Axiomas de Kolmogorov

Ω espacio muestral

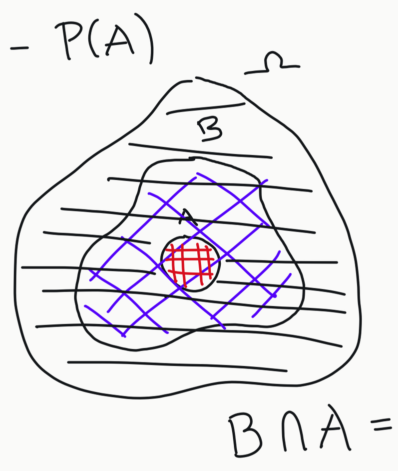
Una funcion P: Eventos $\to [0,1]$ es una funcion de probabilidad si verifica

1. $P(A) \geq 0 \quad \forall$ evento A
2. $P(\Omega) = 1$
3. si $\{A_k\}$ es una sucesion de eventos incompatibles dos a dos (no pueden ocurrir simultaneamente)

$$P\left(\bigcup_{k=1}^{\infty} A_k\right) = \sum_{k=1}^{\infty} P(A_k)$$

ej: si A y B son incompatibles $\Rightarrow P(A \cup B) = P(A) + P(B)$

8_ (a) $A \subset B \Rightarrow P(B \setminus A) = P(B) - P(A)$



$B \cap A = A$

- $B \setminus A = B \cap A^c$
- $B = (B \cap A) \cup (B \cap A^c)$

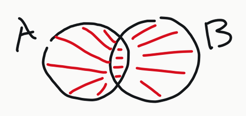
$P(B) = P(\underbrace{(B \cap A)}_{A} \cup (B \cap A^c))$

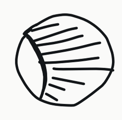
$P(B) = P(B \cap A) + P(B \cap A^c)$

$P(B) = P(A) + P(B \setminus A) \Rightarrow P(B \setminus A) = P(B) - P(A)$ ∎

(b) $P(A \cup B) \geq \max\{P(A), P(B)\}$

$P(A \cup B) = P((A \cap B^c) \cup (A \cap B) \cup (B \cap A^c))$





$P(A \cup B) = P(A \cap B^c) + \cancel{P(A \cap B)} + P(B \cap A^c)$

$P(A \cap B^c) = P(A) - \cancel{P(A \cap B)}$, $\quad P(B \cap A^c) = P(B) - P(A \cap B)$

$P(A \cup B) = P(A) + P(B) - P(A \cap B)$

(I) $P(A \cup B) = P(A) + P(B) - P(B) = P(A)$

(II) $P(A \cup B) = P(A) + P(B) - P(A) = P(B)$

$P(A \cup B) \geq \max\{P(A), P(B)\}$

$P(A \cap B) \leq \min\{P(A), P(B)\}$

$A \cap B \subset A \Rightarrow P(A \cap B) \leq P(A)$

$A \cap B \subset B \Rightarrow P(A \cap B) \leq P(B)$

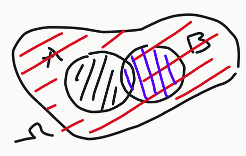
$P(A \cap B) \leq \min\{P(A), P(B)\}$

9_ $P(A) = 1/3 \quad P(B) = 1/2 \quad P(A^c \cap B) = ?$

(a) A y B incompatibles $\quad A^c \cap B = B \Rightarrow P(A^c \cap B) = P(B) = 1/2$

(b) $A \subset B \quad A^c \cap B = B \setminus A \Rightarrow P(A^c \cap B) = P(B \setminus A) = P(B) - P(A) = 1/6$

