensionless
r_T = ratio of hourly to daily total radiation, dimensionless
α = solar altitude angle, degrees
δ = solar declination, degrees
τ_D = $I_{Dn}/I_{on} = I_{Dh}/I_{oh}$ = transmission coefficient for direct solar radiation, dimensionless
τ_d = I_{dh}/I_{oh} = transmission coefficient for

---

[*] This paper is in part the result of researches sponsored by a grant from the National Science Foundation, Washington, D. C. Portions of the material presented in this paper are drawn from the thesis of Benjamin Y. H. Liu prepared in partial fulfillment of the requirements of the degree of Doctor of Philosophy.

[†] Assistant Professor, Department of Mechanical Engineering, University of Minnesota.

[‡] Professor and Head, Department of Mechanical Engineering, University of Minnesota.

Table 1.—Solar Declination, δ, and the ratio, r, of Solar Radiation Intensity at Normal Incidence Outside Earth's Atmosphere to Solar Constant

Month	Day of Month							
	1		8		15		22	
	δ	r	δ	r	δ	r	δ	r
January	−23°04′	1.0335	−22°21′	1.0325	−21°16′	1.0315	−19°51′	1.0300
February	−17°19′	1.0288	−15°14′	1.0263	−12°56′	1.0235	−10°28′	1.0207
March	−7°53′	1.0173	−5°11′	1.0140	−2°26′	1.0103	0°20′	1.0057
April	4°15′	1.0009	6°55′	0.9963	9°30′	0.9913	11°56′	0.9875
May	14°51′	0.9841	16°53′	0.9792	18°41′	0.9757	20°14′	0.9727
June	21°57′	0.9714	22°47′	0.9692	23°17′	0.9680	23°27′	0.9670
July	23°10′	0.9666	22°34′	0.9670	21°39′	0.9680	20°26′	0.9692
August	18°13′	0.9709	16°22′	0.9727	14°18′	0.9757	12°03′	0.9785
September	8°34′	0.9828	5°59′	0.9862	3°20′	0.9898	0°37′	0.9945
October	−2°54′	0.9995	−5°36′	1.0042	−8°14′	1.0087	−10°47′	1.0133
November	−14°11′	1.0164	−16°21′	1.0207	−18°18′	1.0238	−19°58′	1.0267
December	−21°41′	1.0288	−22°39′	1.0305	−23°14′	1.0318	−23°27′	1.0327

diffuse radiation on a horizontal surface, dimensionless

τ_T = I_{Th}/I_{oh}, = transmission coefficient for total radiation on a horizontal surface, dimensionless

ω = hour angle, degrees

ω_s = sunset hour angle, radians

INTRODUCTION

The increase in recent years of the problems with which solar radiation is involved has made solar radiation information frequently needed by workers in many fields. Although solar radiation data are now available for many localities, difficulties are sometimes encountered in utilizing these data since they consist primarily of total (direct plus diffuse) radiation only and a knowledge of the diffuse component is often required. Since the theoretical computation of diffuse radiation is extremely difficult if not impossible at the present time, an attempt is made in this study to investigate, from the limited data now available, the relationships between diffuse and total radiation in order that they may be utilized for the estimation of diffuse radiation for localities where only the total radiation is known. This was first attempted by Parmelee[1] for cloudless days only, but no extension was made to cloudy days.

In this investigation it has also been found necessary to study the characteristic distribution of total radiation. The results obtained serve to indicate the types of parameters to be used in the investigation of other statistical characteristics of solar radiation should the need for these characteristics arise. The present study is restricted to radiation on a horizontal surface only.

Solar Constant

Since a definite value for the solar constant must be used consistently throughout this study, the most recent value of 2.00 ly/min,* or 442 Btu/hr-sq ft, given by Johnson,[2] in terms of the Smithsonian Pyrheliometric Scale of 1932† shall be adopted. The solar constant is the rate at which solar energy is impinging upon a unit surface, normal to sun's rays, in free space, at the earth's mean distance from the sun. It is in general slightly different from the solar radiation at normal incidence at the outer limit of the atmosphere due to the variation of the distance between the earth and the sun. (See Table 1.)

Relationships Between the Intensities of Direct and Diffuse Radiation on Clear Days

As solar radiation penetrates the atmosphere it is depleted by absorption and scattering. Not all of the scattered radiation is lost, since part of it eventually arrives at the surface of the earth in the form of diffuse radiation. The term, diffuse radiation, is used here in the customary way to denote this short wavelength radiation coming from all parts of the sky. It should be distinguished clearly from the atmospheric thermal radiation which, although also diffuse in nature, is of much longer wavelengths.

To facilitate the discussion, the following dimensionless transmission coefficients shall be defined:

τ_D = transmission coefficient for direct solar radiation
= $I_{Dn}/I_{on} = I_{Dh}/I_{oh}$

τ_d = transmission coefficient for diffuse radiation on a horizontal surface = I_{dh}/I_{oh}

These transmission coefficients are functions of the solar altitude, atmospheric water vapor content, dust content, ozone content, and any other radiation de-

---

* 1.0 langley/min = 1.0 gm cal/min-sq cm = 3.687 Btu/min-sq ft = 69.7 milliwatts/ sq cm.

† The 1932 Pyrheliometric Scale of the Smithsonian Institution is 2.5% below the scale of 1913, 1.0% above the Angstrom scale and 0.5% below the recently proposed International Pyrheliometric Scale.[3]

pleting factors. However, in a nonindustrial locality where the atmosphere is relatively clean and the effect of dust small, the daily variation of these transmission coefficients for the sun at a fixed altitude, is primarily due to the variation of the atmospheric water vapor. Thus as the atmospheric water vapor content varies from day to day causing both τ_D and τ_d to vary, a functional relationship between τ_D and τ_d is generated.

The four upper curves of Fig. 1 show the theoretical relationships between τ_D and τ_d for a cloudless and dust free atmosphere for four air masses, 1, 2, 3 and 4 corresponding to solar altitude angles of 90, 30, 19.5 and 14.5 respectively. These relationships can be derived readily from the transmission coefficients computed by Kimball when the assumption is made that the diffuse radiation received on a horizontal surface is half of the solar radiation scattered by the atmospheric constituents.[4] However, due to the fact that Kimball's computations are based upon a zero air mass solar spectrum low in the ultraviolet, the theoretical values of τ_D and τ_d in Fig. 1 have been reduced by 3%. Since these theoretical relationships have been derived without considering the effect of dust and are based upon an assumption which is known to be only approximately correct, they should not be expected to represent the correct relationships between τ_D and τ_d under actual cloudless sky conditions and are derived here to serve merely as guides in the search of experimental relationships.

The experimental points of Fig. 1 are derived from the measurements made at Hump Mountain, North Carolina, by Moore and Abbot[5] whose data appear to be the best available. In computing the experimental transmission coefficients, however, the direct and diffuse radiation intensities from the original data have been reduced by 2.5% in order that the radiation intensities be expressed in the 1932 Smithsonian Pyrheliometric scale upon which the presently adopted solar constant of 2.00 ly/min is based.

The fact that both the theoretical curves and experimental points of Fig. 1 show that the values of τ_d corresponding to a fixed value of τ_D depend only very moderately upon the air mass indicates that, for the degree of accuracy here sought, a relationship which is independent of the air mass is adequate. The following equation of a straight line, obtained by the method of least squares, best fits the experimental points:

$$\tau_d = 0.2710 - 0.2939\tau_D \qquad [1]$$

A total of 149 points representing the data of 28 clear days were used in obtaining this equation. The probable error computed by the equation,

$$\text{Probable error} = 0.6745\sqrt{\frac{v^2}{n-1}}$$

is 0.0052, where v is the difference between the experimental value of τ_d and the value of τ_d given by the straight line at the same value of τ_D, and n the total number of points used.

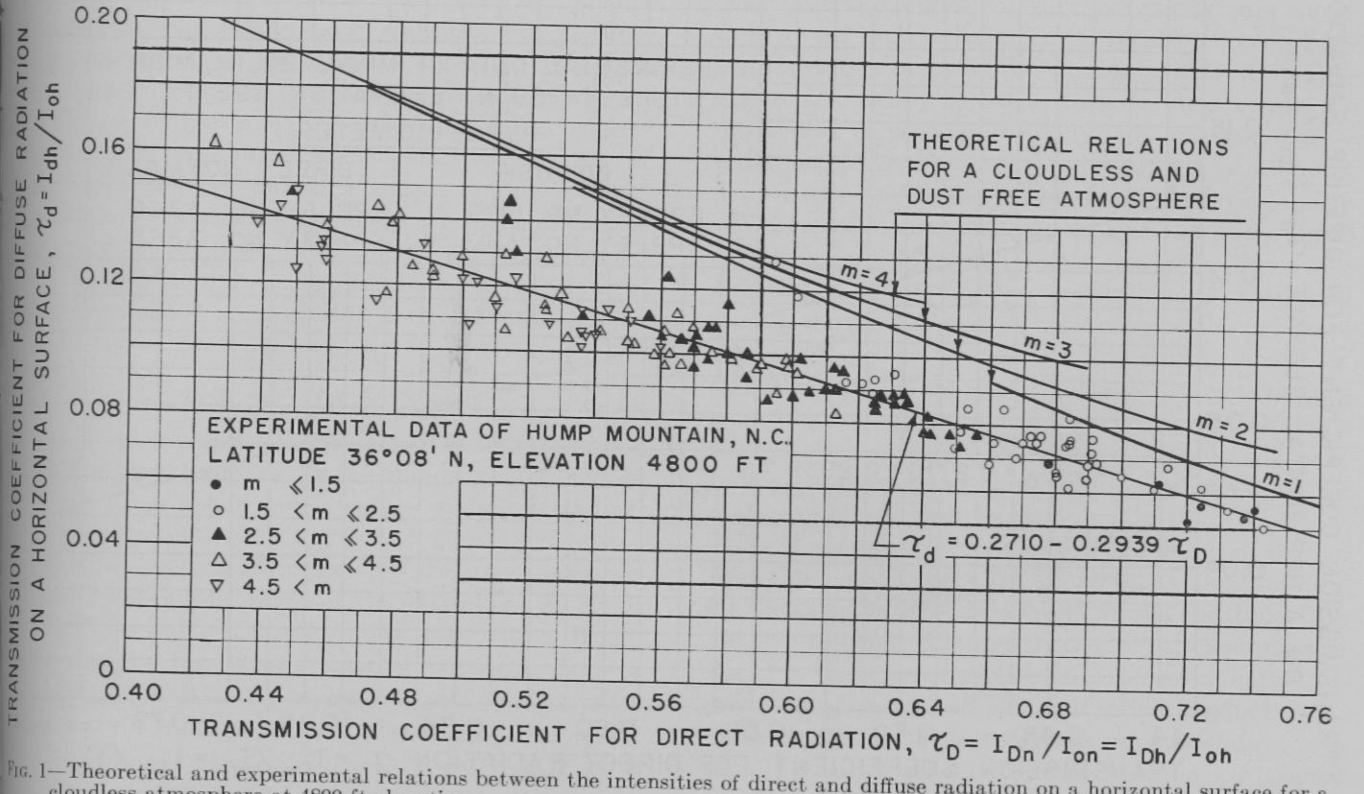


FIG. 1—Theoretical and experimental relations between the intensities of direct and diffuse radiation on a horizontal surface for a cloudless atmosphere at 4800 ft elevation.

