

Distribucion conjunta de variables aleatorias discretas

X, Y variables aleatorias discretas

X toma valores en R_x

Y toma valores en R_y

la funcion de probabilidad conjunta se define como

$$P_{x,y}(x,y) = P(X=x, Y=y) = P(\{X=x\} \cap \{Y=y\})$$

$$\sum_{\substack{x \in R_x \\ y \in R_y}} P_{x,y}(x,y) = 1$$

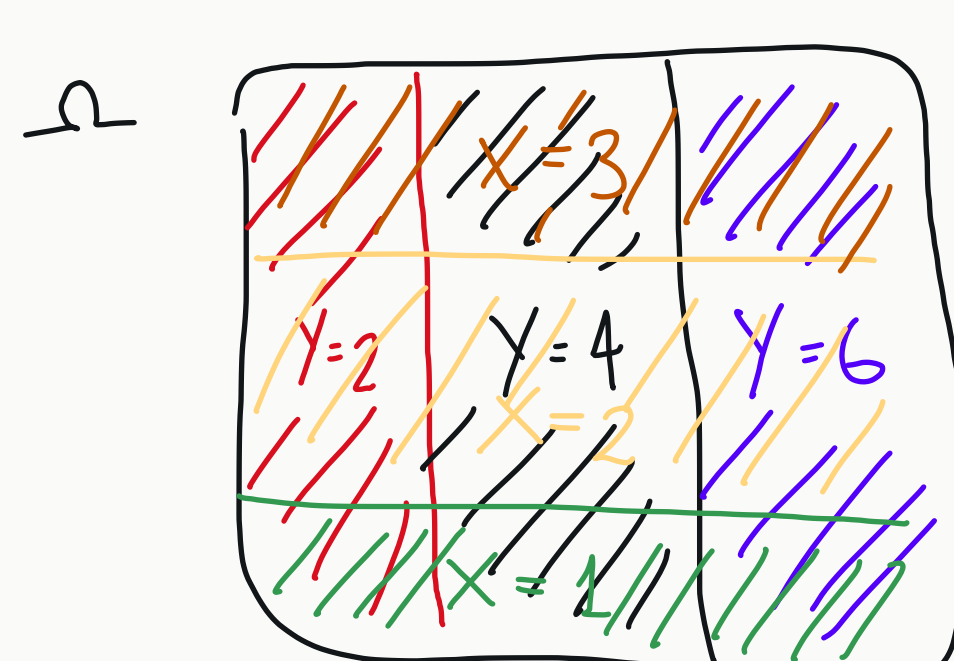
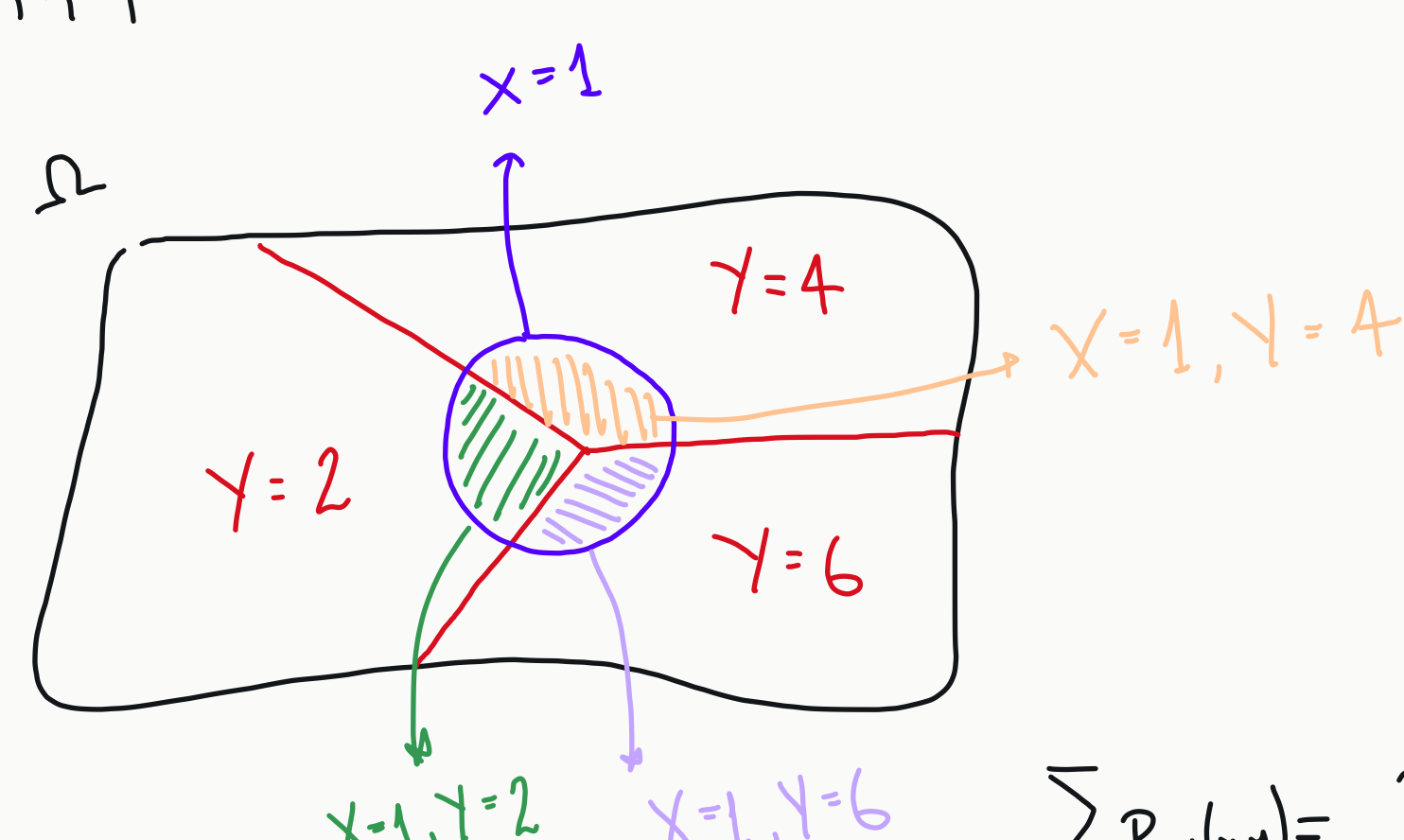
funciones de probabilidad marginales

