Table E.3.1.1. Composition of the Date Fruit and Densities of Food Components at 25°C

Component	*Weight (%)*	*Density (kg/m³)*
Water	22.5	995.7
Carbohydrate	72.9	1592.9
Protein	2.2	1319.6
Fat	0.5	917.15
Ash	1.9	2418.2

where

T_{ref} = reference temperature (60°C) which is a typical temperature of air drying of foods,
R = the ideal gas constant (8.3143 J/mol K),
k_0 = thermal conductivity at moisture $X = 0$ and temperature $T = T_{ref}$ (W/m K),
k_i = thermal conductivity at moisture $X = \infty$ and temperature $T = T_{ref}$ (W/m K),
E_0 = activation energy for heat conduction in dry material at $X = 0$ (J/mol),
E_i = activation energy for heat conduction in wet material at $X = \infty$ (J/mol).

Example 3.1. The composition of date fruit (*Phoenix dactylifera*) and the densities of food components are given in Table E.3.1.1. Determine the thermal conductivity of the fruit at 25°C, using parallel, series, and isotropic Kopelman models.

Solution:

To calculate the thermal conductivity of the fruit using predictive models, thermal conductivity values of food components at 25°C are required. They can be calculated using Eqs. (3.15)–(3.19) (Table E.3.1.2).

Using the composition and density of components data given in the question, the specific volume of each component is calculated:

$$\text{Specific volume of component } i = \frac{\text{Mass fraction of component } i}{\text{Density of component } i}$$

Then, total specific volume is determined by adding the volume of each component and found as 7.14×10^{-4} m³. Volume fractions of components are calculated by dividing the component volume to total volume. Specific volumes and volume fractions of each component are given in Table E.3.1.3.

Table E.3.1.2. Thermal Conductivity Values of Components at 25°C

Component	*Thermal Conductivity Equation*	*Eq. no.*	*k_i (W/m K)*
Water	$k_{water} = 0.57109 + 1.7625 \times 10^{-3} T - 6.7036 \times 10^{-6} T^2$	(3.15)	0.610
Carbohydrate	$k_{CHO} = 0.20141 + 1.3874 \times 10^{-3} T - 4.3312 \times 10^{-6} T^2$	(3.16)	0.233
Protein	$k_{protein} = 0.17881 + 1.1958 \times 10^{-3} T - 2.7178 \times 10^{-6} T^2$	(3.17)	0.207
Fat	$k_{fat} = 0.18071 - 2.7604 \times 10^{-3} T - 1.7749 \times 10^{-7} T^2$	(3.18)	0.112
Ash	$k_{ash} = 0.32961 + 1.4011 \times 10^{-3} T - -2.9069 \times 10^{-6} T^2$	(3.19)	0.363

Table E.3.1.3. Specific Volume and Volume Fraction of Components in Date Fruit

Component	*Specific Volume (m^3/kg)*	*Volume Fraction of Components (X_i^v)*
Water	2.26×10^{-4}	0.320
Carbohydrate	4.58×10^{-4}	0.640
Protein	1.67×10^{-5}	0.023
Fat	5.45×10^{-6}	0.0076
Ash	7.86×10^{-6}	0.011

Thermal conductivity of date fruit is calculated by using volume fractions and thermal conductivity values of components using the parallel model (Eq. 3.22):

$$k_{pa} = \sum_{i=1}^{n} k_i X_i^v \tag{3.22}$$

$$k_{pa} = (0.61)(0.32) + (0.233)(0.64) + (0.207)(0.023) + (0.112)(0.0076) + (0.363)(0.011) = 0.353 \text{ W/mK}$$

Using the series model (Eq. 3.25):

$$\frac{1}{k_{se}} = \sum_{i=1}^{n} \frac{X_i^v}{k_i} \tag{3.25}$$

$$= \frac{0.32}{0.61} + \frac{0.64}{0.233} + \frac{0.023}{0.207} + \frac{0.0076}{0.112} + \frac{0.011}{0.363}$$

$$= 3.48 \text{ mK/W}$$

$$k_{\text{se}} = \frac{1}{3.48} = 0.287 \text{ W/mK}$$

In the case of the isotropic model of Kopelman (Eqs. 3.31–3.33), the k value of date fruit is calculated using the order of water (1), carbohydrate (2), protein (3), fat (4), and ash (5) in the iteration.