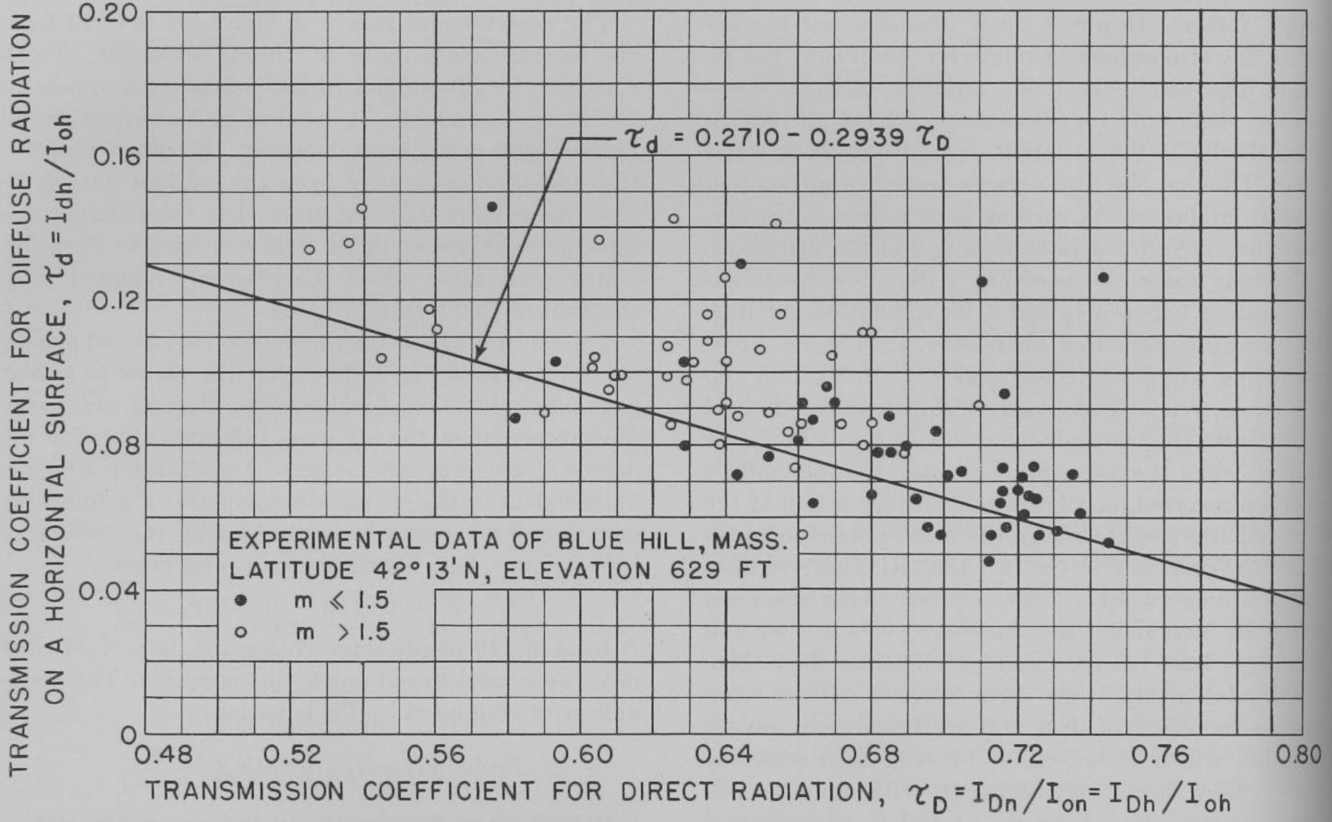


FIG. 2—Comparison of the empirical relation between the intensities of direct and diffuse radiation on a horizontal surface derived from the data for Hump Mountain, N. C., with the data for Blue Hill, Mass.

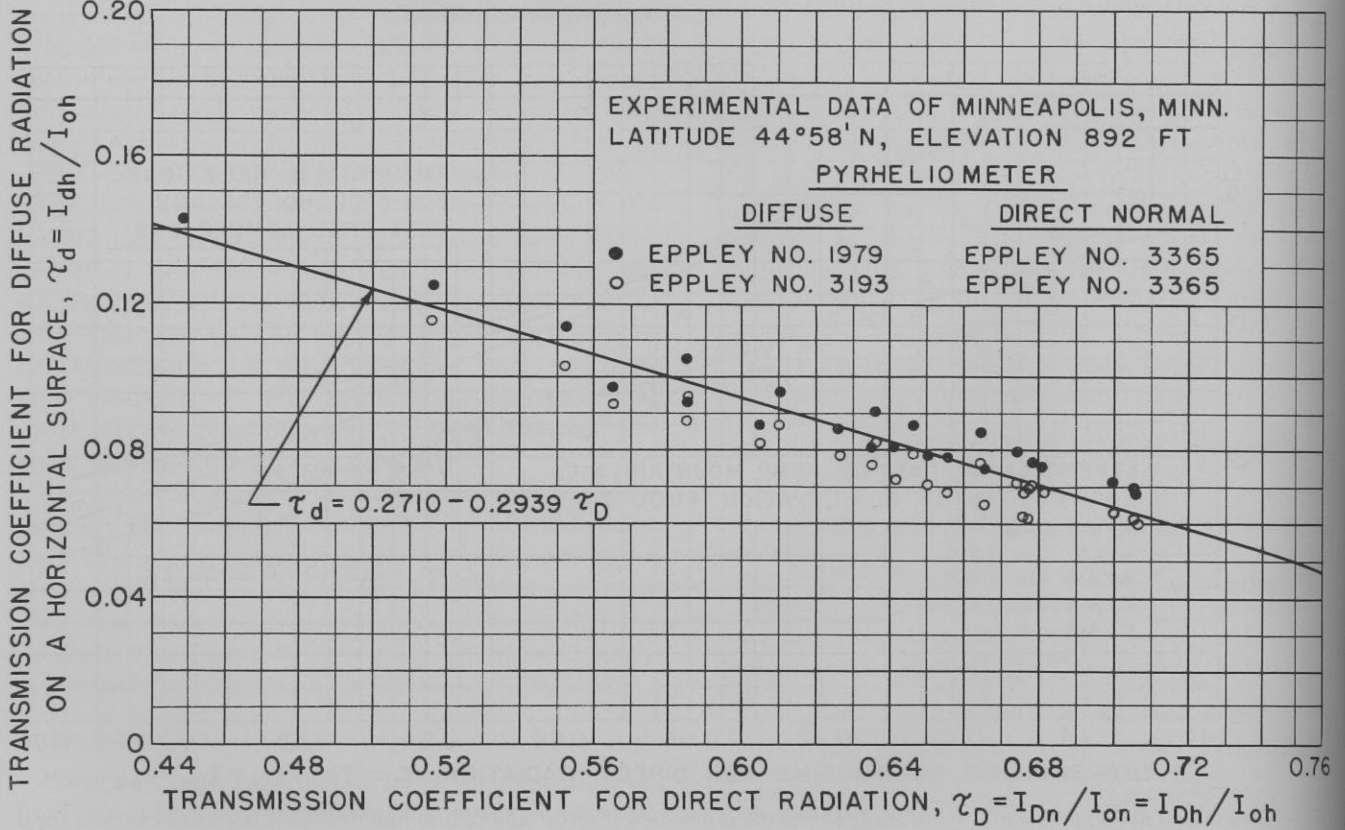


FIG. 3—Comparison of the empirical relation between the intensities of direct and diffuse radiation on a horizontal surface derived from the data for Hump Mountain, N. C., with the data for Minneapolis, Minn.

The reasons that the observed diffuse radiation values are lower than the theoretical values computed for a dust free atmosphere, under the assumption that half of the scattered radiation reaches the earth's surface, are not immediately apparent. Since the theoretical relationships do not take into consideration the effects of dust and since an atmosphere is never completely dust free, the observed diffuse radiation intensities would be expected to be higher than those predicted if the usual assumption is made that dust particles scatter but do not absorb radiation.[4,6] The effect of terrain reflection which was not considered in obtaining the theoretical relationships would also tend to increase the diffuse radiation, since the radiation thus reflected is in turn scattered by the atmospheric constituents and a fraction of this scattered radiation again arrives at the earth's surface. Furthermore, the assumption that half of the scattered radiation reaches the earth's surface is true for pure Rayleigh scattering by air molecules only. The more intensely forward scattered components usually associated with water vapor scattering probably should increase the observed diffuse radiation intensity to values even greater than those predicted. Therefore, it appears that in travelling toward the earth's surface a substantial fraction of the scattered radiation must have been absorbed by the atmospheric water vapor or other absorbing media in the atmosphere. Nevertheless, this factor alone does not seem to have an effect of sufficient magnitude to account for the differences between the theoretical and experimental diffuse radiation intensities, as shown in Fig. 1. However, regardless of the reasons for this difference, Equation [1] provides a means for estimating the intensity of diffuse radiation on a horizontal surface under a cloudless atmosphere when the intensity of direct radiation at normal incidence is known. Since the intensity of total radiation on a horizontal surface is the sum of the intensities of direct and diffuse radiation on a horizontal surface, the following relation between τ_d and τ_T can be readily derived from Equation [1]

$$\tau_d = 0.3840 - 0.4160\tau_T \qquad [2]$$

where τ_T is the ratio of the intensity of total radiation on a horizontal surface to the intensity of radiation incident upon a horizontal surface on top of the atmosphere.

To test whether Equation [1] also represents the correct relationship between the intensities of direct and diffuse radiation at localities other than Hump Mountain, it has been compared with the data of Hand[7] for Blue Hill, Massachusetts, and measurements made by the authors at the University of Minnesota, Minneapolis, Minnesota. These are shown in Figs. 2 and 3. Even though the data for Blue Hill show considerable scattering and the diffuse radiation measurements are higher than those for Hump Mountain, the agreement is still quite satisfactory. The agreement of Equation [1] with the experimental data for Minneapolis is extremely good and the small differences are entirely within the experimental uncertainties of the diffuse radiation measurements by the Eppley pyrheliometers. (Examination of Fig. 3 shows that diffuse radiation measured by two different Eppley pyrheliometers differ by about 10%.) Therefore, it is felt that Equation [1] and hence Equation [2] are of general validity and should be applicable to many localities where the albedo (reflectivity) of the surrounding terrain and the atmospheric contamination by dust are not greatly different from those at Hump Mountain, Blue Hill, and Minneapolis.

Relationship Between Daily Diffuse and Daily Total Radiation on Cloudy Days

Due to the extremely variable cloudiness the intensities of direct and diffuse radiation under sky conditions not completely cloudless will also be highly variable and their values at any one instant are impossible to predict. Therefore, any attempt to establish a relationship between diffuse radiation and total radiation during cloudy days must involve statistical averages which can be obtained from experimental data covering a sufficiently long period of time.

If one month is taken as a period during which the solar declination does not vary excessively, and consequently the daily solar radiation incident upon a horizontal surface outside the atmosphere at a locality also remains fairly constant, then during a month the day to day variation of the daily total and daily diffuse radiation received on a horizontal surface at a locality is primarily due to the variation of cloudiness, and to much lesser extents, the variation of the atmospheric water vapor, dust, and ozone contents. Since the amount of total radiation received on a horizontal surface during a day is an indication of the degree of atmospheric cloudiness whose variability is largely responsible for the observed diffuse radiation variations from day to day, it is expected that, when suitable statistical averages are taken, a relationship will exist between the daily total and daily diffuse radiation for each month at a given locality.

Fig. 4 shows such relationships derived from the ten year (1947–1956) data for Blue Hill, Massachusetts,[8] for the three months, December, March and June. The value of the daily diffuse radiation, D, at each point in Fig. 4 is the average of the daily diffuse radiation received on days with daily total radiation equal to H. However, due to the fact that the number of days with exactly equal values of daily total radiation is extremely small even within the same month, the value of H at

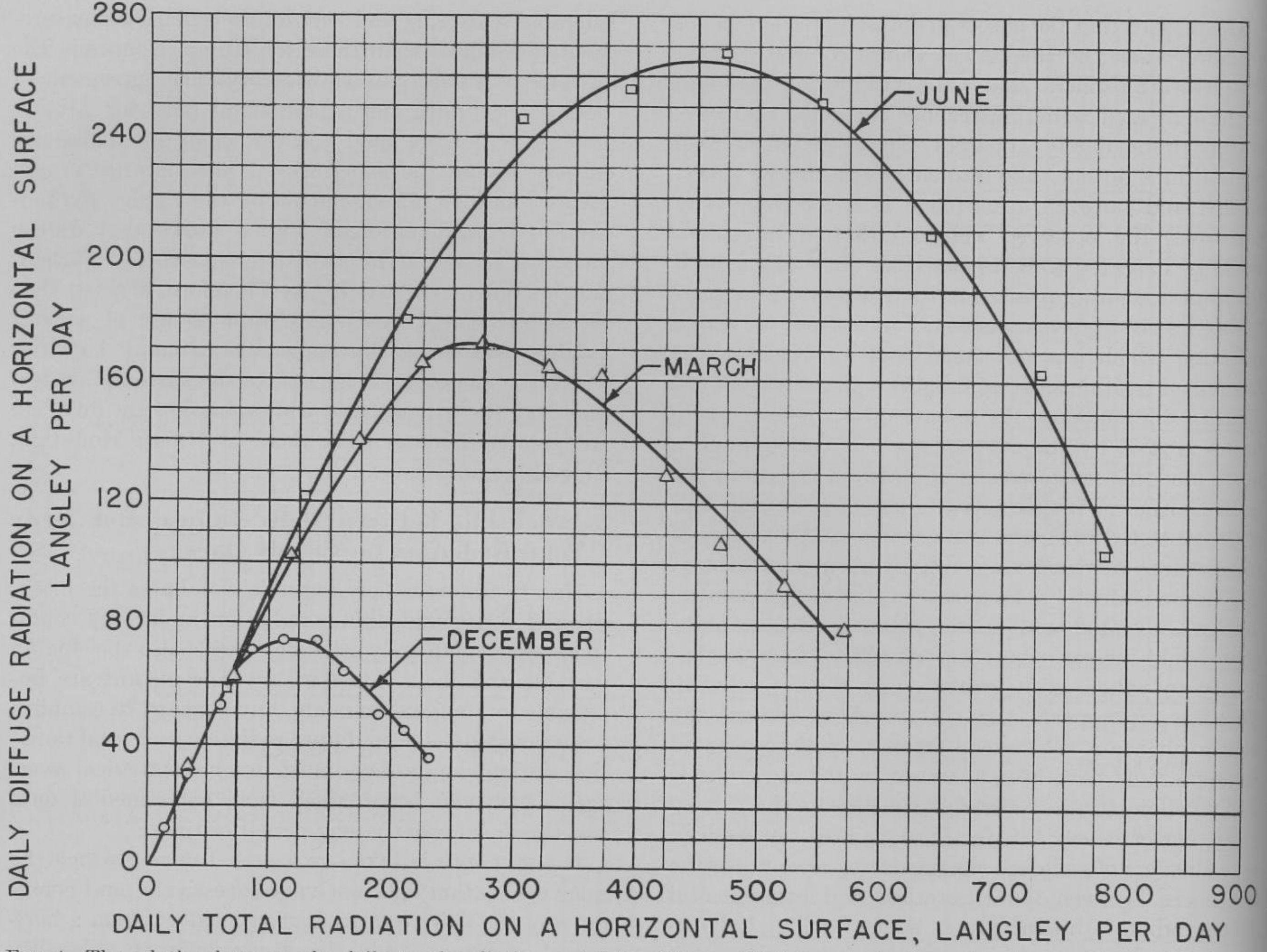


FIG. 4—The relation between the daily total radiation and daily diffuse radiation on a horizontal surface for December, March and June at Blue Hill, Mass.

each point in Fig. 4 is actually the average of the daily total radiation for days whose daily total radiation fall within a small interval of values (the size of the interval is indicated by the difference in the values of H of the neighboring points in Fig. 4). Since ten year data for each month are used and approximately ten points are obtained for each month, each point in Fig. 5 represents approximately the average of thirty days.

The results of Fig. 5 show that a fairly smooth relationship exists between the daily diffuse and daily total radiation for each of the three months. Furthermore, the similarities between the forms of the three curves indicate the possibility of obtaining a unique representation of these relations through normalizing the coordinates. That this indeed is the case can be seen by referring to Fig. 5 showing the relationship between the daily diffuse and daily total radiation for the entire twelve months in the normalized coordinates K_d and K_T where

$$K_d = D/H_o \qquad [3]$$

$$K_T = H/H_o \qquad [4]$$

and H_o, the extraterrestrial daily insolation received on a horizontal surface, is computed from the following equation

$$H_0 = \frac{24}{\pi} r I_{sc}(\cos L \cos \delta \sin \omega_s + \omega_s \sin L \sin \delta) \qquad [5]$$

with the use of the mean solar declination for each month. The sunset hour angle, ω_s, i.e. the hour angle at which the sun sets in the west, in Equation [5] can be determined as follows:

$$\cos \omega_s = -\tan L \tan \delta \qquad [6]$$

The solar declination for the selected days of each month and the ratio, r, of the intensity of radiation at normal incidence outside the atmosphere to the solar constant are given in Table 1. A graphical representation of Equation [6] is shown in Fig. 6 with H_o plotted against the latitude and with the month as a parameter.

Despite a more than threefold increase in the value of H_o from December to June at the latitude (42°13′N) of Blue Hill, as an examination of Fig. 6 will show, the results of Fig. 5 show that a fairly definite relationship exists between the two dimensionless quantities K_d and K_T. It is of interest to note that on days which are relatively cloud free ($K_T = 0.75$) the diffuse radiation received on a horizontal surface is approximately 12% ($K_d = 0.12$) of the solar radiation outside the atmosphere. On certain partly cloudy days ($K_T = 0.40$), however, the diffuse radiation is over twice the clear day value and reaches as much as 25% of the extraterrestrial insolation. With increasing cloudiness, as K_T approaches zero, K_d also approaches zero as would be expected.

The curves of Fig. 5 thus offer an interesting possibility for a method of estimating, to an average accuracy of approximately ±5%, the daily diffuse radiation at localities where measurements of daily total radiation are made, provided, of course, that the relationships of Fig. 5 also hold for other localities. Since additional data is sparse, it has not been possible to extend the studies to other locations. However, it will be shown in subsequent developments that the results derived assuming the validity of these results continue to compare favorably with all available experimental evidence.

A special comment is needed for the few points with $K_T > 0.75$ as they appear to deviate from the general trend of the other points. It should be recalled that in Equations [3] and [4], H_o was computed using the average solar declination for each month. Thus the values of K_T (or K_d) for each point in Fig. 5 truly represent the fraction of the extraterrestrial daily insolation transmitted through the atmosphere as total radiation (or as diffuse radiation), and thus accurately provide an indication of the degree of cloudiness, only if the value of H_o so computed is the true average of the extraterrestrial insolation for the days whose data are used in obtaining each point. Since the extraterrestrial daily insolation does vary to some extent during a month, this then requires that the days whose data are used to obtain a point distribute themselves symmetrically with respect to the middle of the month. In analyzing the data for Blue Hill it was found that this requirement is nearly obeyed for all points with $K_T < 0.75$ but not for the few points with $K_T > 0.75$. In fact the large values of K_T for these points are direct consequences for the fact that the values of H_o used are smaller than the true average. A much better

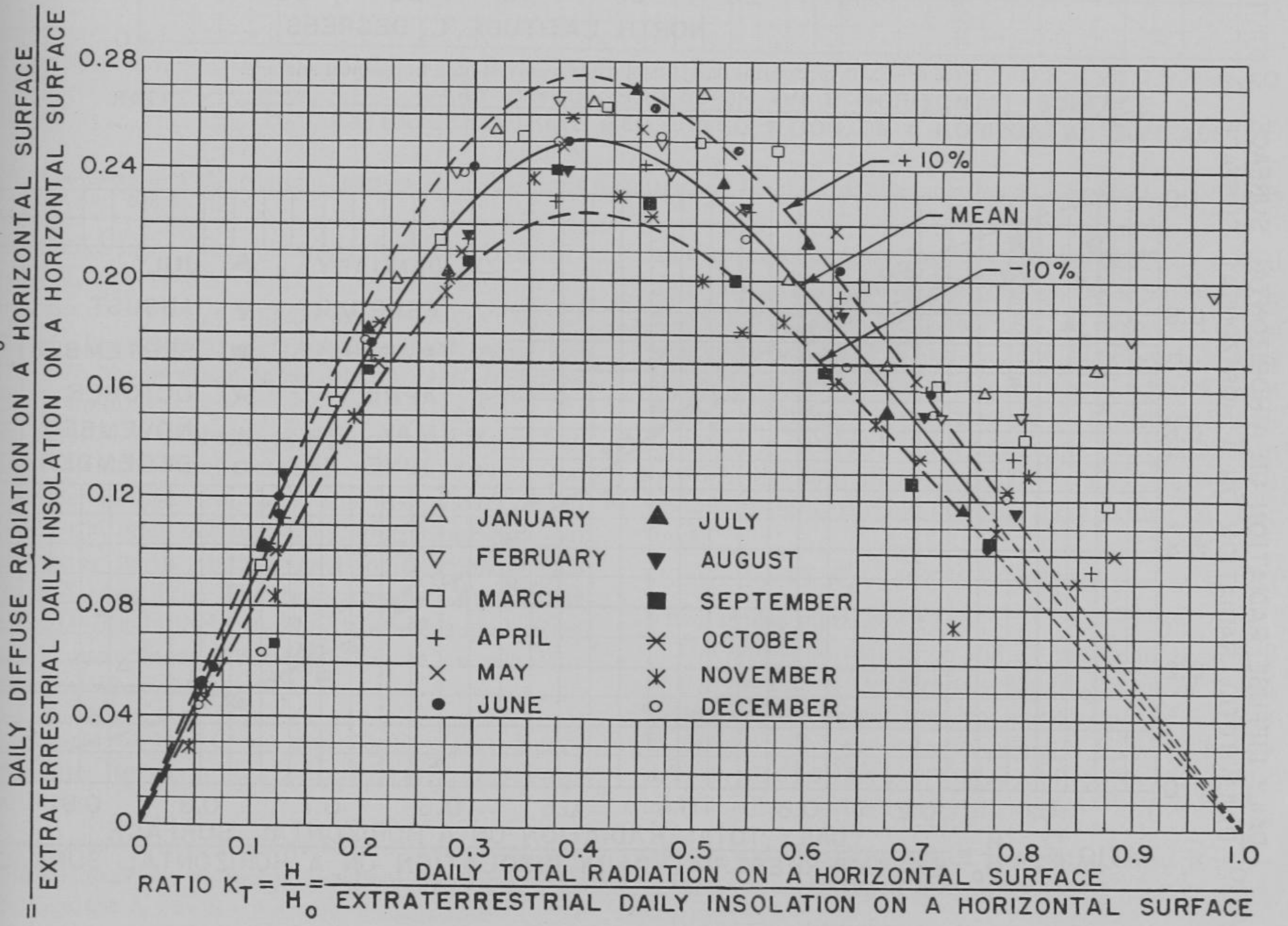


Fig. 5—The relation between the daily total radiation and daily diffuse radiation on a horizontal surface.

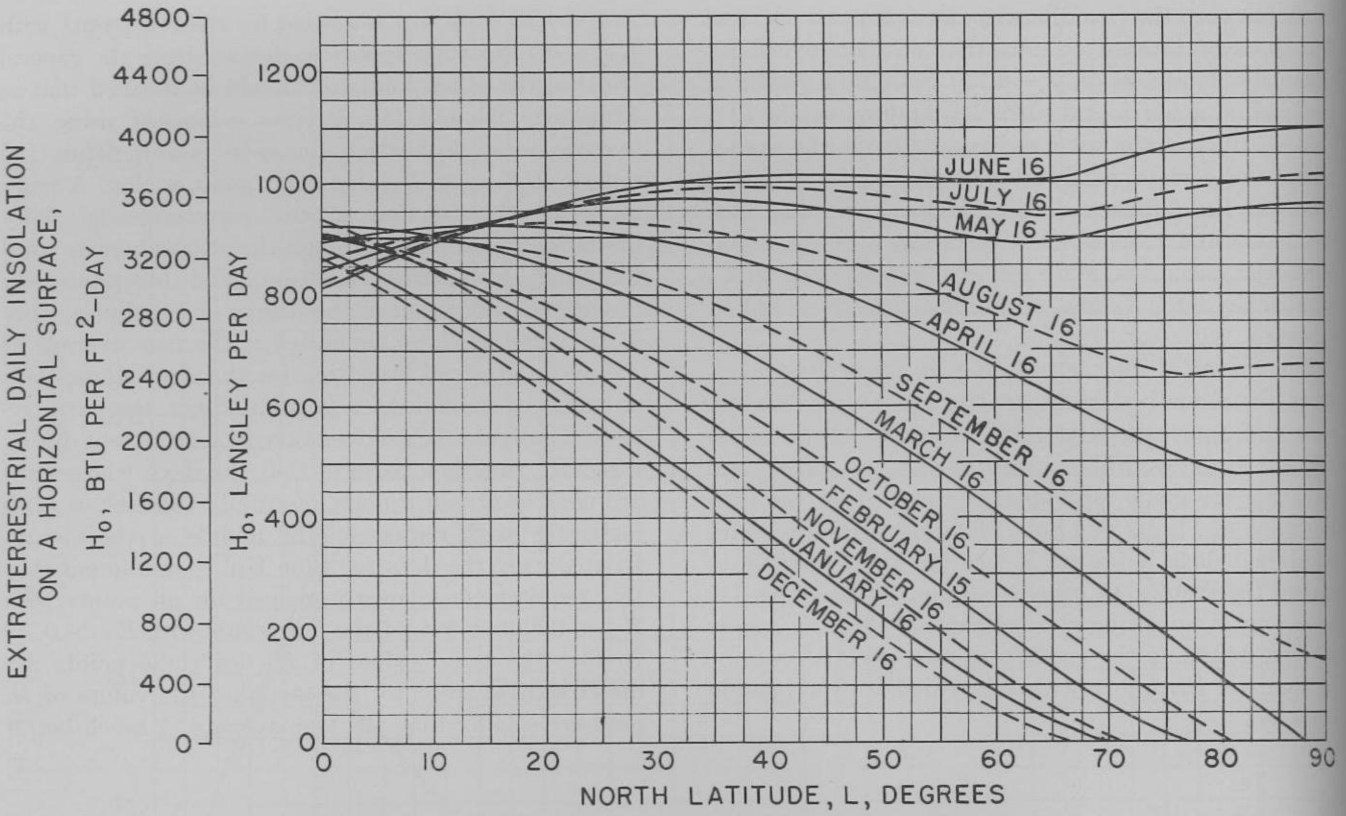


Fig. 6—Extraterrestrial daily insolation received on a horizontal surface.

