Variable aleatoria discreta

$X : \Omega \to \mathbb{R}$

X tomaba valores en un conjunto discreto

* funcion de probabilidad puntual

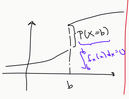
$p_X : \mathbb{R} \to [0,1]$

$p_X(x) = P(X = x)$

* funcion de distribucion acumulada

$F_X : \mathbb{R} \to [0,1]$

$F_X(k) = P(X \leq k) = \sum_{i \leq k} p_X(i)$

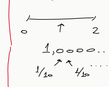


Variable aleatoria (absolutamente) continua

$X : \Omega \to \mathbb{R}$

X toma valores en un conjunto que no es discreto

por ejemplo un intervalo



$P(X = 1) = 0$

$P(X = x) = 0$

1,0000....

1/10 1/100

* funcion de densidad de probabilidad

$f : \mathbb{R} \to \mathbb{R}$ tal que

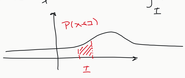
→ Integrable

→ Positiva $f(x) \geq 0 \ \forall x$

→ Integra 1 $\left(\int_{-\infty}^{+\infty} f(x)\,dx = 1 \right)$

* X es absolutamente continua si existe una funcion de densidad

f_X tal que $P(X \in I) = \int_I f_X(x)\,dx$



* funcion de distribucion acumulada

$F_X : \mathbb{R} \to [0,1]$

$F_X(y) = P(X \leq y) = \int_{-\infty}^{y} f_X(x)\,dx$

→ $F_X' = f_X$

→ F_X es continua

$P(a < X \leq b) = P(a \leq X \leq b) = F_X(b) - F_X(a)$
$= P(a < X < b)$

3_ $f_X = \begin{cases} 0 & \text{si } x < 0 \\ bx & \text{si } x \in [0,1) \\ ae^{-x} & \text{si } x \geq 1 \end{cases}$ $\quad P(X \in [0,2]) = 2P(X \in [2,4])$

$$\int_{-\infty}^{+\infty} f_X(x)\,dx = 1$$

$$\int_{-\infty}^{+\infty} f_X(x)\,dx = \int_{-\infty}^{0} 0\,dx + \int_0^1 bx\,dx + \int_1^{+\infty} ae^{-x}$$

$$= b\frac{x^2}{2}\Big|_0^1 + (-ae^{-x})\Big|_1^{+\infty} = \frac{b}{2} + \frac{a}{e}$$

$$\boxed{\frac{b}{2} + ae^{-1} = 1} \quad (I)$$

$$P(X \in [0,2]) = \int_0^2 f_X(x)\,dx = \int_0^1 bx\,dx + \int_1^2 ae^{-x}\,dx$$

$$= \frac{b}{2} - ae^{-2} + ae^{-1}$$

$$P(X \in [2,4]) = \int_2^4 ae^{-x}\,dx = -ae^{-4} + ae^{-2}$$

$$\frac{b}{2} - ae^{-2} + ae^{-1} = -2ae^{-4} + 2ae^{-2}$$

$$\boxed{\frac{b}{2} + ae^{-1} - 3ae^{-2} + 2ae^{-4} = 0} \quad (II)$$

$(II) - (I) \Rightarrow -3ae^{-2} + 2ae^{-4} = -1 \Rightarrow a = \frac{1}{3e^2 - 2e^4}$

$$\frac{b}{2} + \frac{e^{-1}}{3e^2 - 2e^4} = 1 \Rightarrow b = 2\left(1 - \frac{e^{-1}}{3e^2 - 2e^4}\right)$$

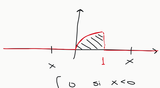
4_ $f_1 = \begin{cases} c_1\sqrt{x} & \text{si } x \in (0,1) \\ 0 & \text{si } x \notin (0,1) \end{cases}$

$$\int_{-\infty}^{+\infty} f_1(x)\,dx = 1 \Rightarrow \int_{-\infty}^{+\infty} f_1(x)\,dx = \int_{-\infty}^{0} 0\,dx + \int_0^1 c_1\sqrt{x}\,dx + \int_1^{+\infty} 0\,dx$$

$$= \int_0^1 c_1 x^{1/2}\,dx = c_1\frac{x^{3/2}}{3/2}\Big|_0^1 = \frac{c_1}{3/2}$$

$$1 = \frac{c_1}{3/2} \Rightarrow \boxed{c_1 = \frac{3}{2}}$$

$F_X(x) = P(X \leq x) = \int_{-\infty}^{x} f_1(x)\,dx$



caso $x < 0$

$F_X(t) = \int_{-\infty}^{t} 0\,dt = 0$

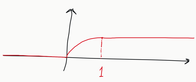
caso $x \in (0,1)$

$F_X(t) = \int_{-\infty}^{0} 0\,dt + \int_0^x \frac{3}{2}t^{1/2}\,dt = \frac{3}{2}\frac{t^{3/2}}{3/2}\Big|_0^x = x^{3/2}$