(c) $P(A \cap B) = 1/8$



$P(A^c \cap B) = P(B) - P(A \cap B) = 3/8$

otra forma

$B = (A \cap B) \cup (A^c \cap B)$

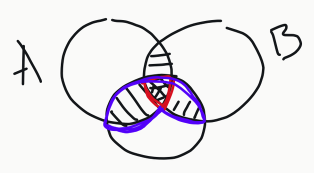
$P(B) = P(A \cap B) + P(A^c \cap B)$

10_ $P(A) = 3/8 \quad P(B) = 1/2 \quad P(A \cap B) = 1/4$

(a) $P(A^c) = P(\Omega) - P(A) = 1 - P(A) = 5/8$

(b) $P(A \cup B) = P(A) + P(B) - P(A - B)$

11_ (a) $P(A \cup B \cup C) = P(A) + P(B) + P(C) - P(A \cap B) - P(B \cap C) - P(C \cap A) + P(A \cap B \cap C)$



$P((A \cup B) \cup C) = P(A \cup B) + P(C) - P((A \cup B) \cap C)$

$\hookrightarrow P(A) + P(B) - P(A \cap B)$

$P((A \cup B) \cap C) = P((A \cap C) \cup (B \cap C)) = P(A \cap C) + P(B \cap C) - \underbrace{P(A \cap C \cap B \cap C)}_{P(A \cap B \cap C)}$

$P(A \cup B \cup C) = P(A) + P(B) + P(C) - P(A \cap B) - \left(P(A \cap C) + P(B \cap C) - P(A \cap B \cap C)\right)$

(b) $P\left(\bigcup_{i=1}^{n} A_i\right) = \sum_{1 \leq i \leq n} P(A_i) - \sum_{1 \leq i < j \leq n} P(A_i \cap A_j) + \dots + (-1)^{n-1} P(A_i \cap \dots \cap A_n)$

caso base $\quad n = 2 \quad P(A_1 \cup A_2) = P(A_1) + P(A_2) - P(A_1 \cap A_2)$ ✓

paso inductivo (H) $P\left(\bigcup_{i=1}^{n} A_i\right) = \sum_{1 \leq i \leq n} P(A_i) - \sum_{1 \leq i < j \leq n} P(A_i \cap A_j) + \dots + (-1)^{n-1} P(A_i \cap \dots \cap A_n)$

(T) $P\left(\bigcup_{i=1}^{n+1} A_i\right) = \sum_{1 \leq i \leq n+1} P(A_i) - \sum_{1 \leq i < j \leq n+1} P(A_i \cap A_j) + \dots + (-1)^{n} P(A_i \cap \dots \cap A_{n+1})$

$P\left(\bigcup_{i=1}^{n+1} A_i\right) = P\left(\bigcup_{i=1}^{n} A_i \cup A_{n+1}\right) = P\left(\bigcup_{i=1}^{n} A_i\right) + P(A_{n+1}) - P((A_1 \cup A_2 \cup \dots \cup A_n) \cap A_{n+1})$

$P\left(\bigcup_{i=1}^{n} A_i\right) = \sum_{1 \leq i \leq n} P(A_i) - \sum_{1 \leq i < j \leq n} P(A_i \cap A_j) + \dots + (-1)^{n-1} P(A_i \cap \dots \cap A_n)$

$P((A_1 \cup A_2 \cup \dots \cup A_n) \cap A_{n+1}) = P((A_1 \cap A_{n+1}) \cup (A_2 \cap A_{n+1}) \cup \dots \cup (A_n \cap A_{n+1}))$

$= P((A_1 \cap A_{n+1}) \cup \dots \cup (A_{n-1} \cap A_{n+1})) + P(A_n \cap A_{n+1}) - P(((A_1 \cap A_{n+1}) \cup \dots \cup (A_{n-1} \cap A_{n+1})) \cap (A_n \cap A_{n+1}))$

$\sum_{1 \leq i \leq n} P(A_i) + P(A_{n+1}) = \sum_{1 \leq i \leq n+1} P(A_i)$

$\Rightarrow \sum_{1 \leq i < j \leq n+1} P(A_i \cap A_j)$