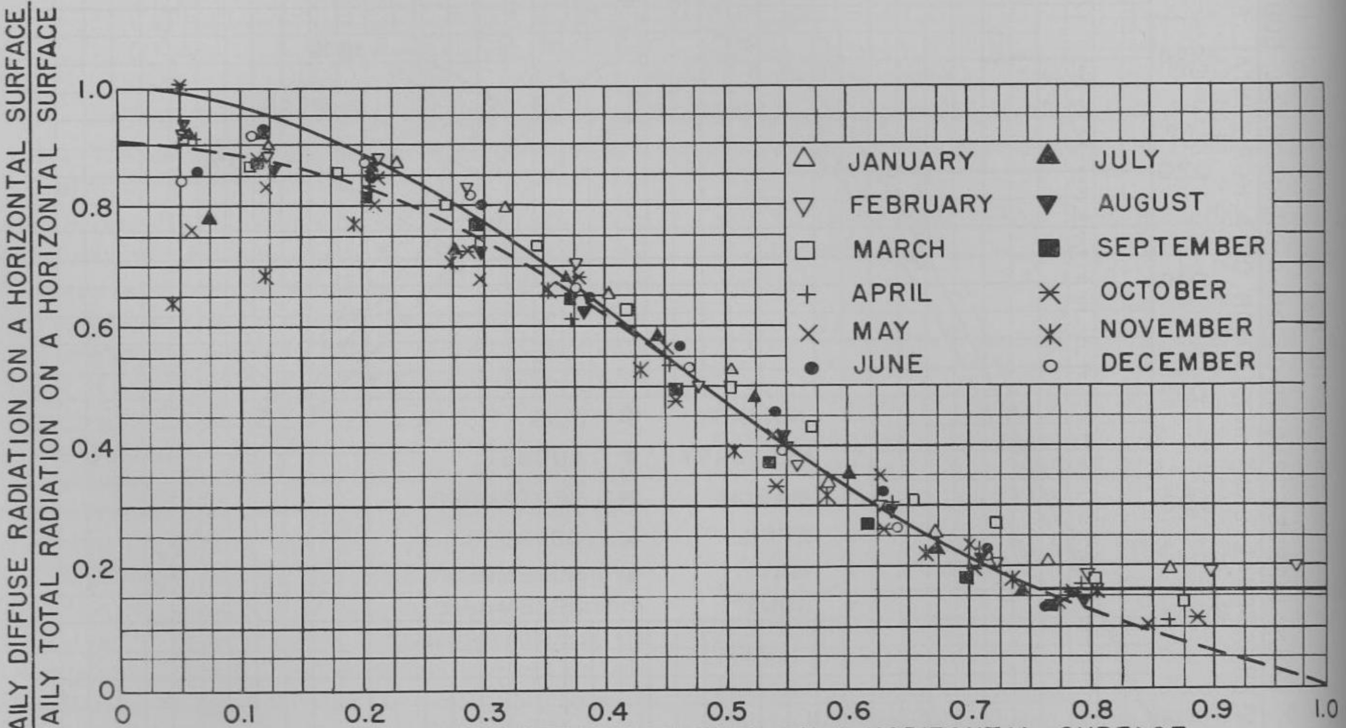


Fig. 7—The ratio of the daily diffuse radiation to the daily total radiation as a function of the cloudiness index K_T.

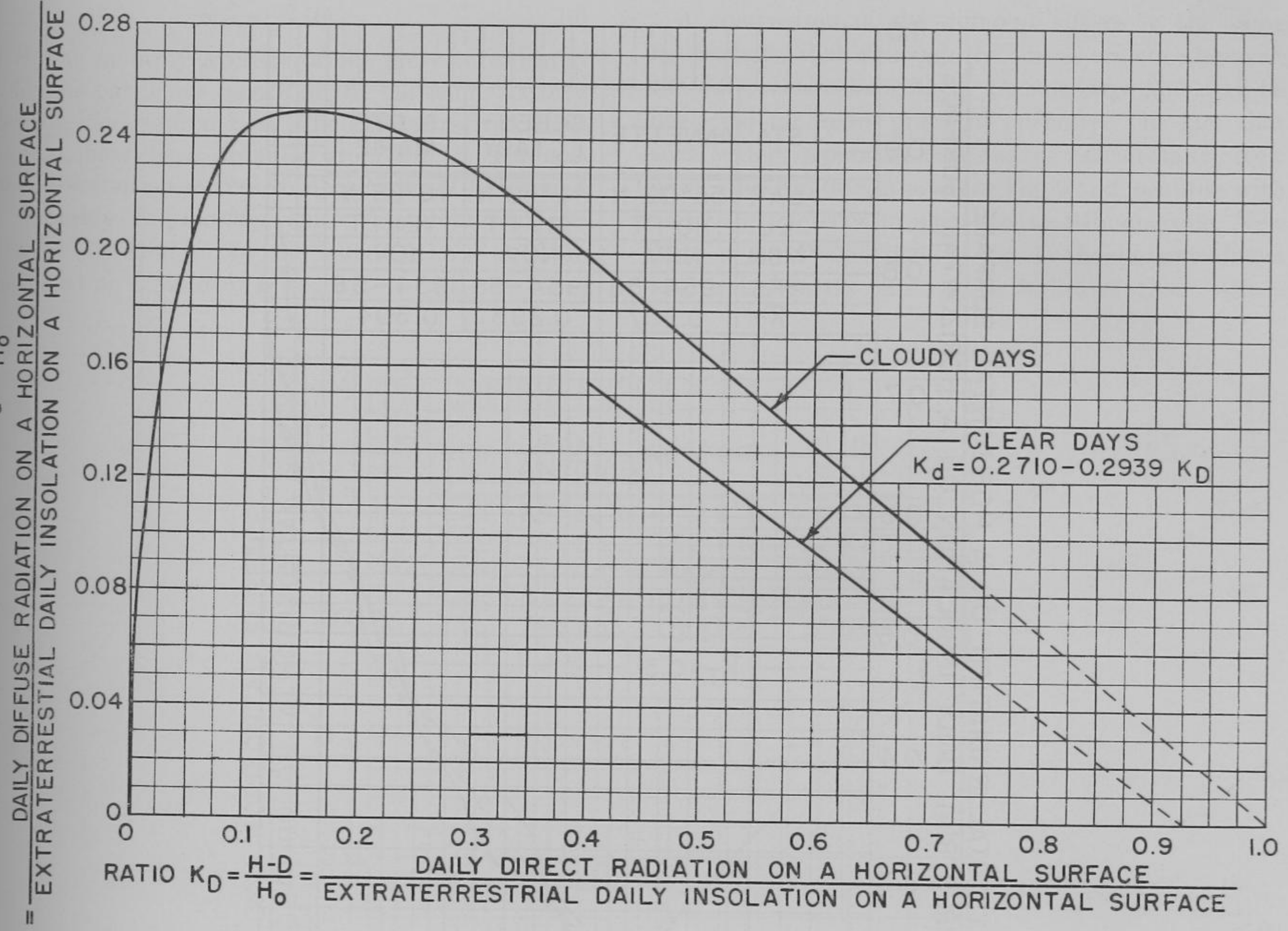


FIG. 8—Comparison of the relations between the daily direct radiation and the daily diffuse radiation on a horizontal surface on clear and cloudy days.

result would have been obtained had the "correct" average H_o been employed. This was not done in order that H_o for each month might have a definite value. To overcome this difficulty it is recommended that when $K_T > 0.75$ the average ratio 0.16 of D/H for these points be used.

The ratio D/H is shown as a function of K_T in Fig. 7. Since the diffuse and total radiation should be equal for completely overcast days, the ratio D/H should approach the limit one when K_T approaches zero. An inconsistency in the data is seen for the points with values of K_T near zero. The solid curve is so drawn that the ratio D/H is brought to unity when K_T equals zero, and it is recommended that this curve be used for practical application.

Comparison of the Clear and Cloudy Day Relationships Between the Daily Direct and Diffuse Radiation

A comparison between the clear and cloudy day diffuse radiation is shown in Fig. 8 where K_d of Fig. 5 is plotted against K_D with

$$K_D = (H - D)/H_o = K_T - K_d \qquad [7]$$

and represents the fraction of the extraterrestrial daily insolation transmitted through the atmosphere as direct radiation. Since K_d and K_D are respectively the weighed averages of τ_d and τ_D, and since on clear days the relation between τ_d and τ_D is linear, when K_d and K_D are substituted for τ_d and τ_D in Equation [1], an equation is obtained which represents the relationship between the daily direct and diffuse radiation received on a horizontal surface on clear days. Thus, on clear days,

$$K_d = 0.2710 - 0.2939K_D \qquad [8]$$

which is also plotted in Fig. 8.

The effects of clouds on diffuse radiation is clearly seen from Fig. 8 which shows that the value of K_d on cloudy days is higher than the corresponding value of K_d on clear days at the same values of K_D. The higher diffuse radiation on cloudy days is obviously due to the additional scattering effects of clouds.