$$k_{\text{comp},i+1} = \frac{k_i(1 - Q_{i+1})}{1 - Q_{i+1}\left[1 - \left(X_{d,i+1}^v\right)^{1/3}\right]} \tag{3.31}$$

where

$$Q_{i+1} = \left(X_{d,i+1}^v\right)^{2/3}\left[1 - \frac{k_{i+1}}{k_i}\right] \tag{3.32}$$

$$X_{d,i+1}^v = \frac{V_{i+1}}{\sum_{i}^{i+1} V_i} \tag{3.33}$$

Starting with the water continuous and carbohydrate (CHO) dispersed phase, the thermal conductivity of water–CHO system is calculated as:

$$X_{d,\text{CHO}}^v = \frac{V_{\text{CHO}}}{V_{\text{water}} + V_{\text{CHO}}} = \frac{4.58 \times 10^{-4}}{2.26 \times 10^{-4} + 4.58 \times 10^{-4}} = 0.669$$

$$Q_{\text{CHO}} = \left(X_{d,\text{CHO}}^v\right)^{2/3}\left[1 - \frac{k_{\text{CHO}}}{k_{\text{water}}}\right]$$

$$Q_{\text{CHO}} = (0.669)^{2/3}\left[1 - \frac{0.233}{0.610}\right] = 0.473$$

$$k_{\text{water-CHO}} = \frac{k_{\text{water}}(1 - Q_{\text{CHO}})}{1 - Q_{\text{CHO}}\left[1 - \left(X^v_{d,\text{CHO}}\right)^{1/3}\right]}$$

$$\text{k}_{\text{water-CHO}} = \frac{0.61(1 - 0.473)}{1 - 0.473\left[1 - (0.669)^{1/3}\right]} = 0.342\ \text{W/m K}$$

Then, the water–carbohydrate phase is taken as the continuous phase and protein as the dispersed phase.

$$X^v_{d,\text{prot}} = \frac{V_{\text{prot}}}{V_{\text{water}} + V_{\text{CHO}} + V_{\text{prot}}} = \frac{1.67 \times 10^{-5}}{2.26 \times 10^{-4} + 4.58 \times 10^{-4} + 1.67 \times 10^{-5}} = 0.024$$

$$Q_{\text{prot}} = \left(X^v_{d,\text{prot}}\right)^{2/3}\left[1 - \frac{k_{\text{prot}}}{k_{\text{water-CHO}}}\right]$$

$$Q_{\text{prot}} = (0.024)^{2/3}\left[1 - \frac{0.207}{0.342}\right] = 0.033$$

$$k_{\text{water-CHO-prot}} = \frac{k_{\text{water-CHO}}(1 - Q_{\text{prot}})}{1 - Q_{\text{prot}}\left[1 - \left(X^v_{d,\text{prot}}\right)^{1/3}\right]}$$

$$k_{\text{water-CHO-prot}} = \frac{0.342(1 - 0.033)}{1 - 0.033\left[1 - (0.024)^{1/3}\right]} = 0.338\ \text{W/m K}$$

The same procedure is followed throughout all phases to find the thermal conductivity of date fruit. The calculated values in the iterative procedure are shown in Table E.3.1.4. The thermal conductivity of date fruit is found to be 0.337 W/m K using the isotropic model of Kopelman.

3.2.2 Measurement of Thermal Conductivity

Measurement of thermal conductivity can be done by either steady-state or transient-state methods. There are a number of experimental measurement techniques under each of these two categories. Mohsenin (1980) and Rahman (1995) previously reviewed various thermal properties measurement methods for food materials.

Table E.3.1.4. Results Obtained in the Iterative Procedure

System	X^v_d	*Q*	*k (W/m K)*
Water–CHO	0.669	0.473	0.342
Water–CHO–protein	0.024	0.033	0.338
Water–CHO–protein–fat	0.008	0.026	0.337
Water–CHO–protein–fat–ash (date fruit)	0.011	−0.004	0.337