caso $x > 1$

$F_X(t) = \int_{-\infty}^{0} 0\,dt + \int_0^1 \frac{3}{2}t^{1/2}\,dt + \int_1^{+\infty} 0\,dt = \frac{3}{2}\frac{t^{3/2}}{3/2}\Big|_0^1 = 1$

$$F_X = \begin{cases} 0 & \text{si } x < 0 \\ x^{3/2} & \text{si } x \in (0,1) \\ 1 & \text{si } x > 1 \end{cases}$$



$P(0{,}3 < X < 0{,}6) = \int_{0{,}3}^{0{,}6} \frac{3}{2}\sqrt{x}\,dx =$

$P(0{,}3 < X < 0{,}6) = F_X(0{,}6) - F_X(0{,}3) = 0{,}6^{3/2} - 0{,}3^{3/2}$

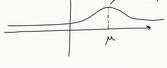
$P(X > 2) = 1 - P(X \leq 2) = 1 - \underbrace{F_X(2)}_{1} = 0$

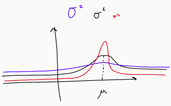
$P(X > 2) = \int_2^{+\infty} 0\,dx = 0$

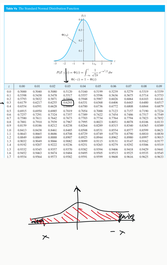
$P(1/2 < X < 3/2) = F_X(3/2) - F_X(1/2) = 1 - (1/2)^{3/2}$

Distribucion normal

$X \sim \mathcal{N}(\mu, \sigma^2) \qquad f_X(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{(x-\mu)^2}{2\sigma^2}}$







z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	0.5000	0.5040	0.5080	0.5120	0.5160	0.5199	0.5239	0.5279	0.5319	0.5359
0.1	0.5398	0.5438	0.5478	0.5517	0.5557	0.5596	0.5636	0.5675	0.5714	0.5753
0.2	0.5793	0.5832	0.5871	0.5910	0.5948	0.5987	0.6026	0.6064	0.6103	0.6141
0.3	0.6179	0.6217	0.6255	0.6293	0.6331	0.6368	0.6406	0.6443	0.6480	0.6517
0.4	0.6554	0.6591	0.6628	0.6664	0.6700	0.6736	0.6772	0.6808	0.6844	0.6879
0.5	0.6915	0.6950	0.6985	0.7019	0.7054	0.7088	0.7123	0.7157	0.7190	0.7224
0.6	0.7257	0.7291	0.7324	0.7357	0.7389	0.7422	0.7454	0.7486	0.7517	0.7549
0.7	0.7580	0.7611	0.7642	0.7673	0.7704	0.7734	0.7764	0.7794	0.7823	0.7852
0.8	0.7881	0.7910	0.7939	0.7967	0.7995	0.8023	0.8051	0.8078	0.8106	0.8133
0.9	0.8159	0.8186	0.8212	0.8238	0.8264	0.8289	0.8315	0.8340	0.8365	0.8389
1.0	0.8413	0.8438	0.8461	0.8485	0.8508	0.8531	0.8554	0.8577	0.8599	0.8621
1.1	0.8643	0.8665	0.8686	0.8708	0.8729	0.8749	0.8770	0.8790	0.8810	0.8830
1.2	0.8849	0.8869	0.8888	0.8907	0.8925	0.8944	0.8962	0.8980	0.8997	0.9015
1.3	0.9032	0.9049	0.9066	0.9082	0.9099	0.9115	0.9131	0.9147	0.9162	0.9177
1.4	0.9192	0.9207	0.9222	0.9236	0.9251	0.9265	0.9279	0.9292	0.9306	0.9319
1.5	0.9332	0.9345	0.9357	0.9370	0.9382	0.9394	0.9406	0.9418	0.9429	0.9441

→ F_X no tiene una formula

$Z \sim \mathcal{N}(0,1)$

$F_Z(x) = P(X \leq z) = \Phi(x)$

$P(Z \leq 0{,}33) = 0{,}6293$

$x > 0$

$P(Z \geq x) = 1 - P(Z \leq x) = 1 - \Phi(x)$

$P(a \leq Z < b) = P(Z \leq b) - P(Z \leq a) = \Phi(b) - \Phi(a)$

$x < 0$

$P(Z \leq x) = P(Z \geq -x) = 1 - P(Z \leq -x) = 1 - \Phi(-x)$