$$P_x(x) = P(X=x) = \sum_{y \in R_y} P_{x,y}(x,y)$$

$$P_y(y) = \sum_{x \in R_x} P_{x,y}(x,y)$$

$$R_x = \{1, 2, 3\} \quad P_x(1) = P(X=1) = P[\{X=1\} \cap \{Y=2\} \cup \{X=1\} \cap \{Y=4\} \cup \{X=1\} \cap \{Y=6\}]$$

$$R_y = \{2, 4, 6\} \quad P(X=1, Y=2) + P(X=1, Y=4) + P(X=1, Y=6)$$



$$\sum_{\substack{y \in R_y \\ x \in R_x}} P_{x,y}(x,y) = P(X=3, Y=2) + P(X=3, Y=4) + P(X=3, Y=6) + \dots + P(X=1, Y=6) = P(\Omega) = 1$$

X e Y son independientes $\Leftrightarrow P(X=x, Y=y) = P(X=x) P(Y=y)$

$$\Rightarrow P_{x,y}(x,y) = P_x(x) P_y(y)$$

1 - 9 personas (2 contadores, 3 abogados, 4 "otros")

Se eligen 5 X = cant. de contadores
 Y = " de abogados

$$\textcircled{1} R_x = \{0, 1, 2\}$$

$$R_y = \{0, 1, 2, 3\}$$

\textcircled{2}

$X \setminus Y$	0	1	2	3	P_x
0	$\frac{C_2^3 C_3^2 C_4^1}{C_5^5}$	$\frac{C_2^3 C_3^1 C_4^2}{C_5^5}$	$\frac{C_2^2 C_3^2 C_4^3}{C_5^5}$	$\frac{C_2^1 C_3^3 C_4^4}{C_5^5}$	$P_x(0)$
1	$\frac{C_2^2 C_3^2 C_4^3}{C_5^5}$	$\frac{C_2^2 C_3^1 C_4^4}{C_5^5}$	$\frac{C_2^1 C_3^2 C_4^4}{C_5^5}$	$\frac{C_2^3 C_3^1 C_4^4}{C_5^5}$	$P_x(1)$
2	$\frac{C_2^1 C_3^3 C_4^4}{C_5^5}$	$\frac{C_2^1 C_3^2 C_4^4}{C_5^5}$	$\frac{C_2^2 C_3^1 C_4^4}{C_5^5}$	$\frac{C_2^3 C_3^2 C_4^4}{C_5^5}$	$P_x(2)$

$$P_x(0) = P(X=0) = P(X=0, Y=0) + P(X=0, Y=1) + P(X=0, Y=2) + P(X=0, Y=3)$$

$$P\{X=Y\} = P(X=0, Y=0) + P(X=1, Y=1) + P(X=2, Y=2)$$

$$X \text{ e } Y \text{ indep.} \Leftrightarrow P(X=x, Y=y) = P(X=x) P(Y=y) \quad \forall x \in R_x, y \in R_y$$

$$\left. \begin{array}{l} P(X=0, Y=0) = 0 \\ P(X=0) \neq 0 \quad P(Y=0) \neq 0 \end{array} \right\} \underbrace{P(X=0, Y=0)}_{=0} \neq \underbrace{P(X=0) P(Y=0)}_{\neq 0}$$

2 -

$X \setminus Y$	2	4	6
-1	0,2	0,25	0,15
1	0,1	α	0,25

$$\sum_{\substack{x \in \{-1, 1\} \\ y \in \{2, 4, 6\}}} P_{x,y}(x,y) = 1$$

$$0,2 + 0,25 + 0,15 + 0,1 + \alpha + 0,25 = 1 \Rightarrow \boxed{\alpha = 0,05}$$

$$P\{X < 1, Y \leq 4\} = P(X=-1, Y=2) + P(X=-1, Y=4) = 0,45$$

$$X + 2Y = U \quad \text{recorrido de } U: \{3, 7, 11, 5, 9, 13\}$$

$$P_u(x) = \begin{cases} 0,2 & \text{si } x=3 \\ 0,1 & \text{si } x=5 \\ 0,25 & \text{si } x=7 \\ 0,05 & \text{si } x=9 \\ 0,15 & \text{si } x=11 \\ 0,25 & \text{si } x=13 \end{cases}$$

$$P_u(3) = P(U=3) = P(X+2Y=3) = P(X=-1, Y=2) = 0,2$$

$$U = |X - Y| \quad \text{recorrido de } U: \{1, 3, 5, 7\}$$

$$P_u(x) = \begin{cases} 0,1 & \text{si } x=1 \\ 0,25 & \text{si } x=3 \\ 0,5 & \text{si } x=5 \\ 0,15 & \text{si } x=7 \end{cases}$$

$$P_u(1) = P(U=1) = P(|X-Y|=1) = P(X=-1, Y=2) = 0,1$$

$$P_u(3) = P(U=3) = P(|X-Y|=3) = P(X=1, Y=4) + P(X=-1, Y=2) = 0,25$$