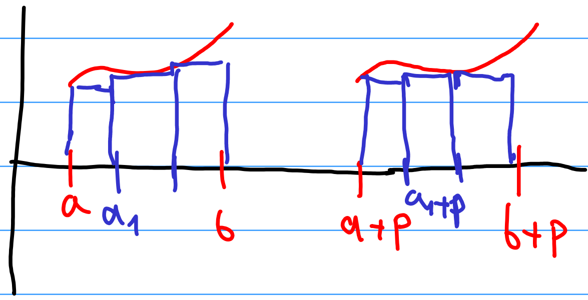
3.5.1. $f:\mathbb{R}\to\mathbb{R}$ integrable, $a<b$

a) $p\in\mathbb{R}$

$$\int_a^b f(t) = \int_{a+p}^{b+p} f(t-p)\,dt$$



Llamo $g(t)=f(t-p)$

Si P es una partición de $[a,b]$, $P+\{p\}$ es una partición de $[a+p,b+p]$ y

1er paso Probar $S^*(f,P) = S^*(g,P+\{p\})$

$S_*(f,P) = S_*(g,P+\{p\})$

2º paso Pasar al supremo e ínfimo

$$I^*(f,[a,b]) = I^*(g,[a+p,b+p])$$

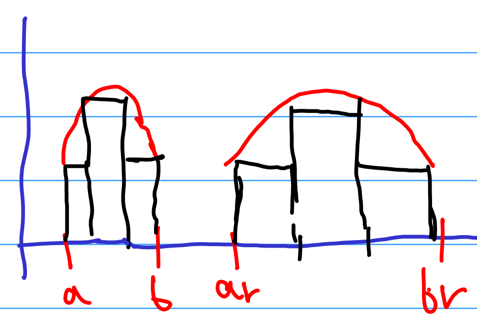
$$I_*(f,[a,b]) = I_*(g,[a+p,b+p])$$

3er paso Concluir

$h(t)=f\left(\frac{t}{r}\right)$

b) $r\int_a^b f(t)\,dt = \int_{ar}^{br} f\left(\frac{t}{r}\right)dt$

$r>0$



$P\mapsto rP$ y $rS^*(f,P)=S^*(h,rP)$

Cambio de variable lineal en la práctica:

$a,b,p,q \in \mathbb{R}$,
$$\int_a^b f(px+q)\,dx = \int_{\underbrace{pa+q}_{u(a)}}^{pb+q = u(b)} f(u)\,\underbrace{\frac{1}{p}\,du}_{dx}$$

$\rightarrow u = px+q$

$du = p\,dx$

$dx = \frac{1}{p}\,du$

Ejemplo
$$\int_2^4 \frac{1}{3x+1}\,dx = \int_{7 = 3\cdot 2+1}^{13 = 3\cdot 4+1} \frac{1}{u}\,\frac{1}{3}\,du$$

$u = 3x+1$

$du = 3\,dx$

$\log = \ln$

$$= \frac{1}{3}\int_7^{13} \frac{1}{u}\,du = \frac{1}{3}\left(\log(13) - \log(7)\right)$$
$$= \frac{1}{3}\log\left(\frac{13}{7}\right)$$

Integrales conocidas:
$$\int_a^b x^n\,dx = \left.\frac{x^{n+1}}{n+1}\right|_a^b = \frac{b^{n+1}}{n+1} - \frac{a^{n+1}}{n+1}$$

válido para $n \neq -1$, probado en el curso para $n = 0, 1, 2, \frac{1}{2}$

$a,b > 0$
$$\int_a^b \frac{1}{x}\,dx = \log(x)\Big|_a^b = \log(b) - \log(a) = \log\left(\frac{b}{a}\right)$$

Propiedades

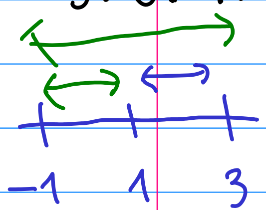
$$\int_a^b (f+g)(x)\,dx = \int_a^b f(x)\,dx + \int_a^b g(x)\,dx$$

$$\int_a^b k\cdot f(x)\,dx = k\int_a^b f(x)\,dx$$

$$\int_a^c f(x)\,dx = \int_a^b f(x)\,dx + \int_b^c f(x)\,dx$$

$a > b$ $$\int_a^b f(x)\,dx = -\int_b^a f(x)\,dx$$

3.6.1.a) $h: \mathbb{R} \to \mathbb{R}$ $\quad \int_{-1}^{1} h(t)\,dt = 0, \quad \int_{-1}^{3} h(t)\,dt = 6$



$$\int_1^3 h(t)\,dt = \int_{-1}^{3} h(t)\,dt - \int_{-1}^{1} h(t)\,dt =$$

$$6 - 0 = 6$$

4. a) $$\int_1^4 3x-2\,dx = \int_1^4 3x\,dx + \int_1^4 -2\,dx$$

$$= 3\int_1^4 x\,dx - 2\int_1^4 1\,dx = 3\left.\frac{x^2}{2}\right|_1^4$$

$$-2x\Big|_1^4 = 3\left(\frac{16}{2} - \frac{1}{2}\right) - 2(4-1)$$

$$= 3\left(8 - \frac{1}{2}\right) - 6 = \frac{45}{2} - 6$$

$$= \frac{33}{2}$$

d) $\int_0^2 2x^2+x-3\,dx = 2\frac{x^3}{3}+\frac{x^2}{2}-3x\Big|_0^2$

$= 2\frac{2^3}{3}+\frac{2^2}{2}-3\cdot 2 - 0 = \frac{16}{3}+2-6 =$

$= \frac{16}{3}-4 = \frac{4}{3}$

g) $\int_0^2 (u+3)(u+1)\,du = \int_0^2 u^2+4u+3\,du =$

$\frac{u^3}{3}+4\frac{u^2}{2}+3u\Big|_0^2 = \frac{8}{3}+8+6$

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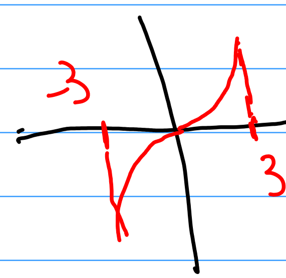
5. a) $\int_{-3}^{3} u^{101}\,du$



$\int_{-3}^{3} u^{101}\,du = \int_{-3}^{0} u^{101}\,du + \int_0^3 u^{101}\,du =$

$\boxed{t=-u}$, $u=-t$

$dt=-du$

$\int_3^0 (-t)^{101}(-1)\,dt + \int_0^3 u^{101}\,du = 0$

$\int_3^0 + t^{101}(-\cancel{1})\,dt + \int_0^3 u^{101}\,du = -\int_0^3 t^{101}\,dt + \int_0^3 u^{101}\,du$

$$8.\ g)\ \int_3^5 \sqrt{16-2x}\,dx = \int_{10}^{6} \sqrt{u}\ \left(-\frac{1}{2}\right) du$$

$$u = 16-2x$$
$$du = -2\,dx$$

$$= -\frac{1}{2}\int_{10}^{6} \underbrace{\sqrt{u}}_{u^{1/2}}\,du = -\frac{1}{2}\,\frac{u^{\frac{1}{2}+1}}{\frac{1}{2}+1}\Bigg|_{10}^{6} =$$

$$-\frac{1}{2}\left(\underbrace{6^{3/2}}_{3/2} - \underbrace{10^{3/2}}_{3/2}\right) = -\frac{1}{2}\left(\underbrace{\sqrt{216}}_{3/2} - \underbrace{\sqrt{1000}}_{3/2}\right)$$

$$= -\frac{1}{3}\left(\sqrt{216} - \sqrt{1000}\right)$$

$$9.\ a)\ \int_0^3 |2u-2|\,du = \int_0^1 |2u-2|\,du + \int_1^3 |2u-2|\,du$$

$$2u-2 \geq 0 \iff u \geq 1$$

$$= \int_0^1 -2u+2\ du + \int_1^3 2u-2\ du =$$

$$-2\frac{u^2}{2} + 2u\Big|_0^1 + 2\frac{u^2}{2} - 2u\Big|_1^3 =$$

$$-2\frac{1}{2} + 2 - 0 + 2\frac{3^2}{2} - 2\cdot 3 - \left(2\frac{1}{2} - 2\right)$$