Statistical Distribution of the Daily Total Radiation on a Horizontal Surface

In the majority of cases it is more important to have a knowledge of the monthly average of the daily dif-

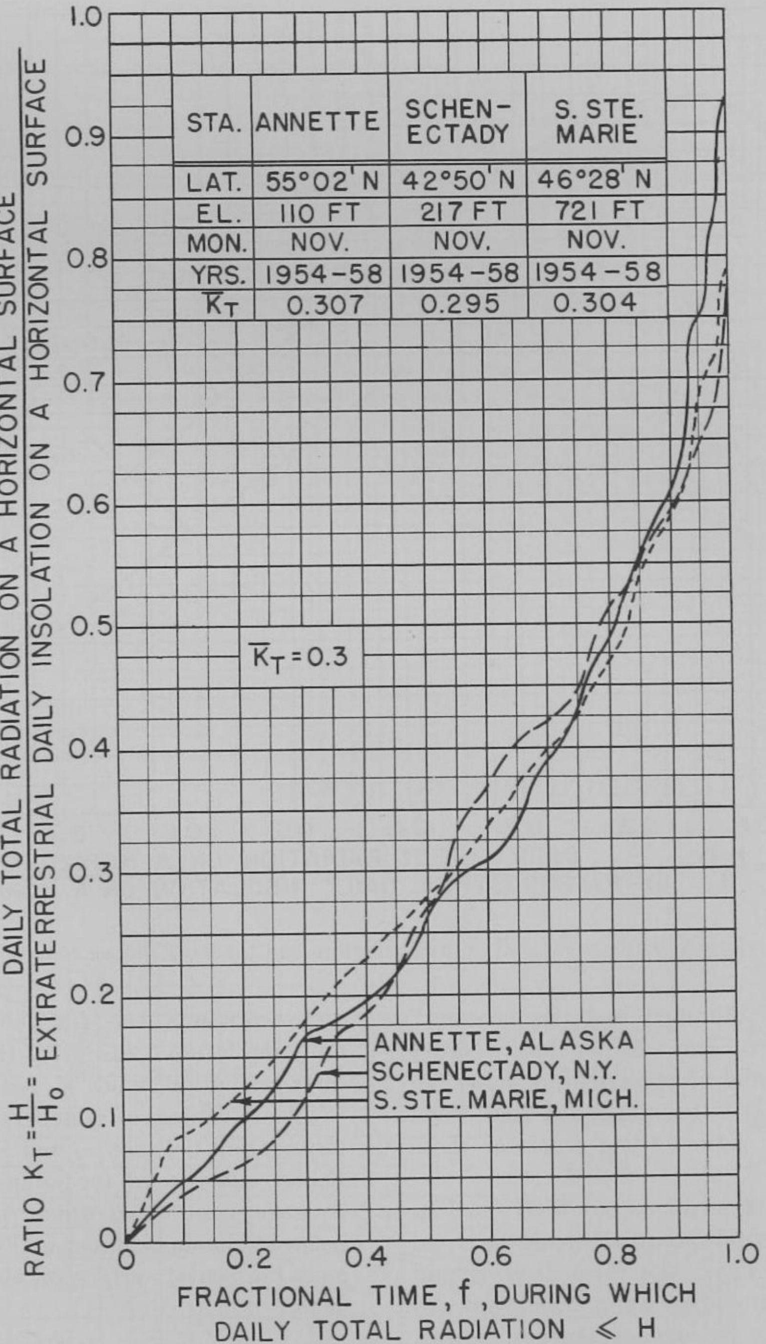


FIG. 9—Monthly K_T curves for $\bar{K}_T = 0.3$.

fuse radiation, $\bar{D}$. However, in order to compute $\bar{D}$, with the use of the relationship of Fig. 7, the statistical distribution of the daily total radiation must be known.

For reasons to be discussed it has been suspected that a correlation possibly exists among the statistical distribution curves of different localities. To test whether this indeed is the case, statistical distribution curves of daily total radiation for widely separated localities have been constructed and compared. Typical examples of these comparisons are shown in Figs. 9, 10 and 11.

Each of the curves of Fig. 9, 10 and 11 have been constructed using five year (primarily 1954–1958) data of daily total radiation from the Weather Bureau publication, Climatological Data, National Summary, and the curves have been so arranged that the values of $\bar{K}_T$ for curves on the same graph are approximately equal where

$$\bar{K}_T = \bar{H}/H_o \qquad [9]$$

and $\bar{H}$, the monthly average of the daily total radiation for the particular month at the particular locality under consideration, and H_o, the extraterrestrial daily insolation, have been obtained from Fig. 6. The statistical distribution curves so obtained will be termed the "monthly K_T curves" since they represent the statistical distribution of the quantity K_T and are constructed on a monthly basis.

A comparison of the different curves in the same figure shows that although the curves are not identical, the differences among them are not large and may be neglected for many practical purposes. The fact that such widely separated localities as Schenectady, New York, and Annette, Alaska, in Fig. 9 and localities with as much difference in elevation as Albuquerque, New Mexico, and Wake Island in Fig. 11 should have almost identical monthly K_T curves indicates that such a

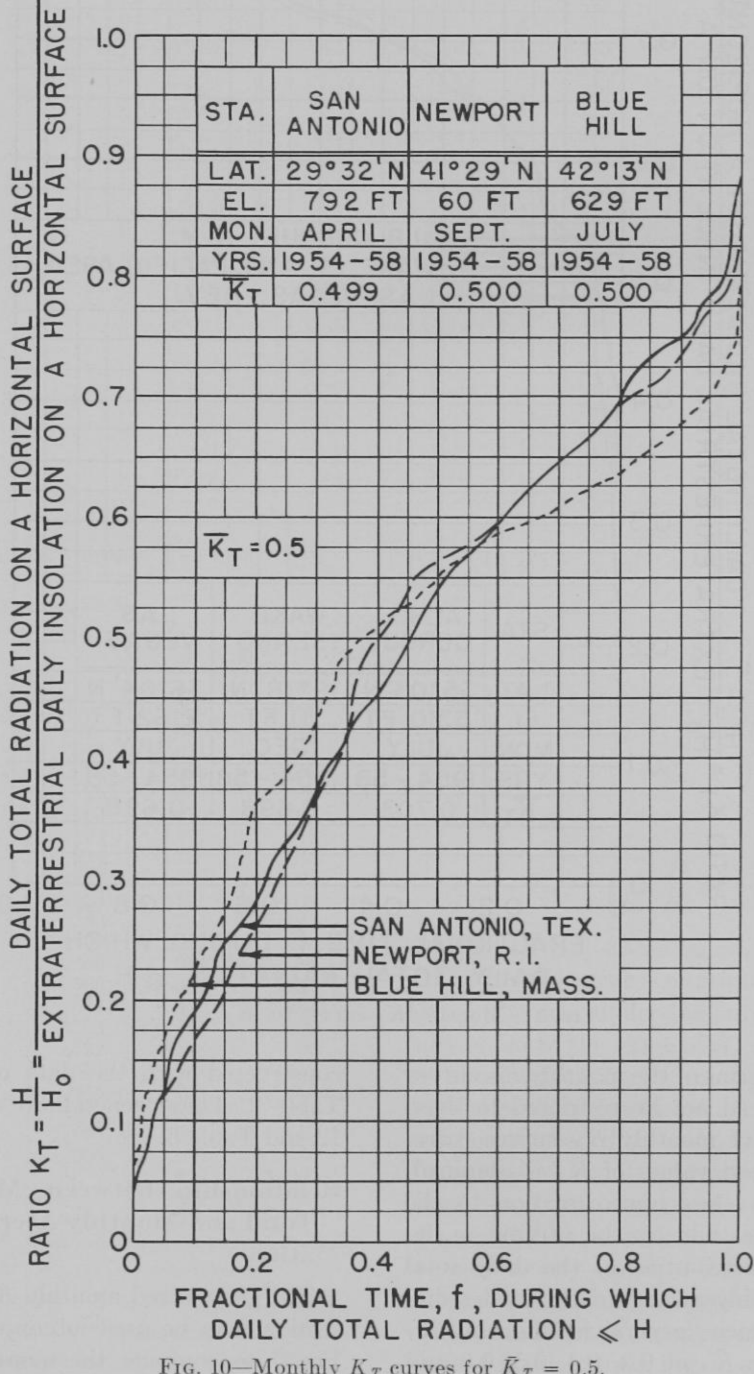
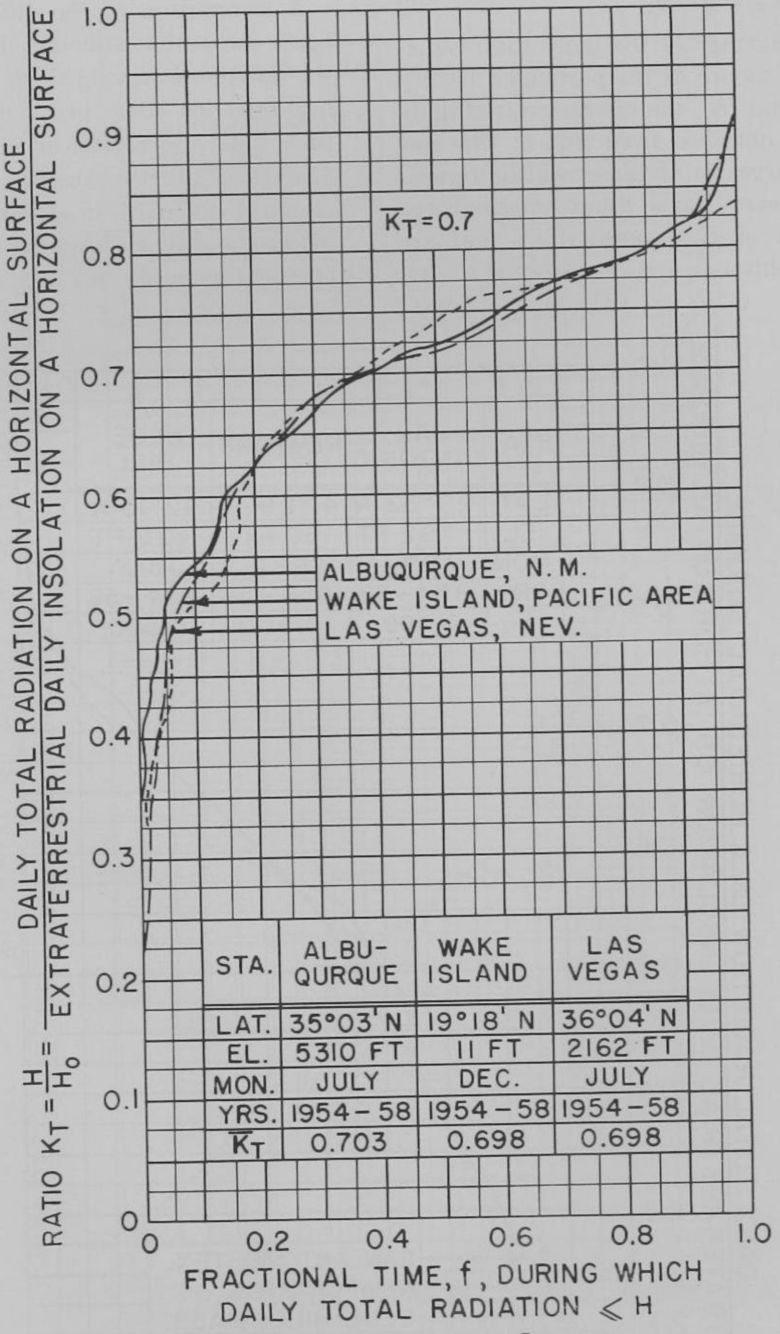


Fig. 10—Monthly K_T curves for $\bar{K}_T = 0.5$.



STA.	ALBU-QURQUE	WAKE ISLAND	LAS VEGAS
LAT.	35°03' N	19°18' N	36°04' N
EL.	5310 FT	11 FT	2162 FT
MON.	JULY	DEC.	JULY
YRS.	1954–58	1954–58	1954–58
$\bar{K}_T$	0.703	0.698	0.698

FIG. 11—Monthly K_T curves for $\bar{K}_T = 0.7$.

correlation between the forms of the monthly K_T curves and the values of $\bar{K}_T$ need not be restricted to these localities. Thus if a set of monthly K_T curves corresponding to different mean values of $\bar{K}_T$ is obtained, they may be used as a close approximation to the actual monthly K_T curves and can be utilized to determine the statistical distribution of the daily total radiation when the monthly average daily total radiation is known. A set of such "generalized monthly K_T curves" corresponding to $\bar{K}_T$ of 0.3, 0.4, 0.5, 0.6 and 0.7 has been obtained from the monthly K_T curves constructed with the data of the localities shown in Table 2. These generalized curves are shown in Fig. 12 and Table 3.

Relationship Between Monthly Average Daily Total and Monthly Average Daily Diffuse Radiation

The generalized monthly K_T curves of the preceding section may be used in conjunction with the curve in Fig. 7 to compute the monthly average of the daily diffuse radiation as follows.

TABLE 2.—Stations Selected for the Construction of the Generalized Monthly K_T Curves

Station	Latitude (North)	Elevation (ft.)	Month	$\bar{H}$ (lg/day)	$\bar{K}_T$	Period of Data
Annette, Alaska	55°02′	110	Nov.	199	.307	1954–1958
Schenectady, N. Y.	42°50′	217	Nov.	380	.295	1954–1958
S. Ste. Marie, Mich.	46°28′	721	Nov.	326	.304	1954–1958
Boston, Mass.	42°22′	15	Dec.	298	.399	1955–1958
Cleveland, O.	41°30′	787	Dec.	310	.400	1955–1958
Indianapolis, Ind.	39°44′	793	Nov.	434	.401	1954–1958
Oak Ridge, Tenn.	36°01′	905	Jan.	429	.399	1952, '54, '55, '57 '58
Put-in-Bay, O.	41°39′	575	Nov.	404	.396	1950–1953
State-College, Pa.	40°48′	1175	Jan.	357	.406	1954–1958
Atlanta, Ga.	33°39′	975	Jan.	468	.498	1954–1958
Blue Hill, Mass.	42°13′	629	July	994	.500	1954–1958
Newport, R. I.	41°29′	60	Sept.	730	.500	1954–1958
Ottawa, Ont.	45°20′		Apr.	815	.498	1954–1958
San Antonio, Tex.	29°32′	792	Apr.	902	.499	1954–1958
Seattle, Wash.	47°36′	14	Aug.	858	.502	1951, '52, '54, '55, '58
Apalachicola, Fla.	29°44′	13	Mar.	778	.600	1952, 1955–1958
Bismarck, N. D.	46°46′	1650	Aug.	862	.599	1954–1958
Cleveland, O.	41°30′	787	May	963	.597	1951, 1955–1958
Grand Lake, Colo.	40°15′	8389	Apr.	850	.598	1950–1953, 1957
Midland, Tex.	32°01′	2854	Oct.	661	.601	1954–1958
Rapid City, S. D.	44°09′	3165	Apr.	825	.600	1954–1958
Albuquerque, N. M.	35°03′	5310	July	997	.703	1954–1958
Grand Junction, Colo.	39°06′	4849	June	1019	.699	1954, '55, '57, '58
Las Vegas, Nev.	36°04′	2162	July	998	.698	1954–1958
Riverside, Calif.	33°58′	1050	Sept.	797	.700	1954–1958
Santa Maria, Calif.	34°56′	238	July	997	.700	1954–1958
Wake Islands, P.A.	19°18′	11	Dec.	635	.698	1954–1958

TABLE 3—The Generalized Monthly K_T Curves

K_T	Value of f for $\bar{K}_T$ =				
	.3	.4	.5	.6	.7
.04	.073	.015	.001	.000	.000
.08	.162	.070	.023	.008	.000
.12	.245	.129	.045	.021	.007
.16	.299	.190	.082	.039	.007
.20	.395	.249	.121	.053	.007
.24	.496	.298	.160	.076	.007
.28	.513	.346	.194	.101	.013
.32	.579	.379	.234	.126	.013
.36	.628	.438	.277	.152	.027
.40	.687	.493	.323	.191	.034
.44	.748	.545	.358	.235	.047
.48	.793	.601	.400	.269	.054
.52	.824	.654	.460	.310	.081
.56	.861	.719	.509	.360	.128
.60	.904	.760	.614	.410	.161
.64	.936	.827	.703	.467	.228
.68	.953	.888	.792	.538	.295
.72	.967	.931	.873	.648	.517
.76	.979	.967	.945	.758	.678
.80	.986	.981	.980	.884	.859
.84	.993	.997	.993	.945	.940
.88	.995	.999	1.000	.985	.980
.92	.998	.999		.996	1.000
.96	.998	1.000		.999	
1.00	1.000			1.000	

It should be observed that the area under any monthly K_T curve is numerically the same as the value of $\bar{K}_T$, since from the definitions of $\bar{K}_T$ and "f" in Figs. 9, 10, 11 or 12,

$$\bar{K}_T = \bar{H}/H_0 = \frac{1}{H_0}\int_{f=0}^{f=1} H\,df = \int_{f=0}^{f=1} \frac{H}{H_0}\,df = \int_{f=0}^{f=1} K_T\,df \qquad [10]$$

However, since the ratio of D/H is a unique function of K_T as shown in Fig. 7, when $\bar{K}_d$ is defined as

$$\bar{K}_d = \bar{D}/H_o \qquad [11]$$

it is seen that

$$\bar{K}_d = \int_{f=0}^{f=1} \frac{D}{H_0}\,df = \int_{f=0}^{f=1} \frac{D}{H} K_T\,df \qquad [12]$$

Equation [12] states that when the ordinate, K_T, of a monthly K_T curve is multiplied by the ratio D/H from Fig. 7, the area under the curve so obtained is numerically equal to the value of $\bar{K}_d$. The value of $\bar{D}$ can then be obtained easily by multiplying $\bar{K}_d$ by the extra-terrestrial daily insolation H_o from Fig. 6.

When the coordinates K_T of the generalized monthly K_T curves of Fig. 12 are multiplied by the ratio D/H from Fig. 7, the curves of Fig. 13 are obtained. The areas under these curves, according to Equation [12], are numerically equal to the values of $\bar{K}_d$. A graphical integration produces the result shown in Table 4.

The value of $\bar{K}_d$ corresponding to $\bar{K}_T = 0.75$ in Table 4 is taken from Fig. 5, since a locality with $\bar{K}_T = 0.75$ should have almost constant clear weather from day to day and therefore K_d and K_T should remain almost constant from day to day and be respectively equal to $\bar{K}_d$ and $\bar{K}_T$.

It should be noticed that the results of Table 4, in which $\bar{K}_d$ is a single valued function of $\bar{K}_T$, can be obtained only when a correlation between the monthly K_T curves and $\bar{K}_T$ such as those shown in Figs. 9, 10 and 11 exists. Indeed this was the reason which initially led the authors to suspect such a correlation, since the relationship between K_d and K_T of Fig. 5, derived from the data of Blue Hill, strongly suggests the existence of a similar unique relationship between $\bar{K}_d$ and $\bar{K}_T$.

In examining Table 4, it should be noted that a locality may be considered to be extremely cloudy, if on a monthly average basis, the daily extraterrestrial solar radiation transmitted through the atmosphere is only 30% ($\bar{K}_T = 0.30$). However, a locality may be considered to be very sunny if $\bar{K}_T = 0.70$. The results of Table 4 show that irrespective of the markedly dif-

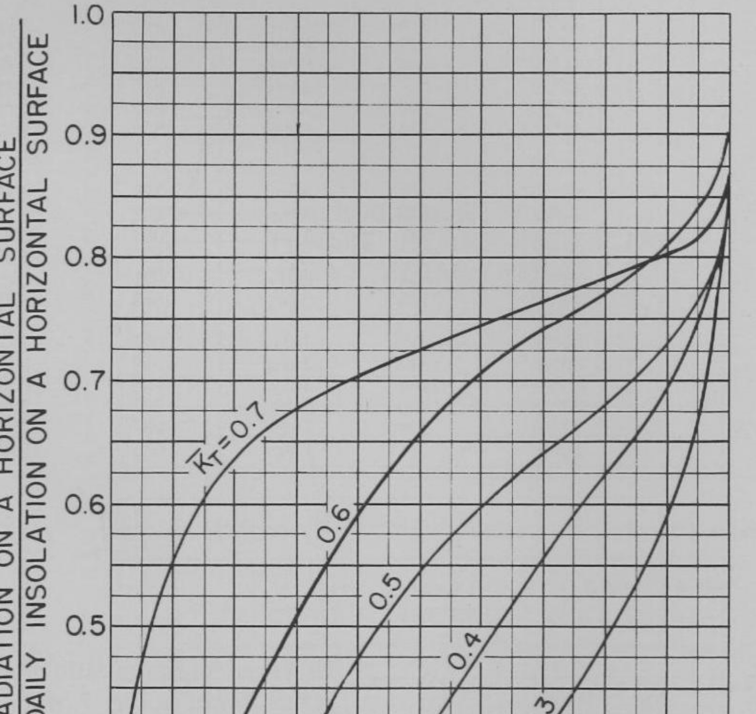


FIG. 12—The generalized monthly K_T curves.

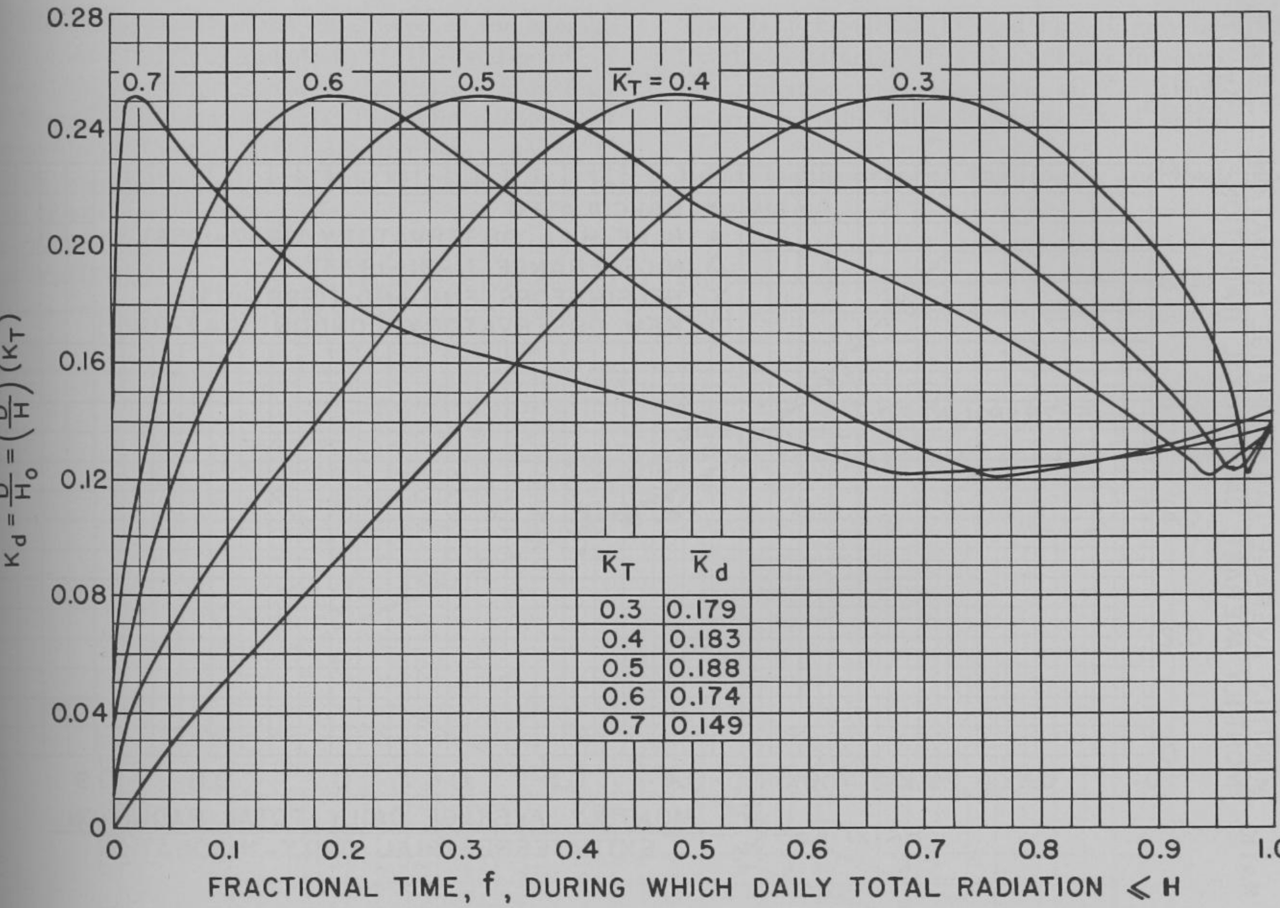


FIG. 13—Curves for the determination of $\bar{K}_d$.

TABLE 4—The Relation Between the Monthly Average Daily Total and the Monthly Average Daily Diffuse Radiation

$\bar{K}_T$	0.3	0.4	0.5	0.6	0.7	(0.75)
$\bar{K}_d$	0.179	0.183	0.188	0.174	0.149	(0.125)

ferent atmospheric conditions associated with the different values of $\bar{K}_T$, on a monthly average basis, the fractions of the extraterrestrial daily insolation transmitted through the atmosphere as diffuse radiation, i.e. the values of $\bar{K}_d$, show only a very moderate variation—from a minimum of 0.125 to a maximum of 0.188. Furthermore an analysis of the data of daily total radiation published by the U. S. Weather Bureau shows that for a great majority (over 70%) of localities and months, the values of $\bar{K}_T$ lie in the range of 0.3 to 0.6. Thus one should expect, from the results of Table 4 alone, that for many localities $\bar{K}_d$ is within the range from 0.174 to 0.188. From the limited diffuse radiation data available, the twelve months average of $\bar{K}_d$ is 0.178 at Blue Hill, Massachusetts, (10 year average);[8] 0.185 at Nice, France, (3 year average);[9] 0.189 at Helsingfors, Finland, (4 year average);[10] and 0.205 at London, England, (5 year average).[11] A further examination of the data of London shows that the reason that the experimental values of $\bar{K}_d$ are higher than those predicted is due, at least in part, to the fact that a "shading ring" correction of 1% to 6% had been added to the measured diffuse radiation. The agreement would have been entirely satisfactory if this correction had not been applied to the measured diffuse radiation as with the data of the other localities.

The ratio $\bar{D}/\bar{H}$ plotted as a function of $\bar{K}_T$ is shown in Fig. 14. The experimental ratio for each month and for each of the four localities are also shown on the same graph for comparison.

Relationship Between Hourly and Daily Diffuse Radiation

A knowledge of the average intensity of diffuse radiation at different times of the day is needed in many problems dealing with solar radiation. Since solar radiation data are not presented for intervals shorter than one hour, the nearest approach to the true average intensity at an instant obtainable from the solar radiation data commonly available is the hourly average intensity. Again it must be emphasized that extremely variable cloudiness precludes the possi-

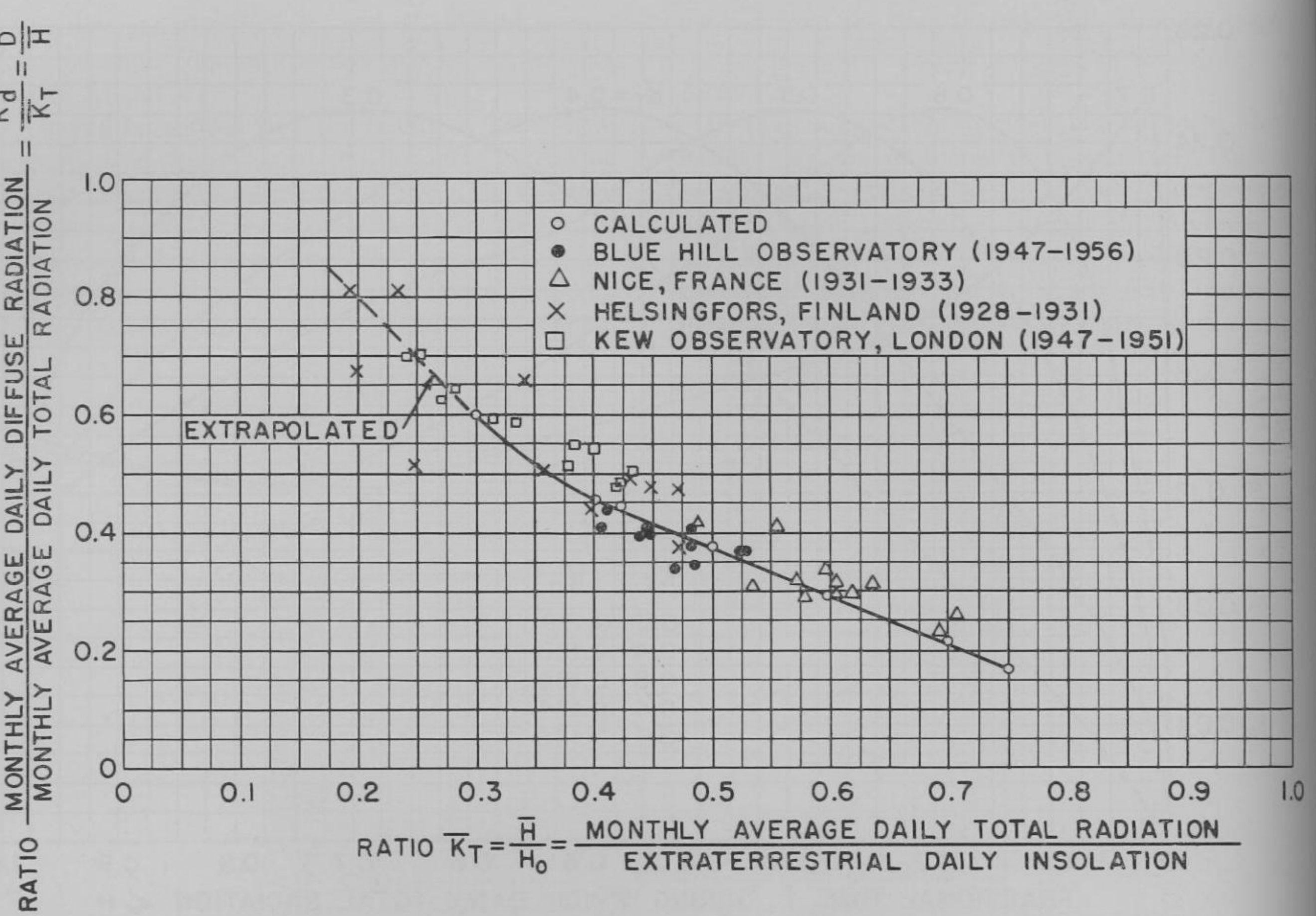


FIG. 14—The ratio of the monthly average daily diffuse radiation to the monthly average daily total radiation as a function of the cloudiness index $\bar{K}_T$.

bility of obtaining a true instantaneous radiation intensity during cloudy days except from direct experimentation.

By Equation [11]

$$\bar{D} = \bar{K}_d H_o \qquad [13]$$

If the assumption is made that the same fraction, $\bar{K}_d$, also represents the ratio of the average intensity of diffuse radiation to the extraterrestrial radiation intensity, i.e.

$$\bar{I}_{dh} = \bar{K}_d I_{oh} \qquad [14]$$

where $\bar{I}_{dh}$ is the average intensity of diffuse radiation received on a horizontal surface, then the ratio r_d of the average intensity of diffuse radiation to the daily diffuse radiation is

$$r_d = \bar{I}_{dh}/\bar{D} = I_{oh}/H_o \qquad [15]$$

The correctness of this assumption can be tested only when the ratio computed by means of Equation [15] is compared with the ratio derived from the experimental data.

An expression for H_o is given in Equation [5], and the instantaneous radiation intensity can be shown to be

$$I_{oh} = rI_{sc}(\cos L \cos \delta \cos \omega + \sin L \sin \delta) \qquad [16]$$

Therefore

$$r_d = \frac{\pi}{24} \frac{\cos L \cos \delta \cos \omega + \sin L \sin \delta}{\cos L \cos \delta \sin \omega_s + \omega_s \sin L \sin \delta} \qquad [17]$$

When the sunset hour angle, ω_s, of Equation [6], is substituted into Equation [17], the expression is obtained,*

$$r_d = \frac{\pi}{24} \frac{\cos \omega - \cos \omega_s}{\sin \omega_s - \omega_s \,\mathrm{cps}\, \omega_s} \qquad [18]$$

Using the ten year data for Blue Hill, Massachusetts, and the four year data for Helsingfors, Finland,[12] the experimental ratio of the monthly average hourly to daily diffuse radiation for the hours: 11:00–12:00 a.m. and 12:00–1:00 p.m., 10:00–11:00 a.m. and 1:00–2:00 p.m., etc., are plotted as shown in Fig. 15 against the sunset hour angle ω_s computed by means of Equation [6] with the use of the mean solar declination for each month. The data for the morning and afternoon hours symmetrical with respect to solar noon have been combined in obtaining these ratios. If these average

---

* An equation which is slightly different, but practically the same as Equation [18] was first derived by Whillier in his investigation of the relation between the hourly and daily total radiation on a horizontal surface to be discussed in the following section.

hourly diffuse radiation are considered as the average intensities of diffuse radiation at the mid-point of these hours, a comparison with the theoretical ratio r_d of Equation [18] can be made. The ratio r_d computed by means of Equation [18] using the hour angle at $\frac{1}{2}$, $1\frac{1}{2}$, $2\frac{1}{2}$, etc., hours from solar noon is shown plotted in Fig. 15 as the solid curves.

The excellent agreement between the experimental ratios and those computed by means of Equation [18] substantiates the assumption made in Equation [14].

Thus both Equation [14] and Fig. 15 may be utilized for the determination of the average intensity of diffuse radiation when the value of $\bar{K}_d$, which can be determined from Table 3 and Fig. 14, is known.

Relationship Between the Hourly and Daily Total Radiation

If the subscript "d" for diffuse radiation in Equations [13] and [14] is replaced by the subscript "T" for total radiation, an expression which is identical to r_d

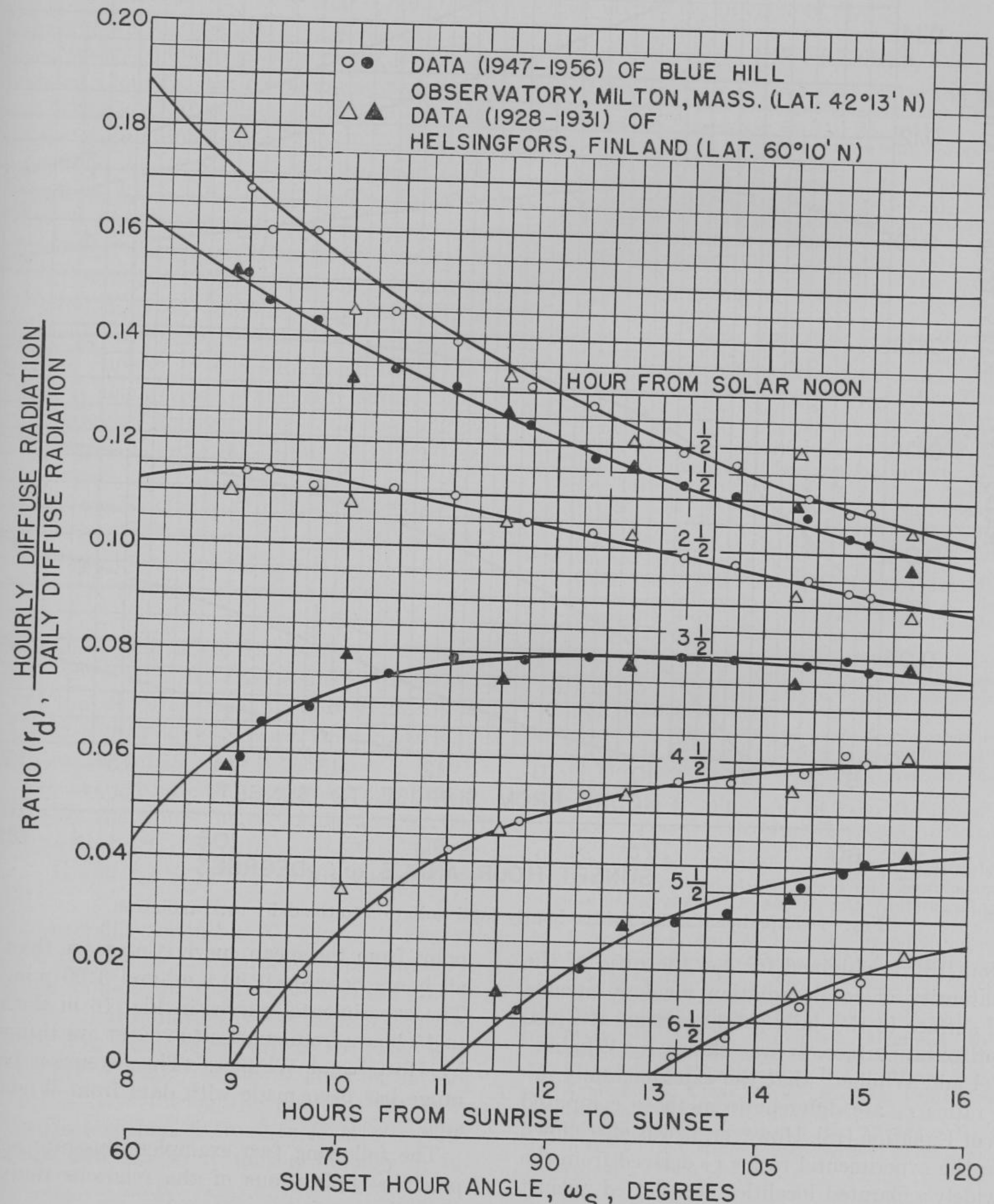


Fig. 15—Theoretical and experimental ratio of the hourly diffuse radiation to the daily diffuse radiation.

