$$\overline{\Gamma, A \vdash A}^{(ax)} \qquad \overline{\vec{u} \Vdash \Gamma, v \Vdash A \vdash v \Vdash A}^{(ax)}$$

ídem: (ex) y (W)

son ≠

hip. de inducción

$$\frac{\Gamma, A, A \vdash B}{\Gamma, A \vdash B}(C) \longrightarrow \frac{(*)\ \vec{u} \Vdash \Gamma \quad v \Vdash A \quad w \Vdash A \vdash e \Vdash B}{}(C)$$

Pero, sustituyendo $\{w/v\}$ en toda la prueba de $(*)$ se llega a una prueba de $\vec{u} \Vdash \Gamma, w \Vdash A, w \Vdash A \vdash e\{w/v\} \Vdash B$

$$(C)\ \overline{\vec{u} \Vdash \Gamma, w \Vdash A \vdash e\{w/v\} \Vdash B}$$

Hip. de ind.

$$\frac{\Gamma \vdash A_1 \qquad \Gamma \vdash A_2}{\Gamma \vdash A_1 \wedge A_2} \qquad \frac{\vec{u} \Vdash \Gamma \vdash e_1 \Vdash A_1 \qquad \vec{u} \Vdash \Gamma \vdash e_2 \Vdash A_2}{\vec{u} \Vdash \Gamma \vdash \underbrace{e_1 \Vdash A_1 \wedge e_2 \Vdash A_2}_{\Updownarrow \text{def.}}}\wedge$$

$e := \langle e_1, e_2 \rangle$

$e = e(\vec{u}, v, F(\Gamma, A, B))\ e \Vdash A_1 \wedge A_2$

$$\frac{\Gamma, A \vdash B}{\Gamma \vdash A \to B} \qquad \frac{\vec{u} \Vdash \Gamma, v \Vdash A \vdash e \Vdash B}{\vec{u} \Vdash \Gamma \vdash v \Vdash A \to e \Vdash B}(\to I)$$

$$(\forall I)\ \frac{\vec{u} \Vdash \Gamma \vdash v \Vdash A \to e \Vdash B}{\vec{u} \Vdash \Gamma \vdash \forall v.(v \Vdash A \to \{\ulcorner \lambda v.e \urcorner\}(v) \Vdash B)}$$

$\ldots v.e^\urcorner\}(v) = e\{v/v\} = e$

$T(\ulcorner \lambda v.e \urcorner, v, k) \wedge e = U(k)$

$\ulcorner \lambda n.0 \urcorner \Vdash \underbrace{S(x)=0}_{\substack{\text{no tiene} \\ \text{realizadores.}}} \to \bot$

$\emptyset \to \emptyset$

$\forall x. \left(\underbrace{x \Vdash S(x)=0}_{\substack{? \\ \bot}} \to \{\ulcorner \lambda n.0 \urcorner\}(x) \Vdash \bot \right)$

$A(x,y) := (y=0 \wedge \exists k.\ T(x,x,k)) \vee (y \neq 0 \wedge \forall k\ \neg T(x,x,k))$

En PA vale $\forall x \exists y\ A(x,y)$ pero no vale que exista e tal que $\forall x.\ A(x,\{e\}(x))$ porque si no $K = \{x \mid \{x\}(x)\downarrow\}$ sería decidible.

(CT_0) es realizable en Heyting:

$\underbrace{\forall x. \exists y. A(x,y)}_{\Vdash\, n}$ Hip

$\forall x. \{n\}(x)\downarrow \Vdash \exists y. A(x,y)$

$\forall x.\ \{n\}(x)\downarrow \wedge (\{n\}(x))^2 \Vdash A(x, (\{n\}(x))^1)$

$r \Vdash \exists e. \forall x\ A(x,\{e\}(x)) \iff r^2 \Vdash \forall x. A(x, \{r^1\}(x))$

$\iff \forall x.\ \{r^2\}(x)\downarrow \Vdash A(x, \{r^1\}(x))$

$\underbrace{\langle \ulcorner \lambda x. (\{n\}(x))^1 \urcorner, \ulcorner \lambda x. (\{n\}(x))^2 \urcorner \rangle}_{\substack{!! \\ r}} \Vdash \exists e. \forall x.\ A(x, \{e\}(x))$ tesis.

Pero $PA \nvdash CT_0$ porque el halting problem es indecidible

Así que $HA \nvdash CT_0$

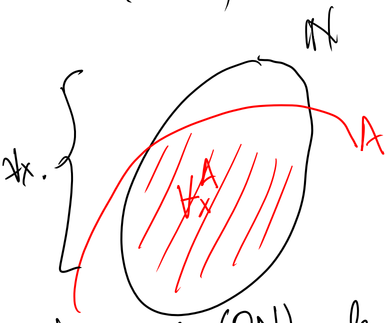
Fórmulas Quasi Negativas

$$A, B ::= \underbrace{\bot \mid t=s}_{\text{atómicas}} \mid \exists x.\, t=s \mid A \wedge B \mid A \to B \mid \forall x.\, A$$

ECT_0 : $\forall^A x.\, \exists y.\, B(x,y) \to \exists e.\, \forall^A x.\, B(x, \{e\}(x))$

A es Quasi negativa

$\forall^A_x B := \forall x.(A(x) \to B)$



Lema 9.1: Para fórmulas A Quasi-Negativas (QN) vale.

(1) $HA \vdash (\exists x.\, x \Vdash A) \to A$.

definido a partir de A

(2) Existem términos γ_A tal que $HA \vdash A \to \gamma_A \Vdash A$

Em part.: $HA \vdash (\exists x.\, x \Vdash A) \to \gamma_A \Vdash A$

$\vdash (\exists x.\, x \Vdash A) \leftrightarrow A$

Dem: