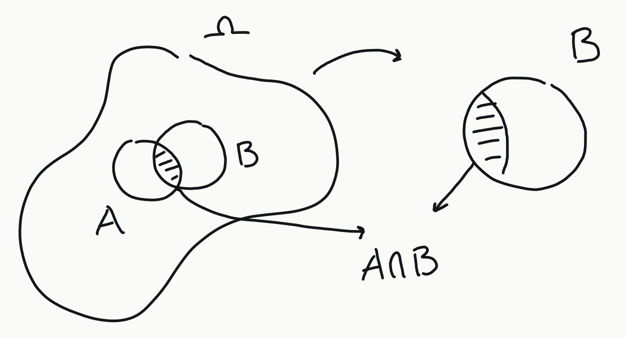
Probabilidad condicional

A y B eventos del espacio muestral Ω, $P(B)>0$

la probabilidad de que ocurra A sabiendo que ocurrio B es $P(A|B)$



$$\boxed{P(A|B)=\frac{P(A\cap B)}{P(B)}}$$

- regla del producto $P(A\cap B)=P(A|B)P(B)$
- formula de probabilidad total A y B eventos $A=(A\cap B)\cup(A\cap B^c)$ union disjunta

$P(A)=P(A\cap B)+P(A\cap B^c)=P(A|B)P(B)+P(A|B^c)P(B^c)$

en general si $C_1,\dots,C_n$ eventos disjuntos tales que

$\bigcup_{i=1}^{n} C_i=\Omega$ entonces $P(A)=P(A|C_1)P(C_1)+P(A|C_2)P(C_2)+\dots+P(A|C_n)P(C_n)$

- A y B son independientes $\iff P(A\cap B)=P(A)P(B)$

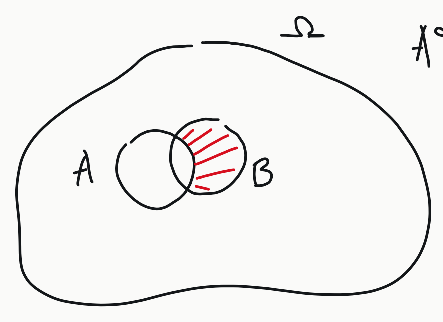
$$P(A|B)=\frac{P(A\cap B)}{P(B)}=\frac{P(A)P(B)}{P(B)}=P(A)$$

4 A y B eventos

① $P(A)=1/2$ $P(B)=1/3$ $P(A\cap B)=1/4$

a- $P(A|B)=\dfrac{P(A\cap B)}{P(B)}=\dfrac{1/4}{1/3}=\dfrac{3}{4}$

c- $P(A^c|B)=\dfrac{P(A^c\cap B)}{P(B)}$



$A^c\cap B=B\setminus(A\cap B)$ $\boxed{A\cap B\subset B}$

$P(A^c\cap B)=P(B)-P(A\cap B)$

$$P(A^c|B)=\frac{P(B)-P(A\cap B)}{P(B)}=1-3/4=1/4$$

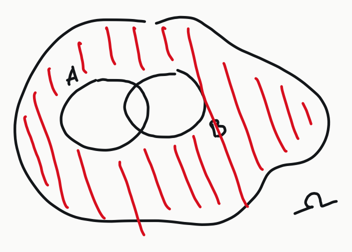
$P(A|B)$

$$P(A^c|B)=\frac{P(B)}{P(B)}-\frac{P(A\cap B)}{P(B)}=1-P(A|B)$$

$P(x|B)$: Eventos $\to[0,1]$ es una funcion de probabilidad
eventos

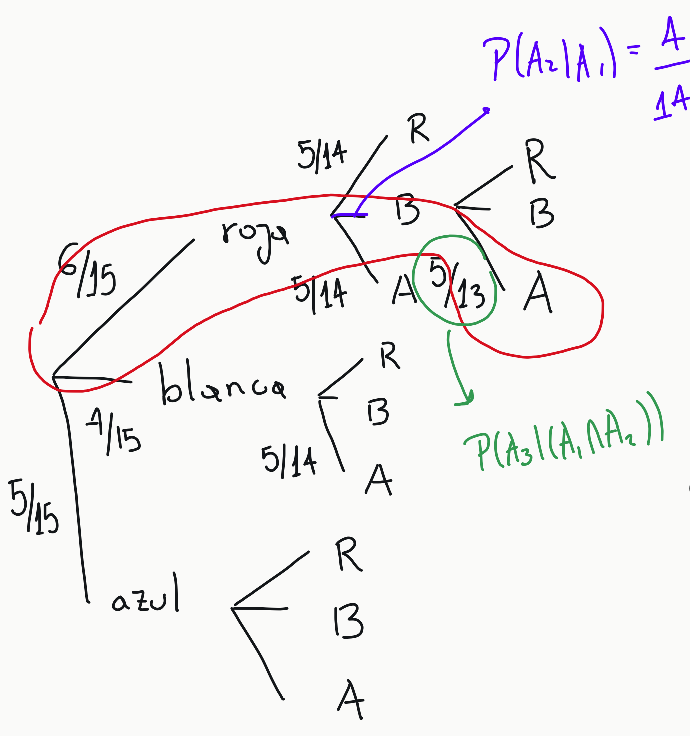
$P(B|x)$ no es una funcion de probabilidad
evento

e- $P(A^c|B^c)=\dfrac{P(A^c\cap B^c)}{P(B^c)}=\dfrac{P((A\cup B)^c)}{P(B^c)}=\dfrac{1-\overset{P(A)+P(B)-P(A\cap B)}{P(A\cup B)}}{1-P(B)}=\dfrac{1+P(A\cap B)-P(A)-P(B)}{1-P(B)}$



Leyes de Morgan $\begin{cases}A^c\cap B^c=(A\cup B)^c\\ A^c\cup B^c=(A\cap B)^c\end{cases}$

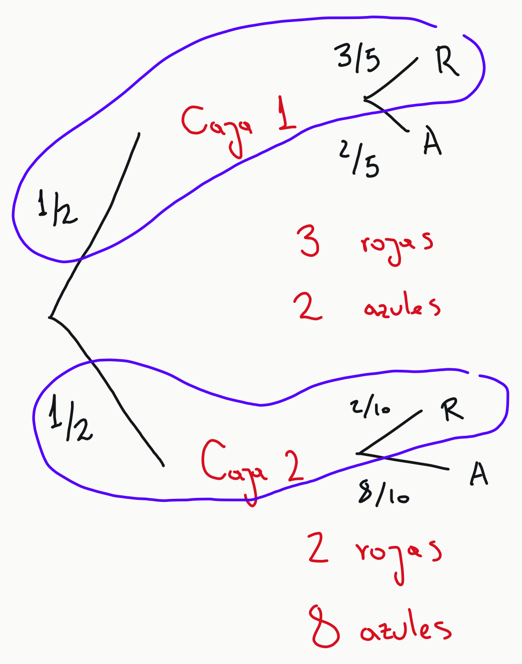
5-



A_1 = la primer bolilla sea roja
A_2 = la segunda " blanca
A_3 = la tercer " azul

el evento que nos interesa es
$A_1\cap A_2\cap A_3$

$$P(A_1\cap A_2\cap A_3)=P((A_1\cap A_2)\cap A_3)=P(A_3|(A_1\cap A_2))P(A_1\cap A_2)$$
$$=P(A_3|(A_1\cap A_2))P(A_2|A_1)P(A_1)=\frac{5}{13}\cdot\frac{4}{14}\cdot\frac{6}{15}$$



R = bolilla extraida roja

P(R)

C_1 = sale caja 1

C_2 = sale caja 2

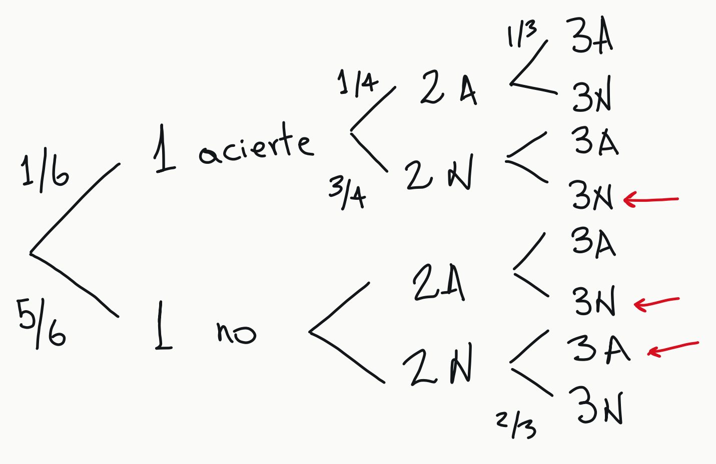
$R=(R\cap C_1)\cup(R\cap C_2)$
eventos disjuntos

$P(C_1\cap R)=P(C_1|R)P(R)$
$=P(R|C_1)P(C_1)$

$P(R)=P(R\cap C_1)+P(R\cap C_2)$
$P(R)=P(R|C_1)P(C_1)+P(R|C_2)P(C_2)$ prob. total
$P(R)=\frac{3}{5}\cdot\frac{1}{2}+\frac{2}{10}\cdot\frac{1}{2}$

$$P(C_1|R)=\frac{P(C_1\cap R)}{P(R)}=\boxed{\frac{P(R|C_1)P(C_1)}{P(R)}=P(C_1|R)}$$ formula de Bayes

6-



A = el blanco es alcanzado una sola vez

$$P(A)=\frac{1}{6}\cdot\frac{3}{4}\cdot\frac{2}{3}+\frac{5}{6}\cdot\frac{1}{4}\cdot\frac{2}{3}+\frac{5}{6}\cdot\frac{3}{4}\cdot\frac{1}{3}$$

otra forma

A_1 = el jugador 1 acierte
A_2 = " 2 "
A_3 = " 3 "
son indep.

$P(A_1)=1/6$ $P(A_1^c)=5/6$

$$A=(A_1\cap A_2^c\cap A_3^c)\cup(A_1^c\cap A_2\cap A_3^c)\cup(A_1^c\cap A_2^c\cap A_3)$$
$$P(A)=P(A_1\cap A_2^c\cap A_3^c)+P(A_1^c\cap A_2\cap A_3^c)+P(A_1^c\cap A_2^c\cap A_3)$$
indep.
$$P(A)=P(A_1)P(A_2^c)P(A_3^c)+P(A_1^c)P(A_2)P(A_3^c)+P(A_1^c)P(A_2^c)P(A_3)$$
$$P(A)=\frac{1}{6}\cdot\frac{3}{4}\cdot\frac{2}{3}+\frac{5}{6}\cdot\frac{1}{4}\cdot\frac{2}{3}+\frac{5}{6}\cdot\frac{3}{4}\cdot\frac{1}{3}$$