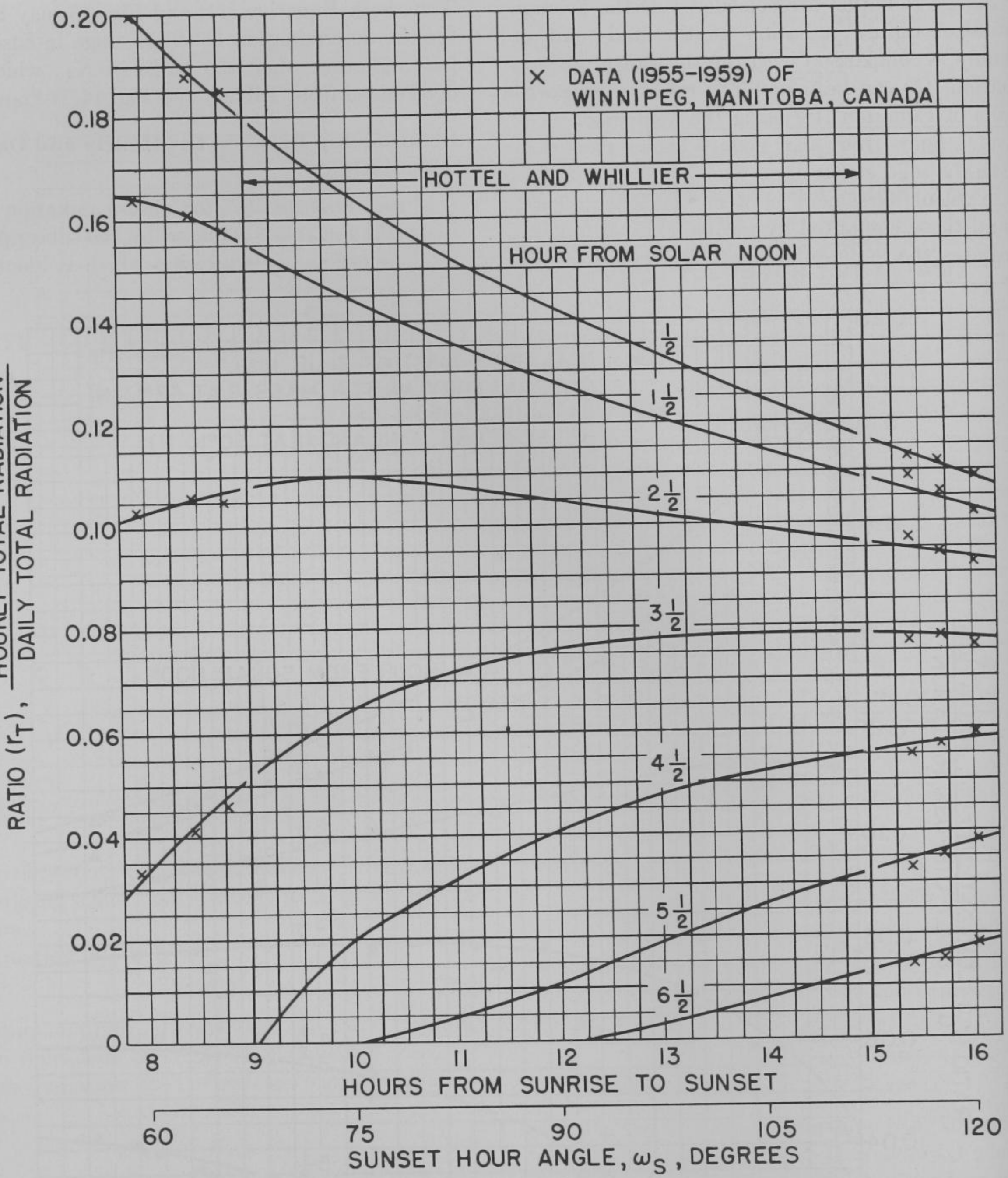


Fig. 16—Experimental ratio of the hourly total radiation to the daily total radiation.

of Equation [18], is obtained for r_T, the ratio of the average intensity of total radiation incident upon a horizontal surface to the daily total radiation received on the horizontal surface. It was shown by Whillier[13] and Hottel and Whillier[14] that the experimentally determined ratios r_T are different from those computed by means of Equation [18]. However, it was also shown that when the experimental ratios r_T derived from the data of widely separated localities are plotted against the sunset hour angle, a mean curve for each hour is obtained such that the deviation of any individual point from the mean curve is no more than ±5% for all hours between 9:00 a.m. and 3:00 p.m. sun time. The experimental curves in Fig. 16 in the range of 9 to 15 hours from sunrise to sunset are those presented by Hottel and Whillier.[14] The extension beyond this range has been made with data from Winnipeg, Canada.[15]

The following two examples illustrate some of the possible applications of the relations derived in the preceding sections.

Example 1: Estimate the intensities of diffuse and total radiation on a horizontal surface at 12:00 noon on June 23 at a locality on 36°N latitude. The direct radiation intensity, I_{Dn}, at normal incidence, according to Threlkeld and Jordan,[16] is 280 Btu/hr-sq ft for the assumed basic atmosphere. The solar altitude is 77.5°.

Solution: From Table 1, on June 23, $r = 0.9670$. Therefore, $I_{on} = rI_{sc} = (0.9670)(442) = 428$ Btu/hr-sq ft and $\tau_D = I_{Dn}/I_{on} = 280/442 = 0.655$. By means of Equation [1], $\tau_d = 0.2710 - (0.2939)(0.655) = 0.079$. Thus $I_{dh} = \tau_d I_{oh} = (0.079)(428)(\sin 77.5°) = 33$ Btu/hr-sq ft and $I_{Th} = I_{Dh} + I_{dh} = (280)(\sin 77.5°) + 33 = 307$ Btu/hr-sq ft.

Had the intensity of total radiation of 307 Btu/hr-sq ft been given, I_{dh} and I_{Dn} can be computed as follows: Since $\tau_T = I_{Th}/I_{oh} = 307/(428)(\sin 77.5°) = 0.734$, and by means of Equation [2] $\tau_d = 0.3840 - (0.4160)(0.734) = 0.079$, $I_{dh} = \tau_d I_{oh} = 33$ Btu/hr-sq ft as before. Therefore, $I_{Dn} = (I_{Th} - I_{dh})/\sin \alpha = (307 - 33)/\sin 77.5° = 280$ Btu/hr-sq ft.

Example 2: The five year (1954–1958) average of the daily total radiation, $\bar{H}$, on a horizontal surface for January in Indianapolis, Indiana, (Lat. 39°44′), is 553 Btu/day-sq ft according to data published by the U. S. Weather Bureau in Climatological Data, National Summary. The average of the daily diffuse radiation and the average intensities of total and diffuse radiation during the hour 11:00–12:00 and 12:00–1:00 are to be estimated.

Solution: By means of Fig. 6 or Equation [5] $H_o = 1370$ Btu/day-sq ft. Therefore $\bar{K}_T = \bar{H}/H_o = 553/1370 = 0.403$, and by means of Fig. 14, $\bar{D}/\bar{H} = 0.454$. Hence $\bar{D} = (\bar{D}/\bar{H})(\bar{H}) = (0.454)(533) = 242$ Btu/day-sq ft.

Since the sunset hour angle, ω_s, is given by Equation [6], and on January 16, the declination is $-21°$, $\cos \omega_s = -(\tan 39°44')(-\tan 21°) = 0.323$, $\omega_s = 71°$ (or $\frac{71}{15} \times 2 = 9.45$ hours between sunrise and sunset). From Figs. 16 and 15, $r_T = 0.172$ and $r_d = 0.161$. Therefore, the average intensities of total and diffuse radiation during the hours 11:00–12:00 and 12:00–1:00 are respectively, $\bar{I}_{Th} = r_T\bar{H} = (0.172)(553) = 95$ Btu/hr-sq ft and $\bar{I}_{dh} = r_d\bar{D} = (0.161)(242) = 39$ Btu/hr-sq ft.

Using the generalized monthly K_T curves of Fig. 12 or Table 3, it is possible to determine approximately, from the given value of $\bar{H}$, the fractional times during which the daily total radiation is less than or equal to certain values. For example, according to Table 3 or Fig. 12, when $K_T = 0.20$, $f = 0.249$. Thus for 25% of the time (or $7\frac{1}{2}$ days during the month), the daily total radiation is less than or equal to $K_T H_o = (0.20)(1370) = 274$ Btu/day-sq ft during January in Indianapolis. Similarly for 76% of the time (or 23 days in the month) the daily total radiation is less than or equal to $(0.60)(1370) = 820$ Btu/day-sq ft. The actual average (1954–1958) number of days in January with radiation below the above values are respectively $6\frac{1}{2}$ and 23 days in Indianapolis.

The generalized monthly K_T curves can also be utilized to determine approximately the statistical distribution of the daily total radiation for localities where only an estimate of the monthly average of the daily total radiation is known[17] since the only information needed is the value of $\bar{H}$.

REFERENCES

1. Parmelee, G. V., "Irradiation of Vertical and Horizontal Surfaces by Diffuse Solar Radiation from Cloudless Skies," *ASHVE Transaction* **60**: 341–358, 1954.
2. Johnson, F. S., "The Solar Constant," *Journal of Meteorology*, **11**: 431–439, December, 1954.
3. Drummond, A. J. and Greer, H. W.: "Fundamental Pyrheliometry," *The Sun at Work* **3**(2): 3–5, 11, June, 1958.
4. Klein, W. H., "Calculation of Solar Radiation and the Solar Heat Load on Man," *Journal of Meteorology*, **5**: 119–129, August, 1948.
5. Moore, A. F. and Abbot, L. H., "The Brightness of the Sky," *Smithsonian Miscellaneous Collection*, **71**(4): 1–36, February, 1920.
6. Fritz, S., "Solar Radiation During Cloudless Days," *Heating and Ventilating*, **46**: 69–74, January, 1949.
7. Hand, I. F., "Methods of Calculating Solar Radiation Values at Blue Hill Observatory, Milton, Massachusetts," *Monthly Weather Review*, **82**: 43–49, February, 1954.
8. Diffuse and total radiation data for Blue Hill, Massachusetts, were obtained from Mr. C. V. Cuniff, U. S. Weather Bureau, Blue Hill Observatory, Milton, Massachusetts. For instrumentation and methods of measurement see: Hand, I. F. and Wollaston, F. A., "Measurements of Diffuse Solar Radiation at Blue Hill Observatory," *Technical Paper No. 18*, U. S. Department of Commerce, Weather Bureau, Washington, D. C., May, 1952.
9. Gorcynski, W., "Enregistrements due Rayonnement Solaire au Moyan des Solarigraphy et des Pyrheliographes," *Nice. Office Meteorologique, Annales*, Tome **II**: 117–165, 1933.
10. Lunelund, H., "Contribution to the Knowledge of Solar Radiation in Finland," *Finska Vetenskaps-Societeten, Helsingfors, Commentationes Physico-Mathematicae*, **7**(11): 1–58, February, 1934.
11. Blackwell, M. J., "Five Years Continuous Recording of Total and Diffuse Radiation at Kew Observatory," *Paper of the Meteorological Research Committee (London)*, No. 895, 1954.
12. Lunelund, H., "Records of Solar Radiation in Helsingfors," *Finska Vetenskaps-Societeten, Helsingfors, Commentationes Physico-Mathematicae* **7**(1): 1–28, 1933.
13. Whillier, A., "The Determination of Hourly Values of Total Solar Radiation from Daily Summations," *Archiv fur Meteorologie, Geophysik und Bioklimatologie*, Vienna, Series B., **7**: 197–244, 1956.
14. Hottel, H. C. and Whillier, A., "Evaluation of Flat-Plate Solar Collector Performance," *Transaction of the Conference on the Use of Solar Energy*: The Scientific Basis, Vol. II, Part I, Section A, pp. 74–104, 1955.
15. Data of total radiation for Winnipeg, Canada, are obtained from Mr. D. M. Robertson, Regional Meteorologist, District Aviation Forecast Office, Winnipeg, Canada.
16. Threlkeld, J. L. and Jordan, R. C., "Direct Solar Radiation Available on Clear Days," *Heating, Piping and Air Conditioning*, **29**:(12): 135–145, December, 1957.
17. Fritz, S. and MacDonald, T. H., "Average Solar Radiation in the United States," *Heating and Ventilating*, **46**(7): 61–64, July, 1949.